Machine Learning - Foundations

Tutorial - Week 2

IIT Madras Online Degree

Outline

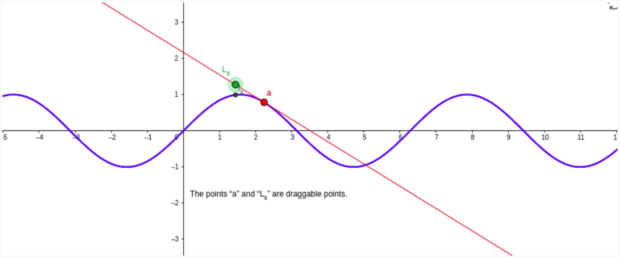
1. Linear approximation
2. Higher order approximations
3. Multivariate linear approximation
4. Directional derivatives

Linear approximation

Linear approximation (Linearization)

Def:

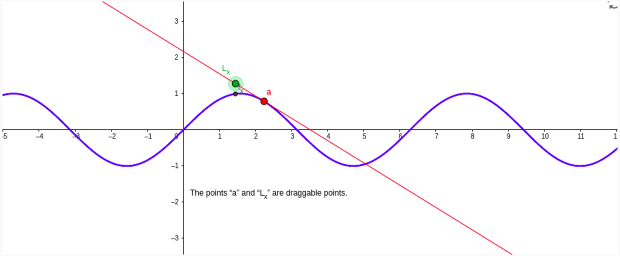
Approximation of any function using a linear function .



Linear approximation (Linearization)

Def:

Approximation of any function using a linear function .



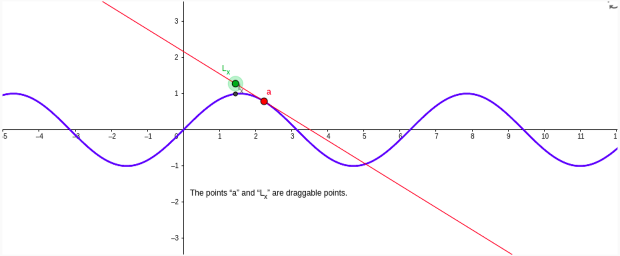
Need:

- Linear functions are easier to work with.

Linear approximation (Linearization)

Def:

Approximation of any function using a linear function .



Need:

- Linear functions are easier to work with.
- Finding approximate values of functions at certain points when exact values are not known.

The equation

The linear approximation $L(x)$ of a function $f(x)$ at point a is given by:

$$\boxed{L(x) = f(a) + f^{'}(a)(x - a)}$$

The equation

The linear approximation $L(x)$ of a function $f(x)$ at point a is given by:

$$\boxed{L(x) = f(a) + f'(a)(x - a)}$$

This is indeed the equation of a tangent line:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y &= y_1 + m(x - x_1) \end{aligned}$$

The equation

The linear approximation $L(x)$ of a function $f(x)$ at point a is given by:

$$\boxed{L(x) = f(a) + f^{'}(a)(x - a)}$$

This is indeed the equation of a tangent line:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y &= y_1 + m(x - x_1) \end{aligned}$$

If $x_1 = a$, $y_1 = f(a)$ and $m = f'(a)$, we get,

$$y = f(a) + f'(a)(x - a)$$

Problem 1

Compute the approximate value of $\sqrt{50}$.

Problem 1

Compute the approximate value of $\sqrt{50}$.

The closest known value to $\sqrt{50}$ is $\sqrt{49}$, so we set $f(x) = \sqrt{x}$ and $a = 49$.

Problem 1

Compute the approximate value of $\sqrt{50}$.

The closest known value to $\sqrt{50}$ is $\sqrt{49}$, so we set $f(x) = \sqrt{x}$ and $a = 49$.

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$$

Problem 1

Compute the approximate value of $\sqrt{50}$.

The closest known value to $\sqrt{50}$ is $\sqrt{49}$, so we set $f(x) = \sqrt{x}$ and $a = 49$.

$$\begin{aligned} f'(x) &= \frac{1}{2}x^{-\frac{1}{2}} \\ f(49) &= \sqrt{49} = 7 \end{aligned}$$

Problem 1

Compute the approximate value of $\sqrt{50}$.

The closest known value to $\sqrt{50}$ is $\sqrt{49}$, so we set $f(x) = \sqrt{x}$ and $a = 49$.

$$\begin{aligned} f'(x) &= \frac{1}{2}x^{-\frac{1}{2}} \\ f(49) &= \sqrt{49} = 7 \\ f'(49) &= \frac{1}{(2)(\sqrt{49})} = \frac{1}{14} \end{aligned}$$

Problem 1

Compute the approximate value of $\sqrt{50}$.

The closest known value to $\sqrt{50}$ is $\sqrt{49}$, so we set $f(x) = \sqrt{x}$ and $a = 49$.

