SOC-GA 2332 Intro to Stats Lab 12

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Logistics

- **Problem set 3** due tonight (11:59pm)
- **Problem set 4** released today
- Lab on Wednesday next week (**11/26**)
- Problem set answer keys

Part 1: Review of the Lecture

1.1 Overview

- We can use longitudinal data where individuals are observed multiple times to make inferences
  - Observe within-unit changes
- There is also longitudinal cross-sectional data, where different individuals are observed in different time periods (ex. to measure changes in national public opinion over time)

1.2 Fixed-effects Models

- FE models control for unit-specific time-invariant characteristics and use within-unit changes for estimation.
  - *Question:* What's an example of a study where this would be useful instead of standard linear regression?
  - *Question:* What assumptions might a standard linear regression violate for longitudinal data?
- FE models tell us how time-*varying* explanatory variables affect a time-*varying* outcome variable while controlling for time-*invariant* explanatory variables. The effect being estimated is essentially an average of within-unit treatment effects.

$$Y_{it} = \alpha + \beta X_{it} + \eta_i + \epsilon_{it}$$

- Where η_i represents unit-specific time-invariant characteristics
  - *Question:* what do we mean by unit-specific time-invariant characteristics?
- We do not impose any restrictions on the relation between η_i and X_{it}
  - For example, in estimating the male marriage premium (i.e. married men tend to make more money than single men), personality that is roughly stable but unobserved can be correlated with whether a man gets married and with his income (without violating any assumptions). In FE model, personality is "absorbed" and controlled by η_i
- We assume that the slope remains the same across units (i.e., we specify β rather than β_i; for recent discussions of unit-variant slopes and new methods, see Brand and Xie (2010)), but we allow units to have their own intercepts. That is, each unit i's intercept would be $\alpha_i = \alpha + \eta_i$
  - We essentially model:

$$Y_{it} = \alpha_i + \beta X_{it} + \epsilon_{it}$$

- Therefore, as we already learned in previous sessions, the model can be estimated by including $n-1$ unit dummy variables into our model
  - This is called Least Squares Dummy Variables (LSDV) estimation
  - This can be cumbersome if there is a large number of units.
- Alternatively, software such as `R` uses a within-estimator. Take the mean of both sides of the equation **for each unit** i:

$$\bar{Y}_i = \alpha_i + \beta\bar{X}_i + \bar{\epsilon}_i$$

- Take the difference of the two equations, we get:

$$\begin{aligned} Y_{it} - \bar{Y}_i &= \alpha_i - \alpha_i + \beta(X_{it} - \bar{X}_i) + (\epsilon_{it} - \bar{\epsilon}_i) \\ &= \beta(X_{it} - \bar{X}_i) + v_{it} \end{aligned}$$

- By comparing each unit's value at time t with its mean value over time, the estimation uses only within-individual changes of both explanatory and dependent variable

- Because we de-mean the data within each unit, any time-invariant variables, such as race, cannot be estimated in an FE model

- Assumptions for causal inference:

  (i) Parametric assumptions (i.e. we have specified the model correctly)
  (ii) The probability of selecting into treatment (Z) is independent of potential outcomes conditional on X and alpha: $Y(1), Y(0) \perp Z | X, \alpha$

- One classic example in family sociology is the male marriage premium (i.e. to determine whether marriage rewards men's income)

  - Comparing the income of married men and unmarried men can lead to biased estimation. *Question:* Why might that be?
  - Instead, we can compare the income of the **same** man before and after marriage using the fixed-effects model
  - However, it does not exclude the possibility of self-selection and anticipation. For example, men who anticipate a higher growth of wage may be more likely to choose marriage soon (check this ASR paper that discusses this problem Ludwig and Bruderl (2018))

- You can also use fixed-effects models to de-mean at a group level. For example, if you're doing a study on a classroom intervention and you think there are school-level traits that might affect your outcome variable, you can also de-mean each individual's outcome by what school they are in (so instead of controlling for individual-specific characteristics, we're controlling for school-level characteristics).

1.3 Random-effects Model

- Random-effects models assume (as most models we have learned about do) that ϵ_i is drawn randomly from a normal distribution. But uniquely, random-effects models also assume that η_i is also drawn from a common normal distribution for each unit.

