Course: Machine Learning - Foundations
Week [redacted]: Test questions

1. (2 points) Two positive numbers have a sum of 60. What is the minimum product of one number times the square of other number?

   A. 0

   B. 900

   C. 60

   D. 240

   Answer: A
   Let the two numbers be x and y
   x+y=60
   objective function from the question will be,
   $f(x) = x^2(60 - x)$
   For optima $f'(x) = 0$, $120x - 3x^2 = 0$
   $x = 0, 40$
   Product is minimum when x=0.

2. (2 points) (Multiple select) The point on $y = x^2 + 1$ closest to (0,2) is

   A. (0.707, 1.5)

   B. (0.707, -1.5)

   C. (-0.707,1.5)

   D. (-0.707, -1.5)

   Answer: A,C
   Objective function $f(x) = (x - 0)^2 + (x^2 + 1 - 2)^2$
   $f(x) = x^4 - x^2 + 1$
   For minima $f'(x) = 0$
   $4x^3 - 2x = 0$
   $x = 0, 0.707, -0.707$
   Corresponding $y = 1, 1.5, 1.5$

3. (2 points) The volume of the largest cone that can be inscribed in a circle of radius 6 m is (correct up to two decimal places)

   _____

   Answer: 268.19 m^3
   $V = \frac{1}{3}\pi r^2 h$
   $r = \sqrt{36 - x^2}$
   $h = 6 + x$

For maxima, $V'(x) = 0$
$-3x^2 - 12x + 36 = 0$
$x = 2, -6$
x can not be nagative.
So $r = 5.65$
$h = 8$
$V = 268.19$

(Questions 5-8 have common data) A firm produces two products A and B. Maximum production capacity is 500 for total production. At least 200 units must be produced every day. Machine hours consumption per unit is 5 hours for A and 3 hours for B. At least 1000 machine hours must be used daily. Manufacturing cost is Rs 30 for A and Rs 20 for B.
Let x_1 = No of units of A produced per day
and x_2 = No of units of B produced per day

4. (1 point) The objective function for above problem is
   A. min $f(x) = 30x_1 + 20x_2$
   B. min $f(x) = 15x_1 + 55x_2$
   C. min $f(x) = 5x_1 + 155x_2$
   D. min $f(x) = 30x_1 - 20x_2$

   Answer: A
   We should minimise cost function.
   Objective function is
   min $f(x) = 30x_1 + 20x_2$

5. (2 points) The constraint due to maximum production capacity is
   A. $x_1 + x_2 \geq 500$
   B. $x_1 + x_2 \leq 500$
   C. $x_1 + x_2 \neq 500$
   D. $x_1 + x_2 = 500$

   Answer: B
   Maximum production capacity is 500.

6. (2 points) The constraint due to minimum production capacity is
   A. $x_1 + x_2 = 200$
   B. $x_1 + x_2 \leq 200$
   C. $x_1 + x_2 \geq 200$
   D. $x_1 + x_2 \neq 200$

   Answer: C
   Minimum production capacity is 200.

7. (2 points) The constraint due to machine hour consumption is

   A. $5x_1 + 3x_2 \leq 1000$

   B. $5x_1 + 3x_2 \neq 1000$

   C. $5x_1 + 3x_2 = 1000$

   D. $5x_1 + 3x_2 \geq 1000$

   Answer: D
   1000 machine hours must be used daily.

   (Questions 9-11 have common data)
   A factory manufactures two products A and B. To manufacture one unit of A, 1 machine hours and 2 labour hours are required. To manufacture product B, 2 machine hours and 1 labour hours are required. In a month, 200 machine hours and 140 labour hours are available. Profit per unit for A is Rs. 45 and for B is Rs. 35.
   Let x_1=Number of units of A produced per month
   and x_2=Number of units of B produced per month

8. (1 point) The objective function for above problem is

   A. $\max f(x) = 45x_1 + 35x_2$

   B. $\min f(x) = 45x_1 + 35x_2$

   C. $\max f(x) = 35x_1 + 45x_2$

   D. $\min f(x) = 35x_1 + 45x_2$

   Answer: A
   We need to maximize profit.

9. (2 points) The constraint for machine hours is

   A. $x_1 + 2x_2 \geq 200$

   B. $x_1 + 2x_2 \leq 200$

   C. $x_1 + 2x_2 \neq 200$

   D. $x_1 + 2x_2 = 200$

   Answer: B
   Total machine hours available=200.

