Course: Machine Learning - Foundations
Week 10 Questions

PRACTICE QUESTIONS

1. (1 point) **(Multiple Select)** Which of the following options is/are always true for three events A, B, and C of a random experiment?

   A. If $A \subset B$ then $P(B|A) = 1$

   B. If $B \subset A$ then $P(B|A) = 1$

   C. If $B \subset A$ then $P(A|B) = 1$

   D. If $P(A|B) > P(A)$ then $P(B|A) > P(B)$ (Assume that events A and B have non zero probabilities.)

Answer: A, C, D

Solution:
Option A is correct, it can be shown as

$$P(B|A) = \frac{P(B \cap A)}{A}$$

As $A \subset B$

So, $P(B \cap A) = P(A)$

Hence, $P(B|A) = \dfrac{P(B \cap A)}{A} = \dfrac{P(A)}{P(A)} = 1$
Similarly option B is correct

As option B is correct, then option C must be false.

Option D is also correct, it can be described as

$$P(A|B) > P(A) = \frac{P(A \cap B)}{P(B)} > P(A)$$

So, $P(A \cap B) > P(A) \times P(B)$

Hence, $P(B|A) > P(B) > P(B)$

2. (1 point) Let the random experiment of selecting a number from a set of integers from 1 to 20, both inclusive. Assuming all numbers are equally likely to occurs. Let A be the event that the selected number is odd, B be the event that the selected number is divisible by 3. Choose the correct options from the following:

   A. A and B are dependent on each other.

   B. A and B are independent on each other.

   C. Can't say

   Answer: B

   Solution:

   $P(A) = \frac{10}{20}$

   $P(B) = \frac{6}{20}$

   $P(A \cap B) = \frac{3}{20}$

   Here, $P(A \cap B) = P(A) \times P(B)$

   Hence, both A and B are independent.

3. (1 point) Mayur rolls a fair die repeatedly until a number that is multiple of 3 is observed. Let the random variable N represent the total number of times the die is rolled. Find the probability distribution of N.

   A.
   $$f_N(k) = \begin{cases} \frac{2}{3} \times (\frac{1}{3})^{k-1}, & \text{for } k = 1, 2, 3, ... \\ 0, & \text{otherwise} \end{cases}$$

   B.
   $$f_N(k) = \begin{cases} \frac{1}{3} \times (\frac{2}{3})^{k-1}, & \text{for } k = 1, 2, 3, ... \\ 0, & \text{otherwise} \end{cases}$$

   C.
   $$f_N(k) = \begin{cases} \frac{1}{2} \times (\frac{1}{2})^{k-1}, & \text{for } k = 1, 2, 3, ... \\ 0, & \text{otherwise} \end{cases}$$

   D.
   $$f_N(k) = \begin{cases} \frac{1}{6} \times (\frac{5}{6})^{k-1}, & \text{for } k = 1, 2, 3, ... \\ 0, & \text{otherwise} \end{cases}$$

   Answer: B

   Solution:

Probability of getting outcomes as multiple of 3 equals $\frac{2}{6}$

It can be considered as $P(S)$

Hence, $P(S) = \frac{1}{3}$

Probability of not getting outcomes as multiple of 3 equals $\frac{4}{6}$

It can be considered as $P(US)$

Hence, $P(US) = \frac{2}{3}$

Hence, option B is correct.

4. (1 point) **(Multiple Select)** Shelly wrote an exam that contains 20 multiple choice questions. Each question has 4 options out of which only one option is correct and each question carries 1 mark. She knows the correct answer of 10 questions, and for the remaining 10 questions, she chooses the options at random. Assume that all the questions are independent. Find the probability that she will score 18 marks in the exam.

   A. ${}^{10}C_8 \times (\frac{1}{4})^8 \times (\frac{3}{4})^2$.

   B. ${}^{10}C_2 \times (\frac{1}{4})^8 \times (\frac{3}{4})^2$.

   C. ${}^{10}C_2 \times (\frac{1}{4})^2 \times (\frac{3}{4})^8$.

   D. ${}^{10}C_8 \times (\frac{3}{4})^8 \times (\frac{1}{4})^2$.

   Answer: A, B

   Solution:

   To score 18 marks she has to correct 8 correct out of 10. Hence,

   Option A and option B is correct.

5. (1 point) Suppose the number of runs scored of a delivery is uniform in $\{1, 2, 3, 4, 5, 6\}$ independent of what happens in other deliveries. A batsman needs to bat till he hits a four. What is the probability that he needs fewer than 6 deliveries to do so? (Answer the question correct to two decimal points.)

   Answer: 0.6

   Answer: Accepted range: 0.58-0.62

Solution:

$P(4) = \frac{1}{6}$

As it resembles geometric distribution. Hence,

$\sum_{n=1}^{5} \frac{1}{6} \times (1 - \frac{1}{6})^5 = 0.6$

6. (1 point) Let X and Y be two random variables with joint PMF $f_{XY}(x, y)$ given in Table 10.1.

x \ y	1	2	3
1	0.25	0.25	0
2	0	0.25	0.25

Table 10.1: Joint PMF of X and Y.

Calculate the Covariance between X and Y .(Answer your question upto two decimals.)

