A Linear Optimization Problem: Minimizing the cost of stew made by UBC Sprouts to provide more free meals to the community.

Risako Kusumoto
Student Number: 31379605
MATH340 201 - Introduction to Linear Programming
Professor Young-Heon Kim
11th April 2023

Abstract

UBC Sprouts Cafe is a student-run cafe that operates on the UBC Life Building. They strive to fight food insecurity both to the UBC and the Vancouver community through their various initiatives including giving 30 free meals a day to the community at the cafe. These 30 free meals are able to be afforded due to the Food Security Initiative funding. Therefore, this study aims to minimize the cost of stew through using linear programming (LP) steps as well as the use of Python's PuLP package on JupyterLab to solve the LP problem. This minimization considers the amount of nutrients contained in one meal as the free meal that Sprouts Cafe provides focuses on being well-rounded, meaning that there is a healthy balance of each nutrient group. The analysis suggests that the use of the vegetables in the recipe are not needed to get the minimal cost of stew, and the optimal value indicates that Sprouts Cafe is able to double the number of free meals in a day to the community with the amount of funding that they receive. Therefore, we concluded that there is an optimal amount of ingredient to allow more free meals to be given in a day.

Introduction

Inflation in Vancouver has been ever-growing, where we saw an 8.8% inflation rate in British Columbia in 2022. Additionally there has been another 10.3% increase on food-related items since September 2022, and this percentage is projected to increase further into 2023 by around 5-7%. With this inflation, many UBC students and community members are struggling to fight food insecurity. As mentioned from the Alma Mater Society at the University of British Columbia, there seems to be around 43% of UBC Students who are food insecure and within that 19% were struggling to have enough food to last a month. Even though these students are also working, the wages seem to be stagnant compared to the steep increases in food, tuition and rent prices.

On that premise, there is a volunteer cooperative on the UBC campus called UBC Sprouts. UBC Sprouts is a student-run cafe and club with the aim to provide affordable and free nutritious meals to the community. They have several initiatives including community eats, the community fridge and distro, which is an initiative to provide free and healthy meals to the homeless community. Furthermore, this is a fully volunteer-run operation which means that any profits made by UBC Sprouts is used to fund their several initiatives as well as to maintain the cafe so that there is enough ingredients, there are new supplies and making sure that all aspects of the cafe are safe, i.e. stable furniture. However, most of the funding for the food cooperative is mainly from the University of British Columbia Food Security Initiative (FSI) where they donated around CAD$30,000 for the 2022/2023 terms. These funds have helped Sprouts provide up to 30 meals per day to the community. Estimating that the number of days in the school year is around 200 days. Then the CAD$30,000 donation would be around $150 per day and with the 30 free meals, the approximate price of producing each stew is around $5.

Acknowledging the aforementioned information, it also needs to be noted that many aspects have to be considered when choosing the recipes that are going to be chosen for the cafe shifts. This includes the requirement of the meal being fully vegan, meaning no animal products, for environmental reasons as well as to cater to everyone in the community. Therefore, I will be investigating a main recipe in the Sprouts Cafe which is the stew. To investigate this, the following criteria will be examined:

1) the nutritional value of each ingredient to ensure that nutrition goals are met for the average human

2) the price of each ingredient as we would like to minimize the cost of a serving of stew (250g)

3) How many more free meals can be provided, given the minimization

Through this research, I hope to establish a better conception of how best to minimize the total cost of each stew that the UBC Sprouts Cafe provides to the UBC community whilst ensuring that the nutritional needs of each individual is met. If the cost is minimized, then the Sprouts Cafe would be able to provide more than 30 free meals a day as the money donated to sprouts by FSI can be used to produce more individual meals for the community.

Furthermore, this information will first be determined by gathering the information for the ingredients i.e. the amount/ proportions needed for the stew, price per gram etc. and this will be the decision variables. Then, I will do research on nutritional needs of the average

person and use all of the information to create a linear programming problem which will then be solved using python via the UBC Syzygy website to find the optimal proportion of each ingredient. The optimal value will be able to estimate how many meals we can produce each day.

Data Collection

Firstly, in the data collection process, the core pieces of information that I had to gather were: 1) sufficient nutrients in one meal for the average human, 2) the price of each ingredient in the recipe (CAD$), 3, the amount of each ingredient in the recipe (grams).

1) According to the National Academy of Medicine, the adequate amount of carbohydrates, protein, fat and fibre for the average human that should be eaten per meal is:

17% Protein
43% Carbohydrate
11% Fibre
9.5% Fat

These values are an average of the nutrients needed per meal for males and females.

