Programming, Data Structures and Algorithms using Python

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Summary of weeks 10 & 11

Content

- String matching
  - Boyer-Moore Algorithm
  - Rabin-Karp Algorithm
  - Automata
  - Knuth-Morris-Pratt Algorithm
  - Tries
  - Regular Expressions
- Linear programming
- Network flows
  - Ford-Fulkerson algorithm
- Reductions
- Checking algorithms
- P, NP and NP-Complete

String matching

- Searching for a pattern is a fundamental problem when dealing with text
- Formal definition:
  - A ***text*** string t of length n
  - A ***pattern*** string p of length m
  - Both t and p are drawn from an ***alphabet*** of valid letters, denoted by $\sum$
  - Find every position i in t such that $t[i : i + m] == p$
- Complexity of naïve algorithm: $O(nm)$

Boyer-Moore Algorithm

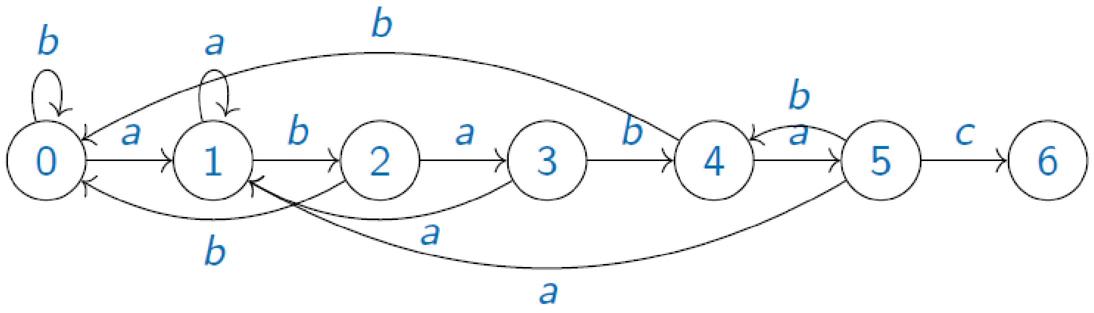
- For each starting position i in t, compare t[i : i + m] with p
  - Scan t[i : i + m] right to left
- If a letter c in t that does not appear in p
  - Shift the next scan to position after mismatched letter c
- If a letter c in t that does appear somewhere in p
  - Align rightmost occurrence of c in p with t
- Use a dictionary last
  - For each c in p, last[c] records right most position of c in p
- Without dictionary, computing last is a bottleneck, complexity is $O(|\Sigma|)$
- The algorithm works well in practice but the worst case complexity remains $O(nm)$

Rabin-Karp Algorithm

- Any string over $\sum$ can be thought of as a number in base 10
- Pattern p is an m digit number n_p
- Each substring of length m in the text t is again an m digit number
- Scan t and compare the number n_b generated by each block of m letters with the pattern number n_p
- Whenever $n_b = n_p$, scan and validate like brute force
  - It can be a false positive (spurious hit)
- In practice number of spurious matches will be small but the worst case complexity remains $O(nm)$
- If $|\sum|$ is small enough to not require modulo arithmetic, overall time is $O(n + m)$ or $O(n)$, since $m \ll n$

Automata

- It is used to keep track of longest prefix that we have matched
- It is a special type of graph where nodes are states and edges describe how to extend the match
- Using this automaton, we can do string matching in $O(n)$
- Bottleneck is precomputing the automaton
  - Overall complexity: $O(m^3.|\sum|)$



Processing *abababac*

$0 \xrightarrow{a} 1 \xrightarrow{b} 2 \xrightarrow{a} 3 \xrightarrow{b} 4 \xrightarrow{a} 5 \xrightarrow{b} 4 \xrightarrow{a} 5 \xrightarrow{c} 6$

