Solve with instructor

session

Mr. Kumar Ramanand
Tutorial Instructor

IIT Madras
BSc Degree

Table of Contents

IIT Madras
BSc Degree

Question-1

$$f(x, y) = x^2 + y^2$$

Which of the following is an stationary point of the above function?

- A. (0,0)
- B. (0,1)
- C. (1,0)
- D. (1,1)

IIT Madras
BSc Degree

Question-2

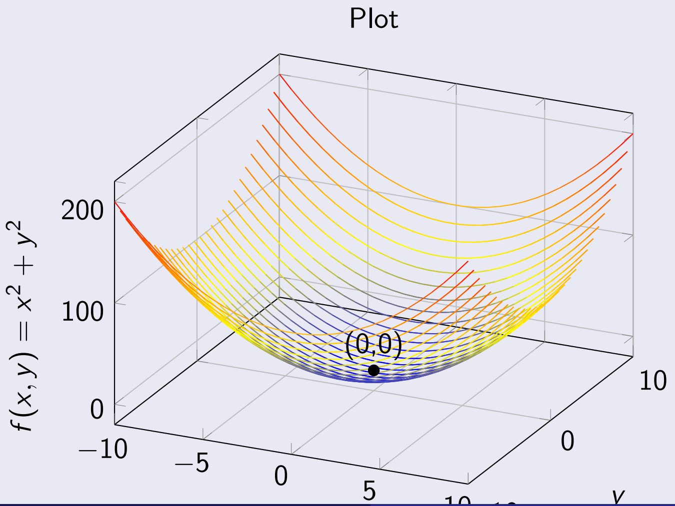
$f(x, y) = x^2 + y^2$
The stationary point of the function is a

- A. Minima
- B. Maxima
- C. Saddle
- D. Can not be determined

IIT Madras
BSc Degree

Question-2 solution

Graphical method



Question-3

$f(x, y) = -x^2 - y^2$

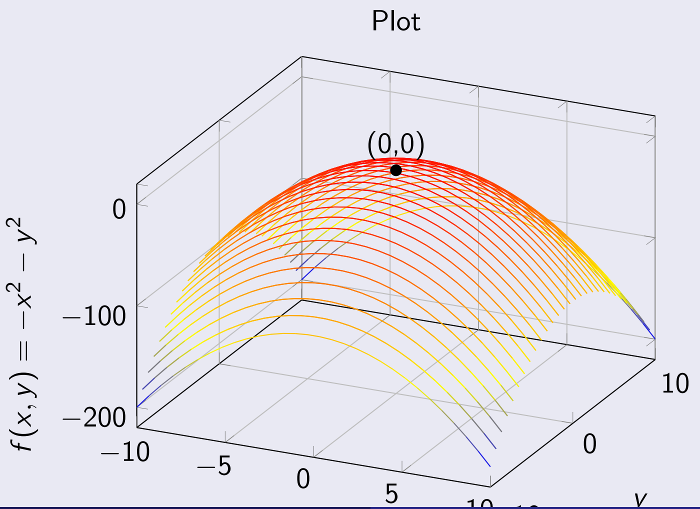
The stationary point of the function is a

