Conditional PDF

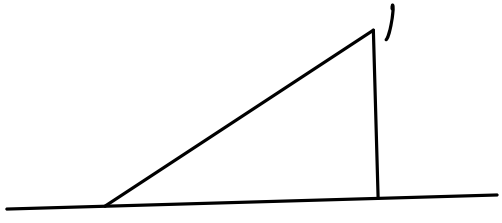
$(\Omega, \mathcal{F}, P)$

X, $A \subseteq \Omega$

$$f_{X|A}(x) = \frac{P(X \in [x, x+dx] \mid A)}{dx}$$

Examples

$$f_X(x) = \begin{cases} x/2 & \text{if } x \in [0,2] \\ 0 & \text{otherwise} \end{cases}$$

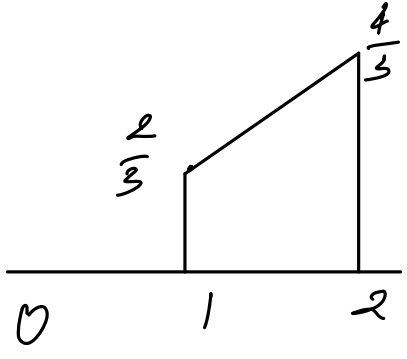


$$A : X > 1$$

$$P(A) = \int_1^2 \frac{x}{2}\,dx = \frac{x^2}{4}\Big|_1^2 = 3/4$$

$$f_{X|A}(x) = \frac{P(X \in [x, x+dx] \mid X > 1)}{dx}$$

$$= \begin{cases} 0 & \text{if } x < 1 \\ \frac{4x}{6} & \text{if } x \in [1,2] \\ 0 & \text{if } x > 2 \end{cases}$$



Functions of Random Variables

X

$Y = X/2$

$Y = X^2$

$Y = |X|$

Examples

$$f_X(x): \begin{cases} 1/2 & \text{if } x \in [-1,1] \\ 0 & \text{otherwise} \end{cases}$$

$$Y = X/2$$

$$f_Y(y): \frac{P(X \in [y, y+dy])}{dy}$$

$$= \frac{P(X \in [2y, 2y+2dy])}{dy}$$

$$: 0 \quad \text{if } 2y \notin [-1,1]$$

$$: 2 \cdot 1/2 \quad \text{if } y \in [-1/2, 1/2]$$

Examples

$Y = |X|$ X is Uniform $([-1,1])$

$$f_Y(y) = \begin{cases} 1 & \text{if } y \in [0,1] \\ 0 & \text{otherwise.} \end{cases}$$