Answer: 0.25

Solution:

$Cov(X, Y) = E(XY) - E(X)E(Y) = \frac{13}{4} - 3 = 0.25$

7. (1 point) Let X and Y be two random variables with joint PMF $f_{XY}(x, y)$ given in Table 10.2.

x \ y	1	2	3
1	0.25	0.25	0
2	0.125	a_1	0.125

Table 10.2: Joint PMF of X and Y.

Calculate $f_{Y|X=2}(2)$. (Answer your question upto two decimals.)

Answer: 0.5

Solution:

$\sum f_{XY}(x, y) = 1$

0.25+ 0.25+ 0.125 + a_1 + 0.125 = 1

Hence, $a_1 = 0.25$

$f_{Y|X=2}(2) = \frac{f_{XY}(2,2)}{f_X(2)} = \frac{0.25}{0.5} = 0.5$

8. (1 point) Two random variables X and Y are jointly distributed with joint pmf

$$f_{XY}(x,y) = \begin{cases} ax + \frac{y}{4}, & \text{for } x, y \in \{0, 1\} \\ 0, & \text{otherwise} \end{cases}$$

Calculate the value of a

Answer: 0.25

Solution:

$\sum f_{XY}(x, y) = 1$

While solving above equation we get $a = 0.25$

9. (1 point) A discrete random variables X has the probability mass function as follows.

$$P(X = x) = \begin{cases} k \times (1 - x)^2, & \text{for } x = 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$$

Evaluate the value of k

Answer: 0.20

Solution:

$\sum P(X = x) = 1$

$k + 4k = 1$

$k = 0.2$

10. (1 point) A discrete random variables X has the probability function as given in Table 10.3, where a, b and c are constants.

x	1	2	3	4
$P(X = x)$	a	b	c	0.3

Table 10.3: Probability distribution

The cdf $F(x)$ is given in table 10.4

x	1	2	3	4
$F(X)$	0.2	0.6	0.7	d

Table 10.4: Cumulative distribution function

Evaluate $a + b + c + d$

Answer: 1.7

Solution:

From both the given table we can get the values as $a = 0.2$, $b = 0.4$, $c = 0.1$, $d = 1$

11. (1 point) A series of four matches is played between India and England. Let the random variable X represent the absolute difference in the number of matches won by India and England. Find the set of possible values that X can take. (Assume that the match does not result in a tie.)

    A. 0 , 2, 4
    B. 0, 1, 2, 4
    C. 0, 1, 2, 3, 4
    D. 0, 1, 3, 4

Answer: A

Solution:

India can win match = {0, 1, 2, 3, 4}

Similarly England can win match = {0, 1, 2, 3, 4}

Now, taking the the absolute difference between these two we get {0, 2, 4}

12. (1 point) There are five multiple choice questions asked in an exam. There is 70% chance that Shelly will solve a question correctly and independent of the rest of the solution. Let X be the random variable that represents the number of questions she solves correctly. Which of the following is the probability mass function of X?

A.

$$P(X = x) = \begin{cases} 0.00243, & \text{for } x = 0 \\ 0.02835, & \text{for } x = 1 \\ 0.1323, & \text{for } x = 2 \\ 0.3087, & \text{for } x = 3 \\ 0.36015, & \text{for } x = 4 \\ 0.16807, & \text{for } x = 5 \end{cases}$$

B.

$$P(X = x) = \begin{cases} 0.00243, & \text{for } x = 0 \\ 0.02835, & \text{for } x = 1 \\ 0.16807, & \text{for } x = 2 \\ 0.3087, & \text{for } x = 3 \\ 0.36015, & \text{for } x = 4 \\ 0.1323, & \text{for } x = 5 \end{cases}$$

C.

$$P(X = x) = \begin{cases} 0.00243, & \text{for } x = 0 \\ 0.02835, & \text{for } x = 1 \\ 0.3087, & \text{for } x = 2 \\ 0.1323, & \text{for } x = 3 \\ 0.36015, & \text{for } x = 4 \\ 0.16807, & \text{for } x = 5 \end{cases}$$

D.

$$P(X = x) = \begin{cases} 0.00243, & \text{for } x = 0 \\ 0.01835, & \text{for } x = 1 \\ 0.1223, & \text{for } x = 2 \\ 0.2987, & \text{for } x = 3 \\ 0.37015, & \text{for } x = 4 \\ 0.19807, & \text{for } x = 5 \end{cases}$$

Answer: A

Solution:

$$P(X = k) =^5 C_k \times 0.7^k \times 0.3^{5-k}$$

13. (points) In a horse race, a player has to cross 12 horses. The probability that he will clear each horses is 2/3. What is the probability that either he will cross all the horses or he will not cross 1 horse.

A. $\frac{14}{3} \times \frac{2^{11}}{3^{11}}$

B. $\frac{7}{3} \times \frac{2^{11}}{3^{11}}$

C. $\frac{7}{3} \times \frac{1^{11}}{3^{11}}$

D. $\frac{14}{3} \times \frac{1^{11}}{3^{11}}$

Answer: A

Solution:

$$= P(\text{He will cross all horses}) + P(\text{He will not cross 1 horse})$$

$$= \frac{2}{3}^{12} +^{12} C_1 \times \frac{2}{3}^{11} \times \frac{1}{3}$$