# Due Dates

- **Part 1:** February 25, 2026 by 11:59 PM
- **Part 2:** March 11, 2026 by 11:59 PM
- **Part 3:** March 25, 2026 by 11:59 PM

# Part 2: Transient Solution - Model I and Model II

## Task 1 - Analytical Transient Solution Derivation

For this part of the lab, you will:

- Simplify the coefficient $b_n$ by evaluating the integral from 0 to L.
- Determine the number of modes to use within the infinite sum for the analytical model.
- Discuss the Fourier number as it pertains to the number of modes to use within the infinite sum.

### Number of Modes in Analytical Solution

The general solution, $u(x,t)$, involves taking the summation of the transient effects from $n = 1$ to $n \to \infty$. It is not practical (or even possible) to program an infinite summation on a computer. Therefore, you are tasked with determining how any modes in the infinite sum are appropriate to resolve an analytical solution of reasonable fidelity. There are different methods of determining the appropriate number of modes to keep in the infinite sum. The method explained here is a crude, but decent method that you may use.

Begin by considering the location along the rod which is subject to the largest temperature gradient. At this point, calculate $u(x,t)$ for different number of modes ranging from $n = 0$ (your steady solution) to $n = 10$ and plot the Temperature variation at this spatial location for different $n$ summations. You should see that the solution will not change much after after a certain number of modes are considered in the solution sum.

### Fourier Number

The **Fourier number** is a non-dimensional number that is calculated as follows:

$$Fo = \frac{\alpha t}{L^2}$$

The Fourier number is a dimensionless time parameter that characterizes the time required for heat to diffuse over a distance. Notice that you can refactor the exponential decay term to be written in terms of the Fourier number.

In Heat Transfer, a generally accepted rule is that for Fourier numbers greater than 0.2, then a *one-term approximation* of the transient solution is 98% accurate.

### Deliverables

In your PDF, include the following:

- **Derivation** showing the full simplification of $b_n$. Your work should clearly show the steps required to integrate this function from 0 to L and simplify the coefficient to a function that is purely in terms of $n$, $L$, and $H$ with **no remaining trigonometric functions**. You will use this simplified form in your calculations of the analytical solution.

  *Hint: Use integration-by-parts, substitute the definition of $\lambda_n$, and apply trigonometric identities.*

- **Plot** which displays the Temperature value ($T$) vs. number of modes considered in the summation term ($n$) at the 8th thermocouple locations ($\text{Th}_8$) for two different times:
  - $t = 1s$
  - $t = 1000s$

  This plot effectively shows the *convergence* of the temperature solution based on the number of required modes.

  For this Task, consider only the following:
  - Aluminum at **25V**, using the values for $H_{analytical}$ and $T_0$ from Part 1.
  - $x$ is the location of the last thermocouple ($\text{Th}_8$)
- **Discussion** of the following:
  1. Report on your Convergence study, stating how many number of modes see reasonable when solving the analytical model.
  2. Does one time require more modes to converge over the other?
  3. Calculate the Fourier number for both times for the entire rod length. Is one-term sufficient for this experiment?

## Task 2: Transient Temperature Solution - Model IA

For this part of the lab, you will:
- Plot the transient response at each thermocouple location using the analytical model (Model IA) and compare to experimental data.

### Deliverables

In your PDF, include the following:

- **Plots** of the analytical solution $u(x,t)$ for each material and voltage case as a function of time for each thermocouple location.
  - For the analytical solution, use the analytical steady state slope calculated in Part 1 - Task 1 ($H_{analytical}$) along with the steady-state y-intercept in Part 1 - Task 1 ($T_0$).
  - Plot each thermocouple location as a single color line and overlay this with the corresponding experimental data for each thermocouple location using a separate color line.
  - There should be 5 plots in total, one for each material at each voltage.
  - Include a legend that clearly states the difference between experimental and analytical solutions.
  - Include a title for the plots which clearly indicate `Model IA`

## Task 3: Transient Temperature Solution - Model IB

For this part of the lab:
- Plot the transient response at each thermocouple location using an updated form of the analytical model (Model IB) and compare to experimental data.

### Method

Ideally, the analytical and experimental steady state temperature solutions should be within approximately $1°C$ of each other. However, this may not be the case for certain materials due to discrepancies between $H_{experiental}$ and $H_{analytical}$. As such, it is useful to use the experimental data to best inform what assumptions are inaccurate in the analytical model. Considering the definition of $H$, it is clear that if the cross-sectional area is constant and the thermal conductivity is also constant, then there is likely an issue with the heat flux assumption. In an attempt to quantify the heat flux discrepancy, substitute the slope of your experimental curve fit into the analytical model.

#### Deliverables

In your PDF, include the following:

- **Plots** of the analytical solution $u(x,t)$ for each material and voltage case as a function of time for each thermocouple location using the updated $H$ value in the analytical solution.
  - For the analytical solution, substitute the experimental steady state slope calculated in Part 1 - Task 1 ($H_{experimental}$)
  - Plot each thermocouple location as a single color line and overlay this with the corresponding experimental data for each thermocouple location using a separate color line.
  - There should be 5 plots in total, one for each material at each voltage.
  - Include a legend that clearly states the difference between experimental and analytical solutions.
  - Include a title for the plots which clearly indicate `Model IB`

### Task 4: Transient Temperature Solution - Model II

For this part of the lab, you will:

- Apply a linear initial condition and re-establish the $b_n$ coefficient.
- Plot the transient response at each thermocouple location using an updated form of the analytical model (Model II) and compare to experimental data.

#### Method

After applying the modification in Task 3, you may have noticed that some of the materials do not satisfy the initial state condition. In an effort to adjust the analytical model to better represent the experimental data set, you will be tasked with implementing the initial state distribution into Model IB. To do this, you need to apply the *linear* initial condition that was found in Part 1 - Task 2 where $T_0$ represented the y-intercept and $M$ represented the slope of the initial condition. Applying this linear initial condition to the original problem will modify the coefficient $b_n$ as follows:

$$b_n = \frac{2}{L}\int_0^L (M - H)x sin(\lambda_n x)dx$$

Integrating and simplifying this coefficient as you did in Part 2 - Task 1 and using the experimental slope ($M_{exp}$) will yield a more accurate analytical solution.

#### Deliverables

In your PDF, include the following:

- **Plots** of the analytical solution $u(x,t)$ for each material and voltage case as a function of time for each thermocouple location using the updated $H$ value in the analytical solution and the updated initial state distribution ($M_{exp}$) in the analytical solution using the updated coefficient definition, $b_n$.
  - For the analytical solution, starting with Model IB, modify the initial state distribution by using the y-intercept determined in Part 1 - Task 2 ($T_0$).
  - Plot each thermocouple location as a single color line and overlay this with the corresponding experimental data for each thermocouple location using a separate color line.
  - There should be 5 plots in total, one for each material at each voltage.
  - Include a legend that clearly states the difference between experimental and analytical solutions.
  - Include a title for the plots which clearly indicate `Model IB`