- A. Minima
- B. Maxima
- C. Saddle
- D. Can not be determined

IIT Madras
BSc Degree

Question-3 solution

Graphical method



Question-4

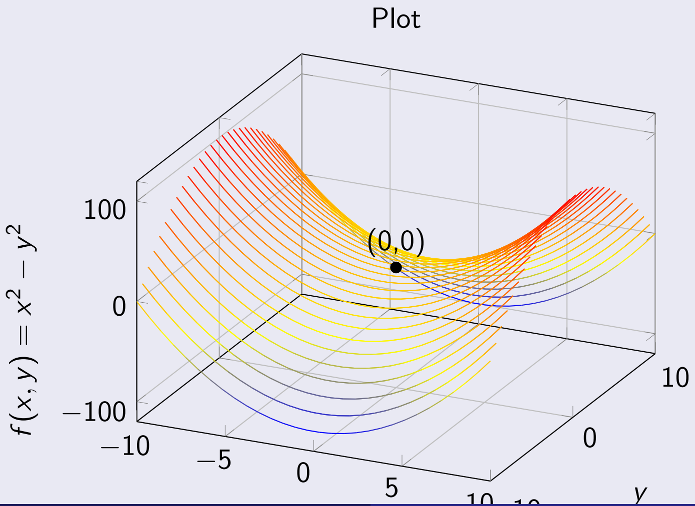
$f(x, y) = x^2 - y^2$

The stationary point of the function is a

- A. Minima
- B. Maxima
- C. Saddle
- D. Can not be determined

Question-4 solution

Graphical method



Question-5

$f(x, y) = -x^2 + y^2$

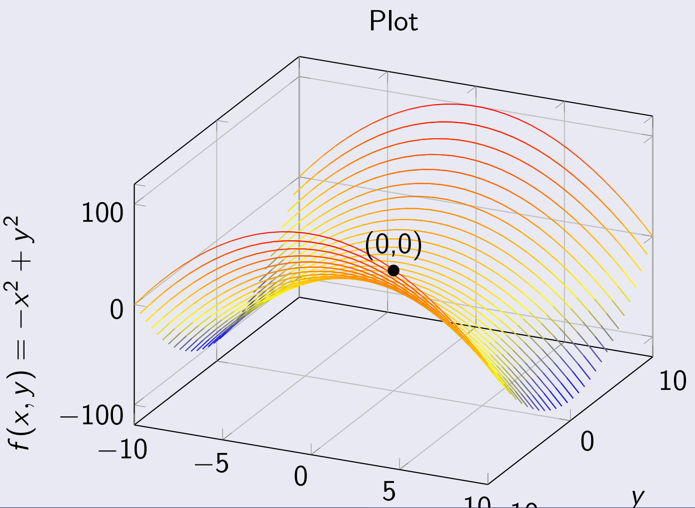
The stationary point of the function is a

- A. Minima
- B. Maxima
- C. Saddle
- D. Can not be determined

IIT Madras
BSc Degree

Question-5 solution

Graphical method



Question-6

Which of the following is an stationary point of the below function?
$f(x, y) = 8x^2 + 12xy + 10y^2$

- A. (0,0)
- B. (0,1)
- C. (1,0)
- D. (1,1)

Question-6 solution

Quadratic form, $f(x, y) = 8x^2 + 12xy + 10y^2$

Derivative method

Check for stationary point:

- First order partial derivatives: $f_x = 16x + 12y, f_y = 12x + 20y$
- First order partial derivatives at point (0,0): $f_x = 0, f_y = 0$
- This means the point (0, 0) is an stationary point for $f(x, y)$

IIT Madras
BSc Degree

Question-7

Which of the following is an stationary point of the below function?
$f(x, y) = 2x^2 - 8x + 4xy - 8y + 2y^2$

- A. (0,0)
- B. (-1,-1)
- C. (1,1)
- D. Can not be determined

IIT Madras
BSc Degree

Question-7 solution

Quadratic form, $f(x, y) = 2x^2 - 8x + 4xy - 8y + 2y^2$

Derivative method

Check for stationary point:

- First order partial derivatives: $f_x = 4x + 4y - 8, f_y = 4x + 4y - 8$
- First order partial derivatives at point (1,1): $f_x = 0, f_y = 0$
- This means the point (1, 1) is an stationary point for $f(x, y)$

IIT Madras
BSc Degree

Question-8

$f(x, y) = 8x^2 + 12xy + 10y^2$

The stationary point of the function is a

- A. Minima
- B. Maxima
- C. Saddle
- D. Can not be determined

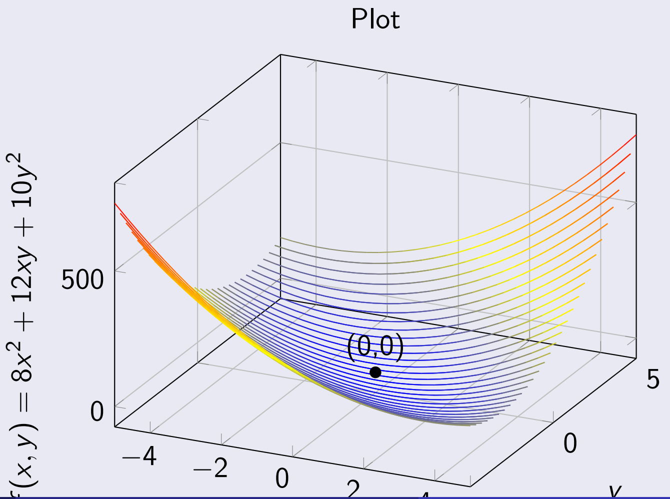
IIT Madras
BSc Degree

Question-8 solution

Derivative method

- Second order partial derivatives, $f_{xx} = 16$, $f_{xy} = 12$, $f_{yy} = 20$
- $D = f_{xx} f_{yy} - {f_{xy}}^2 = 16 * 20 - 12^2 = 176$
- $a > 0$ and $D > 0$,
- The function has a minima at the point $(0, 0)$

