Lecture-18
Algorithmic Mathematics(CSC545)

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Chapter-9

Discrete Random Variables and Probability Distributions

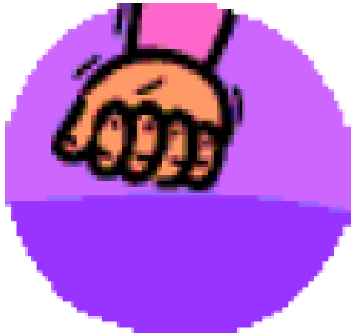
Probability

In a given situation, we would like to be able to calculate the probability of something. To do this we need to make our discussion more formal. We begin with an *experiment* which is simply an activity that results in a definite outcome, which is called sample space.

- The sample space S of statistical experiment is the set of all possible outcomes of an experiment.

For example, consider a set of six balls numbered 1, 2, 3, 4, 5, and 6. If we put the six balls into a bag and without looking at the balls, we choose one ball from the bag, then, this is an experiment which has 6 possible outcomes i.e.

$$S= \{1,2,3,4,5,6\}$$



Random Variables

- A random variable is a numerical value determined by the outcome of an experiment that varies from trial to trial.

- A statistical experiment is any process by which several measurements are obtained.

- Consider a random experiment in which a coin is tossed three times. Let x be the number of heads. Let H represent the outcome of a head and T the outcome of a tail.

- The possible outcomes for such an experiment will be:

  TTT, TTH, THT, THH,
  HTT, HTH, HHT, HHH.

- Thus the possible values of x (number of heads) are

x=0:	TTT	$P(x=0) = 1/8$
x=1:	TTH, THT, HTT	$P(x=1) = 3/8$
x=2:	THH, HTH, HHT	$P(x=2) = 3/8$
x=3:	HHH	$P(x=3) = 1/8$

If the coin is fair

- From the definition of a random variable, x as defined in this experiment, is a *random variable*.

Two different classes of random variables:

1. **A discrete random variable**
2. **A continuous random variable**

- A discrete random variable is a quantitative random variable that can take on only a finite number of values or a countable number of values.

Examples:

- The number of children per family
- The number of cavities a patient has in a year.
- The number of bacteria which survive treatment with some antibiotic.
- The number of times a person had a cold in Gaza Strip.

- **A continuous random variable is a quantitative random variable that can take infinite number of values within an interval**

Example: the amount of rainfall in during the month of January.

Probability distribution

- A probability distribution is the listing of all possible outcomes of an experiment and the corresponding probability.

- Depending on the variable, the probability distribution can be classified into:

  - Discrete probability distribution
  - Continuous probability distribution

Discrete probability distribution

- A discrete probability distribution is a table, graph, formula, or other device used to specify all possible values of a discrete random variable along with their respective probabilities.

Examples:

