

Advanced Cryptography

Assignment #1

1. Evaluate the following

- a. $7503 \bmod 81 \Rightarrow 51$
- b. $-7503 \bmod 81 \Rightarrow 30$
- c. $81 \bmod 7503 \Rightarrow 81$
- d. $-81 \bmod 750 \Rightarrow 7422$

2. Use exhaustive key search to decrypt the following cipher text, which was encrypted using shift cipher:

"BEEAKFYDJXUQYHYJIQRYHTYJQFBQDUYJIIKFUHCQD "

Ans: Here we apply 0-25 key for decrypting the given cipher text. If we found meaningful text then we stop decrypting. For decryption we use following formula,

$$\text{i.e., } d_k(x) = (y - k) \bmod 26$$

First we convert cipher text into corresponding integer sequence which are,

1 4 4 0 10 5 24 3 9 23 20 16 24 7 24 9 8 16 17 24 7
19 24 9 8 16 5 1 16 3 20 24 9 8 8 10 5 20 7 2 16 3

Next we subtract 1 from each value and apply modulo 26 then we get ,

0 3 3 25 9 4 23 2 8 22 19 15 23 6 23 8 7 15 16 23
6 18 23 8 7 15 4 0 15 2 19 23 8 7 7 9 4 19 6 1 15 2

Finally we convert the sequence of integers to alphabetic characters. They are,

addzjexciwtpxgxihpqqxgxihpeapctxihhjetgbpc

Similarly apply key 2,3,4.....

zccyidwbhvsowfwhgopwfrwhgodzobswhggidsfaob
ybbxhcvagurnvevgfnoveqvgfncynarvgffhcrezna
xaawgbuzftqmudufemnudpufembxmzqufeegbqdzmyz

.
. .
. .
. .
. .

At last applying key as 16 i.e, 'Q' then we get,

lookupintheairitsabirditsaplaneitssuperman

That's why the key is 16(Q) and plain text is

look up in the air its a bird its a plane its superman

3. Determine the number of key in affine cipher over Z_m for $m=30, 100$ and 1225 .

Ans:

$30 = 2 \times 3 \times 5$, so $\phi(30) = 1 \times 2 \times 4 = 8$. The affine cipher over Z_{30} and has $30 \times 8 = 240$ keys.

$100 = 2^2 \times 5^2$, so $\phi(100) = (2^2 - 2)(5^2 - 5) = 40$. The affine cipher over Z_{100} and has $100 \times 40 = 4000$ keys.

$1225 = 5^2 \times 7^2$, so $\phi(1225) = (5^2 - 5)(7^2 - 7) = 840$. The affine cipher over Z_{1225} and has $1225 \times 840 = 1029000$ keys.

4. (a) Suppose that π is the following permutation of $\{1, \dots, 8\}$:

x	1	2	3	4	5	6	7	8
$\pi(x)$	4	1	6	2	7	3	8	5

Compute the permutation π^{-1} .

(b) Decrypt the following ciphertext, for a *Permutation Cipher* with $m = 8$, which was encrypted using the key π :

TGEEMNELNNTDROEOAAHDOETCSHAEIRLM.

Ans:

a) \Rightarrow For π^{-1} we should interchange the given two rows and rearranging the column.

First interchanging given rows.

$\pi(x)$	4	1	6	2	7	3	8	5
x	1	2	3	4	5	6	7	8

Now rearrange into ascending order

$\pi(x)$	1	2	3	4	5	6	7	8
$\pi^{-1}(x)$	2	4	6	1	8	3	5	7

b) \Rightarrow given cipher is "TGEEMNELNNTDROEOAAHDOETCSHAEIRLM". Here $m=8$ so we partition into group of 8 letter.

TGEEMNEL | NNTDROEO | AAHDOETC | SHAEIRLM

Now each letter in group is replace according permutation $\pi(x)$. Then we get

GENTLEME | NDNOTRE | ADEACHOT | HERSMAIL

The final plain text is, "gentle men do not read each others mail"

5. Here is how we might cryptanalyze the Hill Cipher using a cipher text only attack. Suppose that we know that $m=2$. Break the cipher text into blocks of length two letters (diagrams). Each such diagrams are the encryption of a plain text diagrams and assume it in the encryption of a common diagrams for example, TH or ST. Each such guess, proceed as I the known plaintext attack, until the correct encryption matrix is found.

