

Ⓐ ←

$$\begin{array}{ll} \min_x & f(x) \\ \text{st} & g(x) \leq 0 \end{array}$$



Given  $\underline{x^*}$  that is claimed to solve Ⓐ [i.e.  $x^*$  is optimal],  
how to check if it is indeed optimal?

①  $\underline{g(x^*)} \leq 0$

② No "descent direction" should be a "feasible direction".

↑  
Any direction that  
reduces the  
function value.

[Property of  
the function  $f$ ]

↑  
Any direction that  
takes to a point  
that is feasible (for some  
step-size)

↓  
Satisfying  
constraint  $g(x) \leq 0$

[Property of the  
function  $g$ ]

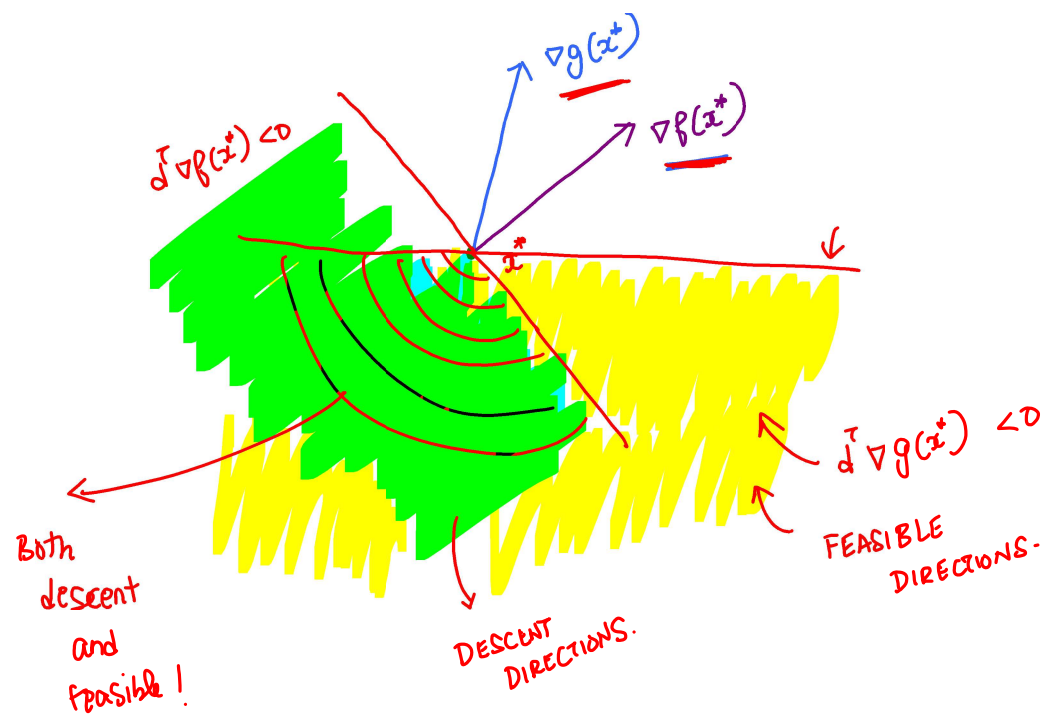
No descent dir is feasible dir

$$g(x^*) \leq 0$$

$$f(x+\eta d) \approx f(x) + \eta d^T \nabla f(x)$$

$\Downarrow$

$$d^T \nabla f(x) < 0 \Rightarrow d \text{ is a descent direction}$$



If  $x^*$  satisfies  $g(x^*) \leq 0$ , then any descent direction for  $f$  is also a "feasible direction" for  $\bar{A}$

$\Rightarrow x^*$  cannot be optimal for the  $\nabla f(x^*), \nabla g(x^*)$  drawn above!