Note Title 22-09-2021

[Sufficiency part] In general

If (x^*, λ^*) satisfies the above conditions. $\Rightarrow$ Local optima.

[and some regularity conditions]

KKT conditions [Karush-Kuhn-Tucker]

$$\min \; f(x)$$
$$h_i(x) \le 0 \quad \forall i = 1, \dots m$$
$$\ell_j(x) = 0 \quad \forall j = 1, \dots, n$$

$$L(x, u, v) = f(x) + \sum_{i=1}^{m} u_i h_i(x) + \sum_{j=1}^{n} v_j \ell_j(x)$$

x ← vector, $u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$, $v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$

(a) $\nabla f(x^*) + \sum_{i=1}^{m} u_i^* \nabla h_i(x^*) + \sum_{j=1}^{n} v_j^* \nabla \ell_j(x^*) = 0 \in$ vector. ↳ [Stationarity]

(b) $u_i^* h_i(x^*) = 0 \quad \forall i$ [C.S condition]

(c) $h_i(x^*) \le 0 \quad \forall i$; $\ell_j(x^*) = 0 \quad \forall j$ [Primal Feasibility]

(d) $u_i^* \ge 0 \quad \forall i$ [Dual feasibility]

Support vector Machine (SVM) ←

Dataset.

$\{ (x_1, y_1), \ldots, (x_n, y_n) \}$

Optimization problem

$$\min_{w} \frac{1}{2} \|w\|^2 \quad \leftarrow \quad \text{objective}$$

Quadratic ⇒ convex. $\|w\|^2 = \sum_i w_i^2$

$$\text{s.t} \quad \underline{w}^T x_i y_i \geq 1 \quad \forall i$$

constraints.

Linear constraints.
⇒ convex.