Why optimisation?

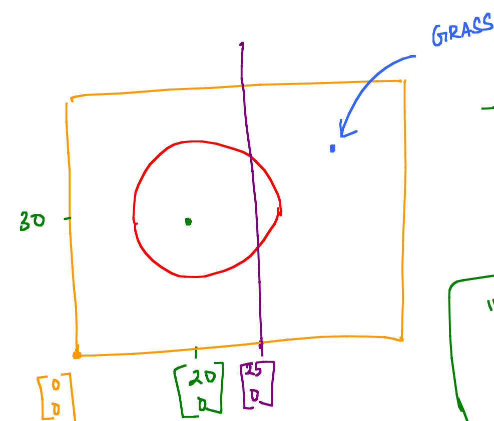
→ We care about finding the "best" classifier!

"Least" Loss

"maximum" reward

Example

- A cow is at $\begin{bmatrix} 20 \\ 30 \end{bmatrix}$
- Cow tied to a 10 unit radius rope.
- Perpendicular fence that passes through $\begin{bmatrix} 25 \\ 0 \end{bmatrix}$



- Grass on the field at $\begin{bmatrix} 40 \\ 40 \end{bmatrix}$

"How close can the cow get to the grass?"

$$\min_{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} \; (x_1 - 40)^2 + (x_2 - 40)^2$$

Rope restriction $\quad (x_1 - 20)^2 + (x_2 - 30)^2 \le 10^2$

Fence restriction $\quad x_1 \le 25$

What are we measuring?

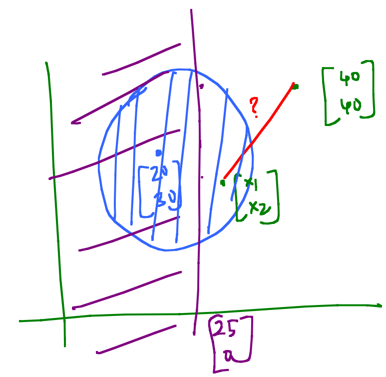
- Distance from grass $\begin{bmatrix} 40 \\ 40 \end{bmatrix}$

Say cow is at $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$d\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \begin{bmatrix} 40 \\ 40 \end{bmatrix} \right) = (x_1 - 40)^2 + (x_2 - 40)^2$$

What to do with distance?

- As "small" as possible.



General form

$$\min_{x \in \mathbb{R}^d} f(x)$$

OBJECTIVE

VARIABLE/PARAMETER.

$$\begin{cases} g_i(x) \le 0 & \forall i = 1, \dots, K \\ h_j(x) = 0 & \forall j = 1, \dots, L \end{cases}$$

CONSTRAINTS

INEQUALITY CONSTRAINTS

EQUALITY CONSTRAINTS.

UNCONSTRAINED OPTIMIZATION

$$\min_{x \in \mathbb{R}} (x-5)^2 = f(x)$$

Solution $x = 5$ Objective value at $x=5$ $= 0.$

$$f'(x) = 2(x^* - 5) = 0$$

$$\Rightarrow \boxed{x^* = 5}$$

$$\min_{x \in \mathbb{R}} 3x^6 + 2x^5 + 3x^3 + 5x^2 + 2$$

$$f'(x) = 18x^5 + 10x^4 + 9x^2 + 10x. = 0$$