2) The list of ingredients in the recipe is as follows:

Ingredients	Price (per gram)
Oil	0.0096
Onion	0.0057
Squash	0.0022
Lentils	0.0073
Brown Rice	0.0064
Water	0.0036

The price of each of the ingredients is obtained from grocery invoices from the UBC Sprouts Cafe and then calculated by dividing the total price of the ingredient by the amount (in grams) of the supply when bought at that price.

For the purpose of this project, I will be neglecting the use of spices as it does not give a significant amount of nutritional value with the amount that is put into the stew. The full recipe is shown in Appendix 1. Furthermore, the price of water is an estimated value

based on the price of the UBC water billing[1] divided by the average amount of water used in a household in Vancouver[2].

3) The nutrition value for each of the ingredients

	Protein (per gram)	Carbohydrates (per gram)	Fibre (per gram)	Fat (per gram)
Oil	0.00	0.00	0.00	1
Onion	0.014	0.10	0.017	0.002
Squash	0.011	0.038	0.014	0.0039
Lentils	0.091	0.20	0.08	0.004
Brown Rice	0.023	0.23	0.018	0.0082
Water	0.00	0.00	0.00	0.00

The values for each ingredient were found on https://www.nutritionix.com/. I divided the amount gram amount for each nutrient to the serving size to get the value per gram.

Setting up the LP Problem

When setting up a linear programming problem, there are several factors that must be considered, identifying the objective, defining the decision variables, formulating the object function and then describing the constraints.

The objective of this linear programming problem is to minimize the cost of making the stew and finding the amounts needed of each of the ingredients.

1. Firstly when setting up an LP Problem, it is important to identify the decision variables. In this case it will be the amount in grams of each ingredient in the stew. This is because the decision variables are crucial to finding the unknown quantities that will answer our question. For this project,

Decision variables ($x_1, x_2, x_3, x_4, x_5, x_6$)

x_1 = the amount of oil used in the stew (in grams)
x_2 = the amount of onion used in the stew (in grams)

---

[1] *Billing UBC Facilities: Energy & Water Services*. University of British Columbia. Available at: https://energy.ubc.ca/community-services/energy-billing/ (Accessed: April 14, 2023).

[2] Honey-Roses, J., Gill, D. and Pareja, C. (2016) *BC Municipal Water Survey 2016 - waterplanninglab.sites.olt.ubc.ca*, *BC Municipal Water Survey 2016*. University of British Columbia. Available at: https://waterplanninglab.sites.olt.ubc.ca/files/2016/03/BC-Municipal-Water-Survey-2016.pdf (Accessed: April 14, 2023).

x_3 = the amount of squash used in the stew (in grams)
x_4 = the amount of lentils used in the stew (in grams)
x_5 = the amount of brown rice used in the stew (in grams)
x_6 = the amount of water used in the stew (in grams)

2. Next, I will formulate the objective function which is to minimize the cost of the stew. The objective function will be:

$$\min 0.0096x_1 + 0.0057x_2 + 0.0022x_3 + 0.0073x_4 + 0.0064x_5 + 0.0036x_6$$

This objective function was formulated with the price per gram of each of the ingredients as the aim is to minimize the cost of the ingredients to produce a serving of stew.

3. The next step after formulating the objective function is to create the constraints for the problem. These constraints will consist of:

$$0.012x_1 + 0.095x_2 + 0.225x_3 + 0.16x_4 + 0.075x_5 + x_6 = 250 \quad \cdots\cdots\cdots\cdots \quad (1)$$

$$0x_1 + 0.014x_2 + 0.011x_3 + 0.091x_4 + 0.023x_5 + 0x_6 \geq 17 \quad \cdots\cdots\cdots\cdots \quad (2)$$

$$0x_1 + 0.1x_2 + 0.038x_3 + 0.2x_4 + 0.23x_5 + 0x_6 \geq 43 \quad \cdots\cdots\cdots\cdots \quad (3)$$

$$0x_1 + 0.017x_2 + 0.014x_3 + 0.08x_4 + 0.018x_5 + 0x_6 \geq 11 \quad \cdots\cdots\cdots\cdots \quad (4)$$

$$1x_1 + 0.002x_2 + 0.0039x_3 + 0.004x_4 + 0.0082x_5 + 0x_6 \geq 9.5 \quad \cdots\cdots\cdots\cdots \quad (5)$$

Each of these constraints have been chosen to answer the projects aim of optimizing the nutrient amount.

For (1), this is the constraint where the sum must be the amount (in grams) of stew per serving which is 250g. Each decision variable is also multiplied by the ratio of the amount in the recipe so it is the amount (in grams) of each ingredient divided by the total amount (in grams) of all the ingredient items.

For (2), this is the constraint for the protein requirement. In the 250g serving, there needs to be at least 17g as calculated in the data collection section. Each decision variable is multiplied by its protein provided per gram.

