**Course: Machine Learning - Foundations**
**Week 10 Questions**

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**TEST QUESTIONS**

1. (1 point) **(Multiple Select)** For three events, A, B, and C, with $P(C) > 0$, Which of the following is/are correct?

   A. $P(A^c|C)$= 1 - $P(A|C)$

   B. $P(\phi|C) = 0$

   C. $P(A|C) \leq 1$

   D. if $A \subset B$ then $P(A|C) \leq P(B|C)$

   **Answer:** A, B, C, D

2. (2 points) **(Multiple Select)** Let the random experiment be tossing an unbiased coin two times. Let A be the event that the first toss results in a head, B be the event that the second toss results in a tail and C be the event that on both the tosses, the coin landed on the same side. Choose the correct statements from the following:

   A. A and C are independent events.

   B. A and B are independent events.

   C. B and C are independent events.

   D. A, B, and C are independent events.

   **Answer:** A, B, C

   Solution:

   $A = \{HT, HH\}$

   $B = \{HT, TT\}$

   $C = \{TT, HH\}$

   $P(A) = \frac{1}{2}$

   $P(B) = \frac{1}{2}$

   $P(C) = \frac{1}{2}$

   $P(A \cap B) = \{HT\}$

   $P(C \cap B) = \{TT\}$

   $P(A \cap C) = \{HH\}$

$P(A \cap B) = P(A) \times P(B)$ Hence, option B is correct

$P(A \cap C) = P(A) \times P(C)$ Hence, option A is correct

$P(C \cap B) = P(C) \times P(B)$ Hence, option C is correct

3. (2 points) **(Multiple Select)** If $A_1, A_2, A_3, , A_n$ are non empty disjoint sets and subsets of sample space S, and a set $A_{n+1}$ is also a subset of S, then which of the following statements are true?

   A. The sets $A_1 \cap A_{n+1}, A_2 \cap A_{n+1}, A_3 \cap A_{n+1}, , A_n \cap A_{n+1}$ are disjoint.

   B. If $A_{n+1}, A_n$ are disjoint then $A_1, A_2, , A_{n-1}$ are disjoint with $A_{n+1}$.

   C. The sets $A_1, A_2, A_3, , A_n, \phi$ are disjoint.

   D. The sets $A_1, A_2, A_3, , A_n, S$ are disjoint.

   **Answer:** A, C

4. (3 points) A triangular spinner having three outcomes can lands on one of the numbers 0, 1 and 2 with probabilities shown in table.

| Outcome | 0 | 1 | 2 |
|---|---|---|---|
| Probability | 0.7 | 0.2 | 0.1 |

Table 1: Table 10.2: Probability distribution

The spinner is spun twice. The total of the numbers on which it lands is denoted by $X$. The the probability distribution of $X$ is.

A.

| $x$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $P(X = x)$ | $\frac{49}{100}$ | $\frac{28}{100}$ | $\frac{1}{100}$ | $\frac{4}{100}$ | $\frac{18}{100}$ |

B.

| $x$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $P(X = x)$ | $\frac{28}{100}$ | $\frac{49}{100}$ | $\frac{18}{100}$ | $\frac{1}{100}$ | $\frac{4}{100}$ |

C.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X = x)$ | $\frac{49}{100}$ | $\frac{28}{100}$ | $\frac{18}{100}$ | $\frac{4}{100}$ | $\frac{1}{100}$ |

D.

| $x$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $P(X = x)$ | $\frac{28}{100}$ | $\frac{49}{100}$ | $\frac{18}{100}$ | $\frac{4}{100}$ | $\frac{1}{100}$ |

**Answer:** C

5. (1 point) When throwing a fair die, what is the variance of the number of throws needed to get a 1?

**Answer:** 30

Solution:

$$= Var(X) = \frac{1-p}{p^2}$$

$$= \frac{1-\frac{1}{6}}{\frac{1}{6}^2}$$

$$= 30$$

6. (1 point) Joint pmf of two random variables $X$ and $Y$ are given in Table

| $x$ \ $y$ | 1 | 2 | 3 | $f_X(x)$ |
|---|---|---|---|---|
| 1 | 0.05 | 0 | $a_1$ | 0.15 |
| 2 | 0.1 | 0.2 | $a_3$ | $a_2$ |
| 3 | $a_4$ | 0.2 | $a_5$ | 0.45 |
| $f_Y(y)$ | 0.3 | 0.4 | $a_6$ | |

Find the value of $f_{Y|X=3}(1)$ i.e ($P(Y = 1|X = 3)$)

**Answer:** 0.22

Solution:

