IIT Madras
ONLINE DEGREE

Tutorial
on
Row space computation using reduced row echelon form
and
solution to Ax=b

Course: MACHINE LEARNING FOUNDATIONS

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Example 2

For the matrix **B** find its row space by computing its reduced row echelon form.

$$B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 6 \\ -2 & 1 & 2 \end{bmatrix}$$

Solution

$$B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 6 \\ -2 & 1 & 2 \end{bmatrix} \xrightarrow[R_3 + 2R_2]{R_2 - 2R_1} \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 3 & 6 \end{pmatrix} \xrightarrow{R_3 - 3R_2} \underbrace{\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}}_{\text{Row echelon form}} \xrightarrow{R_1 - R_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \text{Reduced row echelon form}$$

Row space of B

$$\text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\}$$

Example 3

Consider the system of equations given below:

$$x_1 + 2x_2 - 2x_3 = b_1$$

$$2x_1 + 5x_2 - 4x_3 = b_2$$

$$4x_1 + 9x_2 - 8x_3 = b_3$$

i. Under what condition on b_1, b_2 and b_3, is the system solvable?

ii. Check whether $b_1 = 3$, $b_2 = -1$ and $b_3 = 5$ satisfies the condition or not?

Solution

$$x_1 + 2x_2 - 2x_3 = b_1$$
$$2x_1 + 5x_2 - 4x_3 = b_2$$
$$4x_1 + 9x_2 - 8x_3 = b_3$$

coefficient matrix

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 4 & 9 & -8 \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Augmented matrix $= [A\,b]$

$$\left(\begin{array}{ccc|c} \boxed{1} & 2 & -2 & b_1 \\ 2 & 5 & -4 & b_2 \\ 4 & 9 & -8 & b_3 \end{array}\right) \xrightarrow[R_3 - 4R_1]{R_2 - 2R_1} \left(\begin{array}{ccc|c} 1 & 2 & -2 & b_1 \\ 0 & 1 & 0 & b_2 - 2b_1 \\ 0 & 1 & 0 & b_3 - 4b_1 \end{array}\right) \xrightarrow{R_3 - R_2} \left(\begin{array}{ccc|c} 1 & 2 & -2 & b_1 \\ 0 & 1 & 0 & b_2 - 2b_1 \\ 0 & 0 & 0 & b_3 - b_2 - 4b_1 \end{array}\right)$$

condition : $b_3 - b_2 - 4b_1 = 5 - (-1) - 4(2) = 6 - 6 = 0$ ✓

$b_1 = 3$, $b_2 = -1$ and $b_3 = 5$

$$Ax = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix} \qquad b \in C(A)$$

Thank You