For (3), this is the constraint for the carbohydrate requirement. In the 250g serving, there needs to be at least 43g as calculated in the data collection section. Each decision variable is multiplied by its carbohydrate provided per gram.

For (4), this is the constraint for the fibre requirement. In the 250g serving, there needs to be at least 11g as calculated in the data collection section. Each decision variable is multiplied by its fibre provided per gram.

For (5), this is the constraint for the fat requirement. In the 250g serving, there needs to be at least 9.5g as calculated in the data collection section. Each decision variable is multiplied by its fat provided per gram.

Result

The process of solving the linear programming problem was done using the UBC Syzygy server, by the Pacific Institute for the Mathematical Sciences, Compute Canada and Cybera, specifically programming with Python using the PuLP package.

Below is a screenshot of the optimal solution and the optimal value from the aforementioned linear programming problem that I formulated. The full screenshot of the JupyterLab page displaying the Python code for the linear programming problem is provided Appendix 2.

```
x_1 = 8.52691
x_2 = 0.0
x_3 = 0.0
x_4 = 178.873
x_5 = 31.4146
x_6 = 218.922

Total Cost of Ingredients per stew =  2.376803876
```

This result displayed with the specific ingredients is as following:

Ingredient	Amount (in grams)
Oil	8.527
Onion	0.000
Squash	0.000
Lentils	178.873
Brown Rice	31.415
Water	218.922
Total Cost of 250g of Stew ($): 2.38	

If the total cost of the stew is $2.38 and UBC Sprouts is able to get the same amount of funding for the future, i.e. $30,000, then we can calculate the amount of free meals that Sprouts can offer each day. Again, estimating that the number of days in the school year is

200 days, UBC Sprouts Cafe has around $150 per day for the meals. If we divide this amount by $2.38, then we are totaled with 63 free meals per day.

Discussion

From the previous section, we have obtained the results from programming the objective function and its constraints through the JupyterLab Notebook. From analyzing this information, the results suggest that neither the use of onions or squash is needed to satisfy the nutrient requirement needed by the average human. Furthermore, the ingredient used the most in making the stew was the lentils, which was significantly higher than the use of brown rice. With these ratios of ingredients, UBC Sprouts is able to provide a bit more than double the number of free meals that they would normally give with $30,000.

This result does not seem to be random because of the information collected in the data collection section, showing that lentils and brown rice have a higher nutrient per gram for all food nutrient areas explored (i.e. protein, carbohydrates, fibre and fats). Furthermore, the price per gram of the onion and squash are both generally lower than the price per gram of the lentils and brown rice by around 1/3 for the squash and 2/3 for the onion. From the result, we can then suggest that the price of the vegetables are not a significant indicator to minimizing the cost as these were among the cheapest ingredients from our list of investigative items.

These results should be taken into account when considering how to minimize the cost of each ingredient accounting for nutrients with just these vegetables or similar types of vegetables (e.g. Squash is in the Cucurbitaceae Family). This is because they will most likely have similar levels of nutrient value when specifically looking at the nutrient groups researched in this paper[3].

However, when considering the data and the aim of this paper, it is important to acknowledge several limitations to the research. Firstly, the use of vegetables should not be completely erased as there are other different vitamin and mineral requirements that should be considered. This solely focuses on a limited amount of nutrient groups as there is a plethora of other variables that contributes to a “healthy” meal. Furthermore, these nutrients are an estimate for the average human and does not specifically account for other variables such as gender, age, and possible intolerances/allergies. The definition of “healthy” differs for each person, therefore this project is more generalized and fits a simplified definition of healthy looking at different proportions of food groups. Additionally, taking into consideration that the UBC Sprouts Cafe is a fully volunteer and student run cafe, it is possible that shipments are late or shipments of different vegetables are made, and therefore the vegetables in the recipe are replaced with other vegetables such as carrots, peas etc. This would mean that the optimized values may differ as these ingredients have different prices and nutrients per gram.

---

[3] Diane Rellinger, M.S.U.E. (2018) *Are all vegetables created equal?*, *MSU Extension*. Michigan State University. Available at: https://www.canr.msu.edu/news/are_all_vegetables_created_equal (Accessed: April 14, 2023).

Conclusion

The aim of this project has been to minimize the cost of the stew at UBC Sprouts cafe whilst considering individuals in the community will still obtain enough nutrients in their meal, in order for Sprouts Cafe to provide more free meals to the community and fight food insecurity. Furthermore, through the use of JupyterLab, we have concluded that the most important ingredient to use for the 250g of stew is lentils as it provides the most nutrients in all areas (i.e. protein, carbohydrates, fats and fibre). Conversely, the least important ingredient to use for the 250g of stew are both the vegetables, squash and onion, where the study indicated none of these vegetables were needed to satisfy the nutrient requirement.

