Breadth First Search

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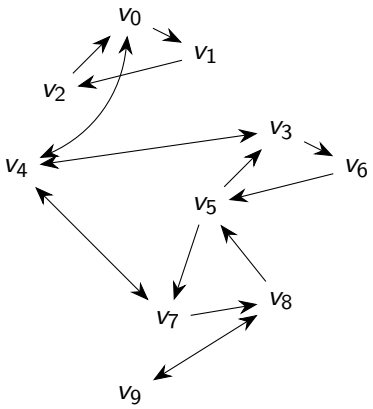
Programming, Data Structures and Algorithms using Python

Week 4

Reachability in a graph

- Mark source vertex as reachable
- Systematically mark neighbours of marked vertices
- Stop when target becomes marked



Reachability in a graph

- Mark source vertex as reachable
- Systematically mark neighbours of marked vertices
- Stop when target becomes marked
- Choose an appropriate representation
  - Adjacency matrix
  - Adjacency list

	0	1	2	3	4	5	6	7	8	9
0	0	1	0	0	1	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0
3	0	0	0	0	1	0	1	0	0	0
4	1	0	0	1	0	0	0	1	0	0
5	0	0	0	1	0	0	0	1	0	0
6	0	0	0	0	0	1	0	0	0	0
7	0	0	0	0	1	0	0	0	1	0
8	0	0	0	0	0	1	0	0	0	1
9	0	0	0	0	0	0	0	0	1	0

0	{1,4}
1	{2}
2	{0}
3	{4,6}
4	{0,3,7}

5	{3,7}
6	{5}
7	{4,8}
8	{5,9}
9	{8}

Reachability in a graph

- Mark source vertex as reachable
- Systematically mark neighbours of marked vertices
- Stop when target becomes marked
- Choose an appropriate representation
  - Adjacency matrix
  - Adjacency list
- Strategies for systematic exploration
  - Breadth first — propagate marks in "layers"
  - Depth first — explore a path till it dies out, then backtrack

	0	1	2	3	4	5	6	7	8	9
0	0	1	0	0	1	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0
3	0	0	0	0	1	0	1	0	0	0
4	1	0	0	1	0	0	0	1	0	0
5	0	0	0	1	0	0	0	1	0	0
6	0	0	0	0	0	1	0	0	0	0
7	0	0	0	0	1	0	0	0	1	0
8	0	0	0	0	0	1	0	0	0	1
9	0	0	0	0	0	0	0	0	1	0

0	{1,4}
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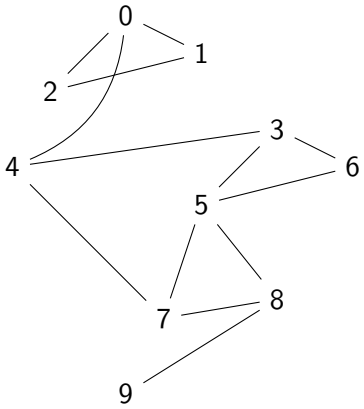
Breadth first search (BFS)

- Explore the graph level by level
  - First visit vertices one step away
  - Then two steps away
  - ...



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- Explore the graph level by level
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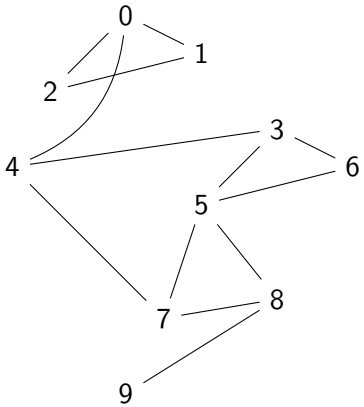
- Each visited vertex has to be explored
  - Extend the search to its neighbours
  - Do this only once for each vertex!



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  - First visit vertices one step away
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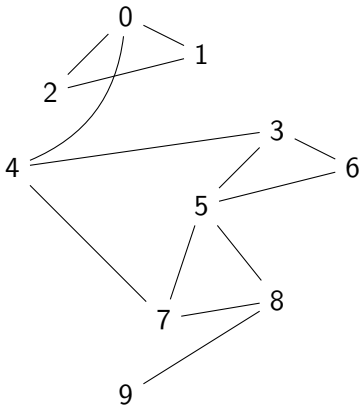
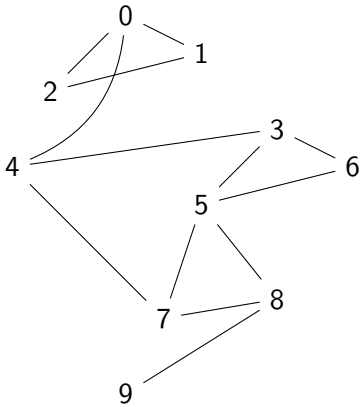
- Each visited vertex has to be explored
  - Extend the search to its neighbours
  - Do this only once for each vertex!
- Maintain information about vertices
  - Which vertices have been visited already
  - Among these, which are yet to be explored



Breadth first search (BFS) ...

- Assume $V = \{0, 1, \ldots, n-1\}$



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- Assume $V = \{0, 1, \ldots, n-1\}$
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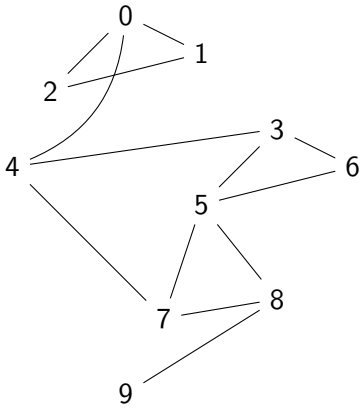
  - Initially, visited(v) = False for all $v \in V$



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- Maintain a sequence of visited vertices yet be explored
  - A queue — first in, first out
  - Initially empty

```
class Queue:
    def __init__(self):
        self.queue = []

    def addq(self,v):
        self.queue.append(v)

    def delq(self):
        v = None
        if not self.isempty():
            v = self.queue[0]
            self.queue = self.queue[1:]
        return(v)

    def isempty(self):
        return(self.queue == [])

    def __str__(self):
        return(str(self.queue))
```

