Outline for Chapter 6 : Probability

6.1 : Discrete Random Variables

6.2 : Continuous Random Variables

1. Random variables
2. Expectation, Variance
3. **Multiple Random Variables**
4. Uniform, Exponential, Normal
5. Convergence in probability. Laws of large numbers: Markov, Chebyshev, Hoeffding, Central limit.

6.3 : Maximum Likelihood and other advanced topics

Joint Distribution/Density Functions

X, Y

$$f_{XY}(x, y) = \frac{P(X \in [x, x+dx], Y \in [y, y+dy])}{dx \cdot dy}$$

$$F_{XY}(x, y) = P(X \leq x, Y \leq y)$$

i) $f_{XY}(x, y) \geq 0$

ii) $\int_x \int_y f_{XY}(x, y)\, dx\, dy = 1$

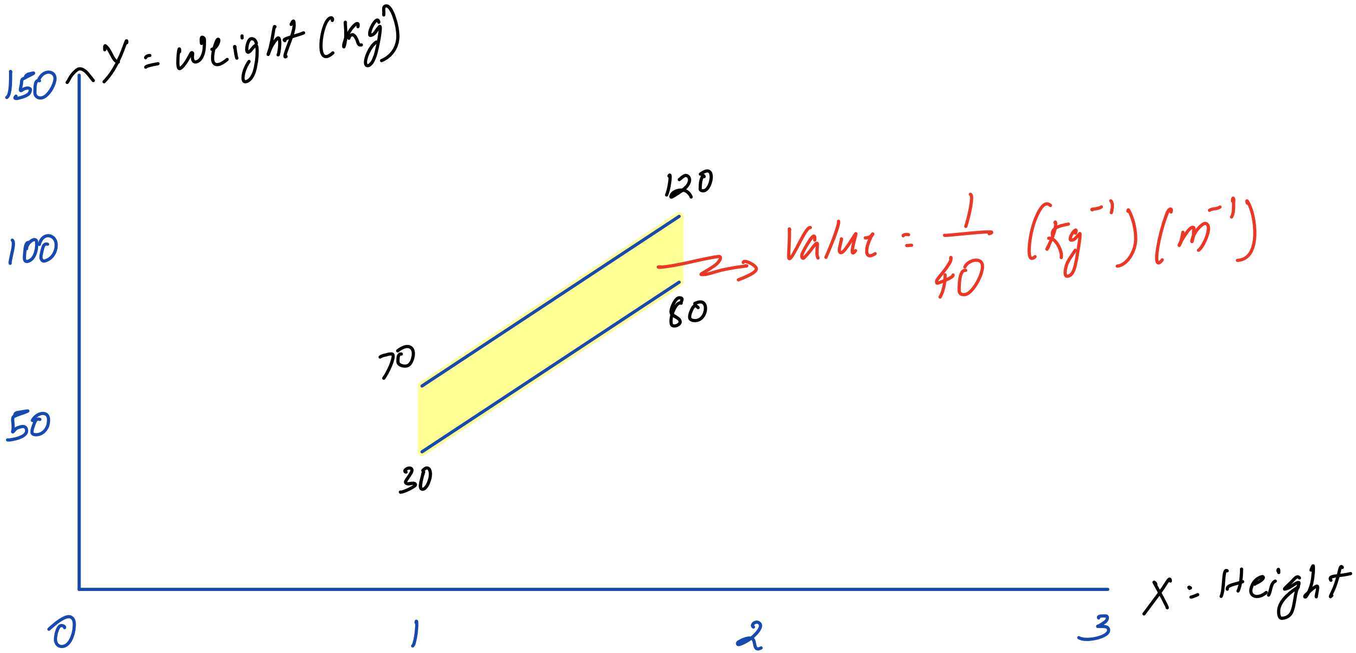
(i) $F_{XY}(-\infty, -\infty) = 0$

(ii) $F_{XY}(\infty, \infty) = 1$

$F_{XY}(x, y)$

Examples

$$f_{XY}(x,y) = \begin{cases} 0 & \text{if } x \notin [1,2] \\ 0 & \text{if } y \notin [50x-20, 50x+20] \\ c & \text{otherwise} \end{cases}$$



