Quicksort

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Programming, Data Structures and Algorithms using Python

Week 3

Shortcomings of merge sort

- Merge needs to create a new list to hold the merged elements
  - No obvious way to efficiently merge two lists in place
  - Extra storage can be costly
- Inherently recursive
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- Merging happens because elements in the left half need to move to the right half and vice versa
  - Consider an input of the form `[0,2,4,6,1,3,5,9]`
- Can we divide the list so that everything on the left is smaller than everything on the right?
  - No need to merge!

Divide and conquer without merging

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  - Sort and pick up the middle element
  - But our aim is to sort the list!
- Instead pick some value in L — pivot
  - Split L with respect to the pivot element

Quicksort [C.A.R. Hoare]

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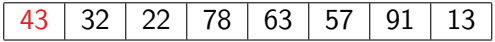

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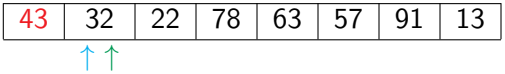

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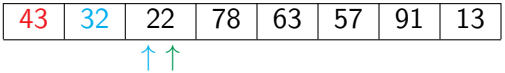

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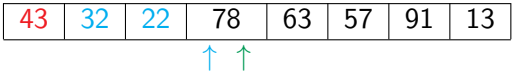

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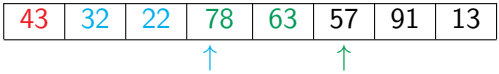

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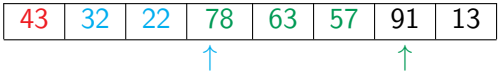

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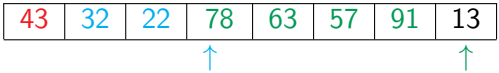

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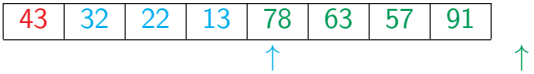

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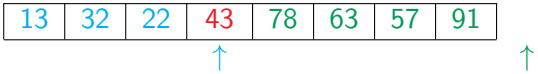

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- Maintain two indices to mark the end of the Lower and Upper segments
- After partitioning, exchange the pivot with the last element of the Lower segment

Quicksort code

- Scan the list from left to right
- Four segments: Pivot, Lower, Upper, Unclassified
- Classify the first unclassified element
  - If it is larger than the pivot, extend Upper to include this element
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```
def quicksort(L,l,r):  # Sort L[l:r]
  if (r - l <= 1):
    return(L)
  (pivot,lower,upper) = (L[l],l+1,l+1)
  for i in range(l+1,r):
    if L[i] > pivot:  # Extend upper segment
      upper = upper+1
    else: # Exchange L[i] with start of upper segment
      (L[i], L[lower]) = (L[lower], L[i])
      # Shift both segments
      (lower,upper) = (lower+1,upper+1)
  # Move pivot between lower and upper
  (L[l],L[lower-1]) = (L[lower-1],L[l])
  lower = lower-1
  # Recursive calls
  quicksort(L,l,lower)
  quicksort(L,lower+1,upper)
  return(L)
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- By partitioning the list carefully, we avoid a merge step
  - This allows an in place sort
- We can also provide an iterative implementation to avoid the cost of recursive calls
- The partitioning strategy we described is not the only one used in the literature
  - Can build the lower and upper segments from opposite ends and meet in the middle
- Need to analyse the complexity of quick sort