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Assignment III
Algorithm and Complexity

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Complexity Theory

A mathematic problem is computable if it can be solved in principle by a computing device. Some common synonyms for "computable" are "solvable", "decidable" & "recursive". There're extensive study & classification of which mathematical problem are computable & which are not. It concerned with the resources, such as time & space, needed to solve computational problem. It is the appropriate setting for the study of such problems.

Complexity Classes

Complexity classes help computer scientists groups problems based on how much time & space required to solve the problem & verify solutions. A big-O notation is necessary to understand the complexity classes. A complexity class is the set of all the computational problems which can be solved using a certain amount of a certain computational resources.

1) P-Class

- This class contains problems that can be solved by non-deterministic TM in polynomial time.
- The class P consists of those problems that are solvable in polynomial time. i.e. these problem can be solved in time $O(n^k)$ in worst-case, where k is constant.
- These problems are called tractable

2) NP-Class

- This class contains problems that are solvable by turing machine in non-deterministic polynomial time. This also includes the problems that are solvable in some polynomial time upto problems that are solvable in expontial time.

- While they can have non-deterministic polynomial time to solve the problem, the result can be verified by the TM in polynomial time.

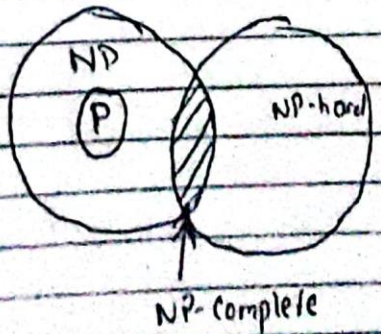
3) NP-hard

- A problem is NP-hard if every problem in NP can be polynomially reduced.

- If a particular problem is hard as the hardest problem in NP-class then we will say that this problem is NP-hard.

4) NP-Complete

- If a problem is both NP & NP-hard then this kind of problem is considered as NP-Complete



Cooks Theorem

- It states that, "the satisfiability problem (SAT) is NP-Complete"

SAT

- A propositional logic formula ϕ is called satisfiable if there is some assignment to its variables that makes it evaluate to true
- $p \wedge q$ is satisfiable if $p=1$ & $q=1$
- $p \wedge \neg q$ is not satisfiable
- Boolean satisfiability or simply SAT is problem of determining if a boolean ~~function~~ formula is satisfiable or not
- The study of boolean functions generally is concerned with the set of true assignments that make the function true.

Vertex Cover

- In mathematical discipline of graph theory, "A vertex cover of a graph is a subset of vertices which covers every edge."
- An edge is covered if one of its endpoint is choosen
- In other words "A vertex cover for a graph G is a ~~subset~~ of vertices incident to every edge in G."
- The vertex cover problem: What is the minimum size vertex cover in G?
- Problem: Given graph $G = (V, E)$ find smallest $V' \subseteq V$ such that if $(u, v) \in E$, then $u \in V'$ or $v \in V$ or both

Clique

- In a graph G, a subset of vertices fully connected to each other, i.e. a complete subgraph of G is called clique
- The maximum clique problem: How large is max. size clique in a graph?
- In another words, given a group of vertices some of which have edges in between them, the maximum clique is the largest subset of vertices in which each point is directly connected to every another vertex in the subset.