ASEN 3802 Lab 2: Heat Conduction Lab

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Task 1: Steady State Temperature Distribution

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Assuming that x_0 lies $1\frac{3}{8}$ inches to the left of the first thermocouple:

- Determine the temperature at the cold end of the rod (T_0) using the experimental data.
- Determine the steady state temperature slope (H_{exp}) using the experimental data.
- Calculate the analytical steady state temperature distribution slope (H_{an}) given the heater power, thermal conductivity, and cross-sectional area.

Deliverables

In your PDF, include the following:

- **Plots** of the steady state temperatures at each location according to the experimental and analytical steady state slopes found above, overlayed onto the experimental data. Include error bars for the experimental results.
- **Table** containing $T_0[°C]$, $H_{exp}[°C/m]$, and $H_{an}[°C/m]$ for each data set.
- **Discussion** on the following:
  1. Compare the experimentally-determined steady-state slope, $H_{exp}[°C/m]$ to the analytically-determined steady-state slope, $H_{an}[°C/m]$ for each data set.
  2. Quantify any differences you observe and comment on the similarity of trends for all of the tested materials.
  3. If some materials do not show good alignment between experimental data and the analytical model, propose several explanations grounded in engineering reasoning.

Fig 1.1: Steady State Temperature Distribution

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Steady State Solution

Because of these boundary conditions, which are valid at any and all times t, the steady state solution of the heat equation, $v(x)$ (shown in Equation (3)), depends only on spatial position:

$$v(x) = T_0 + Hx \qquad \text{where} \qquad H = \frac{\dot{Q}}{kA} \tag{3}$$

Bullet point 3:

Cross sectional area: We know the bar is 1 inch in diameter; so cross sectional area is $\frac{\pi}{4}d^2$, or in this case simply $\frac{\pi}{4}$ in^2, or 0.7853981 in^2.

Table 1 Material Properties for Metal Rods

Material	Properties		
	Density (ρ) [kg/m^3]	Specific Heat Capacity (c_p) [J/(kg·K)]	Thermal Conductivity (k) [W/(m·K)]
Aluminum 7075-T651	2810	960	130
Brass C360	8500	380	115
Stainless Steel T-303 Annealed	8000	500	16.2

Fig 1.2: Materials Properties of Metal Rods

Serial Number (SN)	Materials	T_0 [^{0}C]	H_{exp}[^{0}C/m]	H_{an}[^{0}C/m]
1	Brass 25V, 237 mA	5.9867*E-5	29.35	101.68
2	Brass 30V, 285 mA	3.3996*E-6	35.35	146.73
3	Steel 22V, 203mA	8.8009*E-5	50.31	544.07
4	Aluminum 25V, 250mA	-9.9029*E-6	23.97	94.88
5	Aluminum 30V, 290mA	1.1502*E-5	26.67	132.1

Fig 1.3: Materials with initial temperature (T_0), H_{exp} and H_{an}

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Discussion

Although, there is a large variation between $\mathbf{H}_{exp}$ and $\mathbf{H}_{an}$.But the trends show similar patterns between analytical and experimental values. The H analytical and experimental is larger for Brass (25V, 285 mA) compared to Brass (25V, 237 mA). This shows that they resemble similar trends although values for H analytical and experimental vary by a lot. I.e. for SN1 $\mathbf{H}_{an,\ Brass} = 3.5\mathbf{H}_{exp,\ Brass}$ **and** for SN2 $\mathbf{H}_{an,\ Brass} = 4.15\mathbf{H}_{exp,\ Brass}$

The H analytical and experimental is larger for Aluminum (30V, 290 mA) compared to Brass (25V, 250 mA). This shows that they resemble similar trends although values for H analytical and experimental vary by a lot. I.e. for SN4 $\mathbf{H}_{an,\ Aluminum} = 4\mathbf{H}_{exp,\ Aluminum}$ **and** for SN5 $\mathbf{H}_{an,\ Aluminum} = 5\mathbf{H}_{exp,\ Aluminum}$

The H analytical for the stainless steel is much larger compared to H experimental across any other metal rods. I.e. for SN3 $\mathbf{H}_{an,\ steel} = 10.8\mathbf{H}_{exp,\ steel}$for It shows the highest deviation between H analytical and experimental compared to other metals used. This is because stainless steel has lowest thermal conductivity.

