$\nabla f(x^*)$

$\nabla g(x^*)$

x^*

$\nabla f(x^*)^T d = 0$
$\nabla g(x^*)^T d = 0$.
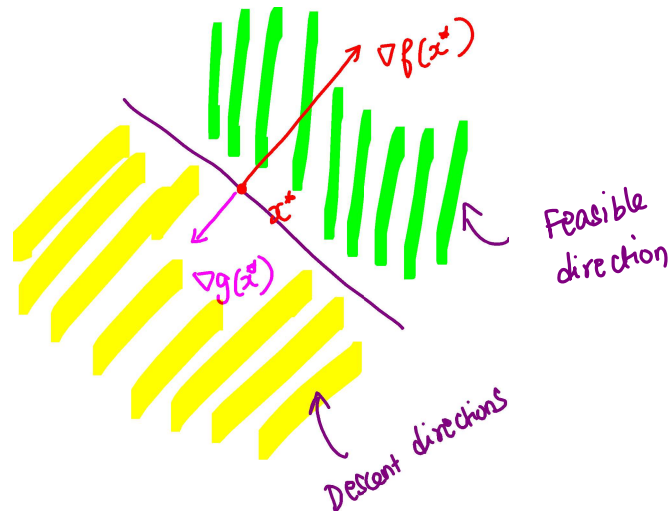
and. $\dfrac{d^T \nabla f(x^*)}{} < 0$
$\dfrac{d^T \nabla g(x^*)}{} < 0$

Every descent direction is
also feasible!

$\Rightarrow$ x^* cannot be optimal for
the configuration above.

$$\min_x \ f(x)$$
$$g(x) \le 0$$

$\nabla f(x^*)$

$\nabla g(x^*)$

Feasible direction

Descent directions
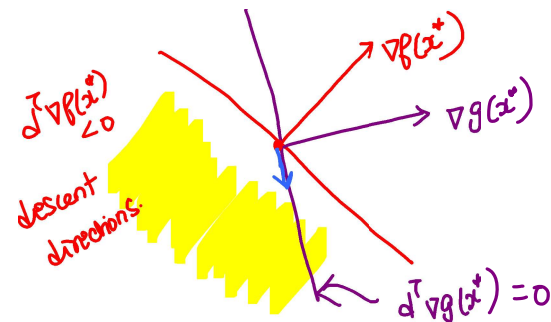
Necessary condition for optimality of x^*.

$$\nabla f(x^*) = -\lambda \, \nabla g(x^*)$$

positive scalar.

$$\min_x \ f(x)$$
$$g(x) = 0$$

$d^T \nabla f(x^*) < 0$

descent directions

$\nabla f(x^*)$

$\nabla g(x^*)$

$d^T \nabla g(x^*) = 0$

$$g(x^*) = 0$$
$$g(x^* + \eta d) \approx g(x^*) + \eta \, d^T \nabla g(x^*)$$
$$0 \approx 0 + \eta \, \underline{d^T \nabla g(x^*)}$$
$$\boxed{d^T \nabla g(x^*) = 0}$$

$\nabla f(x^*)$

$\nabla g(x^*)$

$d^T \nabla g(x^*) = 0$

Additionaly

$\nabla f(x^*)$

$\nabla g(x^*)$

$d^T \nabla g(x^*) = 0$

$d^T \nabla f(x^*) < 0$
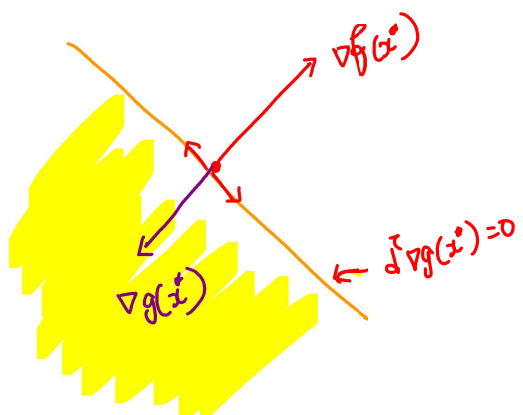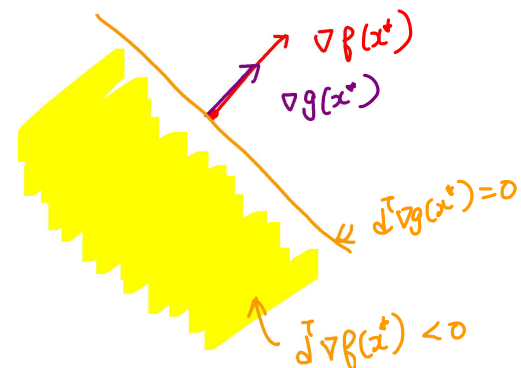
$$\nabla f(x^*) = -\lambda \nabla g(x^*)$$

can be any
arbitrary
scalar.

Lagrange   multiplier.