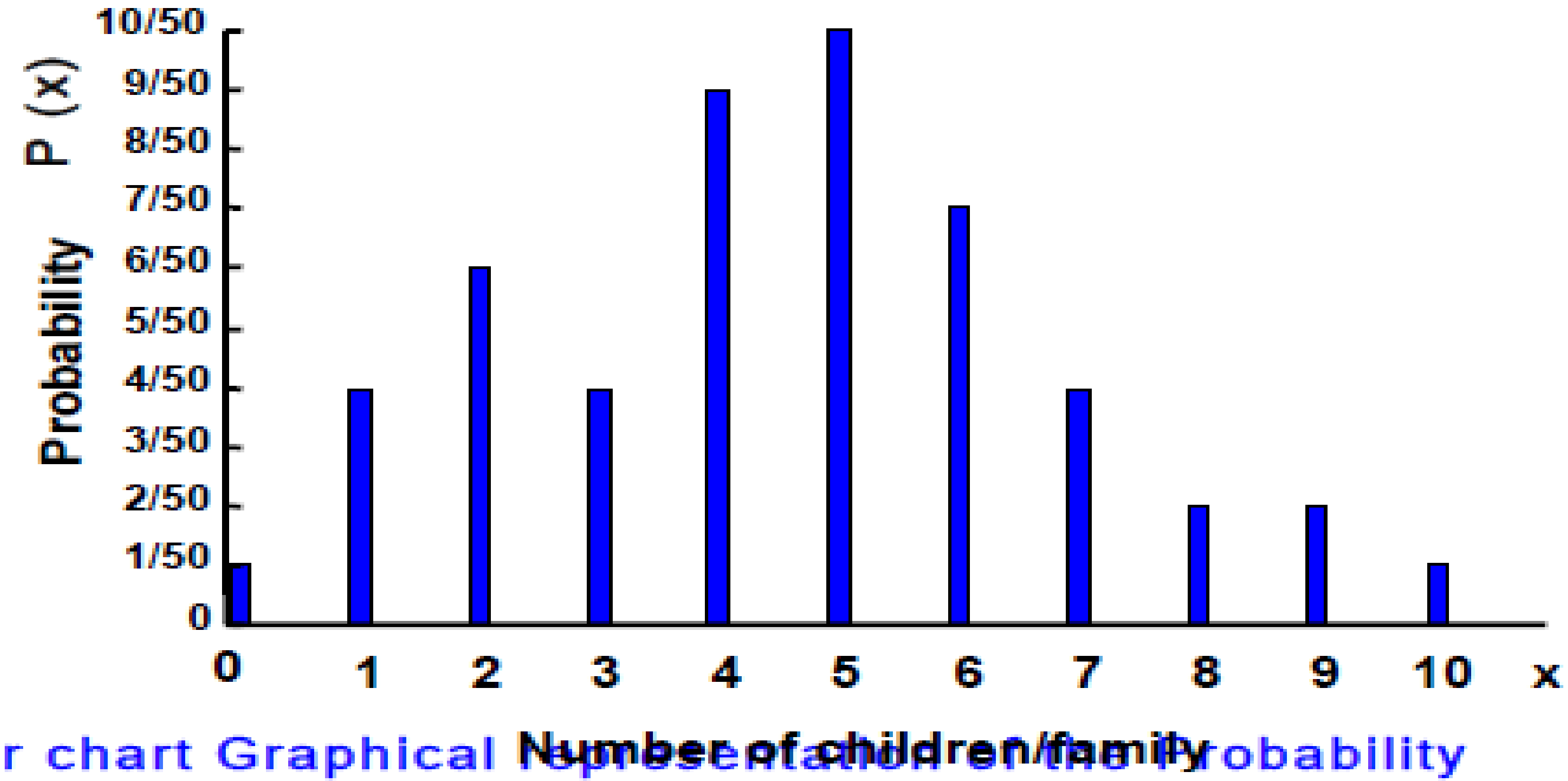
- The number of children per family
- The number of cavities a patient has in a year.
- The number of bacteria which survive treatment with some antibiotic.
- The number of times a person had a cold in Gaza Strip.

Discrete probability distribution

Probability distribution of number of children per family in a population of 50 families

x	Frequency of occurring of x	P(X=x)
0	1	1/50
1	4	4/50
2	6	6/50
3	4	4/50
4	9	9/50
5	10	10/50
6	7	7/50
7	4	4/50
8	2	2/50
9	2	2/50
10	1	1/50
	50	50/50

Discrete probability distribution



Bar chart Graphical representation of the Probability distribution of number of children per family for population of 50 families

Discrete probability distribution

Toss of Two Coins		Roll of a Die		Sex of Three-child Family	
E	*P*(E)	E	*P*(E)	E	*P*(E)
HH	1/4	1	1/6	3 boys•	0.125
HT	1/4	2	1/6	2 boys. 1 girl	.375
TH	1/4	3	1/6	1 boy, 2 girls	.375
TT	1/4	4	1/6	3 girls	.125
	1.0	5	1/6		1.000
		6	1/6		
			1.0		

Continuous probability distribution

- A continuous probability distribution can assume an infinite number of values within a given range – for variables that take continuous values.

  - The distance students travel to class.
  - The time it takes an executive to drive to work.
  - The length of an afternoon nap.
  - The length of time of a particular phone call.