10. (2 points) The constraint for labour hours is

    A. $2x_1 + x_2 = 140$

    B. $2x_1 + x_2 \leq 140$

    C. $2x_1 + x_2 \geq 140$

    D. $2x_1 + x_2 \neq 140$

    Answer: B
    Total labour hour available is 140.

11. (2 points) (Multiple select) Gradient of a continuous and differentiable function

    A. is zero at a minimum

    B. is non zero at a maximum

    C. is zero at a saddle point

    D. decreases as you get closer to minimum

   Answer: A,C,D
   For critical points gradient of a function is 0.
   As we move towards minima gradient decreases.

12. The value of a function at point 10 is 100. The values of the function's first and second order derivatives at this point are 20 and 2 respectively. What will be the function's approximate value correct up to two decimal places at the point 10.5 (Use second order approximation)?

   Answer: 110.25
   According to Taylor's series,
   $f(x+h) = f(x) + hf'(x) + \frac{h^2 f'(x)}{2} + ...$
   Here $x = 10, h = 0.5$
   $\therefore f(x+h) = 110.25$

13. (2 points) For the function $f(x) = x\sin(x) - 1$, with an initial guess of $x_0 = 2.5$, and step size of 0.1, as per gradient descent algorithm, what will be the value of the function after 4 iterations? (Correct up to 3 decimal places)

   Answer: -1.710 (-1.624 to-1.795)
   $x_{n+1} = x_n - \eta f'(x)$
   After first iteration $x_1 = 2.64$
   After second iteration $x_2 = 2.823$
   After third iteration $x_3 = 3.059$
   After fourth iteration $x_4 = 3.355$
   $f(3.355) = -1.710$

14. (2 points) The value of $f(x_1, x_2) = 4x_1^2 - 4x_1x_2 + 2x_2^2$ with an initial guess of (2, 3) after two iterations of gradient descent algorithm will be .... Take the step size $\eta = \frac{1}{t+1}$, where t= 0,1,2....

   Answer: 130
   $x_{n+1} = x_n - \eta \nabla f(x)$
   $$\nabla f = \begin{bmatrix} 8x_1 - 4x_2 \\ -4x_1 + 4x_2 \end{bmatrix}$$
   $$x_1 = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

$x_2 = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$
$f(4, -3) = 130$

15. (2 points) The point of minimum for the function $f(x_1, x_2) = x_1^2 - x_1x_2 + 2x_2^2$ with an initial guess of (3, 2) with step size=0.5 using gradient descent algorithm after second iteration will be .... (correct up to 3 decimal places)

Answer: 2.312 (2.196 to 2.428)
$x_{n+1} = x_n - \eta\nabla f(x)$
$\nabla f = \begin{bmatrix} 2x_1 - x_2 \\ -x_1 + 4x_2 \end{bmatrix}$
$x_1 = \begin{bmatrix} 1 \\ -0.5 \end{bmatrix}$
$x_2 = \begin{bmatrix} -0.25 \\ 1 \end{bmatrix}$
$f(-0.25, 1) = 2.312$

16. (2 points) Suppose we have n data points randomly distributed in space given by $D = \{x_1, x_2, ..., x_n\}$. A function $f(p)$ is defined to calculate the sum of distances of data points from a fixed point, say p. Let $f(p) = \sum_{i=1}^{n}(p - x_i)^2$. What is the value of p so that f(p) is minimum?

    A. $x_1 + x_2 + ... + x_n$

    B. $x_1 - x_2 + x_3 - x_4....$

    C. $\dfrac{x_1 + x_2 + ... + x_n}{n}$

    D. $\dfrac{x_1 - x_2 + x_3 - x_4....}{n}$

Answer: C
$f(p) = \sum_{i=1}^{n}(p - x_i)^2$
$f(p) = (p - x_1)^2 + ... + (p - x_n)^2$
$f'(p) = 2p(p - x_1) + ... + 2p(p - x_n)$
For minima $f'(p) = 0$
$(p - x_1) + (p - x_2) + ... + (p - x_n) = 0$
$np - (x_1 + x_2 + ... + x_n) = 0$
$p = \dfrac{x_1 + x_2 + ... + x_n}{n}$