

Validating Equations

Some open questions:

1. Clarify: is q_{n_t} either the short term or long term moving average?
2. how are N_s and N_l determined? L_l and L_s ? Q_1 and Q_2 ?
3. Are equation 4 inequalities mixed up?
4. Should equation 2 have q_s instead of q_l ?
5. How are the 10 unknowns estimated via the two part sum squared error?
6. Authors say “ q_j = sum of Trenton, Schuylkill, and PST model inflow on day j ” ... is our inflow data already in terms of daily total flow?

10 unknowns below – we would just plug into R to find ad-hocly with two part sum square error?

- a. d_s / d_l = decay coefficient for short-term / long-term moving average flow;
- b. a_s / a_l = constant for short-term / long-term moving average flow relationship;
- c. P_s / P_l = power coefficient for short-term / long-term moving average flow relationship;
- d. C_s / C_l = offset for short-term / long-term-term moving average flow relationship;
- e. τ_s / τ_l = tidal coefficient for short-term / long-term term moving average flow relationship;

10 unknowns below – we need to ask the authors how they were derived?

k_{st} / k_{lt} = short-term / long term moving average flow SC at time t ; -> calculated via equation (2) / (3)

q_{st} / q_{lt} = short-term / long term moving average flow at time t ;

N_s / N_l = number of days for short term / long term

Q_1 / Q_2 = maximum flow at which k_{st} / k_{lt} contributes to K_t ;

L_s / L_l = short-term / long-term maximum 1-day change in SC;

Reading in Ben Franklin Bridge SC data with Delaware River Inflow data to use as a first pass cross check with Figure 3 in Meyer paper

```
library(data.table)
library(tidyr)
library(dplyr)
```

```
##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:data.table':
##
##   between, first, last

## The following objects are masked from 'package:stats':
##
##   filter, lag

## The following objects are masked from 'package:base':
##
##   intersect, setdiff, setequal, union
```

```

dat = fread('../data/processed/training_data.csv')

# use this to pull in actual meyer inputs
meyer_inputs <- fread('../data/raw/meyer_raw_data.csv')
meyer_inputs$Date = as.Date(meyer_inputs$Date)
meyer_water_level = meyer_inputs[, c('Date', 'water_level')]

# use this for SC values
bfb = dat[location == 'del_bfb']
bfb <- bfb %>% drop_na(mean_sc)

# convert inflow data back to cubic feet per second and convert sea level (meters) to feet...
#will revisit this to confirm raw tide data was provided in meters
input_data <- dat %>% group_by(date) %>%
  summarise(inflow = sum(inflow, na.rm = TRUE)/0.0283168, #convert inflow from cmfs to cfps
            reedy_sl = ((reedy_sl- 49.229)/12)) # convert reedy_sl from inches to feet

## `summarise()` regrouping output by 'date' (override with `.groups` argument)
input_data$date = as.Date(input_data$date)
input_data = merge(input_data, meyer_water_level, by.x = 'date', by.y = 'Date')

```

Initialize parameter values based on table of values in Meyer paper

```

#n_s = 10
#n_l = 30
#Q1 = 2800 # they provide Q1 / Q2 in cfps; our raw inflows are in cubic meters per second
#Q2 = 4800

#as = 355000
#Ps = -0.8998
#Cs = 132.08
#ds = .19
#Ls = 3
#tao_s = 35312.54

#al = 2600000
#Pl = -1.06
#Cl = 118.2
#dl = .0426
#Ll = 5.5
#tao_l = -2544.23

n_s = 10
n_l = 30
thresh_10 = 3700 # they provide Q1 / Q2 in cfps; our raw inflows are in cubic meters per second
thresh_30 = 3000

as = 52352.7976
Ps = -0.654734
Cs = 105.45053
ds = .80988481
Ls = 3
tao_s = 35312.54

```

```

a1 = 37912467
P1 = -1.394098647
C1 = -.434095416
d1 = 1.912292411
L1 = 5.5
tao_1 = -2544.23

```

Based on this data, 1964-12-15 is the first day we can make a prediction of K (31 days into the data). So, we need to pick an initial condition to test the PBEM model. Based on the rolling average values of mean sc leading up to those dates, we will choose 640 as the initial k_st value and 884 as the initial k_lt value.

```
library(zoo)
```

```

##
## Attaching package: 'zoo'
## The following objects are masked from 'package:base':
##
##      as.Date, as.Date.numeric
# is it right to calculate the q_st and q_lt values as rolling averages
input_data <- input_data %>% mutate(q_st = rollmean(x = inflow, n_s, align = 'right', fill = NA), # n_s
                                   q_lt = rollmean(x = inflow, n_l, align = 'right', fill = NA), # n_l = 30 days
                                   power_10d = as*lag(q_st)^(Ps) + Cs,
                                   power_30d = al*lag(q_lt)^(Pl) + C1)

