Outline

- Sets and Functions
  - Notations
  - Logic
  - Graphs and visualisations.
- **Univariate Calculus**
  - Continuity and differentiability
  - **Derivatives and Linear approximations**
  - Applications/Advanced rules
- Multivariate Calculus
  - Lines and planes in high dimensional space.
  - Partial derivatives
  - Gradients
  - Linear approximations and Alternate gradient interpretations
  - Applications/Advanced rules

Derivatives and Linear Approximation

Let $f : \mathbb{R} \to \mathbb{R}$ be a diff function

$$f'(x^*) = \lim_{x \to x^*} \frac{f(x) - f(x^*)}{x - x^*}$$

$$f'(x^*) \approx \frac{f(x) - f(x^*)}{x - x^*} \qquad \text{(around } x = x^*\text{)}$$

$$f(x) \approx f(x^*) + f'(x^*)(x - x^*) \qquad \text{(around } x = x^*\text{)}$$

$$L_{x^*}[f](x)$$

$$f(x) \approx L_{x^*}[f](x) \qquad \text{around } x = x^*$$

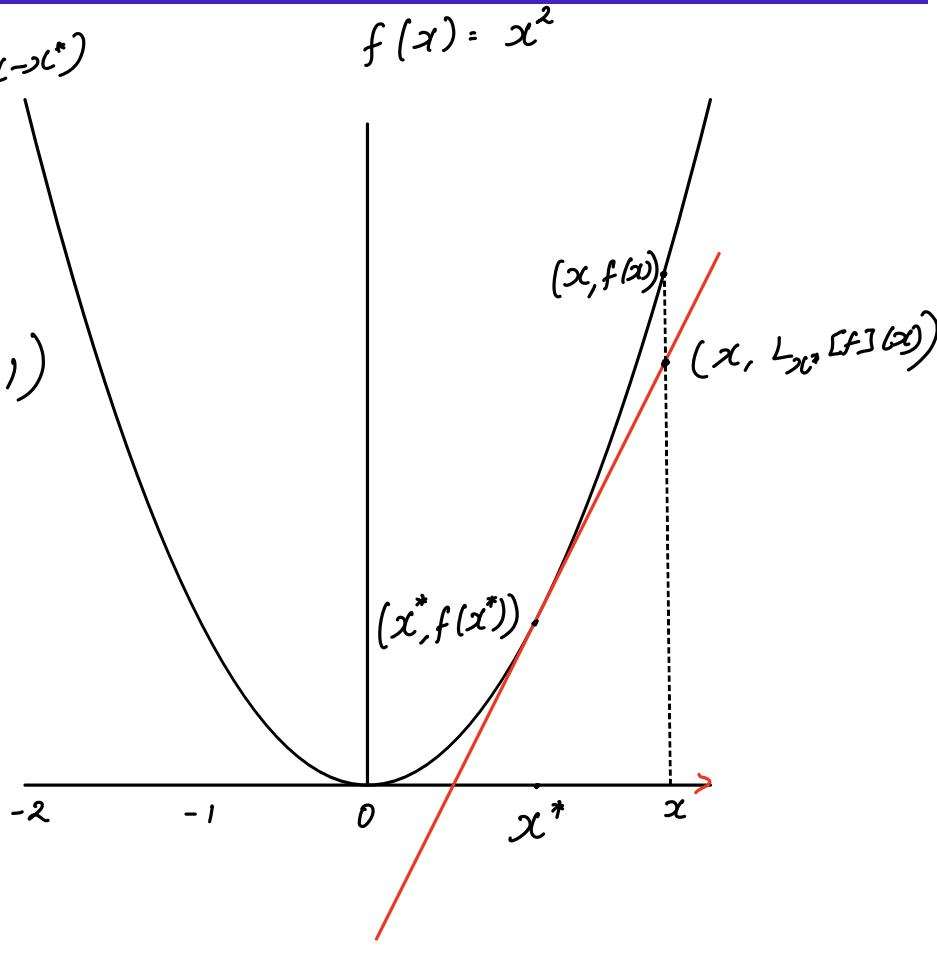
Derivatives and Linear Approximation

$L_{x^*}[f] = f(x^*) + f'(x^*)(x - x^*)$

$= 1^2 + 2(x - 1)$

$= 1 + 2x - 2$

$= 2x - 1$ (around $x = 1$)



