# Simplex Method

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### Optimization & why?

In general optimization means the action of making the best or most effective use of a situation or resources.

Mathematical optimization problems deals with maximizing or minimizing some function relative to some set, often representing a range of choices available in a certain situation.

Application area of optimization: Manufacturing, Production, Inventory control, Transportation, Scheduling, Networks, Finance, Engineering, Mechanics, Economics, Control engineering, Marketing, Policy Modeling, etc

### Optimization technique

It is a technique to solve the given optimization problem to find its best optimal solution (value).

Different technique can be used to solve the optimization problem.

Some of them are:

- 1. Graphical method
- 2. Simplex method

### Some Terminologies

- **1. Linear Programming Problems(LPP):** process which takes certain linear relationships to obtain best possible solution.
- 2. Slack Variable: In any lpp, if a constraint has lesser than or equal to value of sign then in order to make it equal, we need to add something positive to LHS.
- **3. Surplus Variable:** If a constraint has greater than or equal to value of sign then in order to make it equal, we need to subtract something positive to LHS.
- **4. Unrestricted variable:** Any variable  $x_i$  which takes either positive, negative or zero values is called as unrestricted variables.

#### Canonical form of LPP

It is said to be in canonical form if it has the following characteristics.

- -Objective function is of maximization / minimization type.
- -All constraints are of  $\leq$  /  $\geq$  type.
- -All decision variables are of  $\geq 0$ .

The canonical form is:

Max 
$$Z = 3x_1 + 2x_2 + 7x_3$$
  
 $-6x_1 + 2x_2 - 5x_3 \le -5$   
 $-x_1 + 3x_2 - 4x_3 \le 3$   
 $x_1, x_2, x_3 \ge 0$ 

#### Standard form of LPP

A general LPP is said to be in standard form if it has the following characteristics.

- -RHS of each constraint is positive.
- -All constraints are of = type.
- -All decision variable are of  $\geq 0$ .

The Standard form is:

Max 
$$Z = 2x_1 - 3x_2 + 6x_3$$
  
 $x_1 - 3x_2 - s_1 = 4$   
 $2x_1 - 8x_2 + 3x_3 + s_2 = 4$   
 $-x_1 - x_2 + s_3 = 7$   
 $x_1, x_2, x_3, s_1, s_2, s_3 \ge 0$ 