Engineering reasons for variation in analytical and experimental values:

1. Unaccounted Heat loss or leakage through the system → The system might not setup perfectly to called as "adiabatic"
2. The Heat flow assumed to be one Dimensional.
3. The temperature might not be in a steady state when the temperature was measured by each thermocouple. This could bring significant variation in the experimental results and steady-state patterns for each plot.

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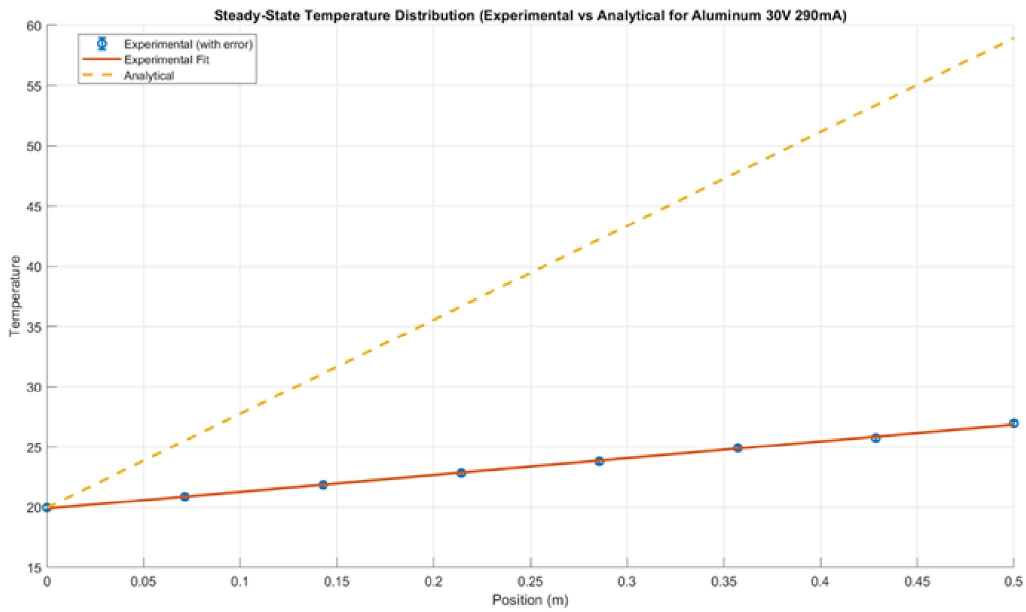

Fig 2.1: Steady State temperature distribution (Experimental vs Analytical for Aluminum 30V, 290mA).

The figure above shows the steady state temperature distribution along the length for aluminum at 30V and 290mA. The experimental data increases gradually and linearly from the chilled end (T= 20°C at x= 0m) to the heated end of the rod (~27°C at x= 0.5m), indicating that the system has reached steady state and that the heat conduction through the rod is 1-D with a constant temperature gradient. However, the analytical solution predicts a greater linear increase in temperature (going from T=20°C at x=0m to ~T=59°C at x=0.5m) than the one seen in the experimental data. This means the theoretical model is predicting a larger temperature rise along the rod than what actually happened. The difference shows that the real system doesn't behave exactly like the idealized model. In specific terms, this is a difference of nearly 80% between H_{exp} and H_{an}.