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```
q = Queue()

for i in range(3):
    q.addq(i)
    print(q)
print(q.isempty())

for j in range(3):
    print(q.delq(),q)
print(q.isempty())
```

```
[0]
[0, 1]
[0, 1, 2]
False
0 [1, 2]
1 [2]
2 []
True
```

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- Exploring a vertex i
  - For each edge (i, j), if visited(j) is False,
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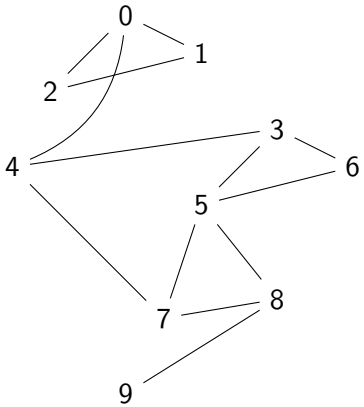

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- Initially
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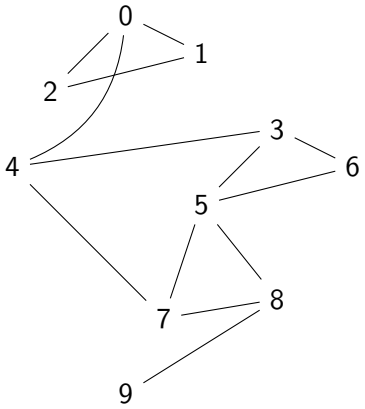
  - Add j to the queue



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  - For each edge (i, j), if visited(j) is False,
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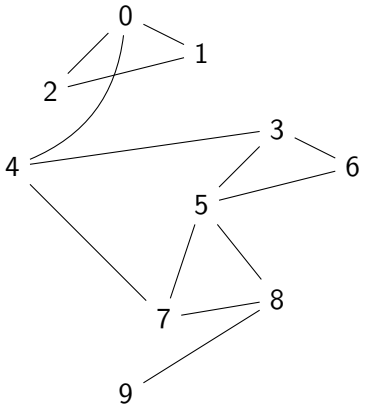

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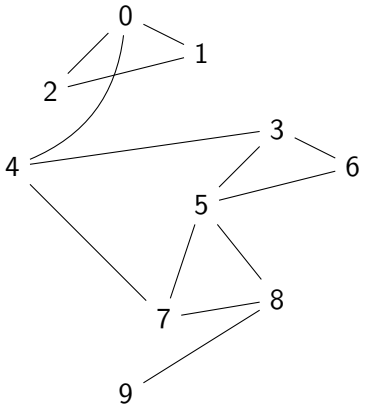
- Stop when queue is empty



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  - Add j to the queue
- Remove and explore vertex i at head of queue
  - For each edge (i, j), if visited(j) is False,
    - Set visited(j) to True
    - Append j to the queue
- Stop when queue is empty

```
def BFS(AMat,v):
    (rows,cols) = AMat.shape
    visited = {}
    for i in range(rows):
        visited[i] = False
    q = Queue()

    visited[v] = True
    q.addq(v)

    while(not q.isempty()):
        j = q.delq()
        for k in neighbours(AMat,j):
            if (not visited[k]):
                visited[k] = True
                q.addq(k)

    return(visited)
```

Breadth first search (BFS) ...

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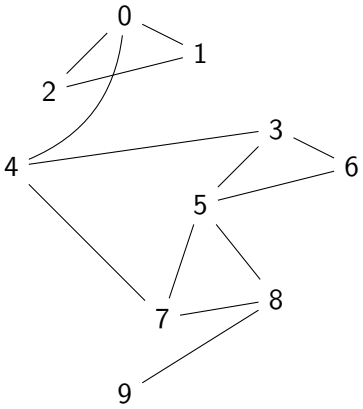
  - visited(v) = False for all $v \in V$
  - Queue of vertices to be explored is empty
- Start BFS from vertex j
  - Set visited(j) = True
  - Add j to the queue
- Remove and explore vertex i at head of queue
  - For each edge (i, j), if visited(j) is False,
    - Set visited(j) to True
    - Append j to the queue
- Stop when queue is empty

```
def BFSList(AList,v):
    visited = {}
    for i in AList.keys():
        visited[i] = False
    q = Queue()

    visited[v] = True
    q.addq(v)

    while(not q.isempty()):
        j = q.delq()
        for k in AList[j]:
            if (not visited[k]):
                visited[k] = True
                q.addq(k)