- In fixed-effects models, we are controlling for unit-specific effects, often because we think they are correlated with the other independent variables and therefore may be confounding our results. In random-effects models, we must assume that unit-specific effects are *not* correlated with any independent variables.

  - *Question:* Why would we add in unit-specific traits if we don't think they are confounders?

- Compared with the FE model, this is a much stronger assumption that is not always met. For one thing, it assumes that the unit-specific effects have no effect on selecting into the treatment group (as well as the other independent variables).

  - This allows you to model both within-unit and between-unit variation

- Recall when we take the mean of both sides of the equation **for each unit** i, we get a between-estimator:

$$\begin{aligned}\bar{Y}_i &= \alpha_i + \beta\bar{X}_i + \bar{\epsilon}_i \\ &= \underbrace{\alpha}_{\text{constant for all units}} + \beta\bar{X}_i + \underbrace{\eta_i + \bar{\epsilon}_i}_{v_i}\end{aligned}$$

- RE model assumes that η_i is uncorrelated with X_{it} for $\forall t = 1, 2, ..., T$; we therefore get $Cov(v_i, \bar{X}_i) = 0$
  - If $\mathbb{E}[v_i|\bar{X}_i] = 0$, OLS estimator $\hat{\beta}$ can be an unbiased and consistent estimator
  - To test whether this assumption holds, you can use the Hausman test (we will go over implementing this in R below).
  - But OLS mixes in two different "random shocks", one that is truly stochastic across units ($\bar{\epsilon}_{it}$), and the other that is specific to each unit η_i
  - To take into account the additional information introduced by η_i, we assume the two "random shocks" are normally distributed **independently**. $v_i = N(0, \sigma_\eta^2) + N(0, \sigma_\epsilon^2)$
  - Therefore, another within-estimator exists:

$$\begin{aligned}Y_{it} - \bar{Y}_i &= \eta_i - \bar{\eta}_i + \beta(X_{it} - \bar{X}_i) + (\epsilon_{it} - \bar{\epsilon}_i) \\ &= \underbrace{\eta_i - \bar{\eta}_i}_{\neq 0} + \beta(X_{it} - \bar{X}_i) + \mu_{it}\end{aligned}$$

- The estimated β takes into account both within- and between-unit variations

- We estimate β through MLE. *Question:* What is MLE?

- By including between-unit variations, RE model allows time-invariant independent variables

- *Question:* Can you think of scenarios where $Cov(v_i, X_{it}) = 0, X_{it}$ for $\forall t = 1, 2, ..., T$ may be satisfied, and RE model is useful?

- *Question:* What is the advantage of RE over FE?

Part 2: Panel Data Structure

- To demonstrate the `R` implementation of FE and RE models, we will use a dataset `wagepan` provided by the R package `wooldridge`. This is a panel of 545 young men from 1980 to 1987 used in the article by F. Vella and M. Verbeek (1998), "Whose Wages Do Unions Raise? A Dynamic Model of Unionism and Wage Rate Determination for Young Men."

- For the purpose of demonstration, this panel data is already organized in the tidy "long-format" with every person-year observation saved in each row. In addition, there is no missing data.

Part 2 Exercise #1 (5 minutes)

- Load the data into your R environment. Look at the data carefully (remember you can use ? in the console to get more information) and answer the following questions
  - (1) Over how many time points were these individuals followed? Is this a balanced, or unbalanced panel (i.e. do all observations have data for every time period)?
  - (2) Among all the variables, which one(s) do you expect to be time invariant?

```r
## load data into the environment
data(wagepan, package = "wooldridge")

# your code here
```

Part 2 Exercise #2 (15 minutes)

- Before estimating models, let's create some descriptive plots for exploratory purposes. Suppose we are interested in the relationship between **labor market experience** and **log(wage)**. Replicate the the plot below following the listed steps.
  - This plot contains two panels, with one illustrating the aggregate relationship and the other the individual-level trajectories.

```r
knitr::include_graphics("graph/exercise1.png")
```