Graphical method



Question-9

$f(x, y) = -8x^2 + 12xy - 10y^2$

The stationary point of the function is a

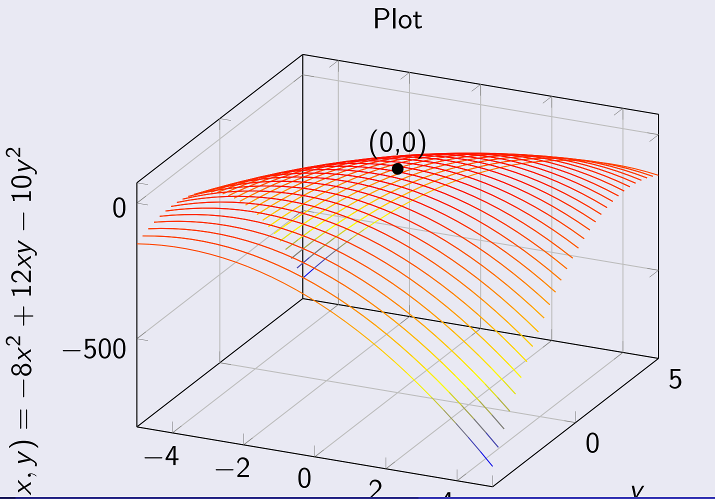
- A. Minima
- B. Maxima
- C. Saddle
- D. Can not be determined

Question-9 solution

Derivative method

- Second order partial derivatives, $f_{xx} = -16, f_{xy} = 12, f_{yy} = -20$
- $D = f_{xx} f_{yy} - {f_{xy}}^2 = -16 * -20 - 12^2 = 174$
- $a < 0$ and $D > 0$,
- The function has a maxima at the point $(0, 0)$

Graphical method



Question-10

$f(x, y) = 4x^2 + 12xy + 5y^2$
The stationary point of the function is a

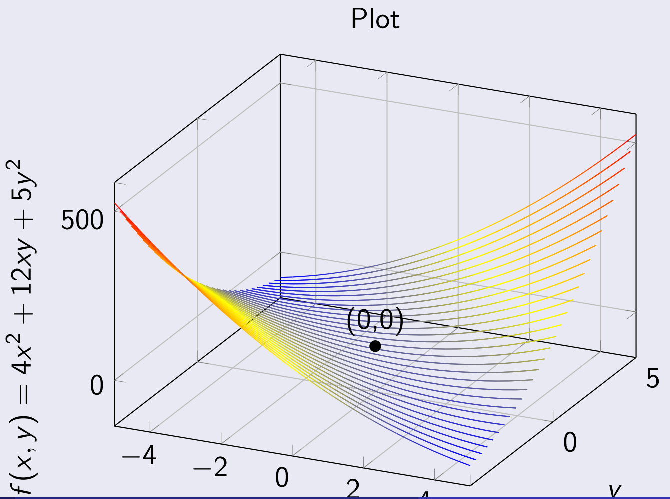
- A. Minima
- B. Maxima
- C. Saddle
- D. Can not be determined

Question-10 solution

Derivative method

- Second order partial derivatives, $f_{xx} = 8, f_{xy} = 12, f_{yy} = 10$
- $D = f_{xx}f_{yy} - {f_{xy}}^2 = 8 * 10 - 12^2 = -64$
- $D < 0$,
- The function has a saddle point at the point $(0, 0)$