Here is a sample of cipher text to decrypt using this method

LMQETXYEAGTXCTUIEWNCTXLZEWUAISPZYVAPEWLMGQWYA
XFTCJMSQCADAGTXLMDXNXSNPJQSYVAPRIQSMHNOCVAXFV

Ans: Not competed

6. Suppose we are told that the plaintext “breathtaking” yields the Ciphertext "RUPOTENTOIFV" where the Hill Cipher is used (but m is not specified). Determine the encryption matrix.

Ans: The given plaintext is "breathtaking".

The cipher text is "RUPOTENTOIFV"

We know encryption method for hill cipher,

$$C=K*P$$

Where K is encryption matrix. Now encryption key K can be calculate by,

$$K= P^{-1} * C$$

P and C matrix can be formed by integer sequence of alphabet. Here we take only 9 alphabet for matrices.

$$P = \begin{bmatrix} 1 & 17 & 4 \\ 0 & 19 & 7 \\ 19 & 0 & 10 \end{bmatrix} \text{ and } C = \begin{bmatrix} 17 & 20 & 15 \\ 14 & 19 & 4 \\ 13 & 19 & 14 \end{bmatrix}$$

Now,

$$K = P^{-1} * C$$

$$= \begin{bmatrix} 1 & 17 & 4 \\ 0 & 19 & 7 \\ 19 & 0 & 10 \end{bmatrix} * \begin{bmatrix} 17 & 20 & 15 \\ 14 & 19 & 4 \\ 13 & 19 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 21 & 20 \\ 4 & 15 & 23 \\ 6 & 14 & 5 \end{bmatrix}$$

$$\text{That's why encryption matrix is } \begin{bmatrix} 3 & 21 & 20 \\ 4 & 15 & 23 \\ 6 & 14 & 5 \end{bmatrix}$$

7. Decrypt the following Ciphertext, obtained from the Autokey Cipher, by using exhaustive key search:

MALVVMAFBHBUQPTSOXALTGVWWRG

Ans: Here we apply 0-25(A-Z) key for decrypting the given cipher text. If we found meaningful text then we stop decrypting. For decryption we use following formula,

$$\text{i.e, } d_z(y) = (y - z) \bmod 26$$

Where z is the set of key stream and the initial value i.e, z_1 is key(K) itself.

First we convert cipher text into corresponding integer sequence which are,

12 0 11 21 21 12 0 5 1 7 1 20 16 15
19 18 14 23 0 11 19 6 21 22 22 17 6

We use key 0 i.e, 'A' for decipher the cipher text. First we generate key stream 'z' for the decryption. So that initial character of keystream is key itself.

$$Z = 0$$

Now we subtract last value of keystream from cipher text character and apply module 26. Then second value would be,

$$Z = 0 \ 12$$

Again subtract last value of keystream i.e, 12 from another value of the cipher text i.e, 0 and apply module 26. Then we get

$$Z = 0 \ 12 \ 14 \quad (\text{where } 0-12 = -12 \bmod 26 \Rightarrow 14)$$

Similarly, applying same method for all character of given cipher text then we get keystream as

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0 12 14 23 24 23 15 11 20 7 0 1 19 23
18 1 17 23 0 0 11 8 24 23 25 23 20 12
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Next we subtract keystream value from each values of cipher text and apply modulo 26 then we get ,

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12 14 23 24 23 15 11 20 7 0 1 19 23
18 1 17 23 0 0 11 8 24 23 25 23 20 12
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Finally we convert the sequence of integers to alphabetic characters. They are,
moxyxpluhabtxsbrxaaliyxxzum

Similarly apply key 1, 2, 3,.....

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zccyidwbhvsowfwhgopwfrwhgodzobswhggidsfaob
ybbxhcvagurnvevgfnoveqvgfncynarvgffhcrezna
xaawgbuzftqmudufemnudpufembxmzqufeegbqdyzmz
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At last applying key 19 i.e, 'T' then we get,

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lookupintheairitsabirditsaplaneitssuperman
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That's why the key is 19(T) and plain text is

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look up in the air its a bird its a plane its superman
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