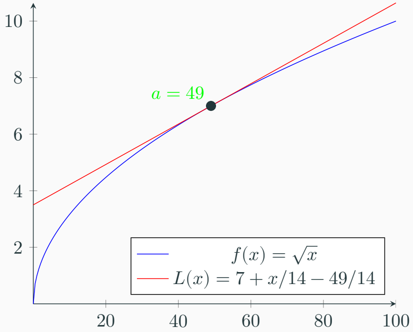
$$\begin{aligned} f'(x) &= \frac{1}{2}x^{-\frac{1}{2}} \\ f(49) &= \sqrt{49} = 7 \\ f'(49) &= \frac{1}{(2)(\sqrt{49})} = \frac{1}{14} \\ L(x) &= f(49) + f'(49)(x - 49) \end{aligned}$$

Problem 1

Compute the approximate value of $\sqrt{50}$.

The closest known value to $\sqrt{50}$ is $\sqrt{49}$, so we set $f(x) = \sqrt{x}$ and $a = 49$.

$$\begin{aligned}
f'(x) &= \frac{1}{2}x^{-\frac{1}{2}} \\
f(49) &= \sqrt{49} = 7 \\
f'(49) &= \frac{1}{(2)(\sqrt{49})} = \frac{1}{14} \\
L(x) &= f(49) + f'(49)(x-49) \\
L(x) &= 7 + \frac{1}{14}(x-49)
\end{aligned}$$

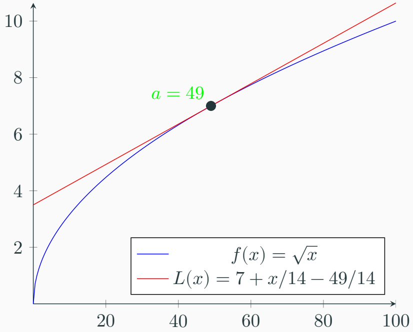


Problem 1

Compute the approximate value of $\sqrt{50}$.

The closest known value to $\sqrt{50}$ is $\sqrt{49}$, so we set $f(x) = \sqrt{x}$ and $a = 49$.

$$\begin{aligned}
f'(x) &= \frac{1}{2}x^{-\frac{1}{2}} \\
f(49) &= \sqrt{49} = 7 \\
f'(49) &= \frac{1}{(2)(\sqrt{49})} = \frac{1}{14} \\
L(x) &= f(49) + f'(49)(x - 49) \\
L(x) &= 7 + \frac{1}{14}(x - 49)
\end{aligned}$$



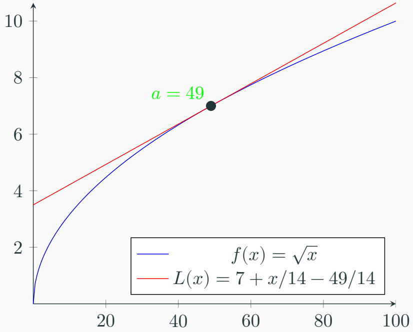
Approximate value of $\sqrt{50} = L(50) = 7 + \frac{1}{14}(50 - 49) = 7 + \frac{1}{14} =$ **7.071**

Problem 1

Compute the approximate value of $\sqrt{50}$.

The closest known value to $\sqrt{50}$ is $\sqrt{49}$, so we set $f(x) = \sqrt{x}$ and $a = 49$.

$$\begin{aligned} f'(x) &= \frac{1}{2}x^{-\frac{1}{2}} \\ f(49) &= \sqrt{49} = 7 \\ f'(49) &= \frac{1}{(2)(\sqrt{49})} = \frac{1}{14} \\ L(x) &= f(49) + f'(49)(x-49) \\ L(x) &= 7 + \frac{1}{14}(x-49) \end{aligned}$$



Approximate value of $\sqrt{50} = L(50) = 7 + \frac{1}{14}(50-49) = 7 + \frac{1}{14} =$ **7.071**

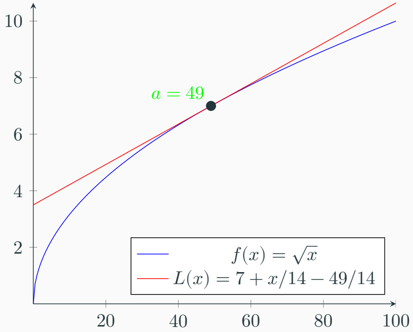
- **Note 1:** Actual value of $\sqrt{50}$ (up to 3 decimal places) is 7.071.

Problem 1

Compute the approximate value of $\sqrt{50}$.

