Solve with Instructors

Week - 5

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The Length of the complex vector $u = \begin{bmatrix} 1+i \\ 1-i \\ 1 \end{bmatrix}$

$$||x||^2 = x^*x = |x_1|^2 + |x_2|^2 + \cdots + |x_n|^2$$

$$||u|| = \sqrt{u^*u} = \sqrt{|u_1|^2 + |u_2|^2 + \cdots + |u_n|^2}$$

$$||u|| = \sqrt{u^*u} = \sqrt{2+2+1} = 2.23$$

The Length of a complex vector and its conjugate remains same. True or False?

The Length of the complex vector $u = \begin{bmatrix} 1+i \\ 1-i \\ 1 \end{bmatrix}$ scaled by the scalar $\eta = 4+3i$ is?

$$||\eta u|| = \sqrt{(\eta u)^*(\eta u)} = |\eta| \ \ ||u||$$

$$= 5 * 2.23 = 11.15$$

Check whether the two vectors $x = \begin{bmatrix} i \\ 3 \\ 1 \end{bmatrix}$ and $y = \begin{bmatrix} i \\ 0 \\ 1 \end{bmatrix}$ are orthogonal?

Find the unit vector that points in the opposite direction of the vector $x = \begin{bmatrix} 1-i \\ 3+i \\ 2 \\ 1+i \end{bmatrix}$

Identify non-Hermitian matrix from the following list of matrices and justify why it is not Hermitian?

1. $A = \begin{bmatrix} 3 & i & 2+i \\ -i & -2 & -7 \\ 2-i & 7*\cos(\pi) & 1 \end{bmatrix}$

3. $C = \begin{bmatrix} -2 & 1-2i \\ 0 & 3 \end{bmatrix}$

2. $B = \begin{bmatrix} 1 & i \\ -i & -i*i \end{bmatrix}$

4. CC^* and C^*C are Hermitian?

Every diagonal matrix is Hermitian.

True or False

Every real diagonal matrix is Hermitian.

True or False

The statement that the singular values are always greater than or equal to zero is?True or False

Find the SVD for the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$.

1. Find $A^T A$

$$\begin{bmatrix} 10 & 20 \\ 20 & 40 \end{bmatrix}$$

2. Eigenvalues of $A^T A$

$$\lambda^2 - 50\lambda = 0 \qquad \lambda_1 = 50, \lambda_2 = 0 \qquad \therefore, \Sigma = \begin{bmatrix} \sqrt{50} & 0 \\ 0 & 0 \end{bmatrix}$$

3. Eigenvectors of $A^T A$

$$\lambda_1 = 50$$

$$N(A^T A - \lambda_1 I) = 0$$

$$x_1 = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} \qquad x_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

4. Normalize the Eigenvectors of $A^T A$ and construct Q_2^T

$$Q_2 = \begin{bmatrix} 0.447 & -0.894 \\ 0.894 & 0.447 \end{bmatrix}$$

5. To find Q_1, either find Eigenvectors of AA^T or use the relation $y_1 = \frac{1}{\sigma_1} A * x_1$

$y_1 = \frac{1}{\sigma_1} A * x_1$

$$y_1 = \frac{1}{\sqrt{50}} \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} * \begin{bmatrix} 0.447 \\ 0.894 \end{bmatrix}$$

$$y_1 = \begin{bmatrix} 0.3162 \\ 0.9487 \end{bmatrix}$$

$y_2 = \frac{1}{\sigma_2} A * x_2$

However, $\sigma_2 = 0$? What is the geometrical meaning of $\sigma_2 = 0$?

$$y_2 \perp y1 \qquad \because [x, y]^T * [-y * x] = 0$$

$$\therefore y_2 = \begin{bmatrix} -0.9487 \\ 0.3162 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 0.3162 & -0.9487 \\ 0.9487 & 0.3162 \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} 0.447 & -0.894 \\ 0.894 & 0.447 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 0.3162 & -0.9487 \\ 0.9487 & 0.3162 \end{bmatrix} \begin{bmatrix} \sqrt{50} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0.447 & 0.894 \\ -0.894 & 0.447 \end{bmatrix}$$

$$\begin{bmatrix} 0.9994 & 1.9989 \\ 2.9986 & 5.9972 \end{bmatrix}$$

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1. https://www.flaticon.com/free-icon/edit_1160515?term=pen%20and%20paper&related_id=1160515