Knuth-Morris-Pratt Algorithm

- It is very similar to what we did in automaton
- Precomputing step can be handled effectively using fail()
- The Knuth-Morris-Pratt algorithm efficiently computes the automaton describing prefix matches in the pattern p
- Complexity of preprocessing the fail() is $O(m)$
- After preprocessing, can check matches in the text t in $O(n)$
- Overall, KMP algorithm works in time $O(m+n)$
- However, the Boyer-Moore algorithm can be faster in practice, skipping many positions

Tries

- A ***trie*** is a special kind of tree deriver from “information re***trie***val”
- Rooted tree
  - Other than root, each node labelled by a letter from Σ
  - Children of a node have distinct labels
- Each maximal path is a word
  - One word should not be a prefix of another
  - Add special end of word symbol $\$$
- Build a trie T from a set of words S with s words and n total symbols
- To search for a word w, follow its path
  - If the node we reach has $\$$ as a successor, $w \in S$
  - If we cannot complete the path, $w \notin S$
- Using a suffix trie we can answer the following:
  - Is w a substring of s
  - How many times does w occur as a substring in s
  - What is the longest repeated substring in s

Regular Expressions

- The automaton we built had a linear structure, with a single path from start to finish
- In general, automata can follow multiple paths and accept multiple words
- The set of words that automata can accept are called regular sets
- Each pattern p describes a set of words, those that it matches
- The sets we can describe using patterns are exactly the same as those that can be accepted by automata
- Those patterns are called regular expressions

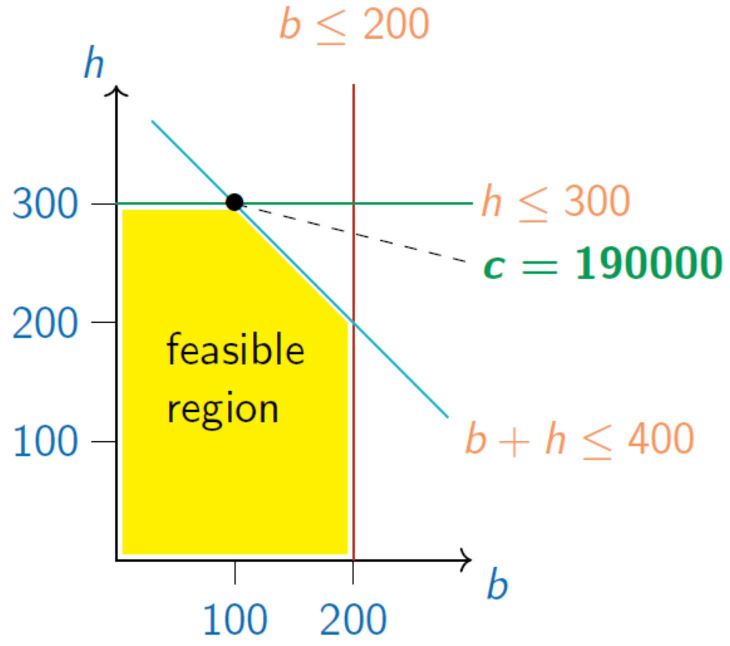
- For every automaton, we can construct a pattern p that matches exactly the words that the automaton accepts
- For every pattern p, we can construct an automaton that accepts all words that match p
- We can extend string matching to pattern matching by building an automaton for a pattern p and processing the text through this automaton
- Python provides a library for matching regular expressions

Linear programming

- Profit for each box of barfis is Rs.100
- Profit for each box of halwa is Rs.600
- Daily demand for barfis is at most 200 boxes
- Daily demand for halwa is at most 300 boxes
- Staff can produce 400 boxes a day, altogether
- What is the most profitable mix of barfis and halwa to produce?