Graphical method



Question-11

$f(x, y) = 2xy$

The stationary point of the function is a

- A. Minima
- B. Maxima
- C. Saddle
- D. Can not be determined

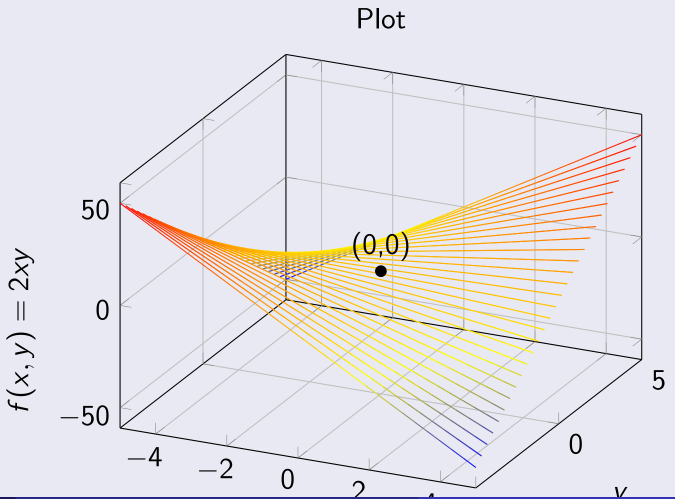
IIT Madras
BSc Degree

Question-11 solution

Derivative method

- Second order partial derivatives, $f_{xx} = 0, f_{xy} = 0, f_{yy} = 0$
- $D = f_{xx}f_{yy} - {f_{xy}}^2 = 0$
- $D = 0$, The second derivative test fails at the point $(0,0)$

Graphical method



Question-12

$f(x, y) = x^2 + 2xy + y^2$

The stationary point of the function is a

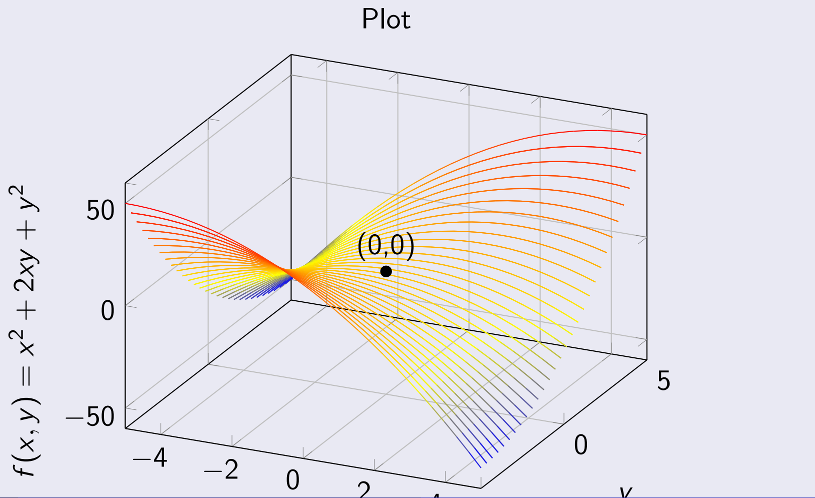
- A. Minima
- B. Maxima
- C. Saddle
- D. Can not be determined

Question-12 solution

Derivative method

- Second order partial derivatives, $f_{xx} = 2, f_{xy} = 2, f_{yy} = 2$
- $D = f_{xx} f_{yy} - {f_{xy}}^2 = 2 * 2 - 2^2 = 0$
- $D = 0$, The second derivative test fails at the point $(0, 0)$

Graphical method



Question-13

$$A = \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix}$$

The above matrix is

- A. Positive definite
- B. Positive semidefinite
- C. Negative definite
- D. Negative semidefinite
- E. Indefinite

IIT Madras
BSc Degree

Question-13 solution

Graphical test

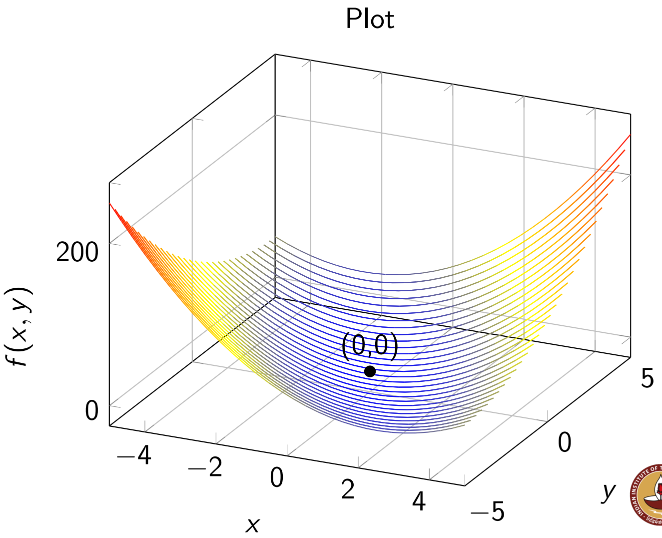
- $f(x, y) > 0$ for all

$$v = \begin{bmatrix} x \\ y \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- A is positive definite