    return(visited)
```

BFS from vertex 7

Visited	
0	False
1	False
2	False
3	False
4	False
5	False
6	False
7	False
8	False
9	False

To explore queue									

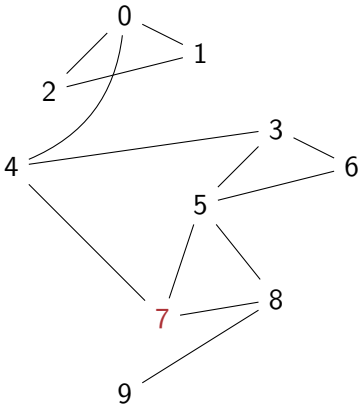


BFS from vertex 7

Visited	
0	False
1	False
2	False
3	False
4	False
5	False
6	False
7	True
8	False
9	False

To explore queue									
7									

- Mark 7 and add to queue

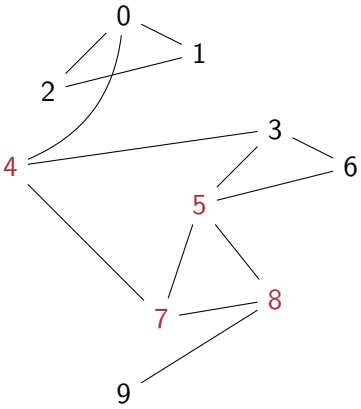


BFS from vertex 7

Visited	
0	False
1	False
2	False
3	False
4	True
5	True
6	False
7	True
8	True
9	False

To explore queue									
4	5	8							

- Mark 7 and add to queue
- Explore 7, visit {4,5,8}

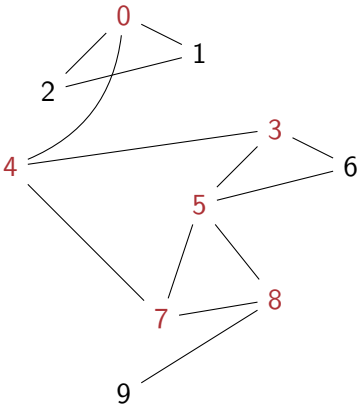


BFS from vertex 7

Visited	
0	True
1	False
2	False
3	True
4	True
5	True
6	False
7	True
8	True
9	False

To explore queue								
5	8	0	3					

- Mark 7 and add to queue
- Explore 7, visit {4,5,8}
- Explore 4, visit {0,3}

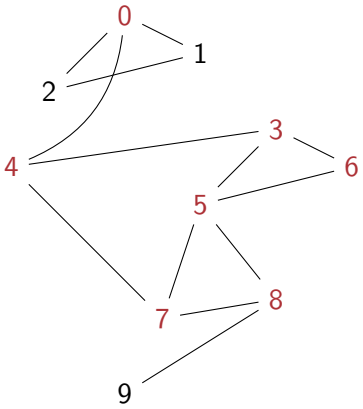


BFS from vertex 7

Visited	
0	True
1	False
2	False
3	True
4	True
5	True
6	True
7	True
8	True
9	False

To explore queue									
8	0	3	6						

- Mark 7 and add to queue
- Explore 7, visit {4,5,8}
- Explore 4, visit {0,3}
- Explore 5, visit {6}

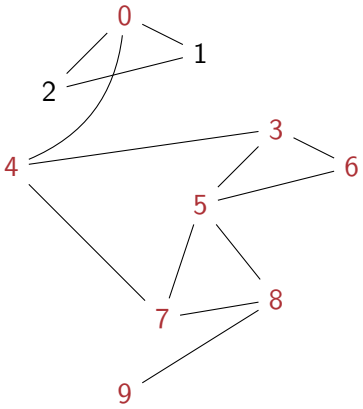


BFS from vertex 7

Visited	
0	True
1	False
2	False
3	True
4	True
5	True
6	True
7	True
8	True
9	True

To explore queue									
0	3	6	9						

- Mark 7 and add to queue
- Explore 7, visit {4,5,8}
- Explore 4, visit {0,3}
- Explore 5, visit {6}
- Explore 8, visit {9}

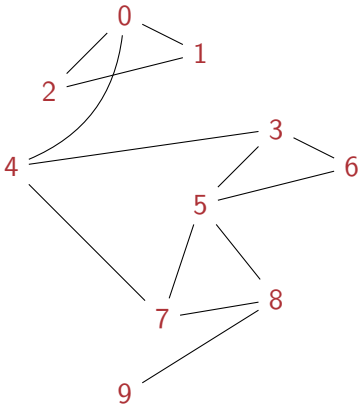


BFS from vertex 7

Visited	
0	True
1	True
2	True
3	True
4	True
5	True
6	True
7	True
8	True
9	True

To explore queue									
3	6	9	1	2					

- Mark 7 and add to queue
- Explore 7, visit {4,5,8}
- Explore 4, visit {0,3}
- Explore 5, visit {6}
- Explore 8, visit {9}
- Explore 0, visit {1,2}



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2	True
3	True
4	True
5	True
6	True
7	True
8	True
9	True

To explore queue									
6	9	1	2						

- Mark 7 and add to queue
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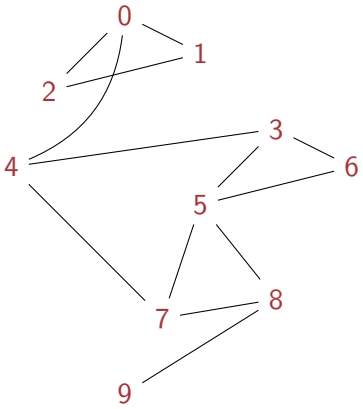
- Explore 8, visit {9}
- Explore 0, visit {1,2}
- Explore 3



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Visited	
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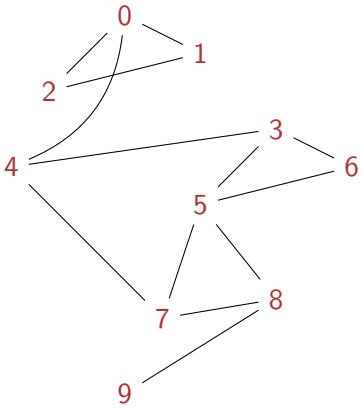
1	True
2	True
3	True
4	True
5	True
6	True
7	True
8	True
9	True

To explore queue									
9	1	2							

- Mark 7 and add to queue
- Explore 7, visit {4,5,8}
- Explore 4, visit {0,3}
- Explore 5, visit {6}
- Explore 8, visit {9}
- Explore 0, visit {1,2}
- Explore 3
- Explore 6



BFS from vertex 7

Visited	
0	True
1	True
2	True
3	True
4	True
5	True
6	True
7	True
8	True
9	True

To explore queue								
1	2							

- Mark 7 and add to queue
- Explore 7, visit {4,5,8}
- Explore 4, visit {0,3}
- Explore 5, visit {6}
- Explore 8, visit {9}
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- Explore 6
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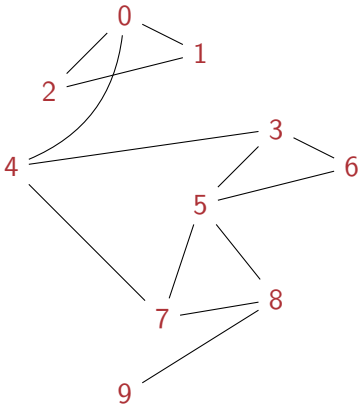
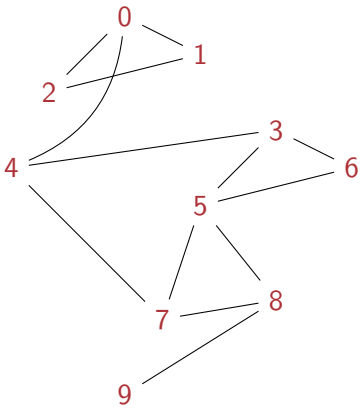


BFS from vertex 7

Visited	
0	True
1	True
2	True
3	True
4	True
5	True
6	True
7	True
8	True
9	True

To explore queue									
2									

- Mark 7 and add to queue
- Explore 7, visit {4,5,8}
- Explore 4, visit {0,3}
- Explore 5, visit {6}
- Explore 8, visit {9}
- Explore 0, visit {1,2}
- Explore 3
- Explore 6
- Explore 9
- Explore 1



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- Explore 5, visit {6}
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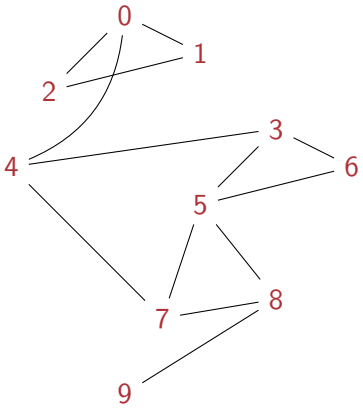