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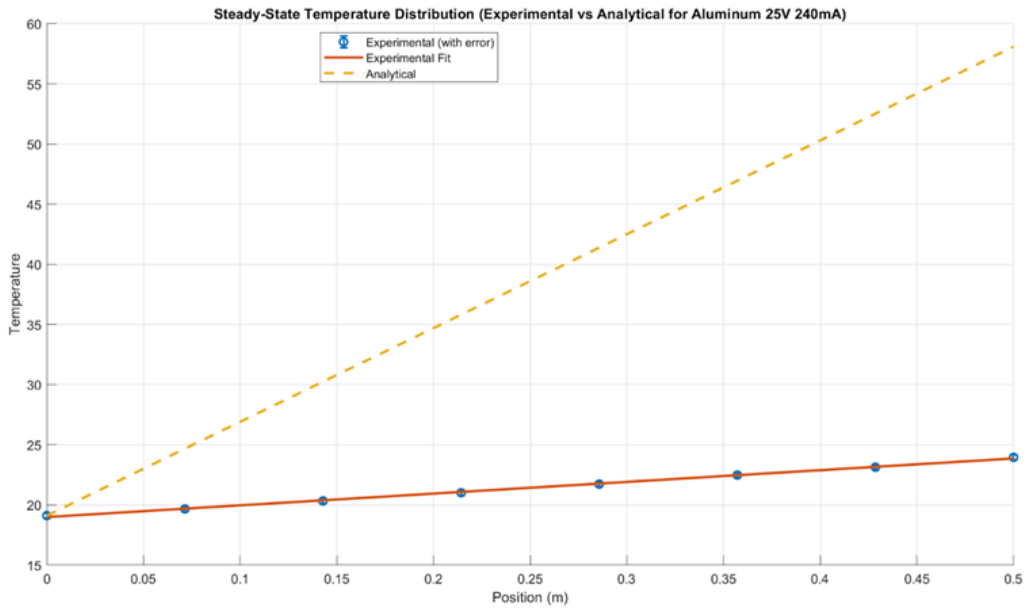

Fig 2.2: Steady State distribution (Experimental vs Analytical for Aluminum 25V, 240mV).

The figure above shows the steady state temperature distribution along the length for aluminum at 25V and 250mA. The experimental data displays a gradual and linear increase in temperature from the chilled end (~19°C at x=0m) to the heated one (~ 24°C at x=0.5m), indicating that the rod has reached steady state conditions and that heat conduction is 1-D. The temperature rise across the rod is relatively small, which is consistent with the aluminum's high thermal conductivity. While the experimental linear fit follows the measured data really closely, the analytical solution predicts a larger temperature gradient and higher temperature at the end of the rod of ~57°C. This suggests that the analytical model overestimated the temperature increase. In specific terms, this is a difference of approximately 74% between H_{exp} and H_{an}.

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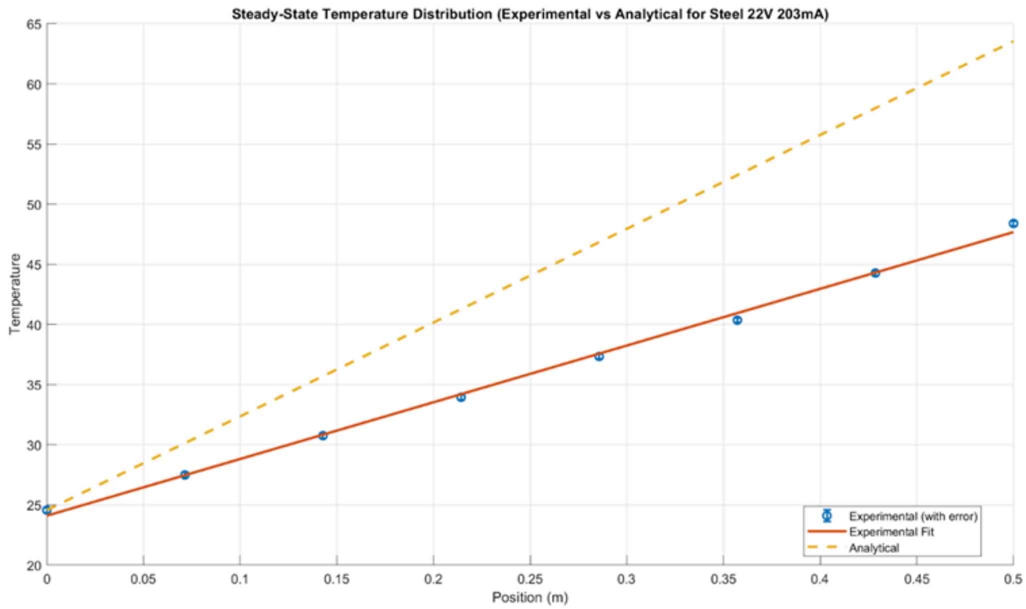

Fig 2.3: Steady State temperature distribution (Experimental vs Analytical for Steel 22V, 203mA).

