Madhavan Mukund

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Programming, Data Structures and Algorithms using Python Week 9

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- "The lecture taught the students to appreciate how the concept of optimal substructures can be used in designing algorithms"

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#### Edit distance

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- In our example, 24 characters inserted, 18 deleted, 2 substituted

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- insert, delete, substitute

- Minimum number of edit operations needed
- In our example, 24 characters inserted, 18 deleted, 2 substituted
- Edit distance is at most 44

- Minimum number of editing operations needed to transform one document to the other
  - Insert a character
  - Delete a character
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  - Genetic similarity of species

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#### Edit distance and LCS

■ Longest common subsequence of u, v

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  - Delete b, i and then insert r, e in bisect

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  - Delete b, i in bisect and r, e in secret
  - Delete b, i and then insert r, e in bisect
- LCS equivalent to edit distance without substitution

$$u = a_0 a_1 \dots a_{m-1}$$

$$v = b_0 b_1 \dots b_{n-1}$$

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### Recall LCS

$$u = a_0 a_1 \dots a_{m-1}$$

$$\mathbf{v} = b_0 b_1 \dots b_{n-1}$$

- Recall LCS
- If  $a_i = b_j$ , LCS(i,j) = 1 + LCS(i+1,j+1)
- If  $a_i \neq b_j$ ,  $LCS(i,j) = \max[LCS(i,j+1), LCS(i+1,j)]$

$$\mathbf{v} = b_0 b_1 \dots b_{n-1}$$

Recall LCS

■ Edit distance — aim is to transform *u* to *v* 

• If 
$$a_i = b_j$$
,  
 $LCS(i,j) = 1 + LCS(i+1,j+1)$ 

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- If  $a_i = b_j$ , nothing to be done

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  - Substitute  $a_i$  by  $b_j$
  - Delete *ai*

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  - Delete ai
  - Insert  $b_j$  before  $a_i$

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  - Substitute  $a_i$  by  $b_i$
  - Delete a;
  - Insert  $b_j$  before  $a_i$
- ED(i,j) edit distance for  $a_i a_{i+1} \dots a_{m-1}, b_j b_{j+1} \dots b_{m-1}$

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  - ED(i, n) = m i for all  $0 \le i \le m$ Delete  $a_i a_{i+1} \dots a_{m-1}$  from u

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  - ED(m,j) = n j for all  $0 \le j \le n$ Insert  $b_j b_{j+1} \dots b_{n-1}$  into u

## Subproblem dependency

■ Subproblems are ED(i,j), for 0 < i < m, 0 < j < n

# Subproblem dependency

- Subproblems are ED(i,j), for  $0 \le i \le m$ ,  $0 \le j \le n$
- Table of  $(m+1) \cdot (n+1)$  values

		0	1	2	3	4	5	6
		s	е	С	r	е	t	•
0	b							
1	i							
2	s							
3	е							
4	С							
5	t							
6	•							

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		s	е	С	r	е	t	•
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1	i				杖	_		
2	S							
3	е			核				
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		s	е	С	r	е	t	•
0	b							6
1	i							5
2	s							4
3	е							3
4	С							2
5	t							1
6	•							0

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		0	1	2	3	4	5	6
		s	е	С	r	е	t	•
0	b						5	6
1	i						4	5
2	s						3	4
3	е						2	3
4	С						1	2
5	t						0	1
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3	е					1	2	3
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5	t					1	0	1
6	•					2	1	0

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3	е		2	3	2	1	2	3
4	С		3	2	2	1	1	2
5	t		4	3	2	1	0	1
6	•		5	4	3	2	1	0

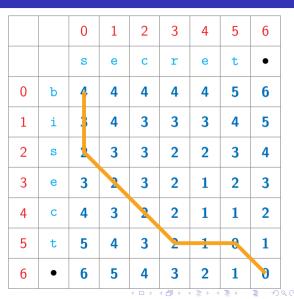
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1	i	3	4	3	3	3	4	5
2	s	2	3	3	2	2	3	4
3	е	3	2	3	2	1	2	3
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5	t	5	4	3	2	1	0	1
6	•	6	5	4	3	2	1	0