```

Some of the sea level days of data are missing, which breaks equations (2) and (3) when we multiply by the tide values. So as a first pass I just imputed the missing days of tide data with the previous non-NA value.

```

input_data <- input_data %>% drop_na(q_lt) %>%
  filter(date >= '1964-11-15' & date <= '2017-06-04') %>%
  fill(reedy_sl)

```

Initialize 1964-12-15 values of k_st and k_lt based on rolling average method and run the for loop to calculate the subsequent values of k_st and k_lt.

```

st_k_values = c(558)
lt_k_values = c(924)

input_data = as.data.table(input_data)
# loop through all rows of data after the first initial row
for (t in (2:nrow(input_data))) {

  # store the previous time step slice of data
  t_minus1 = input_data[t-1]
  t_0 = input_data[t]

  # set k_st equal to equation 2 -- note this assumes that equation (2) is supposed to have q_lt rather
  k_st <- st_k_values[t-1] + min(Ls, ds*(t_0$power_10d - st_k_values[t-1]) + tao_s/t_0$q_st*t_0$reedy_s)

  # set k_lt equal to equation 3
  k_lt <- lt_k_values[t-1] + min(Ll, d1*(t_0$power_30d - lt_k_values[t-1]) + tao_l/t_0$q_st*t_0$reedy_s)

  st_k_values <- append(st_k_values, k_st)
  lt_k_values <- append(lt_k_values, k_lt)
}

```

```
}
```

Add the k_st and k_lt values to our existing data

```
input_data <- input_data %>% mutate(model_10d = st_k_values,
                                     model_30d = lt_k_values)
```

Finally, add the logic to predict capital K depending on the relationship between q and Q

```
# ql_t <= Q1      -> k_st (maybe authors have this swapped?)
# ql_t >= Q2      -> k_lt (maybe authors have this swapped?)
# Q1 < qlt <= Q2 ->
#bf <- bf %>% mutate(K_hat = ifelse(q_lt <= Q1, k_st,
#                                   ifelse(q_lt >= Q2, k_lt, k_st*(q_lt - Q1)/(Q2 - Q1) + k_lt*(Q2 - q_lt)/(Q2 - Q1)),
#                                   inequality = ifelse(q_lt <= Q1, 'first',
#                                                       ifelse(q_lt >= Q2, 'third', 'second'))))

#preds = c()
#for (t in (1:nrow(input_data))) {
#  time_step = input_data[t]
#  p = time_step$model_10d*max(0, min(1, (time_step$q_lt - thresh_30)/ 700)) + time_step$model_30d*min(1, (time_step$qlt - thresh_30)/ 700)
#  preds = append(preds, p)
#}

final_output <- input_data %>%
  select(date, q_lt, model_10d, model_30d) %>%
  group_by(date) %>%
  mutate(model_output = model_10d*max(0, min(1, (q_lt - thresh_30)/ 700)) + model_30d*min(1, (qlt - thresh_30)/ 700))
  ungroup()

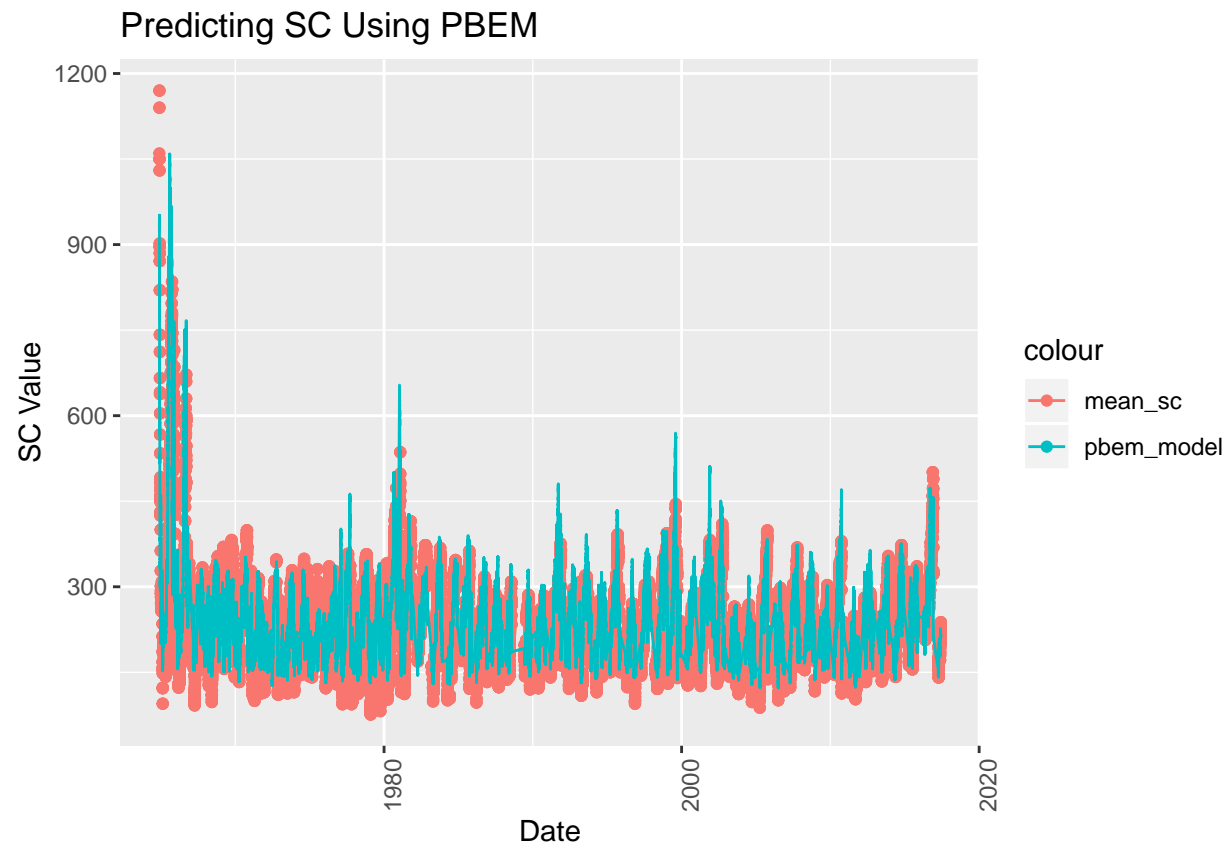
#final_output$model_output = preds
```

