Machine Learning Foundations

Tutorial-Week 4

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Eigenvalues and Eigenvectors

- "Eigen" is a German word literally meaning "own" or "characteristic".
- Solving an Eigen problem is like finding characteristic of a matrix.
- Let's learn geometric interpretation of eigen values and eigen vectors.
- When we apply linear transformation on a set of vectors then most of them are knocked off from their original basis.
- The vectors which hold their original basis when linear transformation is applied are called Eigenvectors.
- The values by which Eigenvectors are scaled during linear transformation are called Eigenvalues.
- We can say that applying linear transformation on an eigenvector is similar to just scaling the eigenvector by the eigen value.

Eigen vector demo

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \qquad |A - \lambda I| = 0$$

$$A - \lambda I = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\lambda & 1 \\ -2 & -3-\lambda \end{bmatrix}$$

$$\begin{vmatrix} -\lambda & 1 \\ -2 & -3-\lambda \end{vmatrix} = 0$$

$$-\lambda(-3-\lambda) + 2 = 0$$

$$3\lambda + \lambda^2 + 2 = 0$$

$$\lambda^2 + 3\lambda + 2 = 0$$

$$\lambda = -1, -2$$

$\lambda = -1$ $\qquad Au = \lambda u \qquad \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\begin{bmatrix} x_2 \\ -2x_1 - 3x_2 \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$x_1 + x_2 = 0 \qquad x_1 = k \quad x_2 = -k$$

$$u = \begin{bmatrix} k \\ -k \end{bmatrix} = k \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad u = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$Au = \begin{bmatrix} -1 \\ 1 \end{bmatrix} = -u$$

$$Av = \begin{bmatrix} -2 \\ 4 \end{bmatrix} = -2v$$

$\lambda = -2 \qquad v = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

The Equation

$$A\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}1\\-5\end{bmatrix}$$

$$A\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}2\\-8\end{bmatrix}$$

$$A\bar{x} = \lambda\bar{x} \tag{1}$$

- A$=$ Transformation matrix
- $\bar{x}$ $=$ *Eigenvector*
- λ $=$ Corresponding Eigenvalue

continued...

$$(A - \lambda I)\bar{x} = 0 \tag{2}$$

$$det(A - \lambda I) = 0 \tag{3}$$

- A = Transformation matrix
- $\bar{x}$ = Eigen Vector
- λ = Corresponding Eigenvalue
- I = Identity matrix
- Eqn 2 is usually used to find eigenvectors, and, Eqn 3 is used to find eigenvalues of a matrix.