Constrains:

- $b \leq 200$
- $h \leq 300$
- $b + h \leq 400$
- $b \geq 0$
- $h \geq 0$



Objective:

- Maximize $100b + 600h$

Linear programming

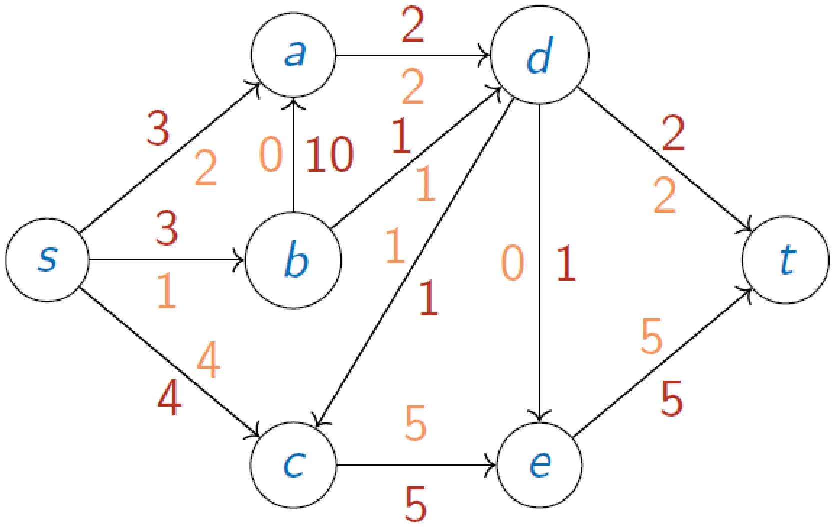
- Constraints and objective to be optimized are linear functions
  - Constrains: $a_1x_1 + a_2x_2 + \ldots + a_mx_m \leq K$, $b_1x_1 + b_2x_2 + \ldots + b_mx_m \geq L$
  - Objective: $c_1x_1 + c_2x_2 + \ldots + c_mx_m$

- Start at any vertex, evaluate objective
- If an adjacent vertex has a better value, move
- If current vertex is better than all neighbors, stop
- Can be exponential, but efficient in practice

- Feasible region is convex
- May be empty – constraints are unsatisfiable, no solutions
- May be unbounded – no upper/lower limit on objective

Network flows

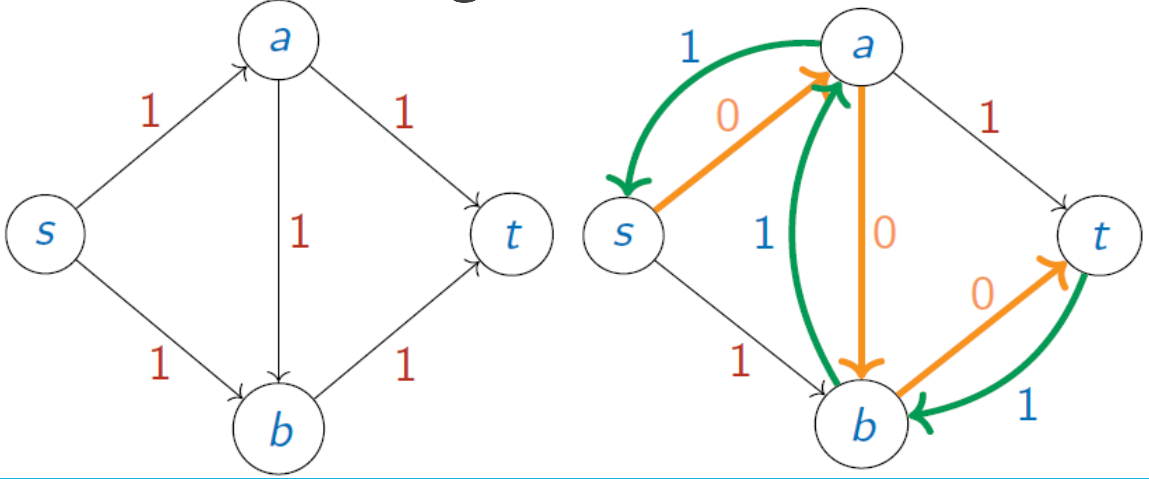
- Network: graph G = (V, E)
- Special nodes: s (source), t (sink)
- Each edge e has capacity c_e
- Flow: f_e for each edge e
  - $f_e \leq c_e$
  - At each node, except s and t, sum of incoming flows equal sum of outgoing flows
- Total volume of flow is sum of outgoing flow from s