$\sum f_{XY}(x, y) = 1$ ............. $(i)$

$f_X(x) = \sum\limits_{y \in R_y} f_{XY}(x, y)$ .............$(ii)$

$f_Y(y) = \sum\limits_{x \in R_X} f_{XY}(x, y)$ ...............$(iii)$

Hence, $a_1 = 0.10$ , $a_2 = 0.40$ , $a_3 = 0.1$, $a_4 = 0.15$, $a_5 = 0.1$, $a_6 = 0.3$

$$f_{Y|X=3}(1) = \frac{f_{XY}(1, 3)}{f_X(3)} = \frac{0.1}{0.45} = 0.22$$

7. (1 point) **(Multiple Select)** Which of the following options is/are correct?

   A. If $Cov[X, Y] = 0$, then $X$ and $Y$ are independent random variables.

   B. $Cov[X, X] = Var(X)$

   C. If $X$ and $Y$ are two independent random variables and $Z = X + Y$ then $f_Z(z) = \sum_x f_X(x) \times f_Y(z - x)$

   D. If $X$ and $Y$ are two independent random variables and $Z = X + Y$ then $f_Z(z) = \sum_y f_X(x) \times f_Y(z - x)$

**Answer:** B, C

Solution:

Option B

$Cov[X, X]$ is the covariance between $X$ and $X$ i.e $Var(X)$

Option C is correct from its definition.

8. (1 point) **(Multiple Select)**A discrete random variables $X$ has the cumulative distribution function is defined as follows.

$$F_X(x) = \begin{cases} \dfrac{x^3 + k}{40}, & \text{for } x = 1, 2, 3 \end{cases}$$

Which of the following options is/are correct for $F(x)$ as given?

   A. $k = 17$

   B. $Var(X) = \dfrac{259}{320}$

   C. $k = 13$

   D. $Var(X) = \dfrac{249}{310}$

**Answer:** B, C

Solution:

For $k$

$F_X(3) = 1$

$\frac{x^3 + k}{40} = 1$

Solving above equation to get $k = 13$

To calculate the variance, first calculate the probability distribution of $X$

We will get

$P(X = 1) = \frac{14}{40}$

$P(X = 2) = \frac{7}{40}$

$P(X = 3) = \frac{19}{40}$

Now easily with $Var(X)$ equation we will get $Var(X) = \frac{259}{320}$

9. (1 point) In a game of Ludo, Player A needs to repeatedly throw an unbiased die till he gets a 6. What is the probability that he needs fewer than 4 throws? (Answer the question correct to two decimal points.)

   Solution:

   $P(6) = \frac{1}{6}$

   As it resembles geometric distribution. Hence,
   $\sum_{n=1}^{3} \frac{1}{6} \times (1 - \frac{1}{6})^5 = 0.6$

10. (1 point) **(Multiple Select)** Let $X$ and $Y$ be two random variables with joint PMF $f_{XY}(x, y)$ given in Table 10.3.

| $x$ \ $y$ | 0 | 1 | 2 |
|---|---|---|---|
| 0 | $\frac{1}{6}$ | $\frac{1}{4}$ | $\frac{1}{8}$ |
| 1 | $\frac{1}{8}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |

Table 10.3: Joint PMF of $X$ and $Y$.

   Which of the following options is/are correct for $f_{XY}(x, y)$ given in Table 10.1.

A. $P(X=0, Y \leq 1) = \dfrac{5}{12}$

B. $P(X=0, Y \leq 1) = \dfrac{7}{12}$

C. $X$ and $Y$ are independent.

D. $X$ and $Y$ are dependent.

**Answer:** A, D

11. (1 point) A discrete random variables $X$ has the probability function as given in table 10.4.

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $P(X)$ | a | a | a | b | b | 0.3 |

Table 2: Table 10.4: Probability distribution

If $E(X) = 4.2$, then evaluate $a + b$

**Answer:** 0.3

$\sum P(X = x) = 1$

$3a + 2b = 0.7$

$E(X) = \sum P(X = x_i) \times x_i$

$6a + 9b = 2.4$

Solving both equations, we get $a = 0.1$ and $b = 0.2$

12. (1 point) A discrete random variable $X$ has the probability function as follows.

$$P(X = x) = \begin{cases} k \times (1-x)^2, & \text{for } x = 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$$

Evaluate $E(X)$

**Answer:** 2.8

Solution:

$\sum P(X = x) = 1$

$k + 4k = 1$

$k = 0.2$

$E(X) = \sum P(X = x_i) \times x_i$

$0.2 \times 2 + 0.8 \times 3$

$0.4 + 2.4 = 2.8$