Uniform Distribution

$$X \sim \text{Unif}(a, b)$$

$$f_X(x) = \begin{cases} 1/b-a & \text{if} \quad x \in [a,b] \\ 0 & \text{o.w} \end{cases}$$

$$E[X] = \frac{a+b}{2}$$

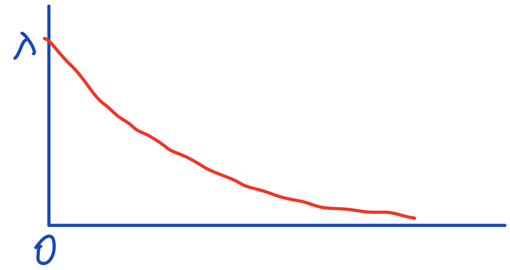
$$Var[X] = \frac{(b-a)^2}{12}$$

Uniform Distribution

Exponential Distribution

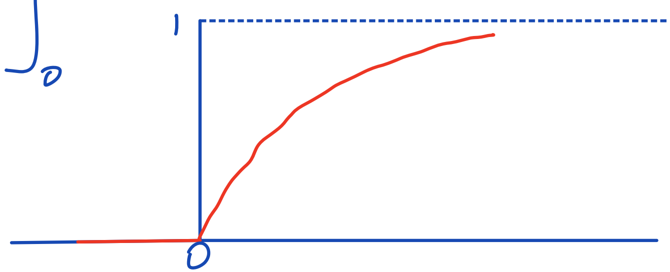
$X \sim \exp(\lambda)$

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$



$$\int_0^\infty \lambda e^{-\lambda x}\,dx = \lambda \left[-\frac{1}{\lambda} e^{-\lambda x}\right]_0^\infty$$

$$= \lambda \cdot \frac{1}{\lambda} = 1$$



$$F_X(x) = P(X \leq x) = \int_0^x f_X(u)\,du = 1 - e^{-\lambda x}$$

Exponential Distribution

Memorylessness: $\quad a > b$

$$P(X \geq a \mid X \geq b) = P(X \geq a-b)$$

L.H.S $= \dfrac{P(X \geq a, X \geq b)}{P(X \geq b)}$

$$= \frac{P(X \geq a)}{P(X \geq b)} = \frac{e^{-\lambda a}}{e^{-\lambda b}} = e^{-\lambda(a-b)}$$

$$= P(X \geq a-b)$$

Exponential Distribution

$X \sim \exp(\lambda)$

$$EX = \int_0^\infty x \cdot \lambda e^{-\lambda x}\, dx = \frac{1}{\lambda}$$

$u = x, \quad du = dx$

$dv = e^{-\lambda x} dx, \quad v = -\frac{1}{\lambda} e^{-\lambda x}$

$$\int_0^\infty x e^{-\lambda x} dx = -\frac{x}{\lambda} e^{-\lambda x}\Big|_0^\infty + \int_0^\infty \frac{1}{\lambda} e^{-\lambda x} dx$$

$$= 0 + \frac{1}{\lambda} \cdot \left[-\frac{1}{\lambda} e^{-\lambda x} \right]_0^\infty = \frac{1}{\lambda^2}$$

Exponential Distribution

$$X \sim \exp(\lambda) \qquad 0.25 \text{ year}^{-1}$$

$$Y \sim \exp(\tau) \qquad 0.25 \text{ year}^{-1}$$

$$Z = \min(X, Y)$$

S.T: $$Z \sim \exp(\lambda + \tau) \qquad 0.5 \text{ year}^{-1}$$

$$F_Z(z) = 1 - (1 - F_X(z))(1 - F_Y(z))$$

$$= 1 - \left(e^{-\lambda z}\right)\left(e^{-\tau z}\right) \qquad \text{if } z \geq 0$$

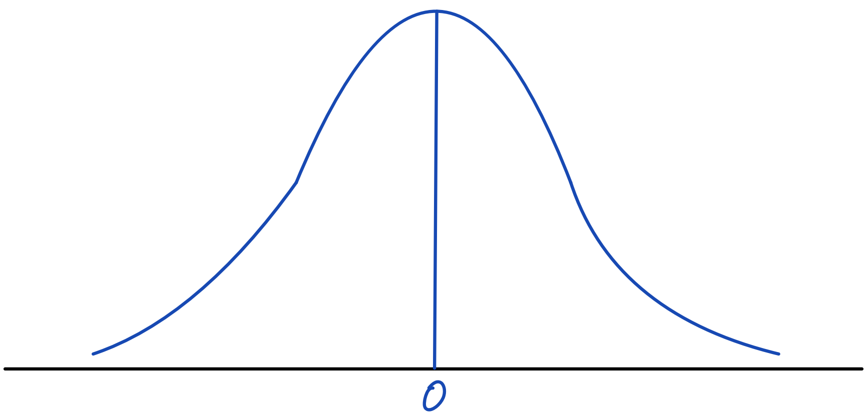
$$= 1 - e^{-(\lambda+\tau) z}$$

$$f_Z(z) = \begin{cases} 0 & \text{if } z < 0 \\ (\lambda+\tau) e^{-(\lambda+\tau) z} & \text{o.w} \end{cases}$$

Normal Distribution

$$Z \sim N(0, 1)$$

$$f_Z(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right)$$



Normal Distribution

$$A = \int_{-\infty}^{\infty} e^{-x^2/2}\, dx$$

$$A^2 = \left(\int_{-\infty}^{\infty} e^{-x^2/2}\, dx \right) \left(\int_{-\infty}^{\infty} e^{-y^2/2}\, dy \right)$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{x^2+y^2}{2}}\, dx\, dy$$

$x = r\cos\theta$

$y = r\sin\theta$

$$= \int_{0}^{\infty} \int_{0}^{2\pi} e^{-r^2/2}\, r\, dr\, d\theta$$

Normal Distribution

$$A^2 = \int_0^{\infty} \int_0^{2\pi} e^{-r^2/2} \, r \, dr \, d\theta$$

$$= \int_0^{2\pi} d\theta \int_0^{\infty} e^{-u} \, du$$

$$= 2\pi$$

$$A = \sqrt{2\pi}$$

Normal Distribution

Gaussian $\qquad Z \sim N(0,1)$

$$X = \sigma Z + \mu \quad ; \quad Z = \frac{X-\mu}{\sigma}$$

$$f_X(x) = ?$$

$$= f_Z\left(\frac{x-\mu}{\sigma}\right) \cdot \frac{1}{\sigma}$$

$$= \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) \cdot \frac{1}{\sigma}$$

Normal Distribution

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2} \cdot \frac{(x-\mu)^2}{\sigma^2}\right)$$

$X \sim N(\mu, \sigma^2)$ $\quad X = \sigma Z + \mu$

$EX = ? = E[\sigma Z + \mu]$

$= \mu + \sigma E[Z]$

$= \mu$

$Var[X] = ? = \sigma^2 \cdot Var[Z] = \sigma^2$