

# Density Estimation

E.g.: Assuming tweets from an account are independently generated randomly. Create a robot account that generates more such tweets.

$f(\text{Tweet}) = \text{Score of the tweet}$

wisdomofchopra.com

It has been said by some that the thoughts and tweets of Mr. Chopra are indistinguishable from a set of profound sounding words put together in a random order, particularly the tweets tagged with "#cosmisconciusness". This site aims to test that claim! Each "quote" is generated from a list of words that can be found in Deepak Chopra's Twitter stream randomly stuck together in a sentence.



"A formless void transforms total mysteries"

RECEIVE MORE WISDOM...



Tweet the wisdom

Disclaimer: This is intended for entertainment purposes only. It in no way reflects the thoughts of any real person.

# Density Estimation



"A formless void transforms total mysteries"

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To generate such sentences randomly, we need to be able to assign a probability score to every possible 128 character sentence, giving high scores to those that are likely to be from the original source.

A density estimation model takes in several samples from a random source, and outputs a model that assigns a probability score to every possible instance.

# Density Estimation

- Data:  $\{\mathbf{x}^1, \mathbf{x}^2, \dots, \mathbf{x}^n\}$

- $\mathbf{x}^i \in \mathbb{R}^d$

- Probability mapping  $P : \mathbb{R}^d \rightarrow \mathbb{R}_+$  that 'sums' to one.

- Goal :  $P(\mathbf{x})$  is large if  $\mathbf{x} \in \text{Data}$ , and low otherwise.

- Loss =  $\frac{1}{n} \sum_{i=1}^n -\log(P(\mathbf{x}^i))$

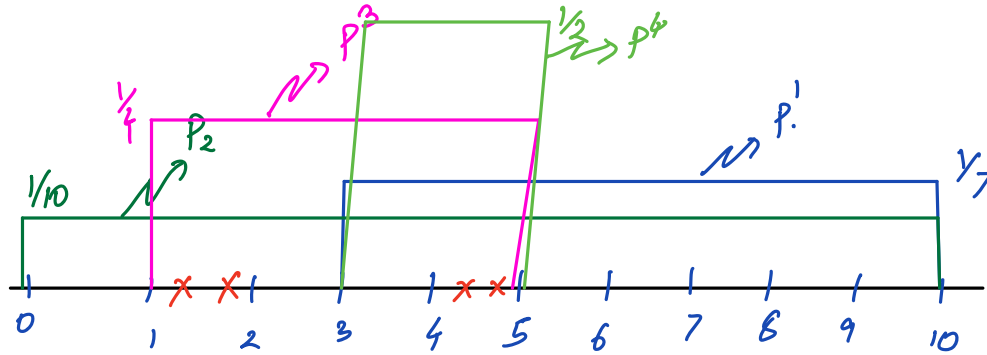
$\sum$   
Sum over  
all tweets i.e.  
{a...z}

$P(\text{tweet}) = 1$

$P(\mathbf{x}^i)$  is large.

$P(\text{anything}) = 10^{-10}$

# Density Estimation Illustration 1



$x^1: [1.2]$   
 $x^2: [1.9]$   
 $x^3: [4.3]$   
 $x^4: [4.8]$

$P^1$  : Uniform in  $[3, 10]$   $0, 0, \frac{1}{7}, \frac{1}{7}$   
 $P^2$  : Uniform in  $[0, 10]$   $\frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}$   
 $P^3$  : Uniform in  $[1, 5]$   $\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}$   
 $P^4$  : Uniform in  $[3, 5]$   $0, 0, \frac{1}{2}, \frac{1}{2}$

$$\text{loss}[P^4] : \text{loss}[P^1] : \infty > \text{loss}[P^2] > \text{loss}[P^3]$$

# Density Estimation Illustration 2

$$d=2$$

$$(1.1, 1.3)$$

$$(0.9, 0.7)$$

$$(2.1, -1)$$

$$(5.1, 0.1)$$

$$(2.2, -0.9)$$

$$(5.1, 0.0)$$

$$(0.9, 1.2)$$

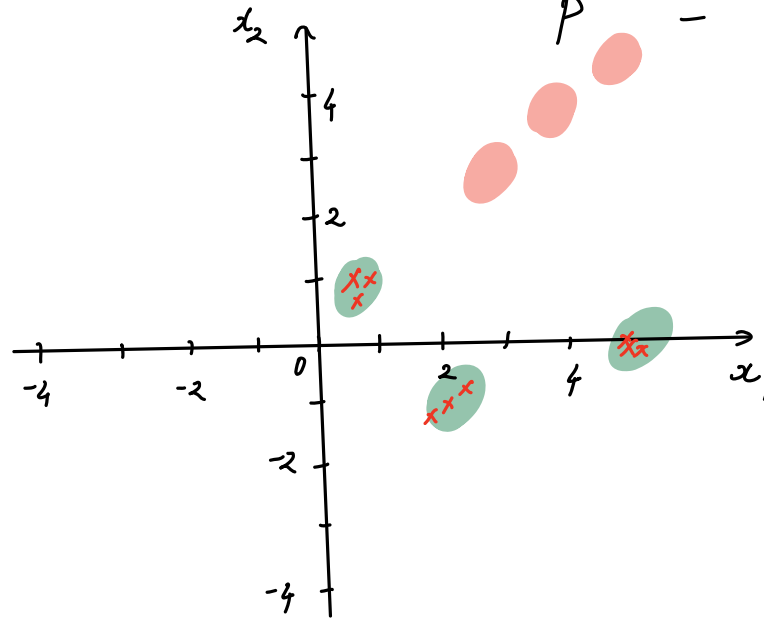
$$(1.9, -1.1)$$

$$(4.8, -0.1)$$

Gaussian Mixture Model

$$P^1 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$



Clustering