Determinant test

- For the function,
  $a = 4, b = 2, c = 2$
- $a > 0, ac - b^2 > 0$
- A is positive definite



Question-14

$$A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

The above matrix is

- A. Positive definite
- B. Positive semidefinite
- C. Negative definite
- D. Negative semidefinite
- E. Indefinite

IIT Madras
BSc Degree

Question-14 solution

Graphical test

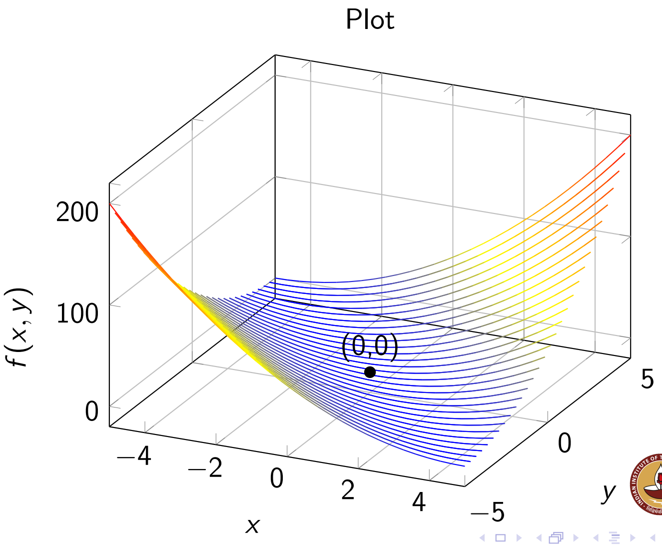
- $f(x, y) \geq 0$ for all

$$v = \begin{bmatrix} x \\ y \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- A is positive semidefinite

Determinant test

- For the function, $a = 2, b = 2, c = 2$
- $a > 0, ac - b^2 = 0$
- A is positive semidefinite



Question-15

$$A = \begin{bmatrix} -4 & 2 \\ 2 & -2 \end{bmatrix}$$

The above matrix is

- A. Positive definite
- B. Positive semidefinite
- C. Negative definite
- D. Negative semidefinite
- E. Indefinite

IIT Madras
BSc Degree

Question-15 solution

Graphical test

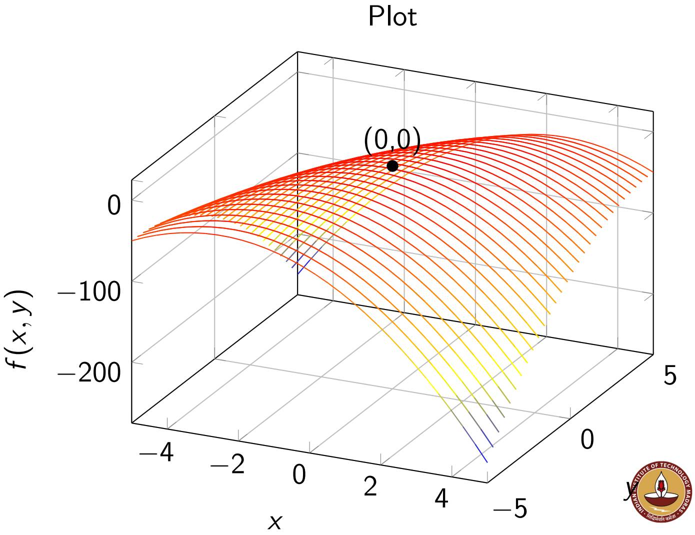
- $f(x, y) < 0$ for all

$$v = \begin{bmatrix} x \\ y \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- A is negative definite

Determinant test

- For the function,
  $a = -4, b = 2, c = -2$
- $a < 0, ac - b^2 > 0$
- A is negative definite



Question-16

$$A = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$$

The above matrix is

- A. Positive definite
- B. Positive semidefinite
- C. Negative definite
- D. Negative semidefinite
- E. Indefinite