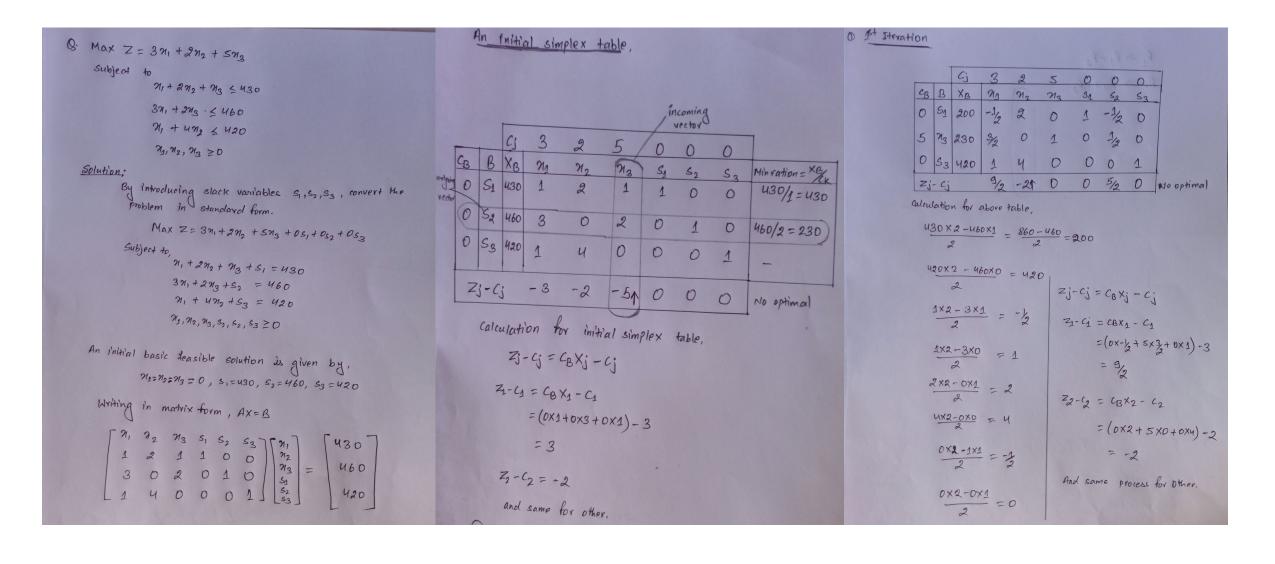
### Simplex Method Algorithm

- 1. Convert the given general LPP into standard LPP.
  - -Objective function of Lpp must be maximized. If it is to be minimized then we have to convert it into a problem of maximization by Max  $Z'=-Min\ Z$
  - -Check all the decision variables are greater than zero.
  - -Express the problem in standard form by introducing slack or surplus variable to convert the inequality constraints into equation.
  - -All the values of right hand side must be positive.
- 2. Write the values of initial basic feasible solution.
- 3. Write the standard form Lpp into matrix form.
- 4. Construct the initial simplex table.
- 5. Calculate the value of  $Z_j C_j = C_B X_j C_j$

### Simplex Method Algorithm

- I. If all  $(Z_i C_i) \ge 0$ , the optimal solution will obtained.
- II. If at least one  $(Z_j C_j)$  is –ve then indicate it by an arrow and this column is called key column.
- III. If more than one  $(Z_i C_i)$  is –ve then choose the most negative of them and this
- IV. column is called key column.
- 6. Calculate minimum positive ratio. ie min.ratio =  $\frac{X_B}{C_k}$ ,  $C_k$  = key column, > 0
- 7. Construct the new simplex table by entering incoming vector.
- 8. Repeat step 5,6.

### Simplex Method Maximization Example



		Ci	3	2	5	0	0	0	1 ×8
Co	B	Xa	211	[212]	Na	SA	So	53	Min-ratio = * Bex
0	(51	200	-1/2	2	0	1	-1/2	0	20/2 = 100)
5	nz	230	3/2	0	1	0	1/2	0	-
0	53	420	1	ч	0	0	0	1	42% = 105
	zj-cj		9/2	-29	0	0	5/2	0	

#### 2 Second Iteration.

								-
	cj	3	2	5	0	0	0	
CB B	XB	211	212	213	Sn	52	53	
2 7/2	100	-1/4	1	0	1/2	-1/4	0	T
5 n3	230	3/2	0	1	0	1/2	0	
0 53	20	2	0	0	-2	1	1	
Zj-C	i	4	0	0	1	2	0	10

