Markov Inequality

X is a positive RV. $EX = \mu$

MI: $P(X \geq t) \leq \dfrac{\mu}{t}$

$$EX = \int_0^\infty x \cdot f_X(x)\,dx$$

$$= \int_0^t x \cdot f_X(x)\,dx + \int_t^\infty x \cdot f_X(x)\,dx$$

$$\geq 0 + t \int_t^\infty f_X(x)\,dx$$

Markov Inequality

$$f_X(x) = \begin{cases} \frac{4}{5} & \text{if } x = 0\,m \\ \frac{1}{5} & \text{if } x = 50\,m \end{cases}$$

$$EX = 10\,m$$

$$P(X \geq 50) = \frac{1}{5} \quad (\text{computation})$$

$$P(X \geq 50) \leq \frac{10}{50} = \frac{1}{5}$$

Chebyshev Inequality

$EX = \mu \quad ; \quad Var[X] = \sigma^2$

CI :
$$P(|X-\mu| \geq t) \leq \frac{\sigma^2}{t^2}$$

$$P(|X-\mu| \geq t) = P((X-\mu)^2 \geq t^2)$$

$$\leq \frac{E[(X-\mu)^2]}{t^2}$$

$$= \frac{Var[X]}{t^2}$$

Chebyshev Inequality

Hoeffding Inequality

$X_1, X_2, \cdots, X_n$ are i.i.d

$E X_1 = \mu$

$a \le X_1 \le b$

$\bar{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i$; $Var[\bar{X}_n] = \frac{1}{n^2} \cdot n \cdot Var[X_i]$

$= \sigma^2 / n$

$E \bar{X}_n = \mu$

$$P(|\bar{X}_n - \mu| \ge \epsilon) \le \frac{Var[\bar{X}_n]}{\epsilon^2} = \frac{\sigma^2}{n\epsilon^2}$$

Hoeffding Inequality

$$P\left(|\bar{X}_n - \mu| \geq \epsilon\right) \leq \frac{Var[\bar{X}_n]}{\epsilon^2} = \frac{\sigma^2}{n\epsilon^2} \quad (CI)$$

$$= O\left(\frac{1}{n}\right)$$

$$P\left(|\bar{X}_n - \mu| \geq \epsilon\right) \leq 2\exp\left(\frac{-2n\epsilon^2}{(b-a)^2}\right) \quad (HI)$$

$$= O\left(e^{-n}\right)$$

Convergence in Probability

$X_1, X_2, \ldots$

$X_n \xrightarrow{P} X$ if (convergence in Prob)

$$\lim_{n\to\infty} P\left(|X_n - X| \geq \epsilon\right) = 0 \quad \forall \epsilon > 0$$

$X_n \xrightarrow{D} X$ if (convergence in dist)

$$\lim_{n\to\infty} \left| F_{X_n}(x) - F_X(x) \right| = 0 \quad \forall x$$

Law of Large Numbers

$X_1, X_2, \ldots$ drawn i.i.d from D.

$E X_1 = \mu$

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i$$

$$\bar{X}_n \xrightarrow{P} \mu$$ (WLLN)

$$P(|\bar{X}_n - \mu| \geq \epsilon) \leq \frac{\sigma^2}{n\epsilon^2}$$ (Chebyshev)

Central Limit Theorem

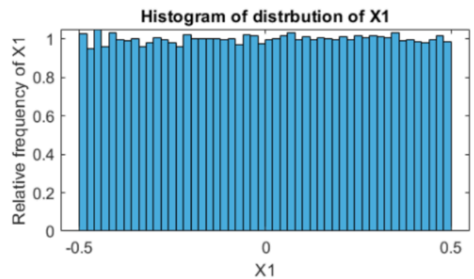
$X_1, X_2, \ldots,$ drawn i.i.d

$EX_1 = \mu, \quad var[X_1] = \sigma^2$

$$Y_n = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} (X_i - \mu)$$

$$Y_n \xrightarrow{D} N(0, \sigma^2)$$

Central Limit Theorem



X_i

$\mu = 0 \;, \sigma^2$