The closest known value to $\sqrt{50}$ is $\sqrt{49}$, so we set $f(x) = \sqrt{x}$ and $a = 49$.

$$
\begin{aligned}
f'(x) &= \frac{1}{2}x^{-\frac{1}{2}} \\
f(49) &= \sqrt{49} = 7 \\
f'(49) &= \frac{1}{(2)(\sqrt{49})} = \frac{1}{14} \\
L(x) &= f(49) + f'(49)(x - 49) \\
L(x) &= 7 + \frac{1}{14}(x - 49)
\end{aligned}
$$



Approximate value of $\sqrt{50} = L(50) = 7 + \frac{1}{14}(50 - 49) = 7 + \frac{1}{14} =$ **7.071**

- **Note 1:** Actual value of $\sqrt{50}$ (up to 3 decimal places) is 7.071.
- **Note 2:** $L(100)$ gives 10.64 while the actual value of $\sqrt{100}$ is 10.

Problem 2

Compute the approximate value of $e^{0.017}$.

Problem 2

Compute the approximate value of $e^{0.017}$.

The closest known value to $e^{0.017}$ is e^0, so we set $f(x) = e^x$ and $a = 0$.

$$f'(x) = e^x$$

Problem 2

Compute the approximate value of $e^{0.017}$.

The closest known value to $e^{0.017}$ is e^0, so we set $f(x) = e^x$ and $a = 0$.

$$\begin{aligned} f'(x) &= e^x \\ f(0) &= 1 \end{aligned}$$

Problem 2

Compute the approximate value of $e^{0.017}$.

The closest known value to $e^{0.017}$ is e^0, so we set $f(x) = e^x$ and $a = 0$.

$$\begin{aligned} f'(x) &= e^x \\ f(0) &= 1 \\ f'(0) &= 1 \end{aligned}$$

Problem 2

Compute the approximate value of $e^{0.017}$.

The closest known value to $e^{0.017}$ is e^0, so we set $f(x) = e^x$ and $a = 0$.

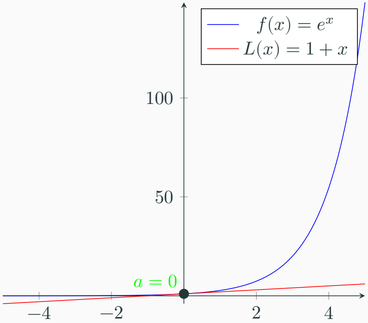
$$\begin{aligned} f'(x) &= e^x \\ f(0) &= 1 \\ f'(0) &= 1 \\ L(x) &= f(0) + f'(0)(x - 0) \end{aligned}$$

Problem 2

Compute the approximate value of $e^{0.017}$.

The closest known value to $e^{0.017}$ is e^0, so we set $f(x) = e^x$ and $a = 0$.

$$\begin{aligned} f'(x) &= e^x \\ f(0) &= 1 \\ f'(0) &= 1 \\ L(x) &= f(0) + f'(0)(x-0) \\ L(x) &= 1 + 1(x) = 1 + x \end{aligned}$$

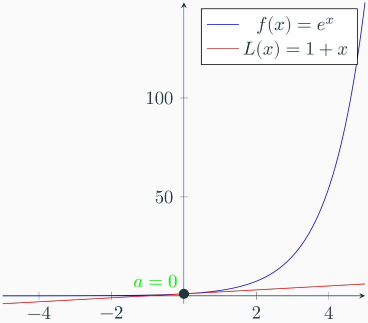


Problem 2

Compute the approximate value of $e^{0.017}$.

The closest known value to $e^{0.017}$ is e^0, so we set $f(x) = e^x$ and $a = 0$.

$$
\begin{aligned}
f'(x) &= e^x \\
f(0) &= 1 \\
f'(0) &= 1 \\
L(x) &= f(0) + f'(0)(x - 0) \\
L(x) &= 1 + 1(x) = 1 + x
\end{aligned}
$$

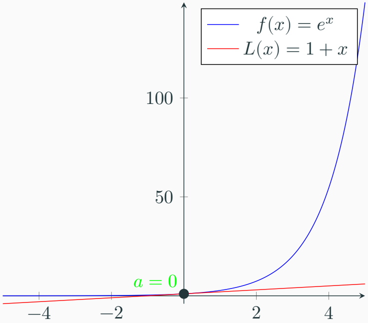


Approximate value of $e^{0.017} = L(0.017) = 1 + 0.017 = 1.017$

Compute the approximate value of $e^{0.017}$.

The closest known value to $e^{0.017}$ is e^{0}, so we set $f(x) = e^x$ and $a = 0$.

$$\begin{aligned} f'(x) &= e^x \\ f(0) &= 1 \\ f'(0) &= 1 \\ L(x) &= f(0) + f'(0)(x - 0) \\ L(x) &= 1 + 1(x) = 1 + x \end{aligned}$$



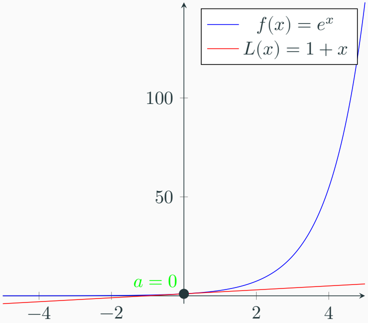
Approximate value of $e^{0.017} = L(0.017) = 1 + 0.017 = 1.017$

- **Note 1:** Actual value of $e^{0.017}$ is also 1.017.

