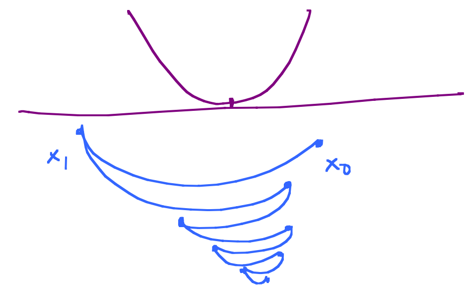
$$x_{t+1} = x_t - \eta_t \underbrace{f'(x_t)}_{\text{direction}}$$

η_t → STEP SIZE (Scalar quantity positive)

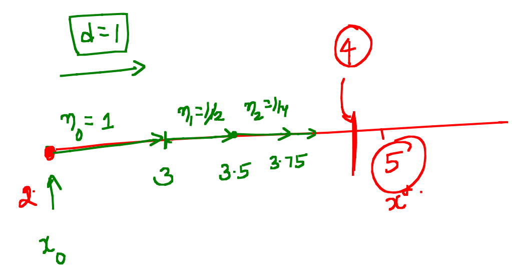


How to choose step size (as a function of t) ?

× First attempt : $\boxed{\eta_0 = 1 \;,\; \eta_1 = 1/2 \;,\; \eta_2 = 1/4 \;,\; \eta_3 = 1/8 \;,\; \ldots}$

$$\boxed{\eta_t = \frac{1}{2^t}}$$

No idea how close x_0 is to x^* ← optimal value.



$\eta_0 \quad \eta_1$

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \cdots$

$\boxed{\eta_t = \frac{1}{t+1}}$ ← good step size sequence. ✓

$$\sum_{t=0}^{\infty} \eta_t = \sum_{t=0}^{\infty} \frac{1}{t+1}$$

$$= \underline{1 + 1/2 + 1/3 + \cdots} = \infty$$

$\boxed{\eta_t = \frac{1}{2^t}}$ ← BAD!

$1, 1/2, 1/4, \cdots$

$$\sum_{t=0}^{\infty} \eta_t = \sum_{t=0}^{\infty} \frac{1}{2^t} = 2$$

$$\underline{1 + 1/2 + 1/4 + 1/8} + \cdots = 2$$

$\boxed{\min_{x \in \mathbb{R}} f(x)}$ ←

$\boxed{\min_{x \in \mathbb{R}^d} f(x)}$

ALGORITHM - GRADIENT DESCENT ALGORITHM.

Initialize at $x_0 \in \mathbb{R}$

for $t = 1, 2, \cdots$

$$x_{t+1} = x_t - \eta_t f'(x_t) \quad \text{where} \quad \boxed{\eta_t = \frac{1}{t+1}}$$

end.