

MACHINE LEARNING - FOUNDATIONS

REVISION (WEEK 2)

IIT Madras Online Degree

Topics

- Continuity
- Differentiability
- Linear Approximation
- · Higher order approximations
- Multivariate Linear Approximation
- · Directional Derivative

Continuity

A function f(x) is continuous at x = a if:

$$f(a) = \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

Differentiability

A function f(x) is differentiable at x = a if:

$$f'(a) = \lim_{x \to a^{-}} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a^{+}} \frac{f(x) - f(a)}{x - a}$$

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Linear Approximation

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If
$$x_1=a$$
, $y_1=f(a)$ and $m=f^\prime(a)$, we get,

$$y = f(a) + f'(a)(x - a)$$

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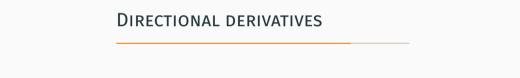
Higher-order Approximations

$$\begin{split} L(x) &= f(a) + f^{(1)}(a)(x-a) + \frac{f^{(2)}(a)}{2}(x-a)^2 + \\ &\quad + \frac{f^{(3)}(a)}{3 \cdot 2}(x-a)^3 + \frac{f^{(4)}(a)}{4 \cdot 3 \cdot 2}(x-a)^4 \dots \end{split}$$

Multivariate linear approximation: Linear approximation of functions involving multiple variables

The linear approximation of a function f of two variables x and y in the neighborhood of (a,b) is:

$$L(x,y) = f(a,b) + \frac{\partial f}{\partial x}(a,b)(x-a) + \frac{\partial f}{\partial y}(a,b)(y-b)$$



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Directional derivative can be considered to be a weighted sum of partial derivatives.

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