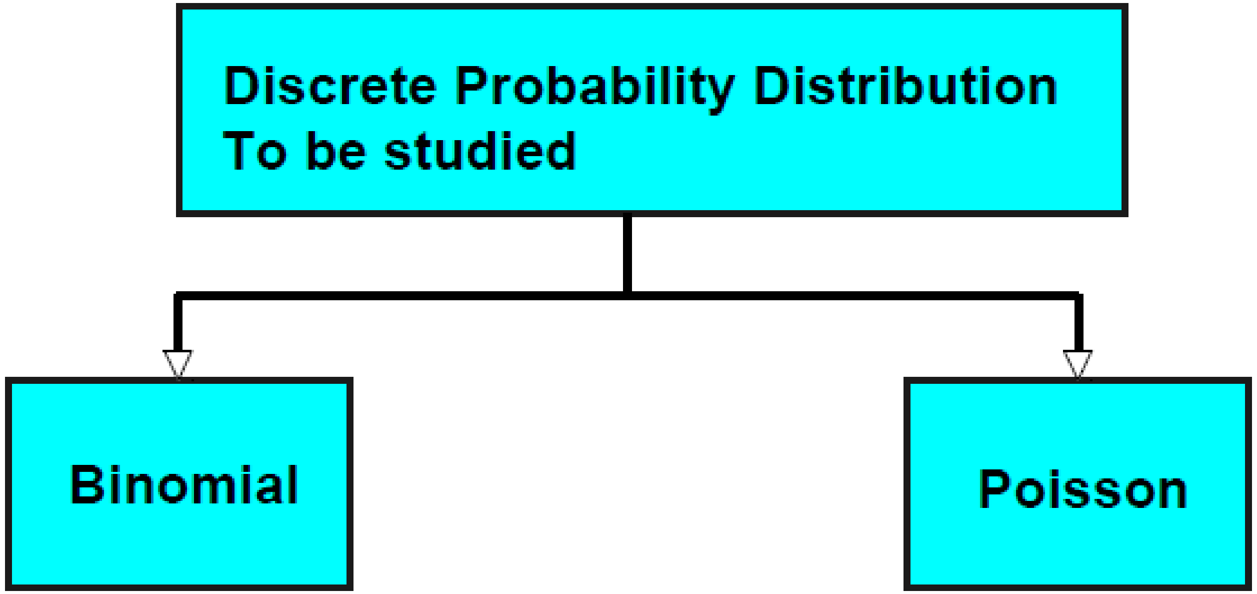
Features of a Discrete Distribution

- The main features of a discrete probability distribution are:
  - The probability of a particular outcome, $P(X_i)$, is between 0 and 1.00.
  - The sum of the probabilities of the various outcomes is 1.00. That is,
    $$P(X_1) + \ldots + P(X_N) = 1$$
  - The outcomes are mutually exclusive. That is,
    $$P(X_1 \text{ and } X_2) = 0 \text{ and}$$
    $$P(X_1 \text{ or } X_2) = P(X_1) + P(X_2)$$

Probability Distributions

Important Probability Distributions:

- **The Binomial Distribution**
- **The Poisson Distribution**
- The Hypergeometric Distribution
- **The Normal Distribution**
- The t Distribution
- The χ^2 Distribution
- The F Distribution


Discrete Probability Distribution
To be studied
Binomial
Poisson

Binomial Distribution

A *binomial* experiment has the following conditions:

1. There are *n* repeated trials

- 2. Each trial has only two possible outcomes-success or failure, girl or boy, sick or well, dead or alive, at risk or not at risk, infected–not infected, white–nonwhite, or simply positive–negative etc

1 1 0 1 0 0 1 0 0 0 0 0 1 0 0 0 00 0 0
1 1 0 0 0 0 0 1 0 1 1 0 0 0 0 1 0 0 1 0

3. The probabilities of the two outcomes remains constant from trial to trial.

The probability of success denoted by p.

The probability of a failure, $1-p$, is denoted by q.

Since each trial results in success or failure,

$$p + q = 1 \text{ and } q = 1 - p.$$

4. The outcome of each trial is independent of the outcomes of any other trial; that is, the outcome of one trial has no effect on the outcome of any other trial.

5. When the conditions of the binomial experiment are satisfied, Our interest is in the number of successes *r* occurring in the *n* trials.

Example

If a certain drug is known to cause a side effect 10% of the time and if ***five*** patients are given this drug, what is the probability that ***four*** or more experience the side effect?

