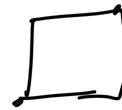
Transformed Random Variables

w, x

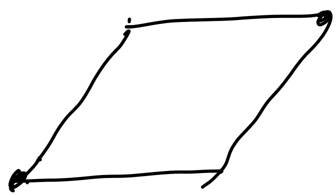
$y = g(w, x) \qquad w = q(y, z)$

$z = h(w, x) \qquad x = r(y, z)$

$f_{YZ}(y, z)\, dy\, dz = P\Big((Y, Z) \in$  $\Big)$

$(y+dy, z+dz)$

(y, z)

$= P\Big((W, X) \in$  $\Big)$

$q(y,z), r(y,z)$

Transformed Random Variables

$$f_{YZ}(y,z) = f_{WX}\big(q(y,z),\, r(y,z)\big)\,|J|$$

$$J = \begin{bmatrix} \frac{\partial q}{\partial y} & \frac{\partial q}{\partial z} \\ \frac{\partial r}{\partial y} & \frac{\partial r}{\partial z} \end{bmatrix}$$

Transformed Random Variables

$W \sim \text{Unif}(0,1)$

$X \sim \text{Unif}(0,1)$

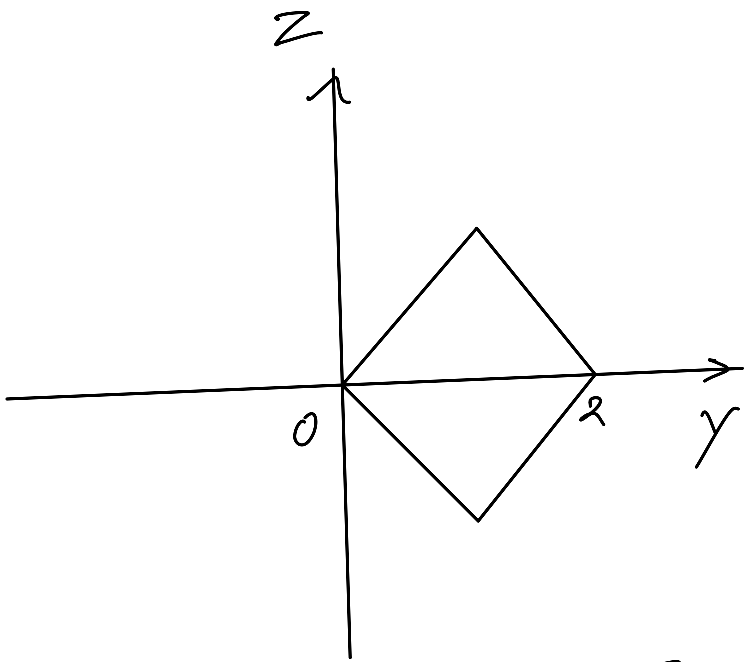
W, X ind.

$Y = W + X$

$Z = W - X$

$W = \frac{Y+Z}{2}$

$X = \frac{Y-Z}{2}$



$$J = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix}$$

$|\text{Det}(J)| = 1/2$

Transformed Random Variables

$$f_{YZ}(y,z) = f_{WX}\left(\frac{y+z}{2}, \frac{y-z}{2}\right)\cdot\frac{1}{2}$$

$$= f_W\left(\frac{y+z}{2}\right)\cdot f_X\left(\frac{y-z}{2}\right)\cdot\frac{1}{2}$$