Examples

Marginals and Conditionals

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dx$$

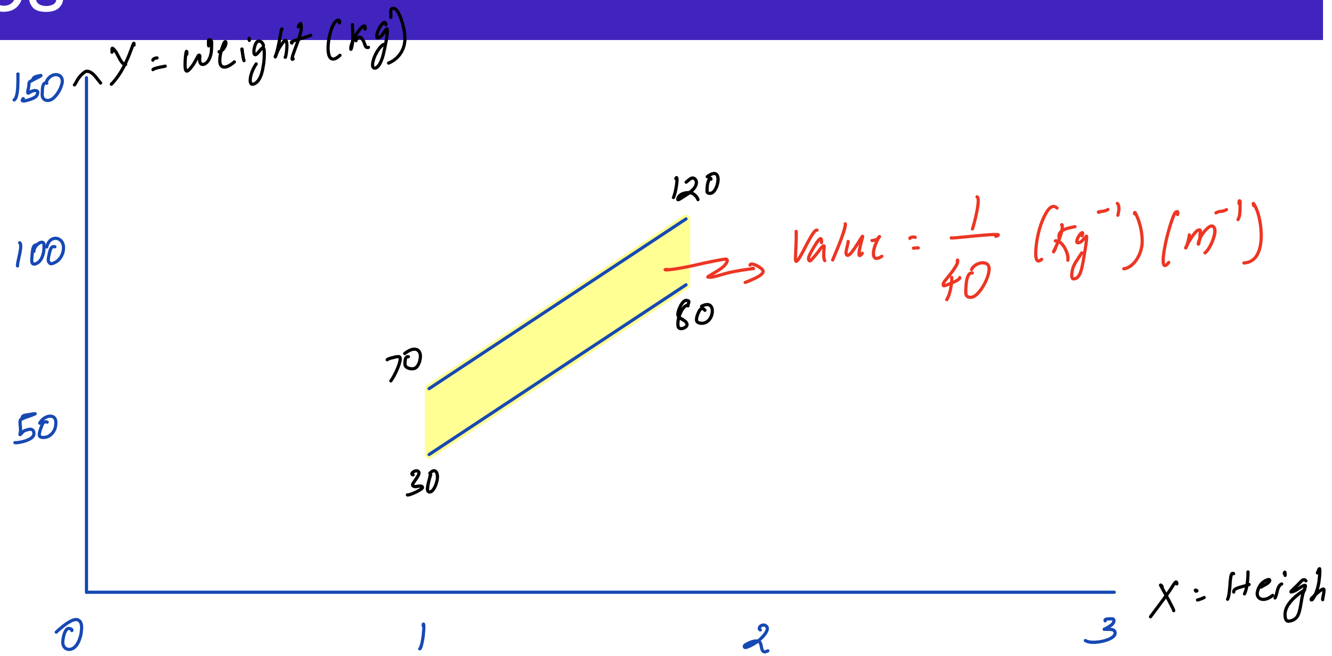
$$f_{X|Y}(x|y) = \frac{f_{XY}(x,y)}{f_Y(y)}$$

Examples

$$f_{XY}(x,y) = \begin{cases} 0 & \text{if } x \notin [1,2] \\ 0 & \text{if } y \notin [50x-20, 50x+20] \\ c & \text{otherwise} \end{cases}$$

$$f_X(x) = \begin{cases} 0 & \text{if } x \notin [1,2] \\ \int_{50x-20}^{50x+20} \frac{1}{40} \cdot dy = 1 & \text{if } x \in [1,2] \end{cases}$$

Examples



$$f_Y(y) : \begin{cases} 0 & y < 30 \\ & 30 \le y < 70 \\ & 70 \le y < 80 \\ & 80 \le y < 120 \\ 0 & y > 120 \end{cases}$$

Examples

X = Height (m)

Y = weight (kg)

$$f_{Y|X}(y|x) = \begin{cases} NA & \text{if } x \notin [1,2] \\ \dfrac{f_{XY}(x,y)}{1} & \text{if } x \in [1,2] \end{cases}$$

case: $x \in [1,2]$

$$= f_{XY}(x,y)$$

$$= \begin{cases} 1/40 & \text{if } y \in [50x-20, 50x+20] \\ 0 & \text{otherwise.} \end{cases}$$

Examples

Independence of Random Variables

$$X, Y$$

$$f_{XY}(x, y) = f_X(x) \cdot f_Y(y)$$

$$\Updownarrow$$

$$\forall \quad g, h; \quad E[g(X)h(Y)] = E[g(X)]\, E[h(Y)]$$

Examples

Height, weight expt.

$$f_{XY}(x,y) \neq f_X(x) \cdot f_Y(y)$$

why?

e.g

$$f_{XY}(1.9, 70) = 0$$

But $f_X(1.9) \neq 0$

$f_Y(70) \neq 0$