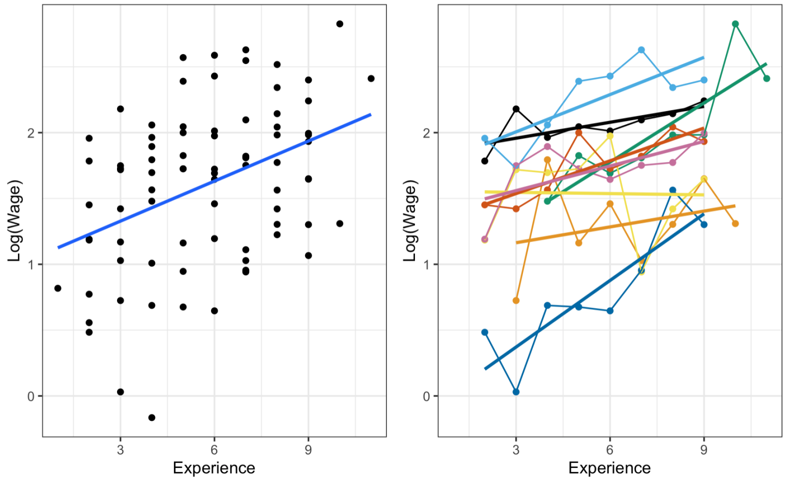


Figure 1: Graphical Demonstration of KOB Decomposition

- 1. Sample ten persons from the dataset;
- 2. Create an "aggregate trend" scatter plot of these individuals across all observation years with an OLS regression line for the variable `exper` and `lwage` (the left panel);
- 3. Similarly, create an "individual trend" scatter plot (the right panel);
- 4. Arrange the two plots using `ggarrange()`;
- 5. How does the relationship between `exper` and `lwage` differ in these two plots? What would be the possible reasons for the difference?
  - Note: Due to sample randomness, it is okay if your plot looks different from the example