The figure above shows the steady state temperature distribution along the length for steel at 22V and 203mA.The experimental data increases steadily and linearly from around 24°C at the chilled end (x= 0m) to about 48°C at x=0.5m, indicating that the system has reached steady state and that the heat conduction is 1-D with a constant temperature gradient. Compared to the past two figures where Aluminum was analyzed, the temperature rise is larger, which is expected as steel has a lower thermal conductivity that causes heat to accumulate more and produce a larger gradient. While the analytical solution follows the same linear trend, it predicts higher temperatures, reaching temperatures ~63°C at the far end of the rod. This means the theory also overestimates the rise in temperature, and this is likely due to assumptions like idealized insulation and no heat loss to the surroundings. This is a difference of 90% between H_{exp} and H_{an}, predicting nearly double the temperature distribution.

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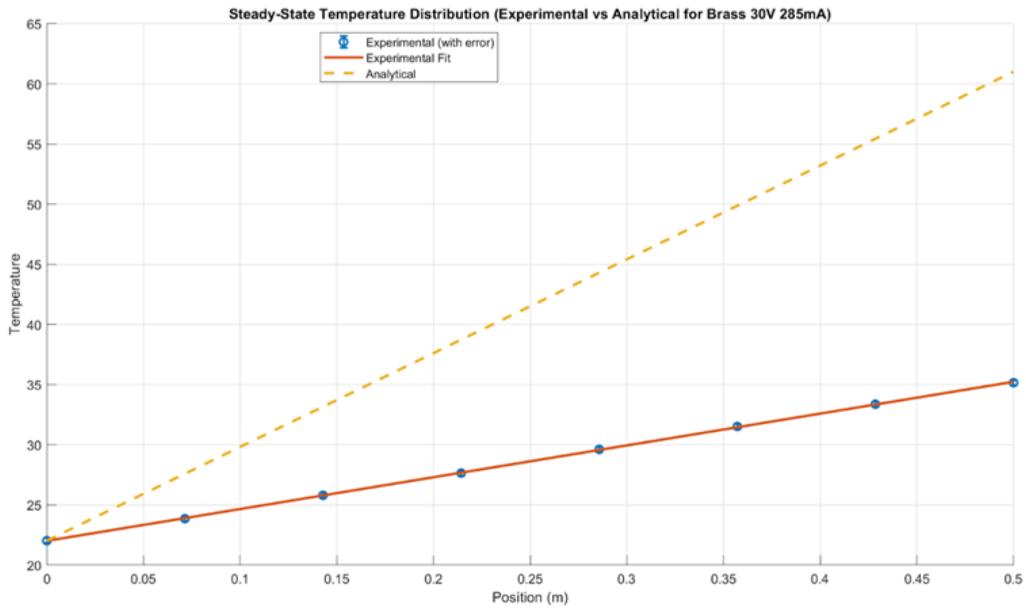

Fig 2.4: Steady State temperature distribution (Experimental Vs Analytical for Brass 30V, 285mA).

The figure above shows the steady state temperature distribution for brass at 30V and 285mA. The experimental data increases linearly from around 22°C at the chilled end (x=0m) to 35°C at x=0.5m, it indicates that the rod has reached steady state conditions and that heat conduction is 1-D with a constant temperature gradient. The slope of the experimental line is moderate, which is consistent with brass having a thermal conductivity between aluminum and steel. However, the analytical solution predicts a much larger temperature rise that reaches a temperature around 61°C at the far end. This means that the theoretical mode overestimated the temperature increase, this is likely due to assuming perfect insulation and no heat losses to the environment. This is a difference of about 75% between H_{exp} and H_{an}.

