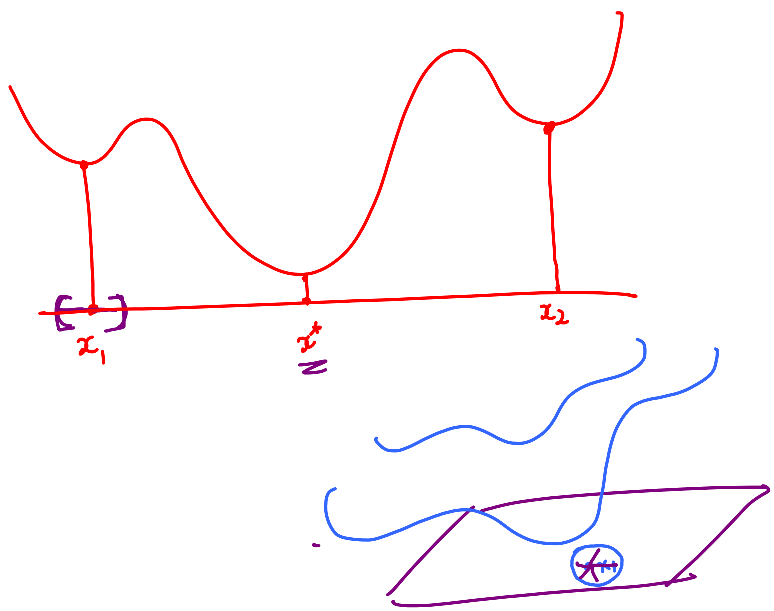
theorem 1 : If f is a convex function, then all local minima of f are also global minima.



x_1 is a local minima ?

$\exists \delta > 0$; $[x_1 - \delta, x_1 + \delta] = B$

$f(x_1) \leq f(z)$
$\forall z \in B$.

In general

$\exists \delta > 0 : \forall z : \|x_1 - z\| \leq \delta$

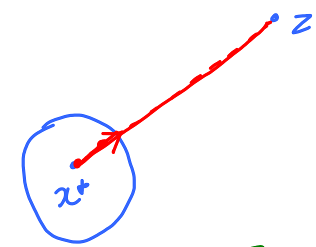
$f(x_1) \leq f(z)$

Proof: Let x^* be the local minimum that is not a global minimum and z be the global minimum.

$$\boxed{f(z) < f(x^*)}$$

By local-min of x^*, $\exists\, \delta > 0$ s.t.

$\forall y : \|x^* - y\| \le \delta \quad f(y) \ge f(x^*)$



$\Rightarrow \exists\, \lambda > 0$ s.t. $f(\lambda x^* + (1-\lambda) z) \ge f(x^*)$

$\boxed{f(x^*)} \le f(\lambda x^* + (1-\lambda) z)$ [local min property of x^*]

$\le \lambda f(x^*) + (1-\lambda) f(z)$ [convexity of f]

$< \lambda f(x^*) + (1-\lambda) f(x^*) = f(x^*)$ $\Rightarrow$ contradiction

$\Rightarrow$ Every local minima is also a global minima.