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Want $\eta \, d \, f'(x) < 0$ $\Rightarrow$ Want d s.t $\boxed{d \, f'(x) < 0}$

η: Small +ve constant

For the choice of $\boxed{d = -f'(x)}$

$$d \, f'(x) = -\left(f'(x)\right)^2 < 0$$

Higher dimensions

$$f(x_1, x_2) = x_1^2 + 4x_2 + 8x_2^2$$

Derivative $\Longleftrightarrow$ Gradient

vector of partial derivatives.

$$\nabla f\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} \left.\frac{\partial f}{\partial x_1}\right|_{x_1 = a} \\ \left.\frac{\partial f}{\partial x_2}\right|_{x_2 = b} \end{bmatrix}$$

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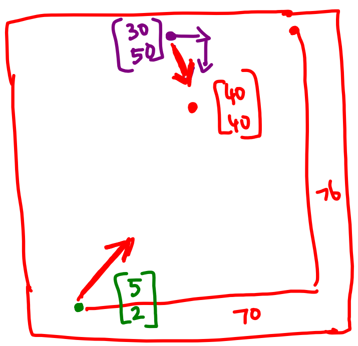
$$f(x_1, x_2) = x_1^2 + 4x_2 + 8x_2^2$$

$$\nabla f\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 \\ 4 + 16x_2 \end{bmatrix}_{x_1 = 1,\ x_2 = 3} = \begin{bmatrix} 2 \\ 4 + 16 \times 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 52 \end{bmatrix}.$$



$$d(x_1, x_2) = (x_1 - 40)^2 + (x_2 - 40)^2$$

$$\nabla d(x_1, x_2) = \begin{bmatrix} 2(x_1 - 40) \\ 2(x_2 - 40) \end{bmatrix} ; \quad \nabla d\left(\begin{bmatrix} 5 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 2(5-40) \\ 2(2-40) \end{bmatrix} = \begin{bmatrix} -70 \\ -76 \end{bmatrix}$$

$$-\nabla d\left(\begin{bmatrix} 30 \\ 50 \end{bmatrix}\right) = -\begin{bmatrix} 2(-10) \\ 2(10) \end{bmatrix} = -\begin{bmatrix} -20 \\ 20 \end{bmatrix} = \begin{bmatrix} 20 \\ -20 \end{bmatrix}$$

$$-\nabla d\left(\begin{bmatrix} 5 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 70 \\ 76 \end{bmatrix}$$

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Gradient descent : $\underbrace{\vec{x}_{t+1}}_{\text{vector}} = \underbrace{\vec{x}_t}_{\text{vector}} + \underbrace{\eta}_{\text{Scalar}} \left(- \underbrace{\nabla f(x_t)}_{\text{vector}} \right)$

$$f(\underbrace{x + \eta d}_{\text{Scalar}}) = f(x) + \eta d f'(x) + \frac{\eta^2 d^2}{2} f''(x) + \dots$$

Higher order Taylor series

$$f(\underbrace{x + \eta d}_{\text{vector}}) = \overbrace{\underbrace{f(x)}_{\text{vector}}}^{\text{scalar}} + \eta \underbrace{d^T \nabla f(x)}_{\text{Scalar}} + \dots$$