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Inverse Z- Transform

We know, z-transform is given by,

$$z(U_n) = \sum_{n=0}^{\infty} U_n z^{-n}$$

$$z(U_n) = \bar{U}(z)$$

Inverse of ~~this~~ this can be written as,

$$z^{-1}(\bar{U}(z)) = U_n$$

- Find Inverse ~~of~~ z-transform of $\frac{z}{z-a}$ where ROC is (i) $|z| > |a|$
  (ii) $|z| < |a|$

Soln:

let, $\bar{U}(z) = \boxed{\frac{z}{z-a}}$

- if we (open) this binomial expansion is accounted. (expand)

For, $|z| > |a|$

$$\frac{|a|}{|z|} < 1$$

Taking z common,

$$\bar{U}(z) = \frac{z}{z(1-a/z)}$$

$\rightarrow (1+x)^{-1}$
$= 1 - x + x^2 - x^3 + \ldots$

Here, $x < 1$

or, $\bar{U}(z) = (1 - a/z)^{-1}$

$$= 1 + \frac{a}{z} + \left(\frac{a}{z}\right)^2 + \cdots + \left(\frac{a}{z}\right)^n + \cdots$$

$$\left[\because (1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots\right]$$

$$= \sum_{n=0}^{\infty} (a/z)^n$$

$$= \sum_{n=0}^{\infty} a^n z^{-n}$$

$$\boxed{Z^{-1}(\bar{U}(z)) = a^n} \quad \boxed{n > 1}$$

(ii) $|z| < |a|$

$$\frac{|z|}{|a|} < 1$$

$$\bar{U}(z) = \frac{z}{z-a}$$

$$= -\frac{1}{a}\left(\frac{z}{1 - z/a}\right)$$

$$= -\frac{z}{a}\left(1 - z/a\right)^{-1}$$

$$= -z/a\left(1 + z/a + (z/a)^2 + \cdots + \frac{z^n}{a^n}\right) + \cdots \infty$$

$$= -\frac{z}{a} - \left(\frac{z}{a}\right)^2 - \left(\frac{z}{a}\right)^3 - \cdots - \infty$$

$$= -\sum_{n=-\infty}^{-1} \left(\frac{a}{z}\right)^n$$

$$= -\sum_{n=-\infty}^{-1} a^n z^{-n}$$

$$\boxed{Z^{-1}(\bar{U}(z)) = -a^n}$$

$$\boxed{n < 1}$$