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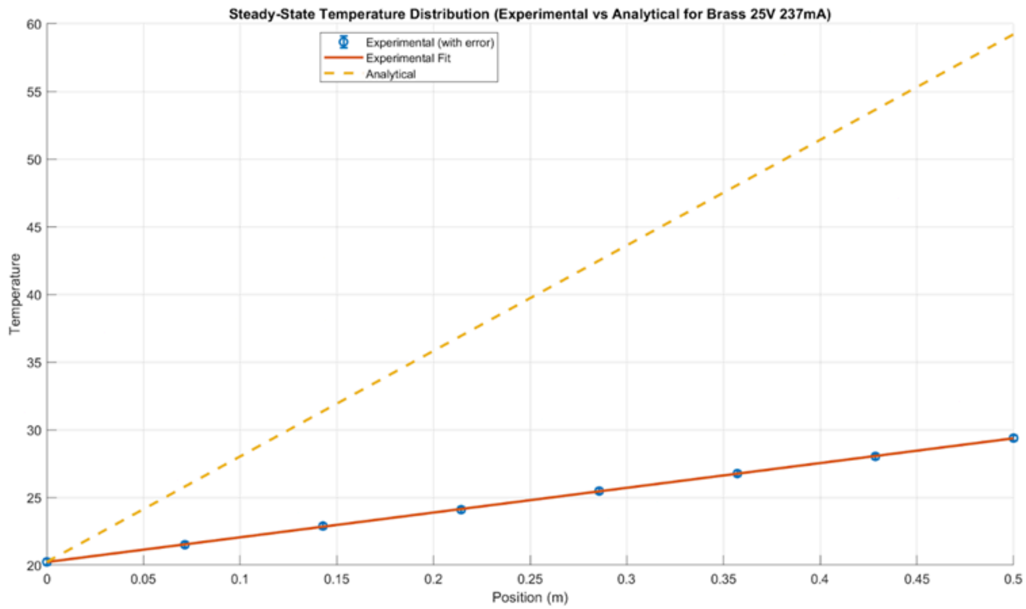

Fig 2.5: Steady State temperature distribution (Experimental VS Analytical for Brass 25V, 237mA).

The figure above shows the steady state temperature distribution for Brass at 25V and 237mA. Similar to figure 4, the experimental data increases gradually and linearly from T=20°C at the chilled end (x=0m) to ~ 29°C at x=0.5m, confirming steady state and 1-D heat conduction. The slope of the experimental line is moderate, which is consistent with brass having a thermal conductivity between aluminum and steel. However, the analytical solution predicts a much larger temperature rise that reaches a temperature around 59°C at the far end. This means that the theoretical mode overestimated the temperature increase, this is likely due to assuming perfect insulation and no heat losses to the environment. In specific terms, this is a difference of 71% between H_{exp} and H_{an}.

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Task 2: Initial Temperature Distribution

For this part of the lab:

- Perform a linear fit on the initial temperature distribution and display your results in a table containing the initial slope $M_{exp}[°C/m]$.
- Generate plots with the initial state temperatures at each thermocouple locations as predicted by your linear fits overlaps onto the raw experimental data.

Deliverables

In your PDF, include the following:

- **Table** containing the initial slope $M_{exp}[°C/m]$ for each material at each voltage.
- **Plots** of the initial temperature of each thermocouple.
- **Discussion** on the following:
    1. Justify the assumptions that the initial temperature of the entire rod was constant for each data set.
    2. If the assumption is not valid, discuss how the altered initial temperature profile might affect the transient solution.
    3. Will a variation in the initial temperature distribution affect the steady state solution?