Compute the approximate value of $e^{0.017}$.

The closest known value to $e^{0.017}$ is e^0, so we set $f(x) = e^x$ and $a = 0$.

$$
\begin{aligned}
f'(x) &= e^x \\
f(0) &= 1 \\
f'(0) &= 1 \\
L(x) &= f(0) + f'(0)(x - 0) \\
L(x) &= 1 + 1(x) = 1 + x
\end{aligned}
$$



Approximate value of $e^{0.017} = L(0.017) = 1 + 0.017 = 1.017$

- **Note 1:** Actual value of $e^{0.017}$ is also 1.017.
- **Note 2:** $L(1)$ gives 2 while the actual value of e is 2.718.

Problem 3

Let $f(x) = \sqrt{x+4}$, what is $f(6)$?

Problem 3

Let $f(x) = \sqrt{x+4}$, what is $f(6)$?

The closest known value to $\sqrt{x+4}$ is $\sqrt{9}$ with $x = 5$, so we set $f(x) = \sqrt{x+4}$ and $a = 5$.

$$f'(x) = \frac{1}{2}(x+4)^{-\frac{1}{2}}$$

Problem 3

Let $f(x) = \sqrt{x+4}$, what is $f(6)$?

The closest known value to $\sqrt{x+4}$ is $\sqrt{9}$ with $x = 5$, so we set $f(x) = \sqrt{x+4}$ and $a = 5$.

$$\begin{aligned} f'(x) &= \frac{1}{2}(x+4)^{-\frac{1}{2}} \\ f(5) &= \sqrt{5+4} = 3 \end{aligned}$$

Problem 3

Let $f(x) = \sqrt{x+4}$, what is $f(6)$?

The closest known value to $\sqrt{x+4}$ is $\sqrt{9}$ with $x = 5$, so we set $f(x) = \sqrt{x+4}$ and $a = 5$.

$$\begin{aligned} f^{'}(x) &= \frac{1}{2}(x+4)^{-\frac{1}{2}} \\ f(5) &= \sqrt{5+4} = 3 \\ f^{'}(5) &= \frac{1}{(2)(\sqrt{9})} = \frac{1}{6} \end{aligned}$$

Problem 3

Let $f(x) = \sqrt{x+4}$, what is $f(6)$?

The closest known value to $\sqrt{x+4}$ is $\sqrt{9}$ with $x = 5$, so we set $f(x) = \sqrt{x+4}$ and $a = 5$.

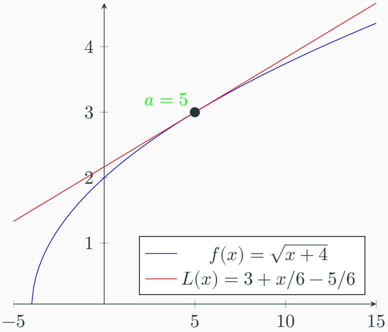
$$
\begin{aligned}
f'(x) &= \frac{1}{2}(x+4)^{-\frac{1}{2}} \\
f(5) &= \sqrt{5+4} = 3 \\
f'(5) &= \frac{1}{(2)(\sqrt{9})} = \frac{1}{6} \\
L(x) &= f(5) + f'(5)(x-5)
\end{aligned}
$$

Problem 3

Let $f(x) = \sqrt{x+4}$, what is $f(6)$?

The closest known value to $\sqrt{x+4}$ is $\sqrt{9}$ with $x = 5$, so we set $f(x) = \sqrt{x+4}$ and $a = 5$.

$$\begin{aligned}
f'(x) &= \frac{1}{2}(x+4)^{-\frac{1}{2}} \\
f(5) &= \sqrt{5+4} = 3 \\
f'(5) &= \frac{1}{(2)(\sqrt{9})} = \frac{1}{6} \\
L(x) &= f(5) + f'(5)(x-5) \\
L(x) &= 3 + \frac{1}{6}(x-5) = 3 + \frac{x-5}{6}
\end{aligned}$$

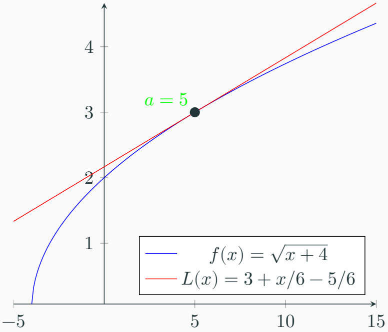


Problem 3

Let $f(x)=\sqrt{x+4}$, what is $f(6)$?