- Explore 0, visit {1,2}
- Explore 3
- Explore 6
- Explore 9
- Explore 1
- Explore 2



Complexity of BFS

- $G = (V, E)$
  - $|V| = n$
  - $|E| = m$
  - If G is connected, m can vary from $n - 1$ to $n(n - 1)/2$

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  - Visit the vertex: add to queue
  - Explore the vertex: remove from queue
  - Visit and explore at most n vertices

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  - Check all outgoing edges
  - How long does this take?

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- Exploring a vertex
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Adjacency matrix

- To explore i, scan $neighbours(i)$
- Look up n entries in row i, regardless of $degree(i)$

Complexity of BFS

- $G = (V, E)$
  - $|V| = n$
  - $|E| = m$
  - If G is connected, m can vary from $n-1$ to $n(n-1)/2$
- In BFS, each reachable vertex is processed exactly once
  - Visit the vertex: add to queue
  - Explore the vertex: remove from queue
  - Visit and explore at most n vertices
- Exploring a vertex
  - Check all outgoing edges
  - How long does this take?

Adjacency matrix

- To explore i, scan $neighbours(i)$
- Look up n entries in row i, regardless of $degree(i)$

Adjacency list

- List $neighbours(i)$ is directly available
- Time to explore i is $degree(i)$
- Degree varies across vertices

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Adjacency list

- List $neighbours(i)$ is directly available
- Time to explore i is $degree(i)$
- Degree varies across vertices

Sum of degrees

- Sum of degrees is $2m$
- Each edge (i, j) contributes to $degree(i)$ and $degree(j)$

Complexity of BFS

BFS with adjacency matrix

- n steps to initialize each vertex
- n steps to explore each vertex
- Overall time is $O(n^2)$

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- $2m$ steps (sum of degrees) to explore all vertices
  - An example of amortized analysis
- Overall time is $O(n + m)$

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- If $m \ll n^2$, working with adjacency lists is much more efficient
  - This is why we treat m and n as separate parameters

Complexity of BFS

BFS with adjacency matrix

- n steps to initialize each vertex
- n steps to explore each vertex
- Overall time is $O(n^2)$

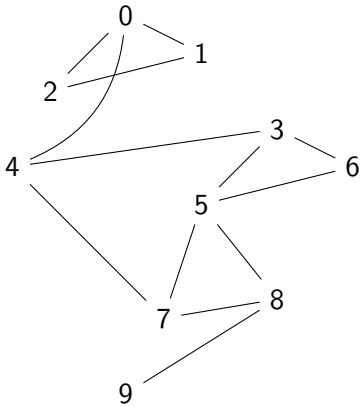
BFS with adjacency list

- n steps to initialize each vertex
- $2m$ steps (sum of degrees) to explore all vertices
  - An example of amortized analysis
- Overall time is $O(n + m)$

- If $m \ll n^2$, working with adjacency lists is much more efficient
  - This is why we treat m and n as separate parameters
- For graphs, $O(m + n)$ is typically the best possible complexity
  - Need to see each each vertex and edge at least once
  - Linear time

Enhancing BFS to record paths

- If BFS from i sets visited(k) = True, we know that k is reachable from i
- How do we recover a path from i to k?



Enhancing BFS to record paths

- If BFS from i sets visited(k) = True, we know that k is reachable from i
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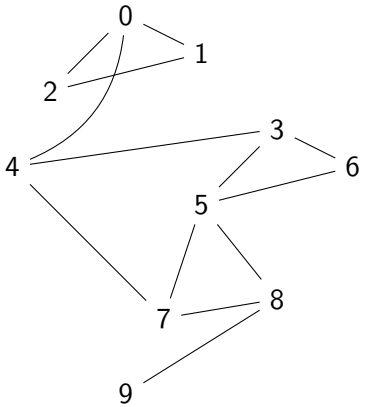
- visited(k) was set to True when exploring some vertex j



Enhancing BFS to record paths

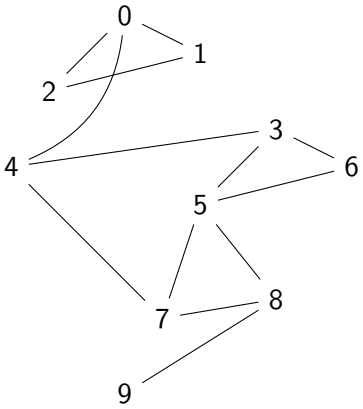
- If BFS from i sets visited(k) = True, we know that k is reachable from i
- How do we recover a path from i to k?
- visited(k) was set to True when exploring some vertex j
- Record parent(k) = j
- From k, follow parent links to trace back a path to i

