Machine Learning - Foundations

Revision (Week 2)

IIT Madras Online Degree

Topics

- Continuity
- Differentiability
- Linear Approximation
- Higher order approximations
- Multivariate Linear Approximation
- Directional Derivative

Continuity

A function $f(x)$ is continuous at $x = a$ if:

$$f(a) = \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

Differentiability

A function $f(x)$ is differentiable at $x = a$ if:

$$f'(a) = \lim_{x \to a^-} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a^+} \frac{f(x) - f(a)}{x - a}$$

Linear Approximation

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If $x_1 = a$, $y_1 = f(a)$ and $m = f'(a)$, we get,

$$y = f(a) + f'(a)(x - a)$$

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Higher-order Approximations

$$\boxed{\begin{gathered} L(x) = f(a) + f^{(1)}(a)(x-a) + \frac{f^{(2)}(a)}{2}(x-a)^2 + \\ + \frac{f^{(3)}(a)}{3 \cdot 2}(x-a)^3 + \frac{f^{(4)}(a)}{4 \cdot 3 \cdot 2}(x-a)^4 \ldots \end{gathered}}$$

Multivariate linear approximation: Linear approximation of functions involving multiple variables

The linear approximation of a function f of two variables x and y in the neighborhood of (a, b) is:

$$\boxed{L(x, y) = f(a, b) + \frac{\partial f}{\partial x}(a, b)(x - a) + \frac{\partial f}{\partial y}(a, b)(y - b)}$$

Directional derivatives

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\begin{aligned}
D_{\vec{u}} f(x, y) &= \nabla f \cdot u \\
&= \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right] \cdot [u_1, u_2] \\
&= u_1 \frac{\partial f}{\partial x} + u_2 \frac{\partial f}{\partial y}
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Directional derivative can be considered to be a weighted sum of partial derivatives.

Thank you.