Cross validate K_hat (predicted SC value) with mean_sc (actual SC values)

```
bfb$date = as.Date(bfb$date)
#copycat = merge(final_output, bfb[, c('date', 'mean_sc')])
risq_attempt = merge(final_output, bfb[, c('date', 'mean_sc')])
```

```
library(ggplot2)
risq_attempt$date = as.Date(risq_attempt$date)
```

```
ggplot()+
  geom_point(data = risq_attempt, aes(x = date, y = mean_sc, color = 'mean_sc')) +
  geom_line(data = risq_attempt, aes(x = date, y = model_output, color = 'pbem_model')) +
  xlab('Date') +
  ylab('SC Value') +
  ggtitle('Predicting SC Using PBEM') +
  theme(axis.text.x = element_text(angle = 90))
```



```
#ylim(-800,800)
#scale_x_continuous(name = "Date", breaks=c('1964-11-01', '1966-11-01'))

drill_down <- risq_attempt %>% filter(date >= '1997-01-01' & date <= '1999-09-01')

ggplot()+
  geom_point(data = drill_down, aes(x = date, y = mean_sc, color = 'mean_sc')) +
  geom_line(data = drill_down, aes(x = date, y = model_output, color = 'pbem_model')) +
  xlab('Date') +
  ylab('SC Value')+
  ggtitle('Predicting SC Using PBEM -- Copied Meyer Water Levels')+
  theme(axis.text.x = element_text(angle = 90))
```

Predicting SC Using PBEM -- Copied Meyer Water Levels



```
#meyer_output = copycat$model_output
#risq_output = risq_attempt$model_output
```

```
#compare = data.frame(meyer_output, risq_output)
```

```
ggplot()+
  geom_point(data = input_data, aes(x = date, y = water_level, color = 'Meyer Water Levels', )) +
  geom_point(data = input_data, aes(x = date, y = reedy_sl, color = 'Risq Tides', alpha = .001, shape =
  xlab('Date') +
  ylab('SC Value')+
  ggtitle('Comparing Water Level Values Across Sources')+
  theme(axis.text.x = element_text(angle = 90))
```

Comparing Water Level Values Across Sources