```
def BFSListPath(AList,v):
    (visited,parent) = ({},{})
    for i in AList.keys():
        visited[i] = False
        parent[i] = -1
    q = Queue()

    visited[v] = True
    q.addq(v)

    while(not q.isempty()):
        j = q.delq()
        for k in AList[j]:
            if (not visited[k]):
                visited[k] = True
                parent[k] = j
                q.addq(k)

    return(visited,parent)
```

BFS from vertex 7 with parent information

	Visited	Parent
0	False	-1
1	False	-1
2	False	-1
3	False	-1
4	False	-1
5	False	-1
6	False	-1
7	False	-1
8	False	-1
9	False	-1

To explore queue									

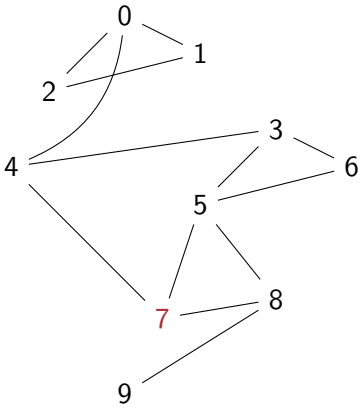


BFS from vertex 7 with parent information

	Visited	Parent
0	False	−1
1	False	−1
2	False	−1
3	False	−1
4	False	−1
5	False	−1
6	False	−1
7	True	−1
8	False	−1
9	False	−1

To explore queue									
7									

- Mark 7, add to queue

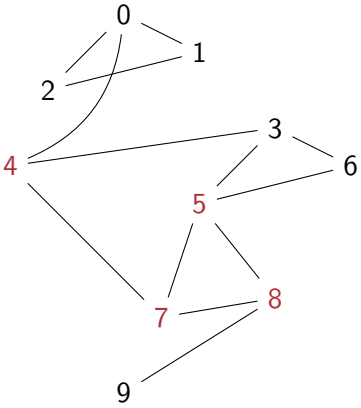


BFS from vertex 7 with parent information

	Visited	Parent
0	False	−1
1	False	−1
2	False	−1
3	False	−1
4	True	7
5	True	7
6	False	−1
7	True	−1
8	True	7
9	False	−1

To explore queue									
4	5	8							

- Mark 7, add to queue
- Explore 7, visit {4,5,8}

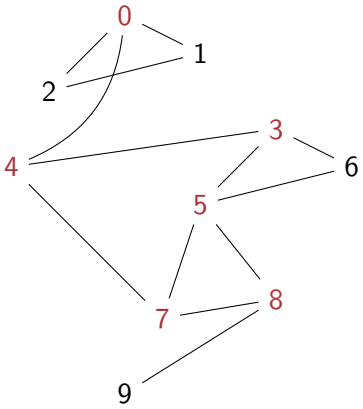


BFS from vertex 7 with parent information

	Visited	Parent
0	True	4
1	False	−1
2	False	−1
3	True	4
4	True	7
5	True	7
6	False	−1
7	True	−1
8	True	7
9	False	−1

To explore queue								
5	8	0	3					

- Mark 7, add to queue
- Explore 7, visit {4,5,8}
- Explore 4, visit {0,3}



BFS from vertex 7 with parent information

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0	True	4
1	False	−1
2	False	−1
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4	True	7
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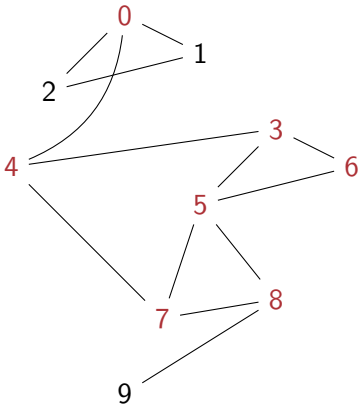
6	True	5
7	True	−1
8	True	7
9	False	−1

To explore queue								
8	0	3	6					

- Mark 7, add to queue
- Explore 7, visit {4,5,8}
- Explore 4, visit {0,3}
- Explore 5, visit {6}



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To explore queue								
0	3	6	9					

- Mark 7, add to queue
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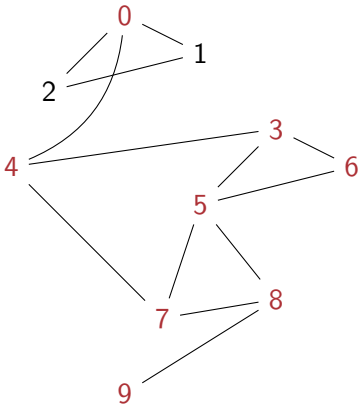
- Explore 4, visit {0,3}
- Explore 5, visit {6}
- Explore 8, visit {9}

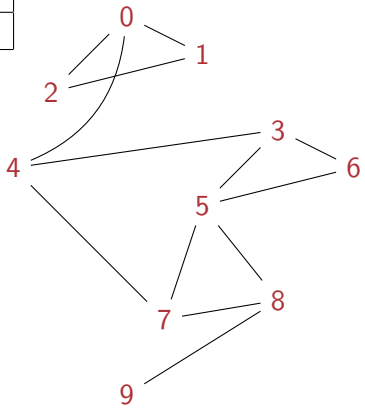


BFS from vertex 7 with parent information

	Visited	Parent
0	True	4
1	True	0
2	True	0
3	True	4
4	True	7
5	True	7
6	True	5
7	True	−1
8	True	7
9	True	8

To explore queue									
3	6	9	1	2					

- Mark 7, add to queue
- Explore 7, visit {4,5,8}
- Explore 4, visit {0,3}
- Explore 5, visit {6}
- Explore 8, visit {9}
- Explore 0, visit {1,2}



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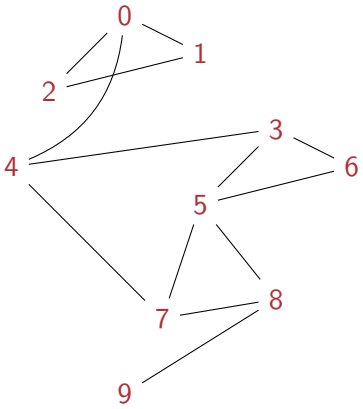
0	True	4
1	True	0
2	True	0
3	True	4
4	True	7
5	True	7
6	True	5
7	True	−1
8	True	7
9	True	8

To explore queue								
6	9	1	2					

- Mark 7, add to queue
- Explore 7, visit {4,5,8}
- Explore 4, visit {0,3}
- Explore 5, visit {6}
- Explore 8, visit {9}
- Explore 0, visit {1,2}
- Explore 3



