Outline

- Sets and Functions
  - Notations
  - Logic
  - Graphs and visualisations.
- **Univariate Calculus**
  - Continuity and differentiability
  - Derivatives and Linear approximations
  - **Applications/Advanced rules**
- Multivariate Calculus
  - Lines and planes in high dimensional space.
  - Partial derivatives
  - Gradients
  - Linear approximations and Alternate gradient interpretations
  - Applications/Advanced rules

Higher Order Approximations

$$f(x) \approx f(x^*) + f'(x^*)(x - x^*) \quad \text{(Linear apx)}$$

$$f(x) \approx f(x^*) + f'(x^*)(x - x^*) + \frac{1}{2} f''(x^*)(x - x^*)^2 \quad \text{(quadratic)}$$

e.g.: 1 $\quad f(x) = x^2$

$f'(x) = 2x$

$f''(x) = 2$

$$x^2 \approx (x^*)^2 + 2x^*(x - x^*) + \frac{1}{2} \cdot 2 \cdot (x - x^*)^2$$

$$= x^2$$

Higher Order Approximations

Approx e^x around $x^* = 0$

$$e^x \approx e^0 + e^0(x - x^*) + e^0 \cdot \frac{1}{2} \cdot (x - x^*)$$

$$= 1 + x + \frac{x^2}{2}$$

Ex: Which is closest to $(1.1)^7$

(a) 1.7 (b) 1.9 (c) 2.1 (d) 2.3

$f(x) = (1+x)^7$, $f'(x) = 7(1+x)^6$, $f''(x) = 42(1+x)^5$

$f'(0) = 7$ $\qquad f''(0) = 42$

$$f(0.1) \approx 1 + 7(0.1) + \frac{1}{2} \cdot 42(0.01)$$

$$= 1.91$$

Higher Order Approximations

Product Rule

$$f(x) = g(x) \cdot h(x)$$

$$f'(x) = ? \qquad x^* = 0$$

$$f(x) \approx (g(0) + x\, g'(0))\,(h(0) + x\, h'(0))$$

$$= g(0)h(0) + x\left[g'(0)h(0) + h'(0)g(0)\right] + x^2 g'(0)h'(0)$$

$$L_x[f] = f(0) + x\, f'(0)$$

$$f'(0) = g'(0)h(0) + h'(0)g(0)$$

Chain Rule

$$f(x) = g(h(x))$$

$$\approx g\left(h(0) + h'(0)x\right)$$

$$\approx g(h(0)) + g'(h(0))\left[h(0) + h'(0)x - h(0)\right]$$

$$= g(h(0)) + g'(h(0))\, h'(0)\, x$$

$$f(x) \approx f(0) + f'(0) \cdot x$$

$$f'(0) = g'(h(0))\, h'(0)$$

Chain Rule

(i) $\dfrac{e^{3x}}{\sqrt{1+x}}$ Give LA around $x=0$

$$\frac{e^{3x}}{\sqrt{1+x}} \approx (1+3x)\left(1-\frac{x}{2}\right)$$

$$\approx 1+\frac{5}{2}x \qquad (\text{around } x=0)$$

(ii) Give Lin. Apx. $e^{\sqrt{1+x}}$ around $x=1$

$$e^{\sqrt{1+x}} \approx e^{\sqrt{2}} + \frac{e^{\sqrt{2}}}{2\sqrt{2}}(x-1) \qquad (\text{around } x=1)$$

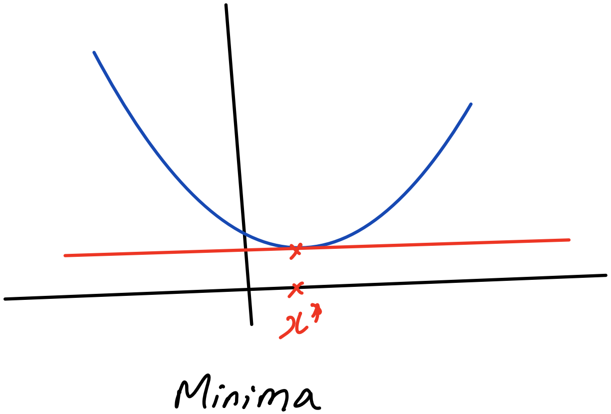
Maxima, minima and saddle points

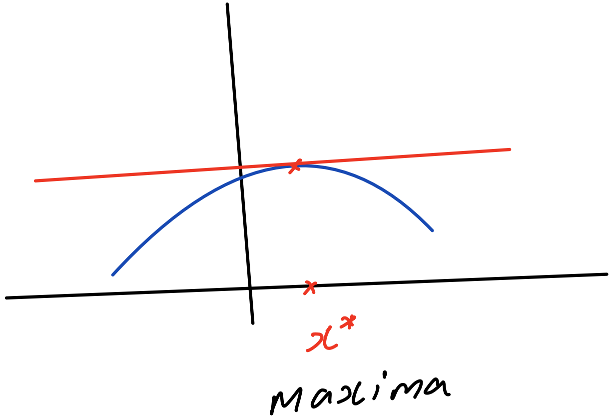
$$L_{x^*}[f] = f(x^*) + f'(x^*)(x - x^*)$$

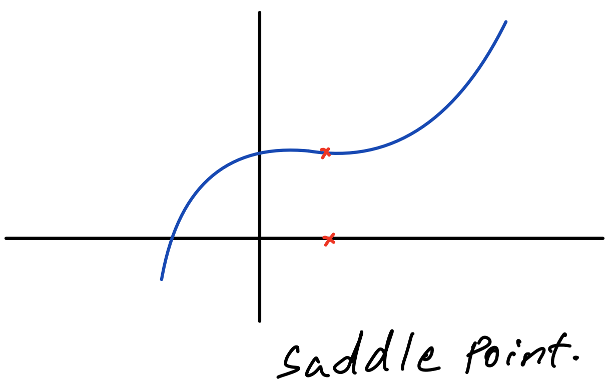
$f'(x^*) = 0 \iff x^*$ is a critical point of f

$$L_{x^*}[f] = ?$$

Maxima, minima and saddle points







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$f : \mathbb{R} \to \mathbb{R}$

$f : \mathbb{R}^d \to \mathbb{R}$