Course: Machine Learning - Foundations
Week 1: Practice questions

1. (2 points) Two positive numbers have a sum of 60. What is the maximum product of one number times the square of other number?

   A. 0

   B. 32000

   C. 60000

   D. 64000

   Answer: B
   Let the two numbers be x and y
   x+y=60
   objective function from the question will be,
   $f(x) = x^2(60 - x)$
   For optima $f'(x) = 0$, $120x - 3x^2 = 0$
   $x = 0, 40$
   Product is maximum when x=40.
   maximum product=32000

2. (2 points) (Multiple select) The point on $y = x^2 + 1$ closest to (0,1.5) is

   A. (0, 1)

   B. (0.707, -1.5)

   C. (-0.707,1.5)

   D. (0, -1)

   Answer: A
   Objective function $f(x) = (x - 0)^2 + (x^2 + 1 - 1.5)^2$
   $f(x) = x^4 + 0.25$
   For minima $f'(x) = 0$
   $4x^3 = 0$
   $x = 0$
   Corresponding $y = 1$

3. (2 points) The volume of largest cone that can be inscribed in a circle of radius 3 m is (correct up to two decimal places)

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   Answer: 33.51 m^3
   $V = \frac{1}{3}\pi r^2 h$
   $r = \sqrt{9 - x^2}$

$h = 3 + x$
For maxima, $V'(x) = 0$
$-3x^2 - 6x + 9 = 0$
$x = 1, -3$
x can not be nagative.
So $r = 2.828$
$h = 4$
$V = 33.5$

4. (2 points) The area of largest rectangle that can be inscribed in a circle of radius 4 is

   A. 16

   B. 8

   C. 32

   D. 20

Answer: C
Let x and y be two sides of the rectangle.
$x^2 + y^2 = 64$
$y = \sqrt{64 - x^2}$
$A = xy = x\sqrt{64 - x^2}$
For maxima $\frac{dA}{dx} = 0$
$x = 4\sqrt{2}$
$y = 4\sqrt{2}$
$A = 32$

(Question 5-8 have common data) A manufacturing plant produces two products M and N. Maximum production capacity is 700 for total production. At least 270 units must be produced every day. Machine hours consumption per unit is 6 hours for M and 5 hours for N. At least 1100 machine hours must be used daily. Manufacturing cost is Rs 25 for M and Rs 35 for N.
Let, x_1 = No of units of M produced per day
and x_2 = No of units of N produced per day

5. (1 point) The objective function for above problem is

   A. min $f(x) = 25x_1 + 55x_2$

   B. min $f(x) = 35x_1 + 25x_2$

   C. min $f(x) = 25x_1 + 35x_2$

   D. min $f(x) = 10x_1 + 35x_2$

Answer: C
We need to minimize the cost of the function.

6. (1 point) The constraint due to maximum production capacity is

   A. $x_1 + x_2 \leq 700$

B. $x_1 + x_2 \geq 700$

C. $x_1 + x_2 \geq 270$

D. $x_1 + x_2 = 700$

Answer: A
Maximum production capacity is 700.

7. (1 point) The constraint due to minimum production capacity is

   A. $x_1 + x_2 \neq 270$

   B. $x_1 + x_2 = 270$

   C. $x_1 + x_2 \leq 270$

   D. $x_1 + x_2 \geq 270$

   Answer: D
   Minimum production capacity is 270.

8. (1 point) The constraint due to machine hour consumption is

   A. $6x_1 + 5x_2 \leq 1100$

   B. $6x_1 + 5x_2 \geq 1100$

   C. $6x_1 + 5x_2 \neq 1100$

   D. $6x_1 + 5x_2 = 1100$

   Answer: B
   At least 1100 hours must be used.

   (Questions 9-11 have common data)
   A factory manufactures two products A and B. To manufacture one unit of A, 3 machine hours and 5 labour hours are required. To manufacture product B, 2 machine hours and 4 labour hours are required. In a month, 270 machine hours and 280 labour hours are available. Profit per unit for A is Rs. 55 and for B is Rs. 15.
   Let x_1=Number of units of A produced per month
   and x_2=Number of units of B produced per month

9. (1 point) The objective function for above problem is

   A. max $f(x) = 55x_1 + 15x_2$

   B. min $f(x) = 55x_1 + 15x_2$

   C. max $f(x) = 15x_1 + 45x_2$

   D. min $f(x) = 15x_1 + 55x_2$

   Answer: A
   We need to maximise profit.

10. (2 points) The constraint for machine hours is

A. $3x_1 + 2x_2 \geq 270$

B. $3x_1 + 2x_2 \leq 270$

C. $3x_1 + 2x_2 \neq 270$

D. $3x_1 + 2x_2 = 270$

Answer: B
270 hours available.

11. (2 points) The constraint for labour hours is

A. $5x_1 + 4x_2 = 280$

B. $5x_1 + 4x_2 \leq 280$

C. $5x_1 + 4x_2 \geq 280$

D. $5x_1 + 4x_2 \neq 280$

Answer: B
280 labour hours is available.

12. (2 points) The value of a function at a point $x = 5$ is 3.2 and the value of the function's derivative at point $x = 5$ is 1.2. What will be the approximate value of the function at a point $x = 5.2$(First order approximation)?

Answer: 3.44
According to Taylor's series,
$f(x + h) = f(x) + hf'(x) + \frac{h^2 f'(x)}{2} + ...$
Here $x = 5, h = 0.2$
$\therefore f(x + h) = 3.44$

13. (2 points) For the function $f(x) = \frac{xsinx-1}{2}$, with an initial guess of $x_0 = -7$, and step size of 0.25, the value of the function after two iterations is (correct up to 3 decimal places)

Answer: -2.471
$x_{n+1} = x_n - \eta f'(x)$
After first iteration $x_1 = -6.258$
After second iteration $x_2 = -5.479$
$f(-5.479) = -2.47$

14. (2 points) The area of the largest rectangle that can be inscribed in a circle of radius 1 is

A. 1

B. 1.5

C. 6

D. 2

Answer: D
Follow same steps as q no 4