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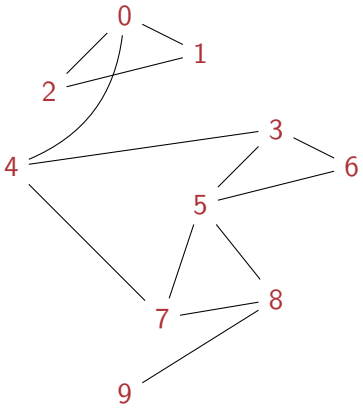
0	True	4
1	True	0
2	True	0
3	True	4
4	True	7
5	True	7
6	True	5
7	True	−1
8	True	7
9	True	8

To explore queue									
9	1	2							

- Mark 7, add to queue
- Explore 7, visit {4,5,8}
- Explore 4, visit {0,3}
- Explore 5, visit {6}
- Explore 8, visit {9}
- Explore 0, visit {1,2}
- Explore 3
- Explore 6



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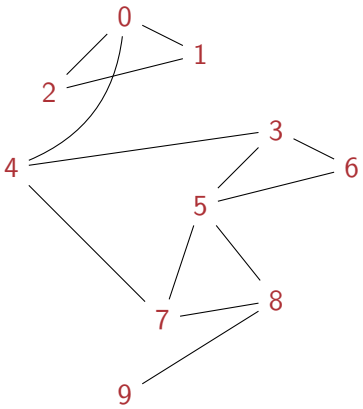
	Visited	Parent
0	True	4
1	True	0
2	True	0
3	True	4
4	True	7
5	True	7
6	True	5
7	True	−1
8	True	7
9	True	8

To explore queue									
1	2								

- Mark 7, add to queue
- Explore 7, visit {4,5,8}
- Explore 4, visit {0,3}
- Explore 5, visit {6}
- Explore 8, visit {9}
- Explore 0, visit {1,2}
- Explore 3
- Explore 6
- Explore 9

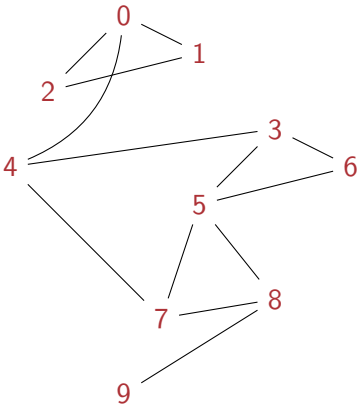


BFS from vertex 7 with parent information

	Visited	Parent
0	True	4
1	True	0
2	True	0
3	True	4
4	True	7
5	True	7
6	True	5
7	True	−1
8	True	7
9	True	8

To explore queue									
2									

- Mark 7, add to queue
- Explore 7, visit {4,5,8}
- Explore 4, visit {0,3}
- Explore 5, visit {6}
- Explore 8, visit {9}
- Explore 0, visit {1,2}
- Explore 3
- Explore 6
- Explore 9
- Explore 1

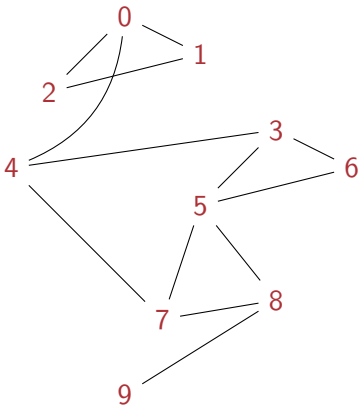


BFS from vertex 7 with parent information

	Visited	Parent
0	True	4
1	True	0
2	True	0
3	True	4
4	True	7
5	True	7
6	True	5
7	True	−1
8	True	7
9	True	8

To explore queue									

- Mark 7, add to queue
- Explore 7, visit {4,5,8}
- Explore 4, visit {0,3}
- Explore 5, visit {6}
- Explore 8, visit {9}
- Explore 0, visit {1,2}
- Explore 3
- Explore 6
- Explore 9
- Explore 1
- Explore 2



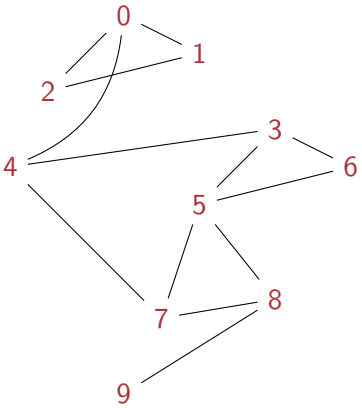
BFS from vertex 7 with parent information

	Visited	Parent
0	True	4
1	True	0
2	True	0
3	True	4
4	True	7
5	True	7
6	True	5
7	True	−1
8	True	7
9	True	8

Path from 7 to 6 is 7–5–6

To explore queue									

- Mark 7, add to queue
- Explore 7, visit {4,5,8}
- Explore 4, visit {0,3}
- Explore 5, visit {6}
- Explore 8, visit {9}
- Explore 0, visit {1,2}
- Explore 3
- Explore 6
- Explore 9
- Explore 1
- Explore 2



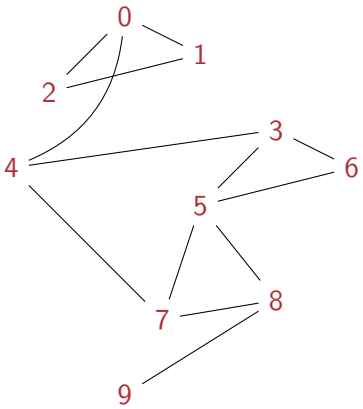
BFS from vertex 7 with parent information

	Visited	Parent
0	True	4
1	True	0
2	True	0
3	True	4
4	True	7
5	True	7
6	True	5
7	True	−1
8	True	7
9	True	8

Path from 7 to 2 is
7–4–0–2

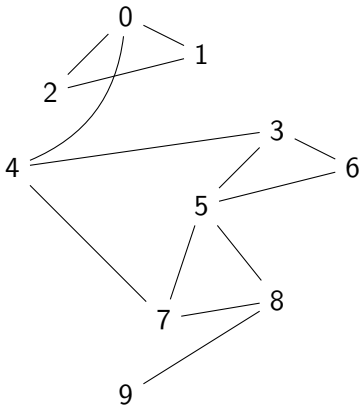
To explore queue									

- Mark 7, add to queue
- Explore 7, visit {4,5,8}
- Explore 4, visit {0,3}
- Explore 5, visit {6}
- Explore 8, visit {9}
- Explore 0, visit {1,2}
- Explore 3
- Explore 6
- Explore 9
- Explore 1
- Explore 2



Enhancing BFS to record distance

- BFS explores neighbours level by level
- By recording the level at which a vertex is visited, we get its distance from the source vertex



Enhancing BFS to record distance

- BFS explores neighbours level by level
- By recording the level at which a vertex is visited, we get its distance from the source vertex
- Instead of visited(j), maintain level)(j)