More Examples

- **# Defective items in a batch of 5 items**
- **# Correct on a 33 question exam**
- **# Customers who purchase out of 100 customers who enter store**

The Binomial probability formula

- The probability of obtaining *r* successes in *n* trials with a probability *p* of success in each trial can be calculated using the formula;

$$p(r\ success) = \left(\frac{n!}{r!(n-r)!}\right) p^r \cdot q^{n-r}$$

where

- n = the number of trials in an experiment
- r = the number of successes
- $n - r$ = the number of failures
- p = the probability of success
- $q = 1 - p$, the probability of failure

$$p(r\ success) = \left(\frac{n!}{r!(n-r)!}\right) p^{r} \cdot q^{n-r}$$

"!" represents the factorial function.

$n! = (n)(n - 1)\ (n - 2)(n - 3) \ldots (1)$. For example,

$4! = (4)(3)(2)(1) = 24$

By definition, $0! = 1$

Binomial Distribution

Example

- What is the probability of having two girls and one boy, 3 boys, and at least one boy in a three-child family if the probability of having a boy is 0.5?

- Solution:

From the calculations in equation

$$p(r\ success) = \left(\frac{n!}{r!(n-r)!}\right) p^{r} . q^{n-r}$$

It can be seen that

Binomial Distribution

For two girls and one boy n = 3, r = 2, p = 1/2, q = 1/2

$P(2G, 1B) =$ $= 3(.125) = .375$

For 3 boys

$$p(r\ success) = \left(\frac{n!}{r!(n-r)!}\right) p^r . q^{n-r}$$

a. considering $n = 3$, $r = 3$, i.e. boy is the success

$P(3B) =$ $= 1 X (.125) X 1 = .125$

b. considering $n = 3$, $r = 0$, i.e. girl is the success or boy is the failure

$P(3B) =$ $= 1 X 1 X (.125) = .125$

For at least 1 boy

$P(3B)) + P(2B, 1G) + P(1B, 2G) = 0.125 + 0.375 + 0.375 = 0.875$

Binomial Distribution

$$p(r\ \text{success}) = \left(\frac{n!}{r!(n-r)!}\right) p^r \cdot q^{n-r}$$

Example:

Ten individuals are treated surgically. For each individual there is a 70% chance of successful surgery. Among these 10 people, the number of successful surgeries follows a binomial distribution with n =10, and p =0.7.

What is the probability of exactly 5 successful surgeries?

n = 10, r = 5, p = 0.7, q = 1 – p = 0.3

$$p(r=5) = \left(\frac{10!}{5!(10-5)!}\right) 0.7^5 \cdot 0.3^{10-5}$$

$$= (252)(0.7^5)(0.3^5) = 0.1029$$

Poisson Probability Distribution

The Poisson probability distribution is often used to model a discrete variable which is applied to experiments with *random* and *independent* occurrences of an event.

The occurrences are considered with respect *to* *(per unit)*

a time interval,

a length interval,

a fixed area

or a particular volume.

Poisson Random Variable

- 1. Our interest is in the Number of events that occur in

  Time interval, length, area, volume

Examples

- The number of serious injuries in a particular factory in a year.
- The number of times a three-old child has an ear infection in a year.
- Number of blood cells in unit area of haemocytometer.
- The number of live insects per square after spraying a large area of land with an insecticide and counting the number of living insects in a randomly selected squares.
- # Customers arriving in 20 minutes
- the number of colonies growing in 1 ml of culture medium.

Formula of Poisson Probability Distribution

The Poisson distribution can be described mathematically using the formula:

$$P(r) = \frac{\lambda e^{-\lambda}}{r!}$$

where

λ (Greek letter lambda) is the mean number of occurrence of an event in a particular interval of time, volume, area, and so forth.

e is the the base of natural logs = 2.71828.

r is the number of occurrence of event (r = 0, 1, 2, 3, ...) in corresponding interval of time, volume, area, and so forth.

EXAMPLE

- An urgent Care facility specializes in caring for minor injuries, colds, and flu. For the evening hours of 6-10 PM the mean number of arrivals is 4.0 per hour. What is the probability of 2 arrivals in an hour?

$$P(r) = \frac{\lambda^r e^{-\lambda}}{r!} = \frac{4^2 e^{-4}}{2!} = .1465$$

Thank You

Any Query??