Task 2: Initial Temperature Distribution

Materials	**[^{0}C/m] M_{exp}**
Brass 25V, 237 mA	18
Brass 30V, 285 mA	10
Steel 22V, 203mA	46
Aluminum 25V, 250mA	10
Aluminum 30V, 290mA	14

Fig 3.1: Materials and M_{exp} value

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Plots

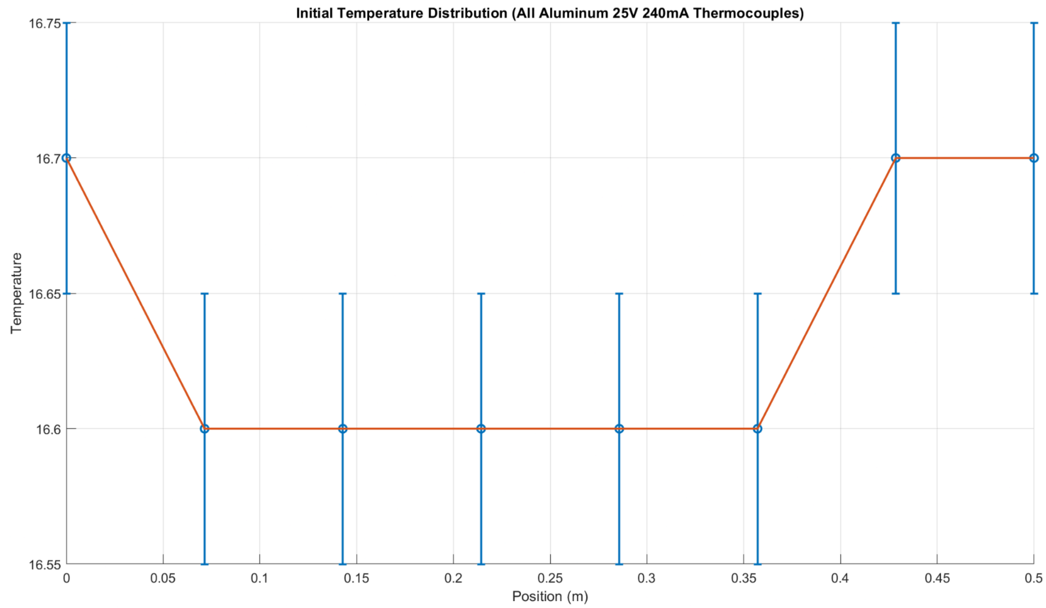

Figure 3.1: Initial Temperature Distribution (All Aluminum 25V 240mA Thermocouples)