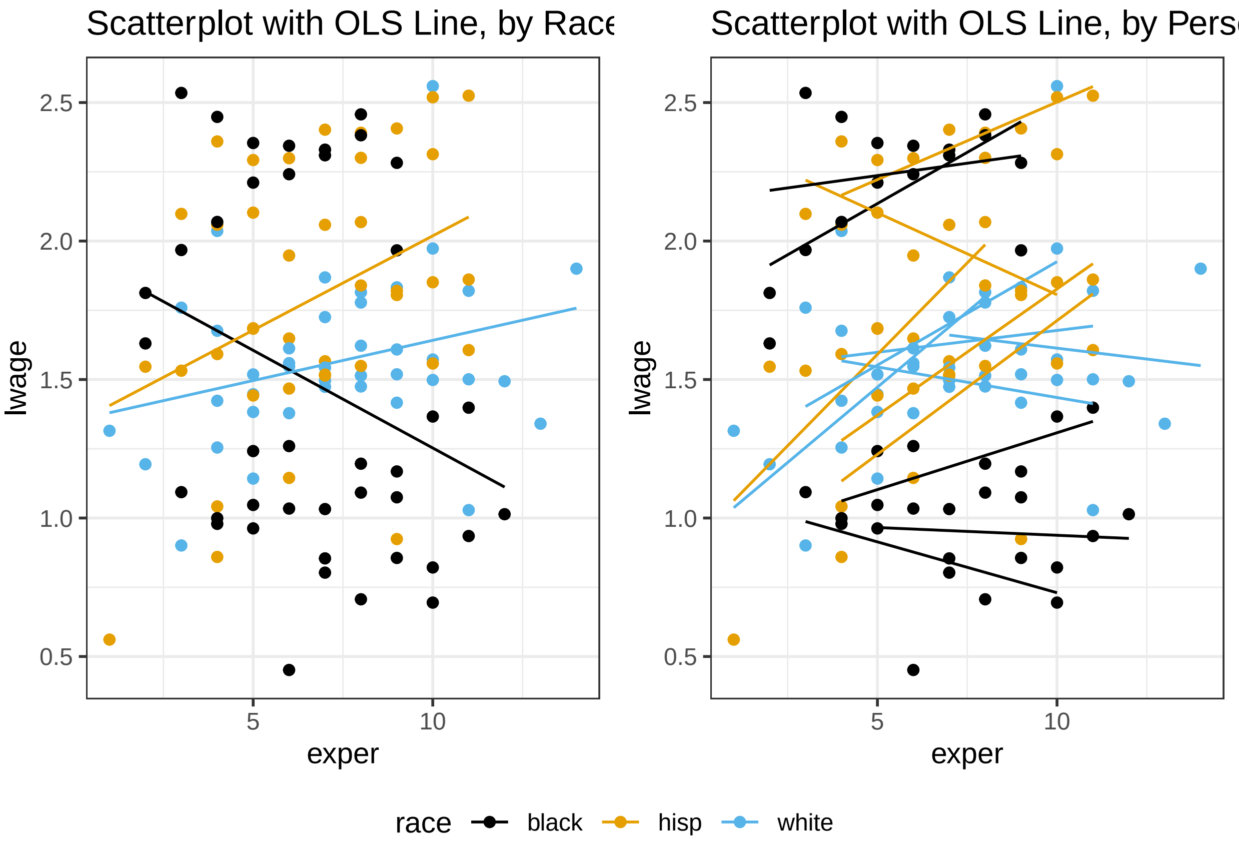
```r
## your code here
```

In the first plot exper is positively correlated to lwage, but looking at the second plot we see this doesn't hold true for all individuals. This might be because individuals take time off work or get new, lower-paying jobs at different points in time.

Part 3: R Implementation of Panel Models

- In addition to a simple bivariate relationship, we can further explore how individual wage trajectories vary by race. For the purpose of demonstration, we can sample 5 individuals from each racial group, and plot their wage trajectories.

```r
## create a character variable "race" for plotting
wagepan <- wagepan %>%
  mutate(race = case_when(black == 1 ~ "black",
                          hisp == 1 ~ "hisp",
                          black == 0 & hisp == 0 ~ "white"))

## sample pid by race
set.seed(123456)
nr_byrace <- wagepan %>%
  # get a list of distinct person id number, keep other variables
  distinct(nr, .keep_all = T) %>%
  # group by race
  group_by(race) %>%
  # sample 5 persons
  sample_n(5) %>%
  ungroup() %>%
  # extract person id number
  pull(nr)

## aggregate trend
fig3 <- wagepan %>%
  filter(nr %in% nr_byrace) %>%
  ggplot(aes(x = exper, y = lwage, color = race, group = race)) +
  geom_point() +
  geom_smooth(method = "lm", se = F, size = 0.5) +
  labs(title = "Scatterplot with OLS Line, by Race") +
  scale_colour_colorblind() +
  theme_bw()

## look at individual trend
fig4 <- wagepan %>%
  filter(nr %in% nr_byrace) %>%
  ggplot(aes(x = exper, y = lwage, color = race, group = as.factor(nr))) +
  geom_point() +
  geom_smooth(method = "lm", se = F, size = 0.5) +
  labs(title = "Scatterplot with OLS Line, by Person and Race") +
  scale_colour_colorblind() +
  theme_bw()

## show the two figures
ggarrange(fig3, fig4, ncol = 2,
          common.legend = TRUE,
          legend = "bottom")
```



- As you may have observed, the relationship between work experience and log wage revealed by the two different estimation approaches are quite different.
- We might even run into the Simpson's paradox, where the OLS model slope of the aggregate data is negative, whereas the OLS model slopes of the individual trends are mostly positive.
- Once we look into the **within-individual** relationship between the outcome and the predictor, we see that a simple OLS model's predictions are not really in line with the data. This is also expected as there is a lot of **between-individual heterogeneity in the trends, which we cannot capture when we pool across all observations**. This is a good motivation for using a FE or RE model!