The closest known value to $\sqrt{x+4}$ is $\sqrt{9}$ with $x=5$, so we set $f(x)=\sqrt{x+4}$ and $a=5$.

$$\begin{aligned} f'(x) &= \frac{1}{2}(x+4)^{-\frac{1}{2}} \\ f(5) &= \sqrt{5+4}=3 \\ f'(5) &= \frac{1}{(2)(\sqrt{9})}=\frac{1}{6} \\ L(x) &= f(5)+f'(5)(x-5) \\ L(x) &= 3+\frac{1}{6}(x-5)=3+\frac{x-5}{6} \end{aligned}$$



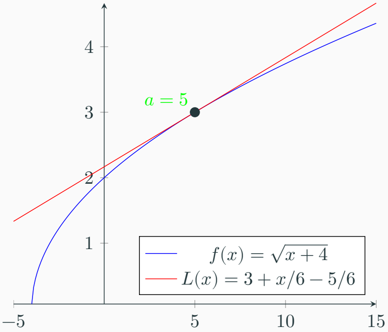
Approximate value of $f(6)=L(6)=3+\frac{6-5}{6}=3+\frac{1}{6}=\frac{19}{6}=3.1666$

Problem 3

Let $f(x) = \sqrt{x+4}$, what is $f(6)$?

The closest known value to $\sqrt{x+4}$ is $\sqrt{9}$ with $x = 5$, so we set $f(x) = \sqrt{x+4}$ and $a = 5$.

$$\begin{aligned}
f'(x) &= \frac{1}{2}(x+4)^{-\frac{1}{2}} \\
f(5) &= \sqrt{5+4} = 3 \\
f'(5) &= \frac{1}{(2)(\sqrt{9})} = \frac{1}{6} \\
L(x) &= f(5) + f'(5)(x-5) \\
L(x) &= 3 + \frac{1}{6}(x-5) = 3 + \frac{x-5}{6}
\end{aligned}$$



Approximate value of $f(6) = L(6) = 3 + \frac{6-5}{6} = 3 + \frac{1}{6} = \frac{19}{6} = 3.1666$

- **Note:** Actual value of $f(6) = \sqrt{10}$ is 3.1622. **Why?**

Higher order approximations

Higher order approximations

Linear Approximation

$$\boxed{L(x) = f(a) + f^{'}(a)(x-a)}$$

Higher order approximations

Linear Approximation

$$\boxed{L(x) = f(a) + f^{'}(a)(x-a)}$$

Quadratic Approximation

$$\boxed{L(x) = f(a) + f^{'}(a)(x-a) + \frac{f^{''}(a)}{2}(x-a)^2}$$

Higher order approximations

Linear Approximation

$$\boxed{L(x) = f(a) + f^{'}(a)(x-a)}$$

Quadratic Approximation

$$\boxed{L(x) = f(a) + f^{'}(a)(x-a) + \frac{f^{''}(a)}{2}(x-a)^2}$$

Higher-order Approximations

$$\boxed{\begin{gathered} L(x) = f(a) + f^{(1)}(a)(x-a) + \frac{f^{(2)}(a)}{2}(x-a)^2 + \\ + \frac{f^{(3)}(a)}{3 \cdot 2}(x-a)^3 + \frac{f^{(4)}(a)}{4 \cdot 3 \cdot 2}(x-a)^4 \ldots \end{gathered}}$$

Problem 4

Let $f(x) = \sqrt{x+4}$, what is $f(6)$?

Problem 4

Let $f(x) = \sqrt{x+4}$, what is $f(6)$?

The closest known value to $\sqrt{x+4}$ is $\sqrt{9}$ with $x = 5$, so we set $f(x) = \sqrt{x+4}$ and $a = 5$.

Problem 4

Let $f(x) = \sqrt{x+4}$, what is $f(6)$?

The closest known value to $\sqrt{x+4}$ is $\sqrt{9}$ with $x = 5$, so we set $f(x) = \sqrt{x+4}$ and $a = 5$.

$$\begin{aligned} f'(x) &= \frac{1}{2}(x+4)^{-\frac{1}{2}} \\ f''(x) &= -\frac{1}{4}(x+4)^{-\frac{3}{2}} \\ f'''(x) &= \frac{3}{8}(x+4)^{-\frac{5}{2}} \end{aligned}$$

Problem 4

Let $f(x) = \sqrt{x+4}$, what is $f(6)$?

