String Matching: Boyer-Moore algorithm

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Programming, Data Structures and Algorithms using Python

Week 10

Speeding up the brute force algorithm

- Text `t`, pattern `p` of of lengths n, m
- For each starting position `i` in `t`, compare `t[i:i+m]` with `p`
  - Scan `t[i:i+m]` right to left

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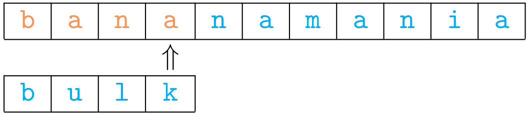

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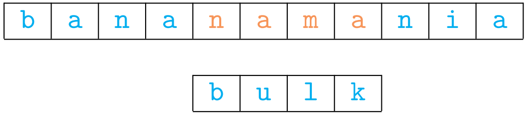
- Shift the next scan to position after mismatched letter
- What if the mismatched letter does appear in `p`?



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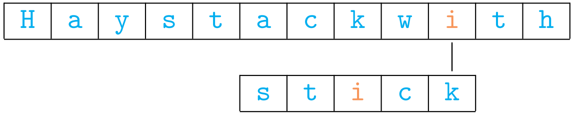

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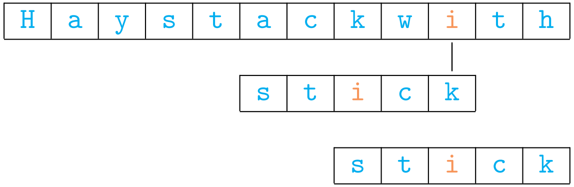

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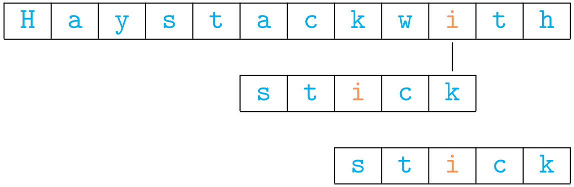

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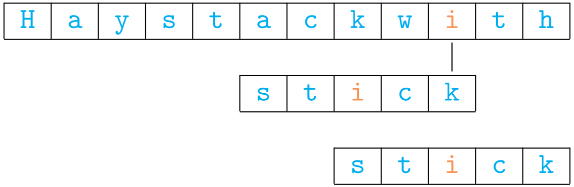

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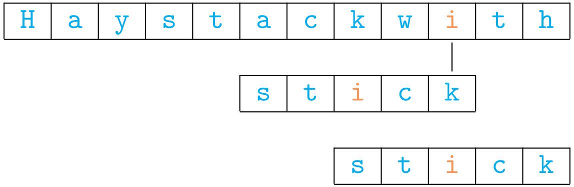
- If `c` not in `p`, shift pattern by `j+1`



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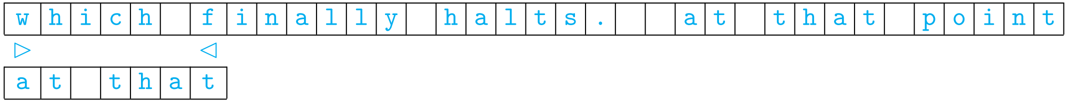
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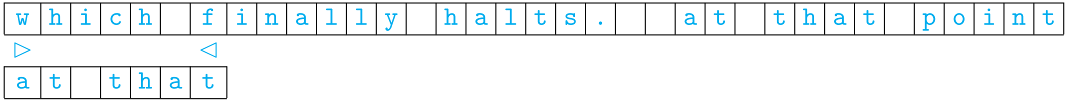

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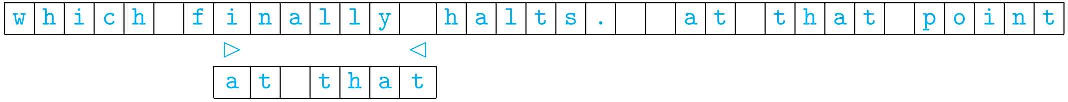

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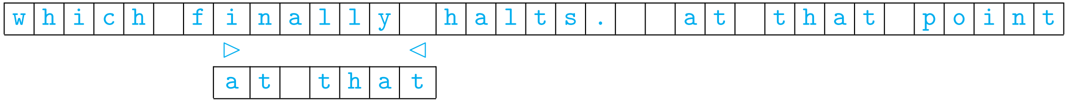

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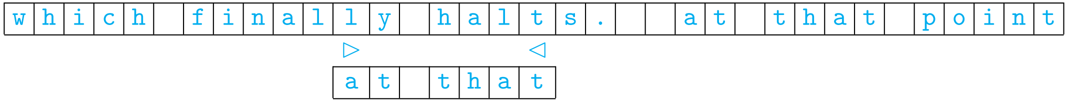

