Priority Queues

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Programming, Data Structures and Algorithms using Python Week 6

Job scheduler

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- How should the scheduler maintain the list of pending jobs and their priorities?

Priority queue

- Need to maintain a collection of items with priorities to optimise the following operations
- delete_max()
 - Identify and remove item with highest priority
 - Need not be unique
- insert()
 - Add a new item to the collection

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 - insert() is O(n)
- Processing *n* items requires $O(n^2)$

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Moving to two dimensions

First attempt

Assume N processes enter/leave the queue

$$N = 25$$

3	19	23	35	58
12	17	25	43	67
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- Assume N processes enter/leave the queue
- Maintain a $\sqrt{N} \times \sqrt{N}$ array

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Moving to two dimensions

First attempt

- Assume N processes enter/leave the queue
- Maintain a $\sqrt{N} \times \sqrt{N}$ array
- Each row is in sorted order

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 - Use size of row to determine

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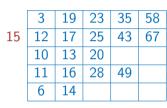
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- Insert into the first row that has space
 - Use size of row to determine
- Insert 15
- Takes time $O(\sqrt{N})$
 - Scan size column to locate row to insert, $O(\sqrt{N})$
 - Insert into the first row with free space, $O(\sqrt{N})$

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Maximum in each row is the last element



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- Identify the maximum amongst these

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- Delete it

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- Maximum in each row is the last element
- Position is available through size column
- Identify the maximum amongst these
- Delete it
- Again $O(\sqrt{N})$
 - Find the maximum among last entries, $O(\sqrt{N})$
 - Delete it, *O*(1)

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- 2D $\sqrt{N} \times \sqrt{N}$ array with sorted rows
 - insert() is $O(\sqrt{N})$
 - delete_max() is $O(\sqrt{N})$
 - Processing *N* items is $O(N\sqrt{N})$

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 - Height $O(\log N)$
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- Flexible need not fix N in advance

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