

Course: Machine Learning - Foundations  
Week 10 Questions

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PRACTICE QUESTIONS

1. (1 point) A discrete random variables  $X$  has the probability function as given in table 10.1.

$x$	1	2	3	4	5	6
$P(X)$	$a$	$a$	$a$	$b$	$b$	$0.3$

Table 1: Table 10.1: Probability distribution

If  $E(X) = 4.2$ , then evaluate  $(a + b)$

$\rightarrow a = 0.1, b = 0.2$   
 $\boxed{a + b = 0.3}$

Answer: 0.3

$$E(x) = \sum x p(x)$$

$$\Rightarrow 4.2 = 1 \times a + 2 \times a + 3a + 4b + 5b + 6 \times 0.3$$

$$\Rightarrow 4.2 = 6a + 9b + 1.8$$

$$\Rightarrow 6a + 9b = 2.4 \quad \text{--- (i) } \checkmark$$

(i-ii)

$$\sum p(x) = 1$$

$$3a + 2b + 0.3 = 1$$

$$2 \quad (3a + 2b = 0.7) \quad \text{--- (ii) } \checkmark$$

$$5b = 1$$

$$b = 0.2$$

$$a = 0.1$$

2. (1 point) Let  $X$  and  $Y$  be two random variables with joint PMF  $f_{XY}(x, y)$  given in Table 10.2.

$x \backslash y$	0	1	2
0	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{8}$
1	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$

Table 10.2: Joint PMF of  $X$  and  $Y$ .

Calculate  $P(X = 0, Y \leq 1)$

$$\Rightarrow X=0 \mid Y: 0, 1$$

$$P(X=0, Y=0) + P(X=0, Y=1)$$

$$\Rightarrow \frac{1}{6} + \frac{1}{4}$$

$$\Rightarrow \frac{2+3}{12} = \frac{5}{12} \quad \text{Ans}$$

3. (1 point) Joint pmf of two random variables  $X$  and  $Y$  are given in Table

$x \backslash y$	1	2	3	$f_X(x)$
1	0.05	0	$a_1$	0.15
2	0.1	0.2	$a_3$	$a_2$
3	$a_4$	0.2	$a_5$	0.45
$f_Y(y)$	0.3	0.4	$a_6$	

Find the value of  $f_{Y|X=2}(2)$  i.e.  $P(Y=2|X=2)$

Properties

- i.)  $f_{XY}(x,y) \geq 0$   
 ii.)  $\sum_{x,y} f_{XY}(x,y) = 1$

$$f_X(x) = \sum_y f_{XY}(x,y)$$

$$f_Y(y) = \sum_x f_{XY}(x,y)$$

$$f_{Y|X=2}(2) = \frac{f_{XY}(2,2)}{f_X(2)}$$

$$= \frac{0.2}{0.4} = \frac{1}{2} = 0.5$$

$$a_1 = 0.1$$

$$0.3 + a_3 = a_2$$

$$(i) - a_3 = a_2 - 0.3$$

$$(ii) - a_4 + a_5 = 0.25$$

$$a_4 = 0.15$$

$$a_5 = 0.1$$

$$-(0.1 + a_3 + 0.1 + 0.4 + 0.15 + 0.15)$$

$$3 \cdot 1 = 0.9 = a_3$$

$$a = 0.1$$

4. (1 point) A discrete random variables  $X$  has the cumulative distribution function is defined as follows.

$$F_X(x) = \begin{cases} \frac{x^2 + k}{40}, & \text{for } x = 1, 2, 3 \end{cases}$$

The value of  $k$  equals.

$$\begin{aligned} F_X(x) &= \sum f(x) \\ &= P(X \leq x) \end{aligned}$$

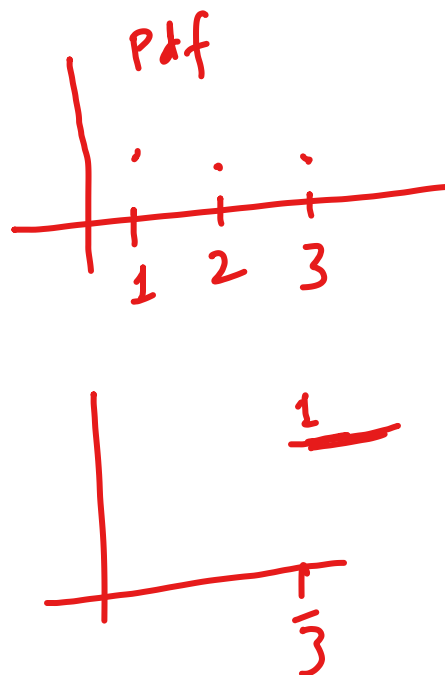
$$F_X(3) = 1$$

$$(x=3) \leftarrow \frac{x^2 + k}{40} = 1$$

$$\Rightarrow \frac{9 + k}{40} = 1$$

$$9 + k = 40$$

$$\boxed{k = 31}$$



5. (1 point) Three friends Shalini, Mohit and Vidhi thought of taking a selfie. Let  $E$  be an event such that Shalini will always be at one end and  $F$  be an event such that Vidhi will be in middle. Then determine  $P(E|F)$

$$E = \{MVS, VMS, SMV, SVM\}$$

$$F = \{MVS, SVM\}$$

$$P(E/F) = \frac{P(E \cap F)}{P(F)}$$

$$E \cap F = \{MVS, SVM\}$$

$$P(E \cap F) = \frac{2}{6} = \frac{1}{3}$$

$$P(F) = \frac{2}{6} = \frac{1}{3}$$

$$P(E/F) = \frac{\frac{1}{3}}{\frac{1}{3}} = 1 \text{ Ans}$$

6. (1 point) Two random variables X and Y are jointly distributed with joint pmf

$$f_{XY}(x, y) = \begin{cases} ax + \frac{y}{4}, & \text{for } x, y \in \{0, 1\} \\ 0, & \text{otherwise} \end{cases}$$

Calculate the value of  $a$

**Answer:** 0.25

$$\sum_{x \in R_X} \sum_{y \in R_Y} f_{XY}(x, y) = 1$$

$$\Rightarrow f_{XY}(0, 0) + f_{XY}(0, 1) + f_{XY}(1, 0) + f_{XY}(1, 1) = 1$$

$ax + \frac{y}{4}$

$$\Rightarrow 0 + \frac{1}{4} + a + a + \frac{1}{4} = 1$$

$$\Rightarrow 2a = 1 - \frac{1}{2} \Rightarrow 2a = \frac{1}{2}$$

$$a = \frac{1}{4} = 0.25$$