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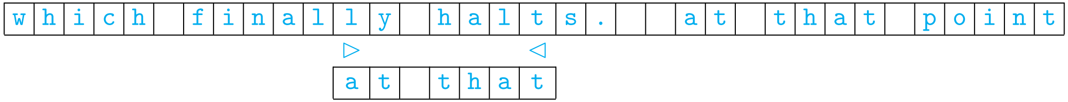
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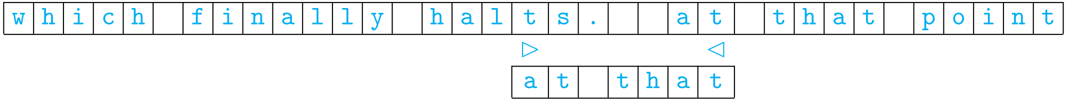

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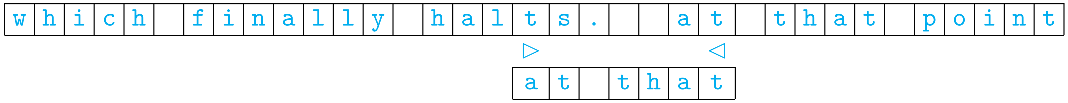

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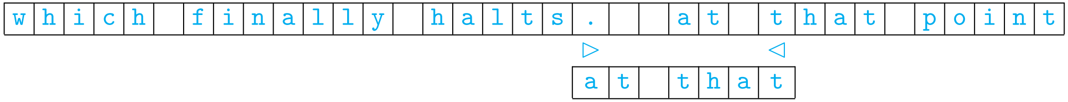

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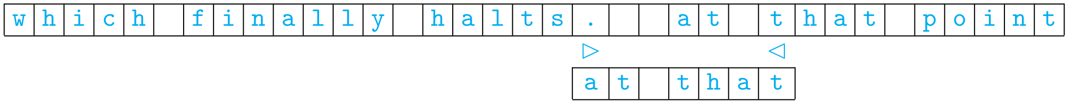

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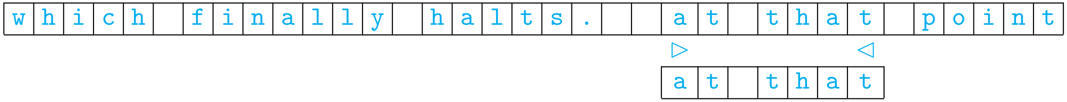

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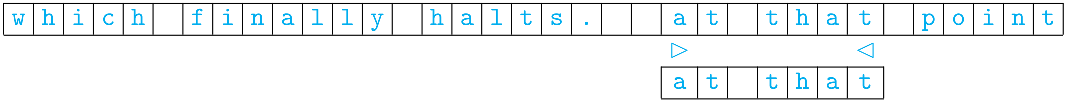

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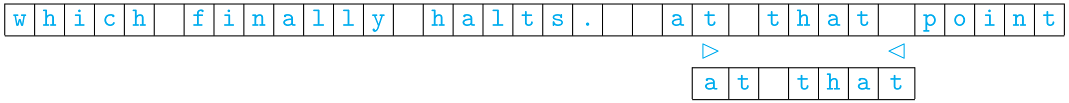

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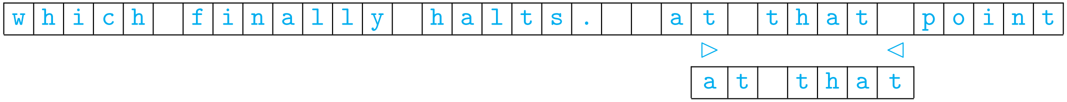

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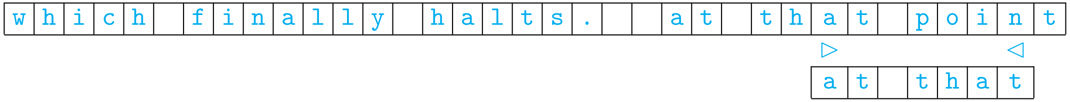

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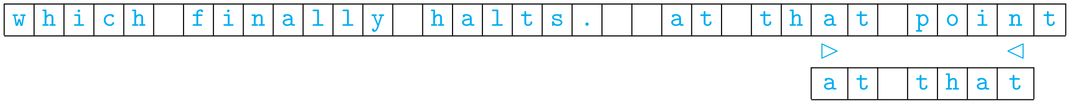

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Implementation

- Initialize `last[c]` for each `c` in `p`
  - Single scan, rightmost value is recorded

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def boyermoore(t,p):
  last = {}                    # Preprocess
  for i in range(len(p)):
    last[p[i]] = i

  poslist,i = [],0             # Loop
  while i <= (len(t)-len(p)):
    matched,j = True,len(p)-1
    while j >= 0 and matched:
      if t[i+j] != p[j]:
        matched = False
      j = j - 1
    if matched:
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    else:
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      if t[i+j] in last.keys():
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- If `p` matches, record and shift by `1`
- We find a mismatch at `t[i+j]`
    - If `j > last[t[i+j]]`, shift by `j - last[t[i+j]]`
    - If `last[t[i+j]] > j`, shift by `1`
        - Should not shift `p` to left!
    - If `t[i+j]` not in `p`, shift by `j+1`

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    - "Sublinear"
    - Experimentally — English text, 5 character pattern, average number of comparisons is 0.24 per character
    - Performance improves as pattern length grows — more characters skipped
- Often used in practice — `grep` in Unix