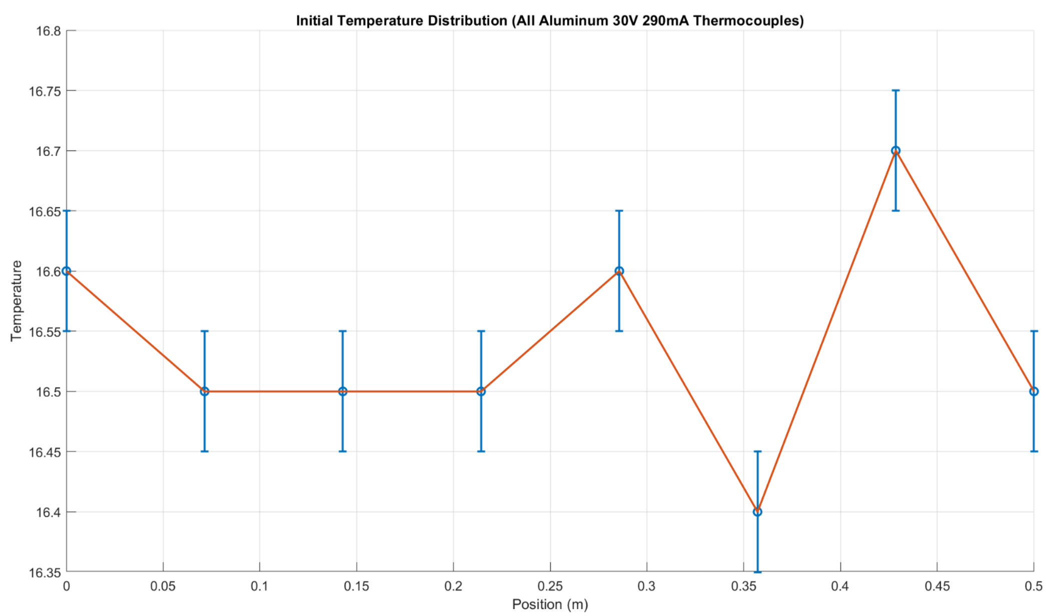

Figure 3.2: Initial Temperature Distribution (All Aluminum 30V 290mA Thermocouples)

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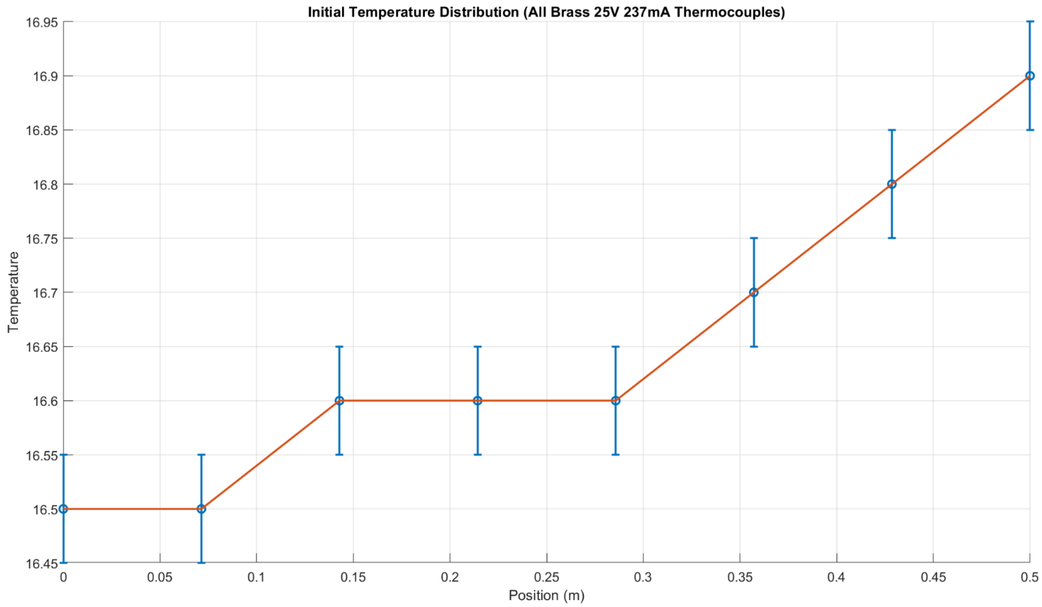

Figure 3.3: Initial Temperature Distribution (All Brass 25V 237mA Thermocouples)