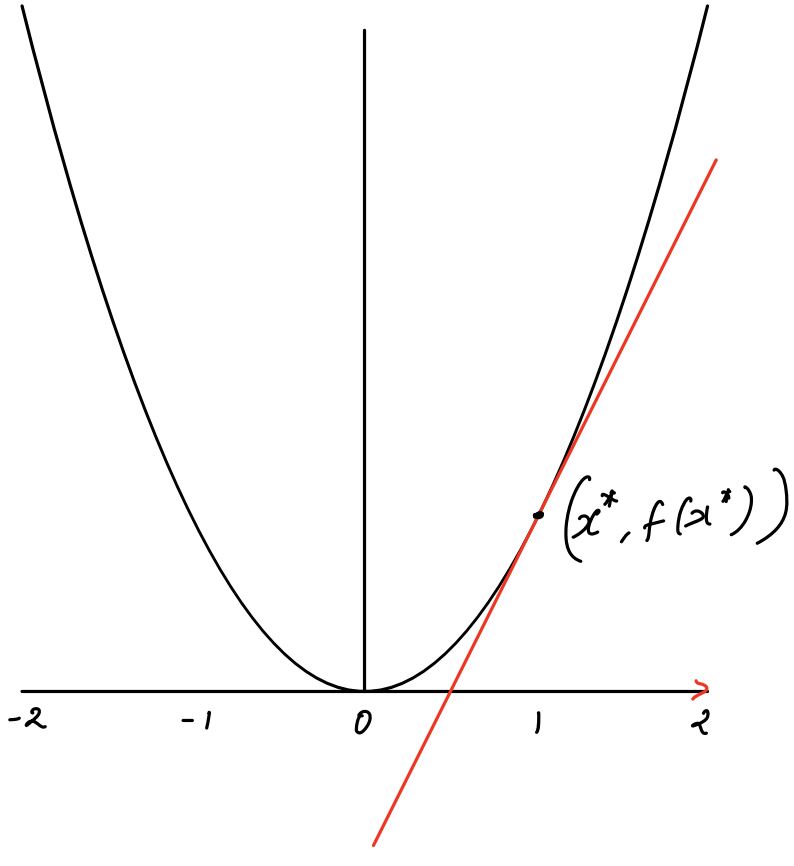
Linear Approximations and Tangent Lines

$G_{L_{x^*}[f]}$ is a

$\cap\!\!|$

$\mathbb{R}^2$

tangent to the graph of f G_f at the point $x^*, f(x^*)$



Derivatives and Linear Approximation

i) Linear approximation of $f(x) = \sin(x)$ around $x^* = 0$

$$f(x) \approx f(x^*) + f'(x^*)(x - x^*)$$

$$= 0 + 1(x - 0)$$

$$= x \quad \text{around } x = 0$$

$$f'(x) = \cos(x)$$

$$f'(x^*) = 1$$

$$f(x^*) = 0$$

$$\sin x \approx x \quad \text{if } x \approx 0$$

(ii) $f(x) = e^x$ around $x^* = 0$

$$e^x \approx e^0 + (x - 0) \cdot 1$$

$$\approx 1 + x \quad \text{around } x = 0$$

Derivatives and Linear Approximation

iii) $\ln(1+x)$ around $x^* = 0$

$\ln(1+x) \approx \ln(1) + 1(x-0)$

$\approx x$ around $x = 0$

$f'(x) = \frac{1}{1+x}$

$f'(x^*) = 1$

$f(x^*) = 0$

iv) $f(x) = (1+x)^r$ around $x^* = 0$

$(1+x)^r \approx 1 + r(x)$

$= 1 + rx$ around $x = 0$.

$f(x^*) = 1$

$f'(x) = r(1+x)^{r-1}$

$f'(x^*) = r$

(v) $(0.99)^7$

(a) 0.95 (b) 0.93

(c) 0.91 (d) 0.9

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