Ford-Fulkerson algorithm

- Start with zero flow
- Choose a path from s to t that is not saturated and augment the flow as much as possible
- Network given in the example has max flow 2
- What if one chooses a bad flow to begin with?
- Add reverse edges to undo flow from previous steps

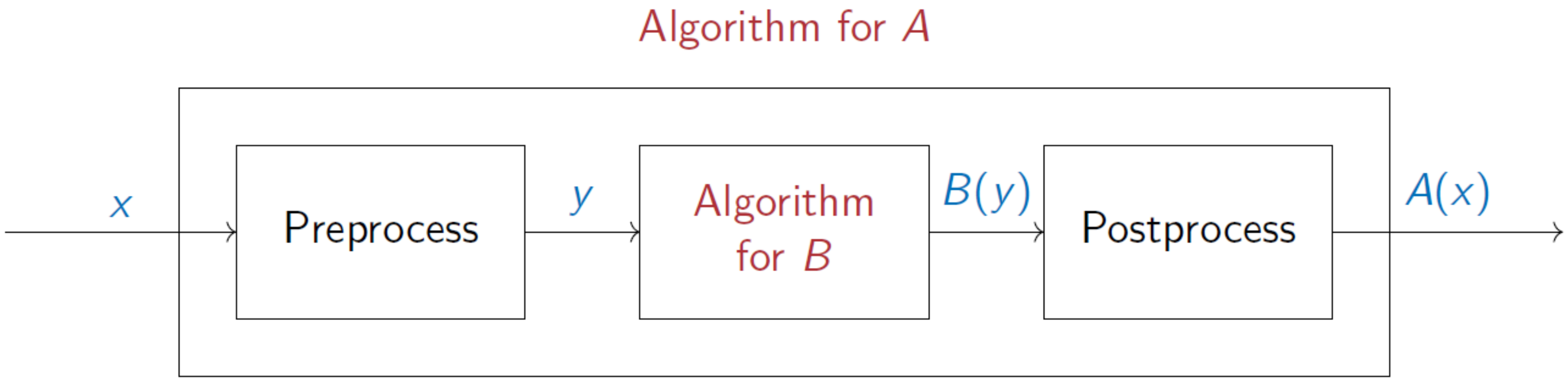
- Residual graph: for each edge e with capacity c_e and current flow f_e
  - Reduce capacity to $c_e - f_e$
  - Add reverse edge with capacity f_e
- Use BFS to find augmenting path with fewest edges



Reductions

- We want to solve problem A
- We know how to solve problem B
- Convert input for A into input for B
- Interpret output of B as output of A
- A reduces to B

- Can transfer efficient solution from B to A
- But preprocessing and postprocessing must also be efficient
- Typically, both should be polynomial time



Reductions

- Bipartite matching reduces to max flow
- Max flow reduces to LP
- Number of variables, constraints is linear in the size of the graph
- Reverse interpretation is also useful
- If A is known to be intractable and A reduces to B, then B must also be intractable
- Otherwise, efficient solution for B will yield efficient solution for A

Checking algorithms

- Factorize a large number that is the product of two primes
- Generate a solution
  - Given a large number N, find p and q such that $pq = N$
- Check a solution
  - Given a solution p and q, verify that $pq = N$
- Examples: satisfiability, travelling salesman, vertex cover, independent set, etc.

- Checking algorithm C for problem P
- Takes an input instance I for P and a solution “certificate” S for I
- C outputs yes if S represents a valid solution for I, no otherwise
- For factorization
  - I is N
  - S is $\{p, q\}$
  - C involves verifying that $pq = N$

P, NP and NP-Complete

- P (**P**olynomial) is the class of problems with regular polynomial time algorithms (worst-case complexity)
- NP (**N**on-deterministic **P**olynomial) is the class of problems with checking algorithms
- An algorithm A is NP-Complete if it satisfies two conditions:
  - A is in NP
  - Every algorithm in NP is polynomial time reducible to A