IIT Madras
BSc Degree

Question-16 solution

Graphical test

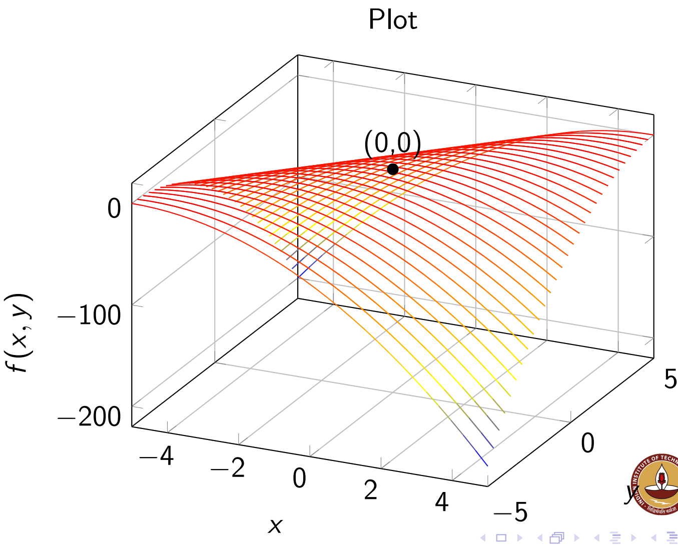
- $f(x, y) \leq 0$ for all

$$v = \begin{bmatrix} x \\ y \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- A is negative semidefinite

Determinant test

- For the function, $a = -2, b = 2, c = -2$
- $a < 0, ac - b^2 = 0$
- A is negative semidefinite



Question-17

$$A = \begin{bmatrix} 2 & 2 \\ 2 & -2 \end{bmatrix}$$

The above matrix is

- A. Positive definite
- B. Positive semidefinite
- C. Negative definite
- D. Negative semidefinite
- E. Indefinite

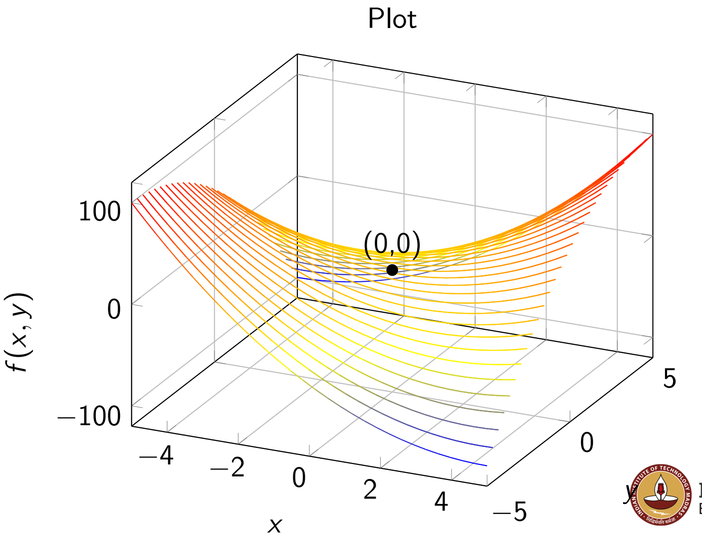
Question-17 solution

Graphical test

- Quadratic form, $f(x, y) = 2x^2 + 4xy - 2y^2$
- $f(x, y) > 0$ at $(1, 0)$, $f(x, y) < 0$ at $(0, 2)$
- A is indefinite

Determinant test

- $a = 2, b = 2, c = -2$
- $ac - b^2 < 0$
- A is indefinite



Question-18

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

The above matrix is

- A. Positive definite
- B. Positive semidefinite
- C. Negative definite
- D. Negative semidefinite
- E. Indefinite

IIT Madras
BSc Degree

Question-18 solution

Eigenvalue test

- Quadratic form,

$$f(x, y, z) = \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- characteristic equation, $(A - \lambda)I = 0$
- $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3$
- A is positive definite

IIT Madras
BSc Degree

Question-18 solution

Determinant test

- $det(A_1) = 1 > 0$
- $det(A_2) = det(\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}) = 2 > 0$
- $det(A_3) = det(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}) = 6 > 0$
- Since, $A_1 > 0, A_2 > 0, A_3 > 0$
- A is positive definite

IIT Madras
BSc Degree

Question-19

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

The above matrix is

- A. Positive definite
- B. Positive semidefinite
- C. Negative definite
- D. Negative semidefinite
- E. Indefinite

IIT Madras
BSc Degree

Question-19 solution

Eigenvalue test

- Quadratic form,

$$f(x,y,z) = \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- characteristic equation, $(A - \lambda)I = 0$
- $\lambda_1 = 1, \lambda_2 = 0, \lambda_3 = 3$
- A is positive semidefinite

IIT Madras
BSc Degree

Question-19 solution

Determinant test

- $det(A_1) = 1 > 0$
- $det(A_2) = det(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}) = 0$
- $det(A_3) = det(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}) = 0$
- Since, $A_1 > 0, A_2 = 0, A_3 = 0$
- A is positive semidefinite