3.1 Estimating FE and RE Models in R

- To estimate fixed effects and random effects models in `R`, we use the `plm` package
  - Another common package is `lme4`
  - For fixed effects models, as we have mentioned earlier, you can also fit a simple linear model with "unit dummies"
    - that is, for n unique persons, create $(n-1)$ dummy variables and include them in regression
    - you can do this using `as.factor(person_id)` when you estimate the model (but keep in mind, this is computationally intensive, especially if you have a lot of units)
- We want to estimate a model predicting mean log wage using years of working experience and race

```
## simple OLS model (for purpose of comparison)
m_ols <- lm(lwage ~ exper + black + hisp, data = wagepan)

## fixed effects model - using person dummies
m_fe_dummy <- lm(lwage ~ exper + as.factor(nr) , data = wagepan)

## fixed effects model
```

```r
## model = "within" indicates fixed effects model
## index = c("nr") is the grouping variable in your fixed effects model
m_fe <- plm(lwage ~ exper, data = wagepan,
            model = "within",
            index = c("nr"))

## random effects model - using plm package
m_re <- plm(lwage ~ exper + black + hisp, data = wagepan,
            model = "random",
            index = c("nr"))

## random effects model - using lme4 package
m_re2 <- lmer(lwage ~ exper + black + hisp + (1 | nr), wagepan)
## only intercepts are random; slopes are constant across units
## you could also set this to (exper | nr) which would allow random intercept and random slope between
## or to (0 + exper | nr) which would create a random slope but not a random intercept

## display modeling results
stargazer(m_ols, m_fe, m_re, m_re2,
          type = "text",
          column.labels = c("OLS", "FE", "RE-plm", "RE-lme4"),
          model.names = F,
          omit.stat = c("f", "ser"))
```

```
##
## ===========================================================
##                                Dependent variable:
##                      ---------------------------------------
##                                       lwage
##                        OLS       FE      RE-plm    RE-lme4
##                        (1)      (2)       (3)       (4)
## -----------------------------------------------------------
## exper               0.035***  0.063*** 0.059***   0.060***
##                      (0.003)  (0.002)   (0.002)   (0.002)
##
## black               -0.170***          -0.182*** -0.182***
##                      (0.025)            (0.053)   (0.055)
##
## hisp                -0.075***           -0.090*   -0.090*
##                      (0.022)            (0.047)   (0.049)
##
## Constant            1.450***           1.299***   1.296***
##                      (0.020)            (0.024)   (0.025)
##
## -----------------------------------------------------------
## Observations          4,360    4,360     4,360     4,360
## R2                    0.044    0.160     0.133
## Adjusted R2           0.043    0.041     0.132
## Log Likelihood                                   -2,323.959
## Akaike Inf. Crit.                                4,659.917
## Bayesian Inf. Crit.                              4,698.198
## ===========================================================
## Note:                           *p<0.1; **p<0.05; ***p<0.01
## you can also compare the coefficients of your person dummy linear model and
## your fixed effects model
```

- You can also look at each unit's fixed-effects with the `fixef` command.

```
m_fe %>% fixef()
```

- **Interpreting fixed-effects coefficients:** how much Y changes, on average per unit (person, stratum, etc.), when X increases by one unit. So we would expect a .063 increase in log wages for each year of work experience gained, on average across individuals.
- **Interpreting random-effects coefficients:** The coefficient tells you that a one-unit increase in experience (within or between individuals) is associated with a 0.059 increase in log wages, on average, holding race/ethnicity constant and accounting for the fact that people differ in their baseline wages.
- Let's check if the random-effect model's assumption of uncorrelated effects holds true. To test this in R, you can use the `phtest()` command from the `plm` package to run a Hausman test. This will tell you whether a fixed-effect or a random-effect model is better for your data.
- The Hausman test compares two estimators:
  - FE estimator (consistent even if η is correlated with X)
  - RE estimator (efficient if η is uncorrelated)
- If RE is valid, both estimators should give similar results. If they differ, it's evidence that the RE assumptions are violated.
- *Question:* Why do we need to do the Hausman test? Why not just test the assumption of no correlation directly with the `corr()` command in R?
- Running the Hausman test on our models above:

```
phtest(m_fe, m_re)
```

```
##
##  Hausman Test
##
## data:  lwage ~ exper
## chisq = 86.742, df = 1, p-value < 2.2e-16
## alternative hypothesis: one model is inconsistent
```

- In the Hausman test, the null hypothesis is that a random-effects model is preferred, and the alternative hypothesis is that a fixed-effects model is preferred. From these results, we can reject the null hypothesis that a random-effects model is preferred.