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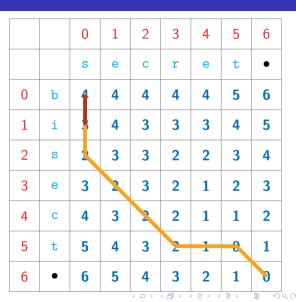
### Reading off the solution

■ Transform bisect to secret



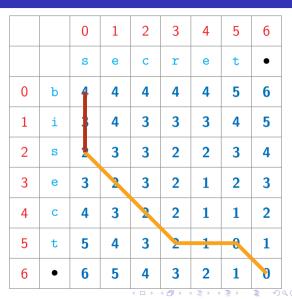
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- Transform bisect to secret
- Delete b



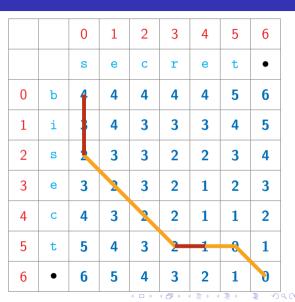
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- Transform bisect to secret
- Delete b , Delete i



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- Transform bisect to secret
- Delete b , Delete i , Insert r



- Subproblems are ED(i,j), for  $0 \le i \le m$ ,  $0 \le j \le n$
- Table of  $(m+1) \cdot (n+1)$  values
- Like LCS, ED(i,j) depends on ED(i+1,j+1), ED(i,j+1), ED(i+1,j)
- No dependency for ED(m, n) start at bottom right and fill by row, column or diagonal

- Transform bisect to secret
- Delete b , Delete i , Insert r , Insert e

		0	1	2	3	4	5	6
		S	е	С	r	е	t	•
0	b	4	4	4	4	4	5	6
1	i	3	4	3	3	3	4	5
2	S		3	3	2	2	3	4
3	е	3	2	3	2	1	2	3
4	С	4	3	2	2	1	1	2
5	t	5	4	3	2	1	-8	1
6	•	6	5	4	3	2	1	0

```
def ED(u,v):
  import numpy as np
  (m,n) = (len(u), len(v))
  ed = np.zeros((m+1,n+1))
 for i in range(m-1,-1,-1):
    ed[i,n] = m-i
 for j in range(n-1,-1,-1):
    ed[m,i] = n-i
 for j in range(n-1,-1,-1):
    for i in range(m-1,-1,-1):
      if u[i] == v[i]:
        ed[i,j] = ed[i+1,j+1]
      else:
        ed[i,j] = 1 + min(ed[i+1,j+1],
                          ed[i,j+1],
                          ed[i+1, j])
 return(ed[0,0])
```

```
def ED(u,v):
  import numpy as np
  (m,n) = (len(u), len(v))
  ed = np.zeros((m+1,n+1))
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 for j in range(n-1,-1,-1):
    ed[m,i] = n-i
 for j in range(n-1,-1,-1):
    for i in range(m-1,-1,-1):
      if u[i] == v[i]:
        ed[i,j] = ed[i+1,j+1]
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        ed[i,j] = 1 + min(ed[i+1,j+1],
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### Complexity

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#### Complexity

Again O(mn), using dynamic programming or memoization

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    ed[i,n] = m-i
  for j in range(n-1,-1,-1):
    ed[m,i] = n-i
  for j in range(n-1,-1,-1):
    for i in range(m-1,-1,-1):
      if u[i] == v[i]:
        ed[i,j] = ed[i+1,j+1]
      else:
        ed[i,j] = 1 + min(ed[i+1,j+1],
                           ed[i,j+1],
                           ed[i+1, j])
 return(ed[0,0])
```

#### Complexity

- Again O(mn), using dynamic programming or memoization
  - Fill a table of size O(mn)
  - Each table entry takes constant time to compute