IIT Madras
BSc Degree

Question-20

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

The above matrix is

- A. Positive definite
- B. Positive semidefinite
- C. Negative definite
- D. Negative semidefinite
- E. Indefinite

IIT Madras
BSc Degree

Question-20 solution

Eigenvalue test

- Quadratic form,

$$f(x, y, z) = \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- characteristic equation, $(A - \lambda)I = 0$
- $\lambda_1 = -1, \lambda_2 = -2, \lambda_3 = -3$
- A is negative definite

IIT Madras
BSc Degree

Question-20 solution

Determinant test

- $det(A_1) = -1$
- $det(A_2) = det(\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}) = 2$
- $det(A_3) = det(\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}) = -6$
- Since, $A_1 < 0, A_2 > 0, A_3 < 0$
- A is negative definite

IIT Madras
BSc Degree

Question-21

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The above matrix is

- A. Positive definite
- B. Positive semidefinite
- C. Negative definite
- D. Negative semidefinite
- E. Indefinite

IIT Madras
BSc Degree

Question-21 solution

Eigenvalue test

- Quadratic form,

$$f(x, y, z) = \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- characteristic equation, $(A - \lambda)I = 0$
- $\lambda_1 = -1, \lambda_2 = -2, \lambda_3 = 0$
- A is negative semidefinite

Question-21 solution

Determinant test

- $det(A_1) = -1 < 0$
- $det(A_2) = det(\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}) = 2$
- $det(A_3) = det(\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}) = 0$
- Since, $A_1 < 0, A_2 > 0, A_3 = 0$
- A is negative semidefinite

IIT Madras
BSc Degree

Question-22

Data points: $x_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, x_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, x_3 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, x_4 = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

For the given data points, perform the following activities:

- The mean vector of the data points
- The covariance matrix, $C = \frac{1}{n}\Sigma_{i=1}^{n}(x_i - \bar{x})(x_i - \bar{x})^T$
- Eigenvalues of the matrix C
- Eigenvectors of the matrix C
- Transformed data points for one dimensional PCA
- The reconstruction error for one dimensional PCA

IIT Madras
BSc Degree

Question-22 solution

- Step-1: Data points: $x_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, x_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, x_3 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, x_4 = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$
- Step-2: Mean vector: $\bar{X} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$
- Step-3: Symmetric matrix, C =
$\frac{1}{4}(\begin{bmatrix} -2 \\ -2 \end{bmatrix} \begin{bmatrix} -2 & -2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} \begin{bmatrix} 0 & 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \end{bmatrix}) = \frac{2}{3} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$
- Step-4: Characteristic equation: $(\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix})u_j = 0$
- Step-5: Eigen values: $\lambda_1 = 3, \lambda_2 = 1$
Eigen vectors:
$\lambda_1 = 3, a_1 = 1, a_2 = 1 => u_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \lambda_2 = 1, a_1 = -1, a_2 = 1 => u_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

IIT Madras
BSc Degree

Question-22 solution

- Step 6: Projecting data points

Projecting x_1 on u_1

  - Projection on $u_1 = \alpha_1 = \frac{1}{\sqrt{2}}(\begin{bmatrix}0 & 0\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}) = 0$

Projecting x_2 on u_1

  - Projection on $u_1 = \alpha_1 = \frac{1}{\sqrt{2}}(\begin{bmatrix}2 & 2\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}) = 2\sqrt{2}$

Projecting x_3 on u_1

  - Projection on $u_1 = \alpha_1 = \frac{1}{\sqrt{2}}(\begin{bmatrix}2 & 4\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}) = 3\sqrt{2}$

Projecting x_4 on u_1

  - Projection on $u_1 = \alpha_1 = \frac{1}{\sqrt{2}}(\begin{bmatrix}4 & 2\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}) = 3\sqrt{2}$

IIT Madras
BSc Degree

Question-22 solution

- Step 7: Choose eigen vectors corresponding to top k eigen values (k=1 here)
  Derive the transformed data points:
  $\tilde{x_1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\tilde{x_2} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $\tilde{x_3} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$ $\tilde{x_4} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$
- Step-8:
  Reconstruction Error =
  $J = \frac{1}{n}\Sigma_{i=1}^{n}||x_i - \tilde{x_i}||^2 = \frac{1}{4}[(0^2 + 0^2) + (0^2 + 0^2) + (1^2 + 1^2) + (1^2 + 1^2)] = 1$,
  Projected Variance $= \lambda_1 = 3$