3.2 Plot Predicted Effects

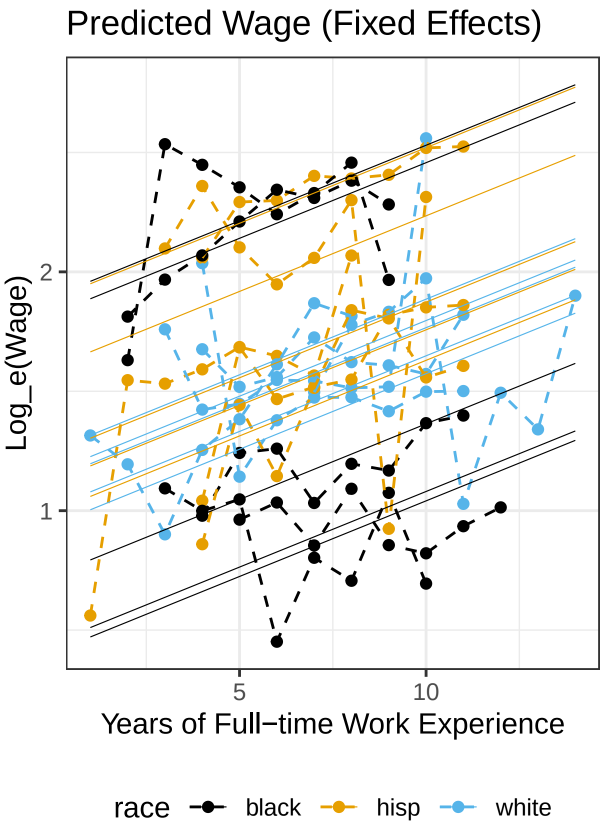
- Let us compare the results of the fixed effects model and the random effects model by plotting the predicted log wage by the predictors. For demonstration purposes, we will use the race-balanced sample ($n = 5$ for each race) obtained earlier.

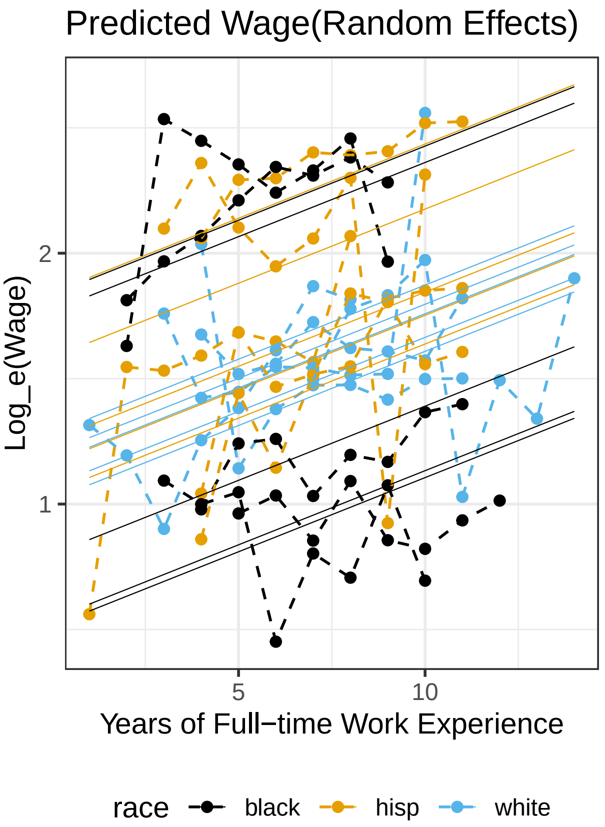
```
## save a subsample of the race-balanced 15 individuals sampled earlier
wagepan_sample <- wagepan %>%
  filter(nr %in% nr_byrace) %>%
  dplyr::select(nr, lwage, exper, black, hisp, race)

## a df that match nr with race
sample_race_key <- wagepan %>%
  dplyr::select(nr, race) %>%
  distinct(nr, .keep_all = T)

## find the range of experience in the subsample
summary(wagepan_sample$exper)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##     1.0     5.0     7.0     6.9     9.0    14.0
```