```
def BFSListPathLevel(AList,v):
    (level,parent) = ({},{})
    for i in AList.keys():
        level[i] = -1
        parent[i] = -1
    q = Queue()

    level[v] = 0
    q.addq(v)

    while(not q.isempty()):
        j = q.delq()
        for k in AList[j]:
            if (level[k] == -1):
                level[k] = level[j]+1
                parent[k] = j
                q.addq(k)

    return(level,parent)
```

Enhancing BFS to record distance

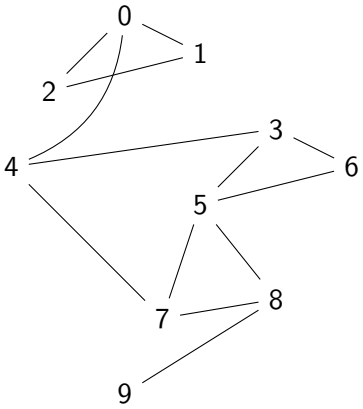
- BFS explores neighbours level by level
- By recording the level at which a vertex is visited, we get its distance from the source vertex
- Instead of visited(j), maintain level)(j)
- Initalize level(j) $= -1$ for all j
- Set level(i) $= 0$ for source vertex
- If we visit k from j, set level(k) to level(j) $+ 1$
- level(j) is the length of the shortest path from the source vertex, in number of edges

```
def BFSListPathLevel(AList,v):
    (level,parent) = ({},{})
    for i in AList.keys():
        level[i] = -1
        parent[i] = -1
    q = Queue()

    level[v] = 0
    q.addq(v)

    while(not q.isempty()):
        j = q.delq()
        for k in AList[j]:
            if (level[k] == -1):
                level[k] = level[j]+1
                parent[k] = j
                q.addq(k)

