Course: Machine Learning - Foundations
Week 11 Questions

GRADED QUESTIONS

1. (1 point) The number of hours Messi spends each day practicing in ground is modelled by the continuous random variable X, with p.d.f. $f(x)$ defined by

$$f_X(x) = \begin{cases} a(x-1)(6-x) & \text{for } 1 < x < 6 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability that Messi will practice between 2 and 5 hours in ground on a randomly selected day.

Answer: 0.80

We know that $\int_{-\infty}^{\infty} f(x)dx = 1$

Solving above equation taking required $f(x)$, value of a can be calculated. i.e $a = \dfrac{6}{125}$

Then calculate $P(2 \leq X \leq 5) = \int_2^5 f(x)dx$

2. (1 point) Let X be a continuous random variable with PDF

$$f_X(x) = \begin{cases} ax & \text{for } 0 < x < 2 \\ a(4-x) & \text{for } 2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Calculate $P(1 \leq x \leq 3)$

Answer: 0.75

We know that $\int_{-\infty}^{\infty} f(x)dx = 1$

Solving above equation taking required $f(x)$, value of a can be calculated. i.e $a = \dfrac{1}{4}$

Then calculate $P(1 \leq X \leq 2) = \int_1^2 f(x)dx$

3. (1 point) The probability density function of X is given by

$$f_X(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2-x & \text{for } 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Calculate $E(X)$

Answer: 1

Solution:
We know that $\int_{-\infty}^{\infty} f(x)dx = 1$

$E(X) = \int_0^2 xf(x)dx$

4. (1 point) The distribution of the lengths of a cricket bat is uniform between 80 cm and 100 cm. There is no cricket bat outside this range. The mean and variance of the lengths of the the cricket ball is a and b. Calculate $a + b$

Answer: 127.33

$$V(X) = \frac{(h-l)^2}{12}$$

$$E(X) = \frac{(h+l)}{2}$$

5. (points) Suppose that random variable X is uniformly distributed between 0 and 10. Then find $P(X + \frac{10}{X} \geq 7)$. (Write answer upto two decimal places)

Answer: 0.7

Solve this quadratic equation, $X + \frac{10}{X} \geq 7$

6. (1 point) **(Multiple Select)** Which of the following option is/are correct?
   A. For a standard normal variate, the value of Standard Deviation is 1.
   B. Normal Distribution is also known as Gaussian distribution.
   C. In Normal distribution, the highest value of ordinate occurs at mean.
   D. The shape of the normal curve depends on its standard deviation.

Answer: A, B, C, D

Let X and Y be continuous random variables with joint density

$$f_{XY}(x,y) \begin{cases} cxy & \text{for } 0 < x < 2,\ 1 < y < 3 \\ 0 & \text{otherwise} \end{cases}$$

From the above information answer questions from 7-13

7. (points) Calculate the value of c

Answer: $\frac{1}{8}$

Solution:
We know that $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{XY}(x,y)dydx = 1$

$\int_0^2\int_1^3 cxydydx = 1$
c can be calculated from above equation.

8. (points) Calculate $P(0 < X < 1, 1 < Y < 2)$

Answer: $\frac{3}{32}$

$P(0 < X < 1, 1 < Y < 2) = \int_0^1\int_1^2 cxydydx$

9. (points) Calculate $P(0 < X < 1, Y > 2)$

Answer: $\frac{5}{32}$

$P(0 < X < 1, Y > 2) = \int_0^1\int_2^3 cxydydx$

10. (points) Calculate $P((X + Y) < 3)$
$= \int_0^2\int_1^{3-x} cxydydx$
Answer: 0.25

11. (points) Calculate $F_X(1)$

Answer: 0.25

$F_X(x) = \int_1^3\int_0^x cxydydx$

12. (points) Calculate $F_Y(2)$

Answer: $\frac{3}{8}$

$F_X(x) = \int_0^2\int_1^y cxydydx$

13. (points) Calculate $F_{X,Y}$

Answer: 0.25

$F_{X,Y} = F_X(x) \times F_Y(y)$

14. (1 point) Suppose a random variable X is best described by a uniform probability distribution with range 1 to 5. Find the value of a such that $P(X \leq a) = 0.5$

Answer: 3

Solution: $P(X \leq 3) = 0.5$, From the area of Uniform distribution curve.

15. (1 point) If X is an exponential random variable with rate parameter λ then which of the following statement(s) is(are) correct.

a) $E[X] = \frac{1}{\lambda}$
b) $Var[X] = \frac{1}{\lambda^2}$
c) $P(X > x + k|X > k) = P(X > x)$ for $k, x \geq 0$.
d) $P(X > x + k|X > k) = P(X > k)$ for $k, x \geq 0$.

Answer: A, B, C

Solution :
Options (a) and (b) are correct for Exponential distribution.

$$P(X > x + k|X > k) = \frac{P((X > x + k) \cap (X > k)}{P(X > k)}$$

$$\Rightarrow \frac{P(X > x + k)}{P(X > k)} = \frac{e^{-\lambda \times (x+k)}}{e^{-\lambda k}}$$

$$\Rightarrow e^{-\lambda x}$$

Hence, Option C is also correct and option (d) is incorrect.