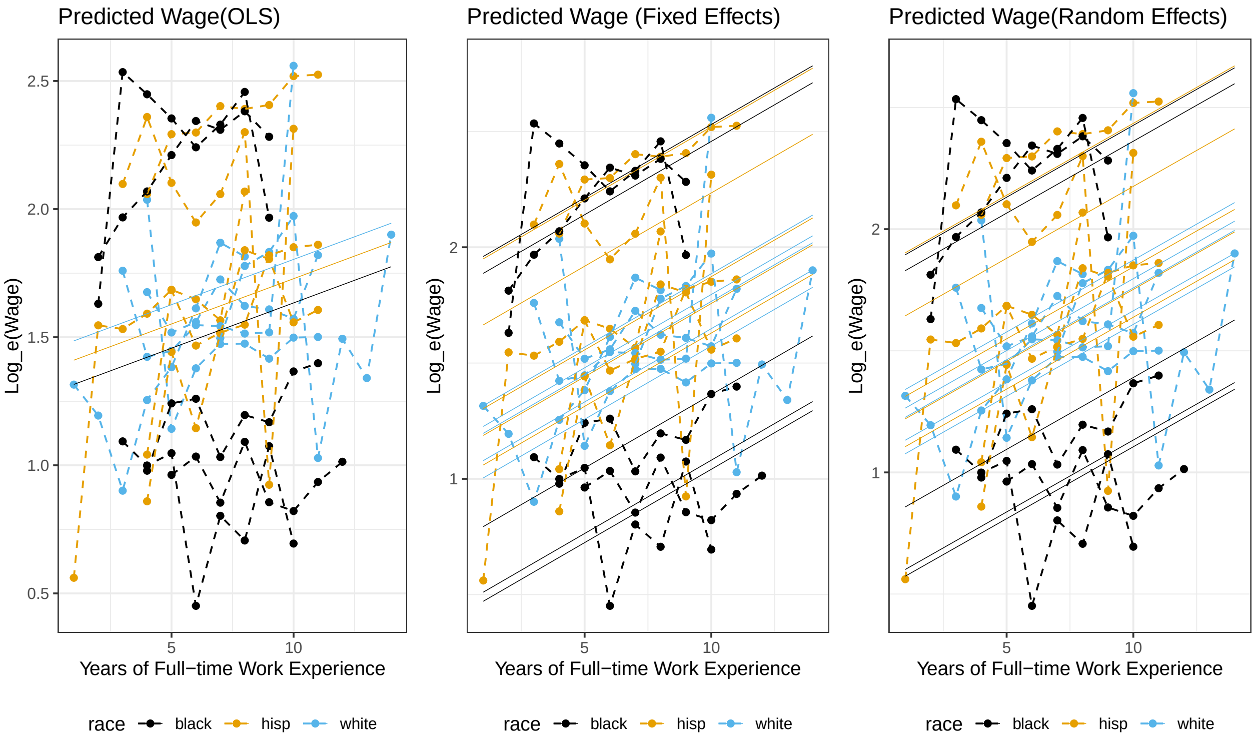
```
# min = 1, max = 14

## create a number sequence for years of experience
exp_seq = seq(1, 14, 1)

#~#~#~#~#~#~# predicted effect of the fixed effects model #~#~#~#~#~#

## create a dataframe with nr and years of experience based on the subsample
IV_fe <- data.frame(
  # each pid will repeat for 14 times for each value of exp
   nr = rep(nr_byrace, length(exp_seq)),
  # the exp seq will repeat 15 times so that it matches with 15 nr
  exper = rep(exp_seq, times = 15)
  )

## get predicted Y using the OLS dummy model
yhat_fe <- predict(m_fe_dummy, newdata = IV_fe, interval = "confidence")

## combine results
predict_fe <- cbind(IV_fe, yhat_fe) %>%
  left_join(sample_race_key, by = "nr")

## plot predicted effect, with original subsample's scatterplot
fig_fe <- ggplot() +
  # observed scatterplot
  geom_point(aes(x = exper, y = lwage, color = race), data = wagepan_sample) +
  # connecting observed dot with dashed lines
  geom_line(aes(x = exper, y = lwage,  group = as.factor(nr), color = race),
            linetype = "dashed", data = wagepan_sample) +
  # fixed effect model curves
  geom_line(aes(x = exper, y = fit, group = as.factor(nr), color = race),
            size = 0.2, data = predict_fe) +
  labs(title = "Predicted Wage (Fixed Effects)",
       x = "Years of Full-time Work Experience",
       y = "Log_e(Wage)") +
  theme_bw() +
  theme(legend.position = "bottom") +
  scale_colour_colorblind()


#~#~#~#~#~#~# predicted effect of the random effects model #~#~#~#~#~#

## dataset to make predictions
IV_re <-  data.frame(
    nr = rep(nr_byrace, each = length(exp_seq)),
    exper = rep(exp_seq, times = 15)
```

```r
    ) %>%
  left_join(sample_race_key, by = "nr") %>%
  mutate(black = ifelse(race == "black", 1, 0),
         hisp = ifelse(race == "hisp", 1, 0))

## here predictions are based on alpha + \betaX_{it}
yhat_temp_re <-  predict(m_re, newdata = IV_re)

# predict \eta using `ranef` function
eta_re <- ranef(m_re) %>%
  cbind(nr = as.numeric(names(.)),
        eta = .) %>%
  as.data.frame() %>%
  filter(nr %in% wagepan_sample$nr)

# merge predicted eta_i to tmp_rand
predict_re <- cbind(IV_re, yhat_temp_re) %>%
  left_join(eta_re, by = "nr") %>%
  # add \eta_i to alpha + \betaX_{it}
  mutate(yhat_re = yhat_temp_re + eta)

## plot
fig_re <- ggplot() +
  geom_point(aes(x = exper, y = lwage, color = race), data = wagepan_sample) +
  geom_line(aes(x = exper, y = lwage,  group = as.factor(nr), color = race),
