

Course: Machine Learning - Foundations  
Week 10 Questions

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PRACTICE QUESTIONS

1. (1 point) Let  $X$  and  $Y$  be continuous random variables with joint density

$$f_{XY}(x, y) \begin{cases} cxy & \text{for } 0 < x < 2, 1 < y < 3 \\ 0 & \text{otherwise} \end{cases}$$

From the above information answer these questions

2. ( points) Calculate the value of  $c$

Answer:  $\frac{1}{8}$

$$\Rightarrow \int_0^2 \int_1^3 cxy \, dx \, dy = 1$$

$$\Rightarrow c \left[ \frac{x^2}{2} \right]_0^2 \cdot \left[ \frac{y^2}{2} \right]_1^3 = 1$$

$$\Rightarrow \frac{c}{4} \cdot [4 - 0] \cdot [9 - 1] = 1$$

$$\Rightarrow \frac{c}{4} \times 4 \times 8 = 1$$

$$\Rightarrow c = \frac{1}{8} \quad \underline{\text{Ans}}$$

3. ( points) Calculate  $P(0 < X < 1, 1 < Y < 2)$

Answer:  $\frac{3}{32}$

$$= \int_0^1 \int_1^2 \frac{xy}{8} dx dy$$

$$= \frac{1}{8} \left[ \frac{x^2}{2} \right]_0^1 \left[ \frac{y^2}{2} \right]_1^2$$

$$= \frac{1}{32} [1-0] [4-1]$$

$$= \frac{3}{32} \text{ Ans}$$

4. ( points) Calculate  $P((X+Y) < 3)$ 

$$= \int_{x=0}^2 \int_{y=1}^{3-x} f(x,y) dx dy \text{ or}$$

$$\int_{y=1}^3 \int_{x=0}^2 f_{xy}(x,y) dx dy$$

$$\Rightarrow \frac{1}{8} \int_0^2 x dx \left[ \frac{y^2}{2} \right]_1^{3-x}$$

$$\Rightarrow \frac{1}{8} \int_0^2 x dx [(3-x)^2 - 1]$$

$$\Rightarrow \frac{1}{16} \int_0^2 [(9-6x+x^2-1)x] dx$$

$$\Rightarrow \frac{1}{16} \left[ \frac{x^4}{4} - 6 \frac{x^3}{3} + 8 \frac{x^2}{2} \right]_0^2$$

$$\Rightarrow \frac{1}{16} [4 - 16 + 16]$$

$$\Rightarrow \frac{1}{4} = 0.25$$



6. ( points) Calculate  $F_Y(2)$ **Answer:** 0.375

$$F_Y(y) = \int_{-\infty}^y \int_{-\infty}^x f_{X,Y}(x,y) dx dy.$$

$$= \frac{1}{8} \left[ \frac{x^2}{2} \right]_0^2 \left[ \frac{y^2}{2} \right]_1^y.$$

$$= \frac{1}{32} \times 4 \times (y^2 - 1) = \frac{y^2 - 1}{8}$$

$$F_Y(y) = \begin{matrix} 0 & y < 1 \\ \frac{y^2 - 1}{8} & 1 < y < 3 \\ 1 & y \geq 3 \end{matrix}$$

$$\left( \frac{2^2 - 1}{8} \right)$$

$$= \frac{3}{8} = 0.375$$