    return(level,parent)
```

BFS from vertex 7 with parent and distance information

	Level	Parent
0	−1	−1
1	−1	−1
2	−1	−1
3	−1	−1
4	−1	−1
5	−1	−1
6	−1	−1
7	−1	−1
8	−1	−1
9	−1	−1

To explore queue									

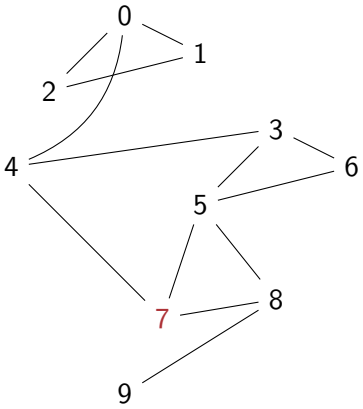


BFS from vertex 7 with parent and distance information

	Level	Parent
0	−1	−1
1	−1	−1
2	−1	−1
3	−1	−1
4	−1	−1
5	−1	−1
6	−1	−1
7	0	−1
8	−1	−1
9	−1	−1

To explore queue									
7									

- Mark 7, add to queue

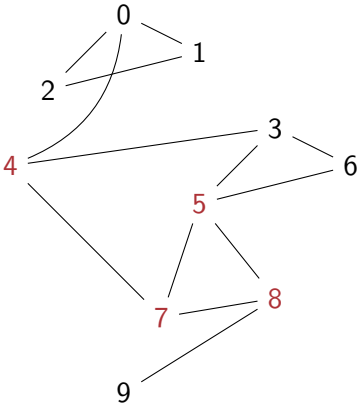


BFS from vertex 7 with parent and distance information

	Level	Parent
0	−1	−1
1	−1	−1
2	−1	−1
3	−1	−1
4	1	7
5	1	7
6	−1	−1
7	0	−1
8	1	7
9	−1	−1

To explore queue									
4	5	8							

- Mark 7, add to queue
- Explore 7, visit {4,5,8}

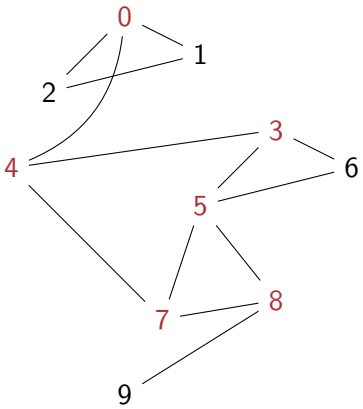


BFS from vertex 7 with parent and distance information

	Level	Parent
0	2	4
1	−1	−1
2	−1	−1
3	2	4
4	1	7
5	1	7
6	−1	−1
7	0	−1
8	1	7
9	−1	−1

To explore queue								
5	8	0	3					

- Mark 7, add to queue
- Explore 7, visit {4,5,8}
- Explore 4, visit {0,3}

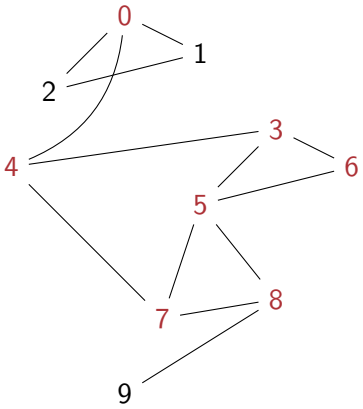


BFS from vertex 7 with parent and distance information

	Level	Parent
0	2	4
1	−1	−1
2	−1	−1
3	2	4
4	1	7
5	1	7
6	2	5
7	0	−1
8	1	7
9	−1	−1

To explore queue								
8	0	3	6					

- Mark 7, add to queue
- Explore 7, visit {4,5,8}
- Explore 4, visit {0,3}
- Explore 5, visit {6}

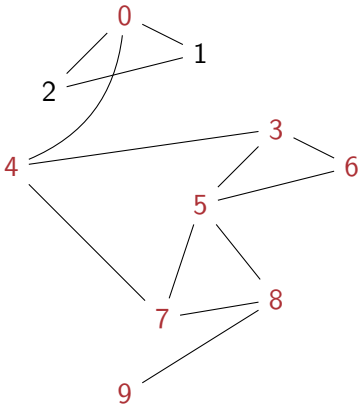


BFS from vertex 7 with parent and distance information

	Level	Parent
0	2	4
1	−1	−1
2	−1	−1
3	2	4
4	1	7
5	1	7
6	2	5
7	0	−1
8	1	7
9	2	8

To explore queue								
0	3	6	9					

- Mark 7, add to queue
- Explore 7, visit {4,5,8}
- Explore 4, visit {0,3}
- Explore 5, visit {6}
- Explore 8, visit {9}

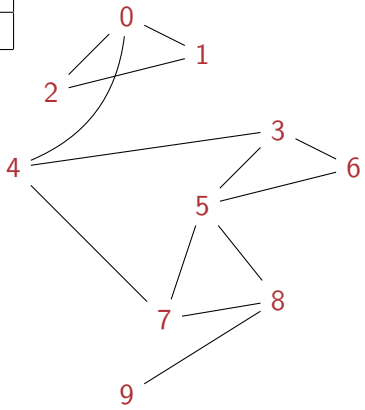


BFS from vertex 7 with parent and distance information

	Level	Parent
0	2	4
1	3	0
2	3	0
3	2	4
4	1	7
5	1	7
6	2	5
7	0	−1
8	1	7
9	2	8

To explore queue									
3	6	9	1	2					

- Mark 7, add to queue
- Explore 7, visit {4,5,8}
- Explore 4, visit {0,3}
- Explore 5, visit {6}
- Explore 8, visit {9}
- Explore 0, visit {1,2}



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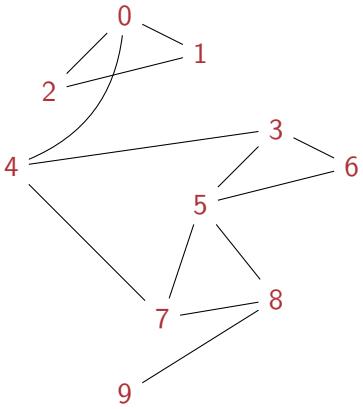
- Explore 3



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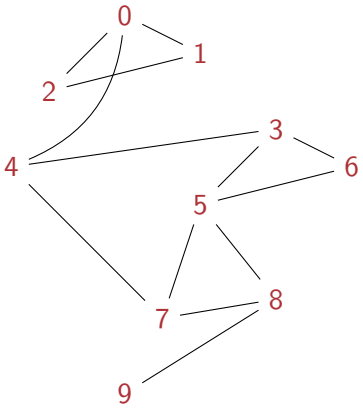

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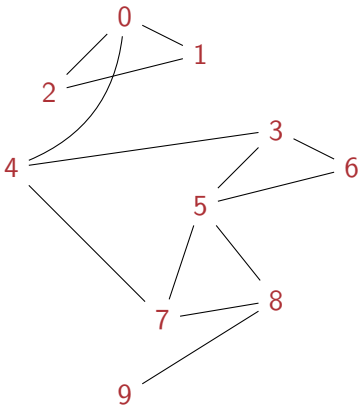

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2									

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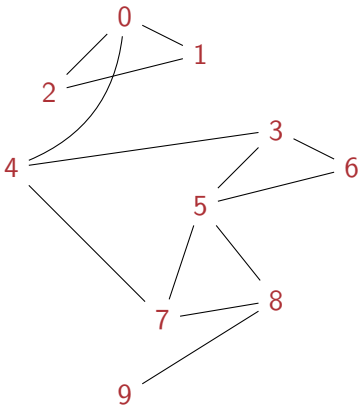
- Explore 6
- Explore 9
- Explore 1



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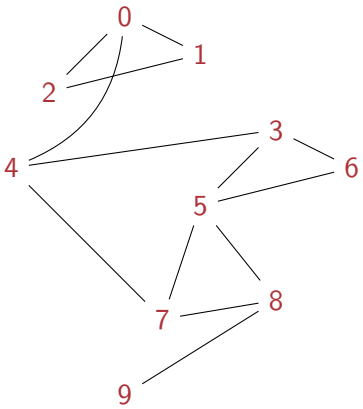
- Explore 6
- Explore 9
- Explore 1
- Explore 2



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  - In general, edges are labelled with a cost (distance, time, ticket price, ...)
  - Will look at weighted graphs, where shortest paths are in terms of cost, not number of edges