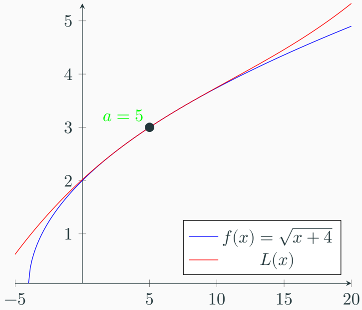
The closest known value to $\sqrt{x+4}$ is $\sqrt{9}$ with $x = 5$, so we set $f(x) = \sqrt{x+4}$ and $a = 5$.

$$\begin{aligned}
f'(x) &= \frac{1}{2}(x+4)^{-\frac{1}{2}} \\
f''(x) &= -\frac{1}{4}(x+4)^{-\frac{3}{2}} \\
f'''(x) &= \frac{3}{8}(x+4)^{-\frac{5}{2}} \\
f(5) &= \sqrt{5+4} = 3 \\
f'(5) &= \frac{1}{(2)(\sqrt{9})} = \frac{1}{6} \\
f''(5) &= -\frac{1}{108} \\
f'''(5) &= \frac{1}{(24)(27)}
\end{aligned}$$

$$
\begin{aligned}
L(x) &= f(5) + f^{'}(5)(x-5) + \frac{f^{''}(5)}{2}(x-5)^2 + \frac{f^{'''}(5)}{(3)(2)}(x-5)^3 + \ldots \\
L(x) &= 3 + \frac{1}{6}(x-5) - \frac{1}{(108)(2)}(x-5)^2 + \frac{1}{(24)(27)(6)}(x-5)^3
\end{aligned}
$$

$$L(x) = f(5) + f^{'}(5)(x-5) + \frac{f^{''}(5)}{2}(x-5)^2 + \frac{f^{'''}(5)}{(3)(2)}(x-5)^3 + \ldots$$

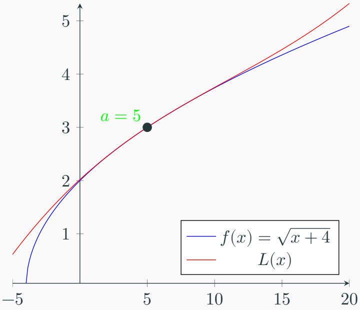
$$L(x) = 3 + \frac{1}{6}(x-5) - \frac{1}{(108)(2)}(x-5)^2 + \frac{1}{(24)(27)(6)}(x-5)^3$$



Approximate value of

$f(6) = L(6) = 3 + \frac{1}{6}(6-5) - \frac{1}{(108)(2)}(6-5)^2 + \frac{1}{(24)(27)(6)}(6-5)^3 = 3.1622$

$$
\begin{aligned}
L(x) &= f(5) + f^{'}(5)(x-5) + \frac{f^{''}(5)}{2}(x-5)^2 + \frac{f^{'''}(5)}{(3)(2)}(x-5)^3 + \ldots \\
L(x) &= 3 + \frac{1}{6}(x-5) - \frac{1}{(108)(2)}(x-5)^2 + \frac{1}{(24)(27)(6)}(x-5)^3
\end{aligned}
$$



Approximate value of

$f(6) = L(6) = 3 + \frac{1}{6}(6-5) - \frac{1}{(108)(2)}(6-5)^2 + \frac{1}{(24)(27)(6)}(6-5)^3 = 3.1622$

Multivariate linear approximation

Linear approximation of functions involving multiple variables

The linear approximation of a function f of two variables x and y in the neighborhood of (a, b) is:

$$\boxed{L(x, y) = f(a, b) + \frac{\partial f}{\partial x}(a, b)(x - a) + \frac{\partial f}{\partial y}(a, b)(y - b)}$$

Problem 5

Find the linearization of $f(x, y) = xe^{xy}$ at $(1, 0)$. Use it to approximate $f(1.1, -0.1)$.

Problem 5

Find the linearization of $f(x, y) = xe^{xy}$ at $(1, 0)$. Use it to approximate $f(1.1, -0.1)$.

$$
\begin{aligned}
\frac{\partial f}{\partial x}(x, y) &= xe^{xy}y + e^{xy} = xye^{xy} + e^{xy} \\
\frac{\partial f}{\partial y}(x, y) &= xe^{xy}x = x^2e^{xy}
\end{aligned}
$$

Problem 5

Find the linearization of $f(x, y) = xe^{xy}$ at $(1, 0)$. Use it to approximate $f(1.1, -0.1)$.

$$\begin{aligned}
\frac{\partial f}{\partial x}(x, y) &= xe^{xy}y + e^{xy} = xye^{xy} + e^{xy} \\
\frac{\partial f}{\partial y}(x, y) &= xe^{xy}x = x^2e^{xy}
\end{aligned}$$

Here $(a, b) = (1, 0)$.

