

$$\text{Eqn \# 1: } z_1 = x_1 \cdot w_1 + b_1$$

$$\text{Eqn \# 2: } a_1 = f(z_1) = \frac{1}{1 + e^{-z_1}}$$

$$\text{Eqn \# 3: } z_2 = a_1 \cdot w_2 + b_2$$

$$\text{Eqn \# 4: } a_2 = f(z_2) = \frac{1}{1 + e^{-z_2}}$$

$$\text{Eqn \# 5: } E = \frac{1}{2} (T - a_2)^2$$

$$1. \frac{\partial E}{\partial w_2} = \frac{\partial E}{\partial a_2} \cdot \frac{\partial a_2}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_2}$$

$$\bullet E = \frac{1}{2} (T - a_2)^2$$

$$\frac{\partial E}{\partial a_2} = 2 \cdot \frac{1}{2} (T - a_2) \cdot (-1) = -(T - a_2)$$

$$\bullet a_2 = \frac{1}{1 + e^{-z_2}} = (1 + e^{-z_2})^{-1}$$

$$\frac{\partial a_2}{\partial z_2} = -1 \cdot (1 + e^{-z_2})^{-2} \cdot e^{-z_2} \cdot (-1)$$

$$= \frac{e^{-z_2}}{(1 + e^{-z_2})^2} = (a_2)^2 \frac{1 - a_2}{a_2} = a_2(1 - a_2)$$

$$\bullet z_2 = a_1 \cdot w_2 + b_2$$

$$\frac{\partial z_2}{\partial w_2} = a_1$$

$$\frac{\partial E}{\partial w_2} = -(T - a_2) \cdot a_2(1 - a_2) \cdot a_1$$

$$2. \frac{\partial E}{\partial b_2} = \frac{\partial E}{\partial a_2} \cdot \frac{\partial a_2}{\partial z_2} \cdot \frac{\partial z_2}{\partial b_2}$$

$$\bullet E = \frac{1}{2}(T - a_2)^2$$

$$\frac{\partial E}{\partial a_2} = 2 \cdot \frac{1}{2}(T - a_2) \cdot (-1) = -(T - a_2)$$

$$\bullet a_2 = \frac{1}{1 + e^{-z_2}} = (1 + e^{-z_2})^{-1}$$

$$\frac{\partial a_2}{\partial z_2} = -1 \cdot (1 + e^{-z_2})^{-2} \cdot e^{-z_2} \cdot (-1)$$

$$= \frac{e^{-z_2}}{(1 + e^{-z_2})^2} = (a_2)^2 \cdot \frac{1 - a_2}{a_2} = a_2(1 - a_2)$$

$$\bullet z_2 = a_1 \cdot w_2 + b_2$$

$$\frac{\partial z_2}{\partial b_2} = 1$$

$$\frac{\partial E}{\partial b_2} = -(T - a_2) \cdot a_2(1 - a_2) \cdot 1$$

$$3. \frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial a_2} \cdot \frac{\partial a_2}{\partial z_2} \cdot \frac{\partial z_2}{\partial a_1} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_1}$$

$$\bullet E = \frac{1}{2}(T - a_2)^2$$

$$\frac{\partial E}{\partial a_2} = -(T - a_2)$$

$$\bullet a_2 = \frac{1}{1 + e^{-z_2}} = (1 + e^{-z_2})^{-1}$$

$$\frac{\partial a_2}{\partial z_2} = a_2(1 - a_2)$$

$$\bullet z_2 = a_1 \cdot w_2 + b_2$$

$$\frac{\partial z_2}{\partial a_1} = w_2$$

$$\bullet a_1 = \frac{1}{1 + e^{-z_1}} = (1 + e^{-z_1})^{-1}$$

$$\frac{\partial a_1}{\partial z_1} = a_1(1 - a_1)$$

$$\bullet z_1 = x_1 \cdot w_1 + b_1$$

$$\frac{\partial z_1}{\partial w_1} = x_1$$

$$\frac{\partial E}{\partial w_1} = -(T - a_2) \cdot a_2(1 - a_2) \cdot w_2 \cdot a_1(1 - a_1) \cdot x_1$$

$$4. \frac{\partial E}{\partial b_1} = \frac{\partial E}{\partial a_2} \cdot \frac{\partial a_2}{\partial z_2} \cdot \frac{\partial z_2}{\partial a_1} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z_1}{\partial b_1}$$

$$\bullet E = \frac{1}{2}(T - a_2)^2$$

$$\frac{\partial E}{\partial a_2} = -(T - a_2)$$

$$\bullet a_2 = \frac{1}{1 + e^{-z_2}} = (1 + e^{-z_2})^{-1}$$

$$\frac{\partial a_2}{\partial z_2} = a_2(1 - a_2)$$

$$\bullet z_2 = a_1 \cdot w_2 + b_2$$

$$\frac{\partial z_2}{\partial a_1} = w_2$$

$$\bullet a_1 = \frac{1}{1 + e^{-z_1}} = (1 + e^{-z_1})^{-1}$$

$$\frac{\partial a_1}{\partial z_1} = a_1(1 - a_1)$$

$$\bullet z_1 = x_1 \cdot w_1 + b_1$$

$$\frac{\partial z_1}{\partial b_1} = 1$$

$$\frac{\partial E}{\partial b_1} = -(T - a_2) \cdot a_2(1 - a_2) \cdot w_2 \cdot a_1(1 - a_1) \cdot 1$$