- unconstrained optimization

If f is convex, $\nabla f(x^*) = 0$ ~~is~~ $\Rightarrow$ x^* is global optimum.

- Constrained optimization.

$$\boxed{\begin{array}{ll} \min\limits_{x} & f(x) \\ \text{st} & h(x) \leq 0 \end{array}}$$

$f : \mathbb{R}^n \to \mathbb{R}$

Lagrangian function

$$L(\underbrace{x}_{\substack{\uparrow \\ \text{vector}}}, \underbrace{\lambda}_{\substack{\uparrow \\ \text{Scalar}}}) = f(x) + \lambda h(x)$$

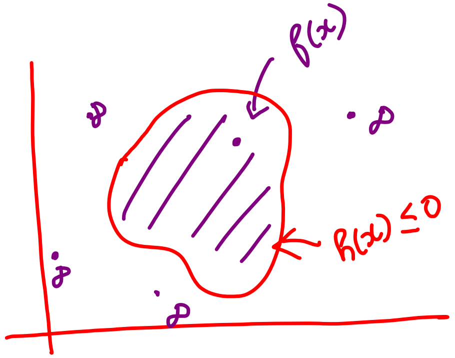
Fix $x \in \mathbb{R}^n$

$f: \mathbb{R}^2 \to \mathbb{R}$

$h: \mathbb{R}^2 \to \mathbb{R}$

$$\max_{\lambda \geq 0} L(x,\lambda) = \boxed{\max_{\lambda \geq 0} f(x) + \lambda h(x)}$$

$$\begin{cases} f(x) & h(x) \leq 0 \\ \infty & h(x) > 0 \end{cases}$$



$$\boxed{\begin{aligned} &\min_{x} \ f(x) \\ &\text{s.t } h(x) \leq 0 \end{aligned}} \equiv \boxed{\min_{x} \left[\max_{\lambda \geq 0} L(x,\lambda) \right]}$$

← can be hard to solve.

$$\boxed{\begin{array}{c}\min_{x} \; f(x) \\ h(x) \leq 0\end{array}} \equiv \min_{x} \left[\max_{\lambda \geq 0} \; L(x, \lambda) \right] \leftarrow \text{PRIMAL PROBLEM}$$

$x^* \to$ primal solution.

$$\max_{\lambda \geq 0} \left[\min_{x} \; L(x,\lambda) \right] = \max_{\lambda \geq 0} \left[\underbrace{\min_{x} \; f(x) + \lambda h(x)} \right]$$

$\lambda \geq 0$ ↑ typically easier.

$\min_x L(x,\lambda)$ ↑ unconstrained.

$$\equiv \boxed{\max_{\lambda \geq 0} \; \underbrace{g(\lambda)}}$$

$g(\lambda)$ → Concave function!

↑ DUAL PROBLEM

$$g(\lambda) = \min_{x} \; L(x, \lambda)$$