Problem 5

Find the linearization of $f(x, y) = xe^{xy}$ at $(1, 0)$. Use it to approximate $f(1.1, -0.1)$.

$$\begin{aligned}
\frac{\partial f}{\partial x}(x, y) &= xe^{xy}y + e^{xy} = xye^{xy} + e^{xy} \\
\frac{\partial f}{\partial y}(x, y) &= xe^{xy}x = x^2e^{xy}
\end{aligned}$$

Here $(a, b) = (1, 0)$.

$$f(1, 0) = e^0 = 1$$

Problem 5

Find the linearization of $f(x,y) = xe^{xy}$ at $(1,0)$. Use it to approximate $f(1.1,-0.1)$.

$$\begin{aligned}
\frac{\partial f}{\partial x}(x,y) &= xe^{xy}y + e^{xy} = xye^{xy} + e^{xy} \\
\frac{\partial f}{\partial y}(x,y) &= xe^{xy}x = x^2e^{xy}
\end{aligned}$$

Here $(a,b) = (1,0)$.

$$\begin{aligned}
f(1,0) &= e^0 = 1 \\
\frac{\partial f}{\partial x}(a,b) &= \frac{\partial f}{\partial x}(1,0) = e^0 = 1
\end{aligned}$$

Problem 5

Find the linearization of $f(x, y) = xe^{xy}$ at $(1, 0)$. Use it to approximate $f(1.1, -0.1)$.

$$\begin{aligned}
\frac{\partial f}{\partial x}(x, y) &= xe^{xy}y + e^{xy} = xye^{xy} + e^{xy} \\
\frac{\partial f}{\partial y}(x, y) &= xe^{xy}x = x^2e^{xy}
\end{aligned}$$

Here $(a, b) = (1, 0)$.

$$\begin{aligned}
f(1, 0) &= e^0 = 1 \\
\frac{\partial f}{\partial x}(a, b) &= \frac{\partial f}{\partial x}(1, 0) = e^0 = 1 \\
\frac{\partial f}{\partial y}(a, b) &= \frac{\partial f}{\partial y}(1, 0) = e^0 = 1
\end{aligned}$$

$$L(x, y) = f(1,0) + \frac{\partial f}{\partial x}(1,0)(x-1) + \frac{\partial f}{\partial y}(1,0)(y-0)$$

$$
\begin{aligned}
L(x,y) &= f(1,0) + \frac{\partial f}{\partial x}(1,0)(x-1) + \frac{\partial f}{\partial y}(1,0)(y-0) \\
&= 1 + 1(x-1) + 1(y) \\
&= x + y
\end{aligned}
$$

$$
\begin{aligned}
L(x,y) &= f(1,0) + \frac{\partial f}{\partial x}(1,0)(x-1) + \frac{\partial f}{\partial y}(1,0)(y-0) \\
&= 1 + 1(x-1) + 1(y) \\
&= x + y \\
f(1.1,-0.1) &= L(1.1,-0.1) \\
&= 1.1 - 0.1 = 1
\end{aligned}
$$

$$
\begin{aligned}
L(x,y) &= f(1,0) + \frac{\partial f}{\partial x}(1,0)(x-1) + \frac{\partial f}{\partial y}(1,0)(y-0) \\
&= 1 + 1(x-1) + 1(y) \\
&= x + y \\
f(1.1,-0.1) &= L(1.1,-0.1) \\
&= 1.1 - 0.1 = 1
\end{aligned}
$$

The actual value of f(1.1, -0.1) = 1.1e$^{-0.11} = \frac{1.1}{1.11628} = 0.98542$

Directional derivatives

Directional Derivative

- $f_x(x, y) = \frac{\partial f}{\partial x}(x, y)$

Directional Derivative

- $f_x(x, y) = \frac{\partial f}{\partial x}(x, y)$ = Rate of change of f as we vary x (keeping y fixed).

Directional Derivative

- $f_x(x, y) = \frac{\partial f}{\partial x}(x, y)$ = Rate of change of f as we vary x (keeping y fixed).
- $f_y(x, y) = \frac{\partial f}{\partial y}(x, y)$

Directional Derivative

- $f_x(x,y) = \frac{\partial f}{\partial x}(x,y)$ = Rate of change of f as we vary x (keeping y fixed).
- $f_y(x,y) = \frac{\partial f}{\partial y}(x,y)$ = Rate of change of f as we vary y (keeping x fixed).

Directional Derivative

- $f_x(x, y) = \frac{\partial f}{\partial x}(x, y)$ = Rate of change of f as we vary x (keeping y fixed).
- $f_y(x, y) = \frac{\partial f}{\partial y}(x, y)$ = Rate of change of f as we vary y (keeping x fixed).
- Directional derivative of $f(x, y)$

Directional Derivative

- $f_x(x, y) = \frac{\partial f}{\partial x}(x, y)$ = Rate of change of f as we vary x (keeping y fixed).
- $f_y(x, y) = \frac{\partial f}{\partial y}(x, y)$ = Rate of change of f as we vary y (keeping x fixed).
- Directional derivative of $f(x, y)$ = Rate of change of f if we allow both x and y to change simultaneously (in some direction (u)).