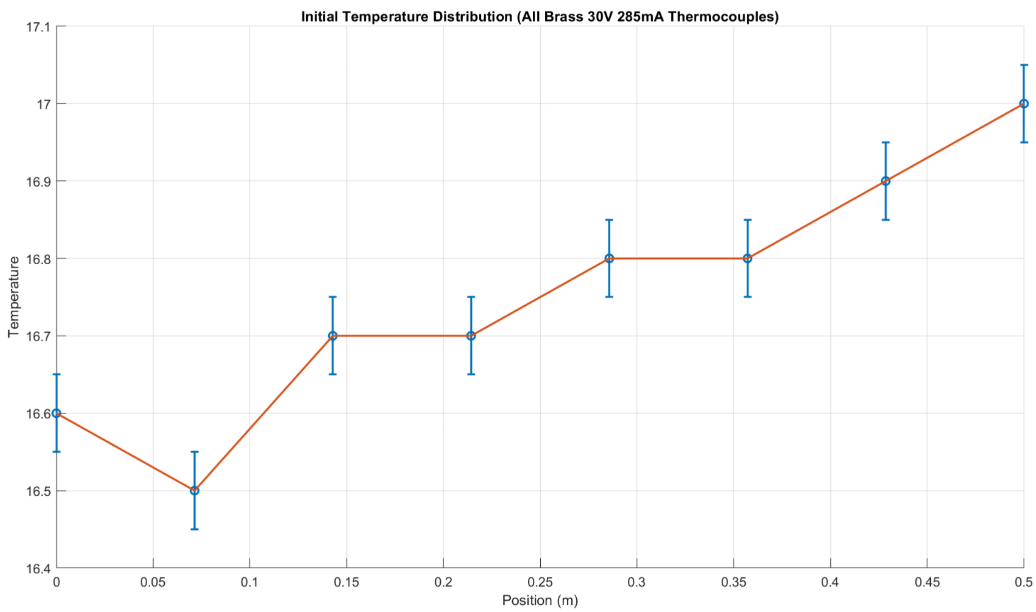

Figure 3.4: Initial Temperature Distribution (All Brass 30V 285mA Thermocouples)

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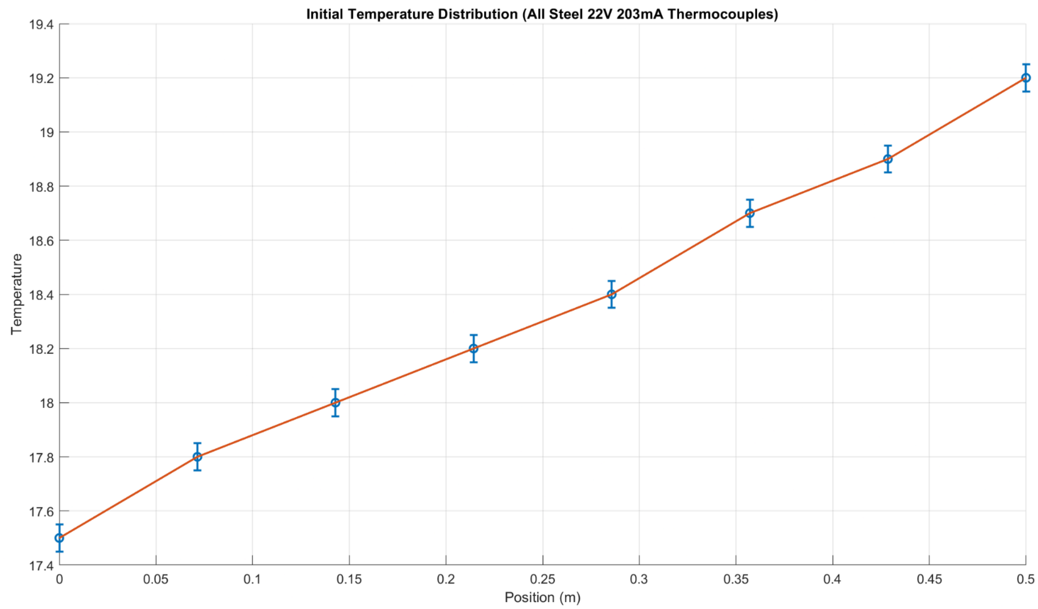

Figure 3.5: Initial Temperature Distribution (All Steel 22V 203mA Thermocouples)

Discussion

The assumption that the initial temperature of the rod was uniform is reasonable as the initial thermocouple readings were nearly the same at all locations, with any small differences falling around a 2°C measurement uncertainty. Indicating the rod was approximately at thermal equilibrium at the start of each trial, making a constant initial temperature a practical assumption for modeling.

If the initial temperature was not uniform, it would affect the early transient response. Since the transient solution depends on the starting temperature distribution, any variation at the initial time would change how the temperature evolves over time. Some sections of the rod might heat up faster or slower than predicted by the model. However, the influence of the initial condition is less over time.

A change in the initial temperature distribution wouldn't affect the steady state solution. The steady state temperature depends only on the boundary conditions and material properties, so regardless of how the system started, the rod will eventually reach the same final temperature distribution.

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