String matching using automata

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Programming, Data Structures and Algorithms using Python

Week 10

Traditional string matching

- Text `t`, pattern `p` of of lengths n, m
- For each starting position `i` in `t`, compare `t[i:i+m]` with `p`
  - Scan `t[i:i+m]` right to left

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- Can skip some positions `i` on mismatch [Boyer,Mooore]
  - Case 1: `t[i+j]` does not appear in `p`
    - Update `i` to `i+j+1`
  - Case 2: `t[i+j]` appears in `p` before `p[j]`
    - Precompute `last[c]` for each `c` in `p`
    - Align `p[last[c]]` with `t[i+j]`
    - Update `i` to `j - last[c]`

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- Worst case remains $O(nm)$: `t = aaa...a`, `p = baaaa`

Remembering previous matches

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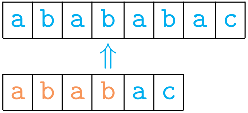

- Keep track of longest prefix of `p[:k]` that we have matched

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a	b	a	b	a	b	a	c

$\Uparrow$

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- To reset, use precomputed values

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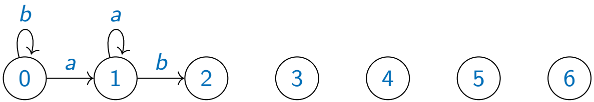

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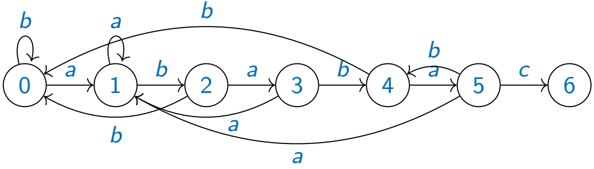

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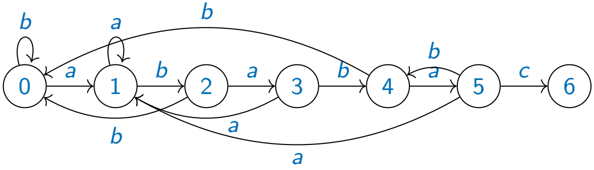
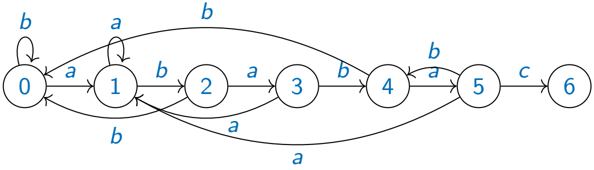
- Brute force, $O(m^2)$ per edge



Pattern matching using automata

- The graph we have constructed is a finite state automaton
  - Nodes are states
  - Edges are tranitions



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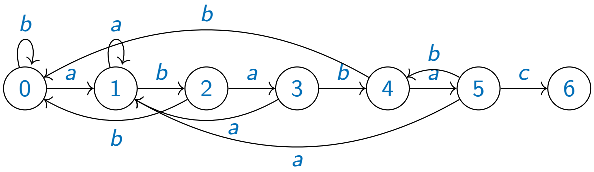

Processing *abababac*

0

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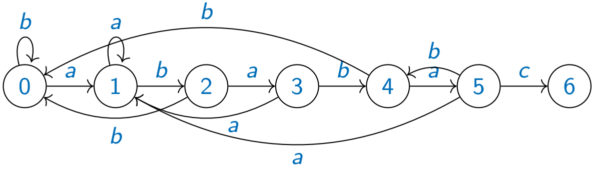

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$0 \xrightarrow{a} 1$

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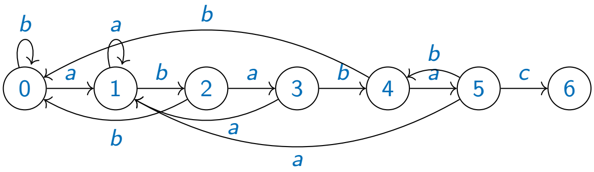

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$0 \xrightarrow{a} 1 \xrightarrow{b} 2$

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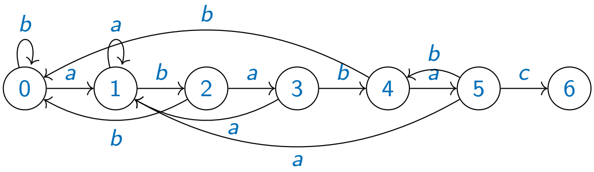

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$0 \xrightarrow{a} 1 \xrightarrow{b} 2 \xrightarrow{a} 3$

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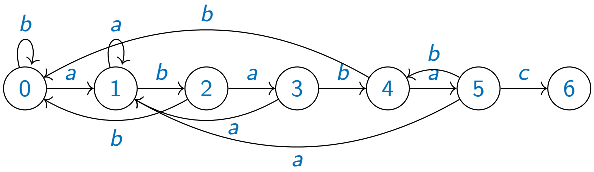

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$$0 \xrightarrow{a} 1 \xrightarrow{b} 2 \xrightarrow{a} 3 \xrightarrow{b} 4$$

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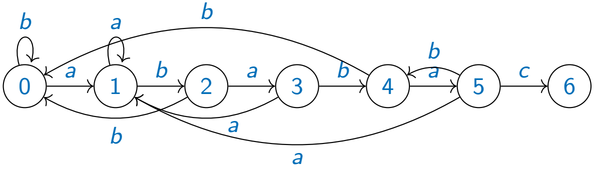

Processing *abababac*

$$0 \xrightarrow{a} 1 \xrightarrow{b} 2 \xrightarrow{a} 3 \xrightarrow{b} 4 \xrightarrow{a} 5$$

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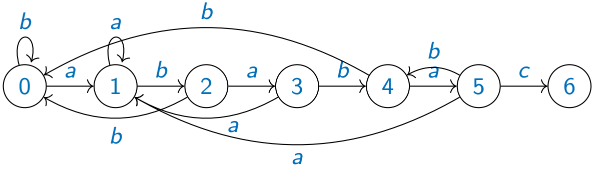

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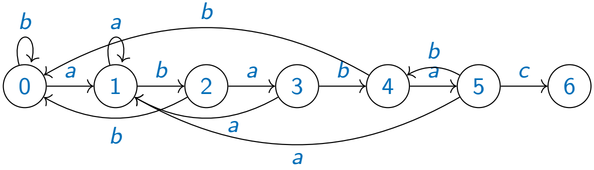

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- Start scanning text in initial state 0
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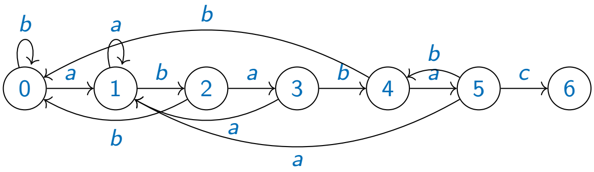
- If we reach the final state m, we have found a full match for `p`



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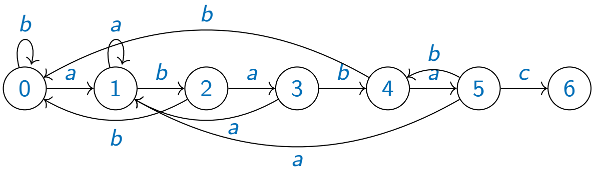
- Single scan of `t` suffices



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  - Computing each edge $i \xrightarrow{a} j$ took $O(m^2)$
  - Do this for each $i \in \{0, 1, \ldots, m\}$ and each $a \in \Sigma$
  - Overall $O(m^3 \cdot |\Sigma|)$

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- Do this in time $O(m)$ — [Knuth, Morris, Pratt]