Directional Derivative

- $f_x(x, y) = \frac{\partial f}{\partial x}(x, y)$ = Rate of change of f as we vary x (keeping y fixed).
- $f_y(x, y) = \frac{\partial f}{\partial y}(x, y)$ = Rate of change of f as we vary y (keeping x fixed).
- Directional derivative of $f(x, y)$ = Rate of change of f if we allow both x and y to change simultaneously (in some direction (u)).

$$
\begin{aligned}
D_{\vec{u}} f(x, y) &= \nabla f \cdot u \\
&= \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right] \cdot [u_1, u_2] \\
&= u_1 \frac{\partial f}{\partial x} + u_2 \frac{\partial f}{\partial y}
\end{aligned}
$$

Directional Derivative

- $f_x(x, y) = \frac{\partial f}{\partial x}(x, y)$ = Rate of change of f as we vary x (keeping y fixed).
- $f_y(x, y) = \frac{\partial f}{\partial y}(x, y)$ = Rate of change of f as we vary y (keeping x fixed).
- Directional derivative of $f(x, y)$ = Rate of change of f if we allow both x and y to change simultaneously (in some direction (u)).

$$
\begin{aligned}
D_{\vec{u}} f(x, y) &= \nabla f \cdot u \\
&= \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right] \cdot [u_1, u_2] \\
&= u_1 \frac{\partial f}{\partial x} + u_2 \frac{\partial f}{\partial y}
\end{aligned}
$$

Directional derivative can be considered to be a weighted sum of partial derivatives.

Problem 6

Find the derivative of $f(x, y) = xcos(y)$ in the direction of $\vec{u} = [2, 1]$.

Problem 6

Find the derivative of $f(x, y) = xcos(y)$ in the direction of $\vec{u} = [2, 1]$.

$$\frac{\partial f}{\partial x} = \cos(y)$$
$$\frac{\partial f}{\partial y} = -x\sin(y)$$

Problem 6

Find the derivative of $f(x,y) = xcos(y)$ in the direction of $\vec{u} = [2,1]$.

$$\frac{\partial f}{\partial x} = \cos(y)$$
$$\frac{\partial f}{\partial y} = -x\sin(y)$$

Unit vector in the direction of $\vec{u}$ = $\left[\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right]$

Problem 6

Find the derivative of $f(x, y) = xcos(y)$ in the direction of $\vec{u} = [2, 1]$.

$$\frac{\partial f}{\partial x} = \cos(y)$$
$$\frac{\partial f}{\partial y} = -x\sin(y)$$

Unit vector in the direction of $\vec{u}$ = $\left[\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right]$

$$\begin{aligned} D_{\vec{u}}f(x, y) &= u_1\frac{\partial f}{\partial x} + u_2\frac{\partial f}{\partial y} \\ &= \frac{2}{\sqrt{5}}\cos(y) - \frac{1}{\sqrt{5}}x\sin(y) \end{aligned}$$

Problem 7

Find the derivative of $f(x, y) = x^2 - xy$ in the direction of $\vec{u} = 0.6i + 0.8j$ at the point $(2, -3)$.

Problem 7

Find the derivative of $f(x, y) = x^2 - xy$ in the direction of $\vec{u} = 0.6i + 0.8j$ at the point $(2, -3)$.

$$\frac{\partial f}{\partial x} = 2x - y$$
$$\frac{\partial f}{\partial y} = -x$$

Problem 7

Find the derivative of $f(x,y) = x^2 - xy$ in the direction of $\vec{u} = 0.6i + 0.8j$ at the point $(2, -3)$.

$$\frac{\partial f}{\partial x} = 2x - y$$
$$\frac{\partial f}{\partial y} = -x$$

$\vec{u}$ is already a unit vector. $u_1 = 0.6, u_2 = 0.8$.

Problem 7

Find the derivative of $f(x,y) = x^2 - xy$ in the direction of $\vec{u} = 0.6i + 0.8j$ at the point $(2,-3)$.

$$\frac{\partial f}{\partial x} = 2x - y$$
$$\frac{\partial f}{\partial y} = -x$$

$\vec{u}$ is already a unit vector. $u_1 = 0.6, u_2 = 0.8$.

$$\begin{aligned} D_{\vec{u}} f(x,y) &= 0.6(2x - y) + 0.8(-x) \\ &= 0.6(2x - y) - 0.8x \end{aligned}$$

Problem 7

Find the derivative of $f(x,y) = x^2 - xy$ in the direction of $\vec{u} = 0.6i + 0.8j$ at the point $(2,-3)$.

$$\frac{\partial f}{\partial x} = 2x - y$$
$$\frac{\partial f}{\partial y} = -x$$

$\vec{u}$ is already a unit vector. $u_1 = 0.6, u_2 = 0.8$.

$$\begin{aligned} D_{\vec{u}} f(x,y) &= 0.6(2x - y) + 0.8(-x) \\ &= 0.6(2x - y) - 0.8x \end{aligned}$$

$$\begin{aligned} D_{\vec{u}} f(2,-3) &= 0.6(2(2) + 3) - 0.8(2) \\ &= 0.6(7) - 1.6 \\ &= 4.2 - 1.6 \\ &= 2.6 \end{aligned}$$

Thank you.