This project illustrates that with limited ingredients, the UBC Sprouts cafe is able to create a stew that still has important nutrients for their community and provide 63 free meals as a result, but it also raises the question of investigating other vegetables that could be contributed into the stew to lower this value and keep the nutrient levels at a healthy amount per meal.

To better understand the implications of the results found in this project, further studies could focus on different families of vegetables that could be added with varying nutrient levels per gram and further, adding more constraints on the vitamin and mineral content as I believe this would change the optimization result.

References

Vercelletto, C. (2019) *How much protein, fat, carbohydrates and fiber each meal should have | Livestrong*, *LIVESTRONG.COM*. Leaf Group. Available at: https://www.livestrong.com/article/518666-how-much-protein-fat-carbohydrates-fiber-should-each-meal-have/ (Accessed: April 14, 2023).

Diane Rellinger, M.S.U.E. (2018) *Are all vegetables created equal?*, *MSU Extension*. Michigan State University. Available at: https://www.canr.msu.edu/news/are_all_vegetables_created_equal (Accessed: April 14, 2023).

Billing UBC Facilities: Energy & Water Services. University of British Columbia. Available at: https://energy.ubc.ca/community-services/energy-billing/ (Accessed: April 14, 2023).

Honey-Roses, J., Gill, D. and Pareja, C. (2016) *BC Municipal Water Survey 2016 - waterplanninglab.sites.olt.ubc.ca*, *BC Municipal Water Survey 2016*. University of British Columbia. Available at: https://waterplanninglab.sites.olt.ubc.ca/files/2016/03/BC-Municipal-Water-Survey-2016.pdf (Accessed: April 14, 2023).

Nutritionix. Syndigo LCC. Available at: https://www.nutritionix.com/ (Accessed: April 14, 2023).

Appendix 1

Lentil Squash Stew

~1 c oil
1.9 kg onion (roughly diced)
½ c garlic powder
¼ c ginger powder
⅓ c curry powder
⅓ c cumin
⅔ c garam masala
3 tbsp turmeric

4.5 kg squash (washed, skin on, cut into 1 inch chunks)
3.2 kg dried lentils
20 L water
1.5 kg brown rice
⅔ c salt
90 g nutritional yeast (1 c)
2 tbsp onion powder
1 tbsp parsley flakes
2 tbsp black pepper
⅓ c cane sugar

1. Prep vegetables
    a. **Onion**: roughly diced
    b. **Squash:** washed, seeds removed, skin on, cut into ~1 inch chunks
2. Wash rice
3. Heat up oil in soup cauldron on setting number **9** (don't need to measure the oil, just approximately)
4. Add left column of ingredients and saute until onions are translucent
5. Add the right column of ingredients and stir
6. **Cover** the cauldron and bring to a boil
7. Turn the heat down to **6** and cook for ~45 minutes or until squash is cooked. **Stir** the stew (especially the bottom) periodically so nothing burns! If the stew is getting too thick, add a little more water.

Appendix 2

project

April 14, 2023

[3]:
```
import sys
!{sys.executable} -m pip install pulp
```

```
Requirement already satisfied: pulp in /opt/conda/lib/python3.10/si
(2.7.0)
```

[4]:
```
import pulp
```

[5]:
```
from pulp import *
```

[6]:
```
prob = LpProblem("Sprouts_Stew_Optimization", LpMinimize)
x_1 = LpVariable("x_1")
x_2 = LpVariable("x_2")
x_3 = LpVariable("x_3")
x_4 = LpVariable("x_4")
x_5 = LpVariable("x_5")
x_6 = LpVariable("x_6")
```

[7]:
```
prob+=0.0096*x_1+0.0057*x_2+0.0022*x_3+0.0073*x_4+0.0064*x_5+0.003

prob+=0.012*x_1+0.095*x_2+0.225*x_3+0.16*x_4+0.075*x_5+x_6==250
prob+=0*x_1+0.014*x_2+0.011*x_3+0.091*x_4+0.023*x_5+0*x_6>=17
prob+=0*x_1+0.1*x_2+0.038*x_3+0.2*x_4+0.23*x_5+0*x_6>=43
prob+=0*x_1+0.017*x_2+0.014*x_3+0.08*x_4+0.018*x_5+0*x_6>=11
prob+=1*x_1+0.002*x_2+0.0039*x_3+0.004*x_4+0.0082*x_5+0*x_6>=9.5
prob+=x_1>=0
prob+=x_2>=0
prob+=x_3>=0
prob+=x_4>=0
prob+=x_5>=0
```