Original	Transformed
$x_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$	$\tilde{x_1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
$x_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$	$\tilde{x_2} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$
$x_3 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$	$\tilde{x_3} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$
$x_4 = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$	$\tilde{x_4} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

Table: Data points

IIT Madras
BSc Degree

Question-23

Data points: $x_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, x_2 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$

For the given data points, perform the following activities:

- The mean vector of the data points
- The covariance matrix, $C = \frac{1}{n}\Sigma_{i=1}^{n}(x_i - \bar{x})(x_i - \bar{x})^T$
- Eigenvalues of the matrix C
- Eigenvectors of the matrix C
- Transformed data points for one dimensional PCA
- The reconstruction error for one dimensional PCA

IIT Madras
BSc Degree

- Step-1: Data points: $x_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, x_2 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$
- Step-2: Mean vector: $\bar{X} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$
- Step-3: Symmetric matrix, C =

$$\frac{1}{3}\left(\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} \right) = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

IIT Madras
BSc Degree

Question-23 solution

- Step-4: Characteristic equation: $(\frac{1}{3}\begin{bmatrix} 2-\lambda & -1 & -1 \\ -1 & 2-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{bmatrix})u_j = 0$
- Step-5: Eigenvalues: $\lambda_1 = 3, \lambda_2 = 3, \lambda_3 = 0$
  Eigenvectors:

$$\lambda_1 = 3, a_1 = -1, a_2 = 1, a_3 = 0 => u_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix},$$

$$\lambda_2 = 3, a_1 = -1, a_2 = 0, a_3 = 1 => u_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda_3 = 0, a_1 = 1, a_2 = 1, a_3 = 1 => u_3 = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

IIT Madras
BSc Degree

Question-23 solution

- Step 6: Projecting data points

Projecting x_1 on u_1

- Projection on u_1 =

$$\alpha_1 = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1 & 2 & 3\end{bmatrix}\begin{bmatrix}-1\\1\\0\end{bmatrix}\right) = \frac{1}{\sqrt{2}}$$

Projecting x_2 on u_1

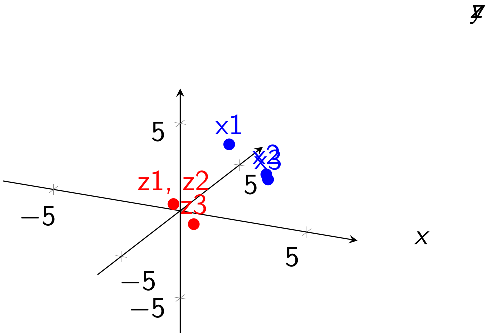
- Projection on u_1 =

$$\alpha_1 = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}2 & 3 & 1\end{bmatrix}\begin{bmatrix}-1\\1\\0\end{bmatrix}\right) = \frac{1}{\sqrt{2}}$$

Projecting x_3 on u_1

- Projection on u_1 =

$$\alpha_1 = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}3 & 1 & 2\end{bmatrix}\begin{bmatrix}-1\\1\\0\end{bmatrix}\right) = \frac{2}{\sqrt{2}}$$



IIT Madras BSc Degree

Question-23 solution

- Step 7: Choose eigenvectors corresponding to top k eigen values (k=1 here)
  Derive the transformed data points:
  $$\tilde{x_1} = \begin{bmatrix} -0.5 \\ 0.5 \\ 0 \end{bmatrix}, \tilde{x_2} = \begin{bmatrix} -0.5 \\ 0.5 \\ 0 \end{bmatrix}, \tilde{x_3} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$
- Step-8:
  Reconstruction Error $= J =$
  $\frac{1}{n}\sum_{i=1}^{n}||x_i - \tilde{x_i}||^2 = \frac{1}{3}[\frac{54}{4} + \frac{54}{4} + 12] = 13,$
  Projected Variance $= \lambda_1 = 3$

Original	Transformed
$x_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$	$\tilde{x_1} = \begin{bmatrix} -0.5 \\ 0.5 \\ 0 \end{bmatrix}$
$x_2 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$	$\tilde{x_2} = \begin{bmatrix} -0.5 \\ 0.5 \\ 0 \end{bmatrix}$
$x_3 = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$	$\tilde{x_3} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

Table: Data points

IIT Madras
BSc Degree

Thank you