            linetype = "dashed", data = wagepan_sample) +
  geom_line(aes(x = exper, y = yhat_re, group = as.factor(nr), color = race),
            size = 0.2, data = predict_re) +
  labs(title = "Predicted Wage(Random Effects)",
       x = "Years of Full-time Work Experience",
       y = "Log_e(Wage)") +
  theme_bw() +
  theme(legend.position = "bottom") +
  scale_colour_colorblind()

## combine plots
ggarrange(fig_fe, fig_re, ncol = 2)
```





```
#~#~#~#~#~# predicted effect of the OLS model #~#~#~#~#~#

## create a dataframe with race and years of experience based on the subsample
IV_ols <- data.frame(
  # each pid will repeat for 14 times for each value of exp
   black = rep(c(rep(1,length(exp_seq)), rep(0,length(exp_seq)), rep(0,length(exp_seq)))),
   hisp = rep(c(rep(0,length(exp_seq)), rep(1,length(exp_seq)), rep(0,length(exp_seq)))),
   white = rep(c(rep(0,length(exp_seq)), rep(0,length(exp_seq)), rep(1,length(exp_seq)))),
  # the exp seq will repeat 15 times so that it matches with 15 nr
  exper = rep(exp_seq, 3)
  )

predict_ols <- IV_ols %>%
  mutate(fit = predict(m_ols,IV_ols)) %>%
  pivot_longer(cols=c("fit"),values_to = "fit")  %>%
  select(-name) %>%
  mutate(race=case_when(
    black==1 ~ "black",
    hisp==1 ~ "hisp",
    white==1 ~ "white"
  ))


## plot
fig_ols <- ggplot() +
  geom_point(aes(x = exper, y = lwage, color = race), data = wagepan_sample) +
  geom_line(aes(x = exper, y = lwage,  group = as.factor(nr), color = race),
            linetype = "dashed", data = wagepan_sample) +
  geom_line(aes(x = exper, y = fit, color = race), size = 0.2, data = predict_ols) +
  labs(title = "Predicted Wage(OLS)",
       x = "Years of Full-time Work Experience",
       y = "Log_e(Wage)") +
  theme_bw() +
  theme(legend.position = "bottom") +
  scale_colour_colorblind()

## combine plots
ggarrange(fig_ols,
          fig_fe,
          fig_re,
          ncol = 3)
```



Part 4 Exercise

How much does a country's per capita income affect how much gasoline they consume? Use the `Gasoline` dataset from the `plm` package to answer this question with either fixed-effects or random-effects.

1. Load in package and explore data
2. What is your time invariant factor? That is to say, what unit are you grouping on for your models?
3. Run a fixed-effects and a random-effects model and compare them with a Hausman test.
4. Create a clean regression table with the best model and interpret the results causally with 1 sentence.