Course: Machine learning Foundations

Solve with Instructor

WEEK 9

IIT Madras
ONLINE DEGREE

Problem 1

Which of the following sets are convex, closed and bounded?

$$A = \{(x, y) \in \mathbb{R}^2; x^2 + y^2 \leq 4\}$$

$$B = \{(x, y) \in \mathbb{R}^2; y = 2x + 4\}$$

$$C = \{(x, y) \in \mathbb{R}^2; x + y \leq 2, x \geq 0, y \geq 0\}$$

$$D = \{(x, y) \in \mathbb{R}^2; y > x^2\}$$

$$E = \{(x, y) \in \mathbb{R}^2; xy > 1, x \geq 0, y \geq 0\}$$

$$F = \{(x, y) \in \mathbb{R}^2; xy > 1, x \leq 0, y \leq 0\}$$

Solution

- https://www.geogebra.org/m/kuk45ab4

Problem 2

Let $f(x) = 3 + x^2 + x$ and $g(x) = e^{-x}$. What can you say about their composition $h_1 = fog$ and $h_2 = gof$?

A. h_1 is convex and h_2 is concave

B. h_1 is concave and h_2 is convex

C. h_1 and h_2 is convex

D. h_1 and h_2 is concave

Solution

- https://www.geogebra.org/m/esqcd4he

Problem 3

Consider the following problem.

Maximize $x + y$ subject to the constraints

$$x \geq 0, \quad y \geq 0, \quad -3x + 2y \leq -1, \quad x - y \leq 2.$$

Is the feasible region bounded or unbounded?

Solution

- https://www.geogebra.org/m/fggdwdsc

Problem 4

- Given below is a set of data points and their labels.

X	y
[1,2]	2.5
[2,1.5]	3
[2,3]	5
[3,4]	7

Perform linear regression on this data set. Let initial w at t=1 be $[0.1, 0.1]^T$. Using the gradient descent update equation with a learning rate as 0.1, compute the value of w at t = 2.

Solution

- $\nabla w = (X^T X)w - (X^T y)$
- $X^T X = \begin{bmatrix} 18 & 23 \\ 23 & 31.25 \end{bmatrix}$
- $X^T y = \begin{bmatrix} 39.5 \\ 52.5 \end{bmatrix}$
- $\nabla w = \begin{bmatrix} -35.4 \\ -47.07 \end{bmatrix}$
- $w^2 = w^1 - \eta \nabla w$
- $w^2 = \begin{bmatrix} 3.64 \\ 4.807 \end{bmatrix}$

Problem 5

- Given below is the primal form of a LP problem.

$$\begin{aligned} &\text{maximize } 3x_1 + 5x_2 + 9x_3 \\ &\text{subject to} \\ &x_1 + x_2 + 2x_3 \leq 6, \\ &2x_1 + 3x_2 + x_3 \leq 8 \text{ and } x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{aligned}$$

The dual objective problem has _____ variables.

Problem 6

- Given below is a primal objective function of a LP problem.

$$
\begin{aligned}
&\text{minimize } 20x_1 + 40x_2 \\
&\text{subject to} \\
&36x_1 + 6x_2 \geq 108, \\
&3x_1 + 12x_2 \geq 36, \\
&20x_1 + 10x_2 \geq 100 \text{ and } x_1 \geq 0, x_2 \geq 0
\end{aligned}
$$

Enter the dual objective function and dual problem constraints.

Solution

$$
\begin{aligned}
&\text{maximize } 108y_1 + 36y_2 + 100y_3 \\
&\text{subject to} \\
&36y_1 + 3y_2 + 20y_3 \leq 20, \\
&6y_1 + 12y_2 + 10y_3 \leq 40, \\
&\text{and } y_1 \geq 0, y_2 \geq 0, y_3 \geq 0
\end{aligned}
$$

Problem 7

- Using KKT conditions solve the following linear program.

minimize

$$f(x) = 2(x_1 + 1)^2 + 2(x_2 - 4)^2$$

subject to

$$x_1^2 + x_2^2 \leq 9$$

$$x_1 + x_2 \geq 2$$

Enter the values of function variables and Lagrange multipliers.

Thank You