Calculation for above table,

$$\frac{420 \times 2 - 200 \times 4}{2} = 20$$

$$\frac{1 \times 2 - (-\frac{1}{2}) \times 4}{2} = 2$$

$$\frac{0 \times 2 - 4 \times 1}{2} = -2$$

$$\frac{0 \times 2 - 4 \times 1}{2} = 1$$

$$\frac{1 \times 2 - 4 \times 0}{2} = 1$$

$$Z_{j}-C_{j}=C_{B}X_{j}-C_{j}$$

$$Z_{1}-C_{1}=C_{B}X_{1}-C_{1}$$

$$=(2X-)_{4}+5\cdot3_{2}+0\times2)-3$$

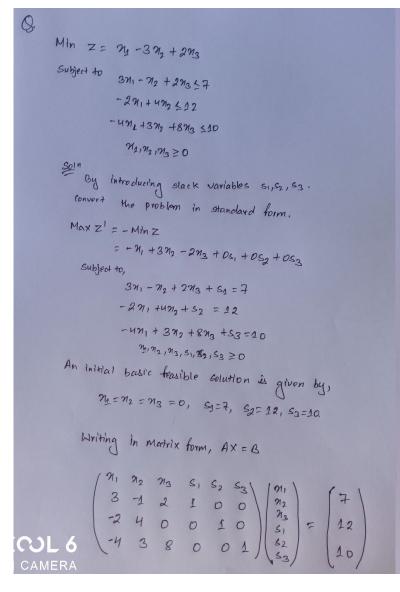
$$=4$$

$$Z_{2}-C_{2}=(BX_{2}-C_{2}$$

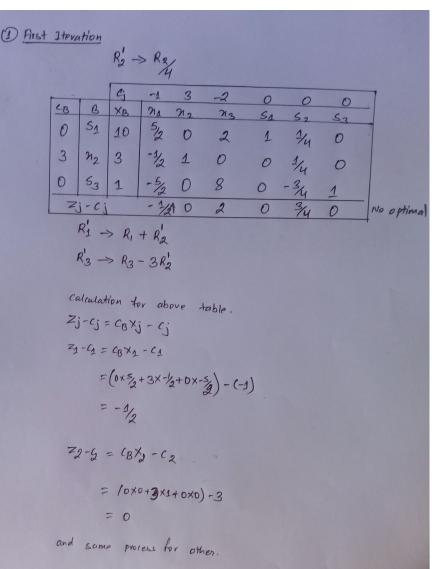
$$=(2\times1+5\times0+0\times0)-2$$
and some for other.

Since all zj-cj zo, the solution is optimum and given by  $N_2 = 100$ ,  $N_3 = 230$ ,  $N_1 = 0$ Max Z = GXB  $=(2\times100+5\times230+0\times20)$ = 1350

### Simplex Method Minimization Example



	itial sim	Trex to	18(6							
					incor	ning v	recto	٧		
	C. 10	Gi	-1	3/	-2	0	0	0		
	CB B 0 S1	XB	211	12	213	SA	52	53	Min ratios XB	
utgoing-	0 52	7	3	THE BELLEVILLE OF	2	Mineral Land Company	A DESIGNATION OF THE PERSON OF		121 - 0	
ector				4	O	0	1	0	12/4 = 3	
	0 53	10	-4	3	8	0	0	1	10/3 = 3.33	
	Zj-Cj 1 -31 2 0 0 0 No optimal									
	calculation for initial simplex table.									
						., 10	10 (6.			
	Zj-G=									
	21-61=	G X1	- 4							
	=	lox3.	+ 0 x -	2+0)	K-4)*	(-1)				
		= 1								
	Z2-C2	= Cp	X2 -	(						
		=(0x1	-1)+0	DX4+	0x3	1 - 2				



		1							
	,	Cj	-1	3	-2	0	0	0	
GB	B	XB	2/2	2	M3	SI	52	53	Min-Ratio = XB/
Co	SI	10	15/2	0	2	1	1/4	0	10/5/9=4)
3	212	3	-1/2	1	0	0	1/4	0	-
0	53	1	-5/2	0	8	0	-3/4	1	-
- 3-G			-1/21	0	2	0	3/4	0	

#### @ Second Iteration

			Ta						
t			14	-1	3	-2	0	0	-
1	B	B	XB	24	7/2	23	SA	6	0
	1	24	14	1	0	4/5	2	2/10	<u>S3</u>
3		2/2	5	0	1		15		0
10		52			_	2/5	75	3/10	0
-	1	3	11	0	0	10	1	-1,	
21-61				0	0	13/2	110	1/2	1
	RI	_	0 .			15	1/5	1/5	0

optimal

 $R_2 \rightarrow R_2 + \frac{1}{2}R_1$   $R_3 \rightarrow R_3 + \frac{5}{2}R_1$ Calculation  $Z_j$ - $C_j$  for above table  $Z_j$ - $C_j = C_B X_j - C_j$ 

$$Z_1 - C_1 = C_B \times_1 - C_1$$
  
=  $(-1 \times 1 + 3 \times 0 + 0 \times 0) - (-1)$   
= 0

$$73^{-1}3 = (-1 \times \frac{1}{5} + 3 \times \frac{3}{5} + 0 \times 10) - 2$$
  
=  $1\frac{3}{5}$ 

and same process for others.

Since, all zj-Cj zo, then solution is optimum.

The optimal solution is given by,  $Max 2' = C_B X_B$   $= (-1 \times U + 3 \times S + 0 \times 11)$ 

$$= -11$$

COL 6:  $n_1 = 4$ ,  $n_2 = 5$ ,  $n_3 = 0$ 

CAMERA

## Thank you!!!