Euclidean Heuristic Optimization

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Introduction

Problem

- Improve the efficiency of finding shortest paths between arbitrary points in a search graph
- Example applications: road networks, virtual worlds

Approach

- Assign each state i in the input graph a point $y_i \in \mathbb{R}^d$
- Use interpoint distances as heuristics for distances in the graph
- Arrange the points in such a way as to minimize the error between the estimated distances and the true distances

Contributions

- Link between heuristic construction and manifold learning
- Promising empirical results on a range of test domains

Euclidean Heuristics

A Euclidean heuristic is a heuristic function h for any state pair that can be computed from distances between points:

$$h(i,j) = \|y_i - y_j\|$$

The arrangement of the points Y defines h.

An optimal Euclidean heuristic minimizes the loss $\mathcal{L}$ between the true distances $\delta(i,j)$ and the heuristics given by the points Y:

$$\begin{aligned} &\underset{Y}{\textbf{minimize}} && \mathcal{L}(Y) \\ &\textbf{subject to} && Y \text{ is admissible and consistent} \end{aligned}$$

Constraints

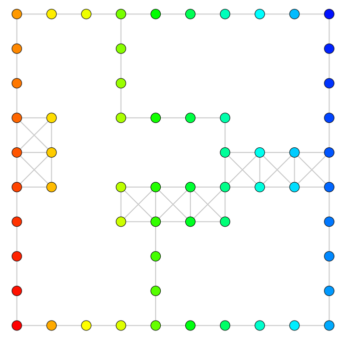
Theorem

Y is admissible between adjacent states

$\Updownarrow$

Y is admissible and consistent

Passino and Antsaklis, 1994



(one constraint per edge)

The Loss Function

Favor larger errors by squaring terms, but admit a weight W_{ij} on the relative importance of each pair (i,j)

$$\mathcal{L}(Y) = \sum_{i,j} W_{ij} \left| \delta(i,j)^2 - \|y_i - y_j\|^2 \right|$$

Admissibility ($\delta(i,j)^2 \geq \|y_i - y_j\|^2$) permits a simpler loss:

$$\mathcal{L}(Y) = \sum_{i,j} W_{ij} \left(\delta(i,j)^2 - \|y_i - y_j\|^2 \right) \equiv -\sum_{i,j} W_{ij} \|y_i - y_j\|^2$$

The resulting optimization problem:

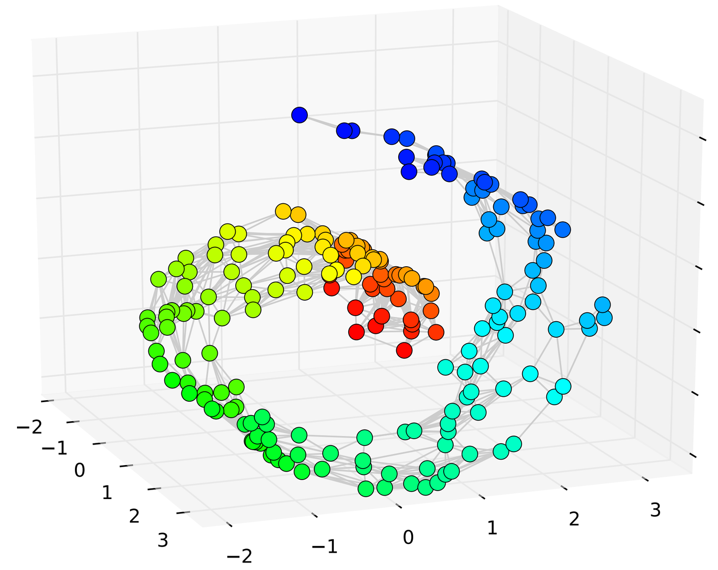
$$\begin{aligned} &\underset{Y}{\textbf{maximize}} && \sum_{i,j} W_{ij} \|y_i - y_j\|^2 \\ &\textbf{subject to} && \forall (i,j) \in E \ \|y_i - y_j\| \leq \delta(i,j) \end{aligned}$$

Connections

Manifold Learning

This is a weighted generalization of MVU (Weinberger *et al.*)

- Nonlinear dimensionality reduction
- MVU's semidefinite reformulation renders the optimization tractable



Differential Heuristics

W can be parametrized to reproduce differential heuristics:

$$W_{\text{diff}} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & (\frac{-1}{n-1}) & (\frac{-1}{n-1}) \\ 1 & (\frac{-1}{n-1}) & 0 & (\frac{-1}{n-1}) \\ 1 & (\frac{-1}{n-1}) & (\frac{-1}{n-1}) & 0 \end{bmatrix}$$

Suppose pivot is state 1:

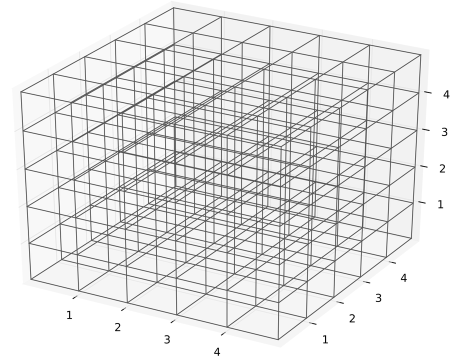
- Push points away from pivot
- Pull points into each other

Experiments

Comparison against differential heuristics using A*:

Cube World

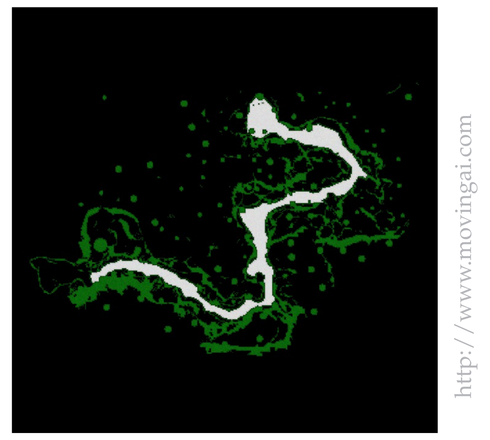
- $20 \times 20 \times 20 = 8{,}000$ states
- Agent increments any/all coordinates by 1; transition costs are the edge lengths
- Multi-dimensional search spaces target a weakness of differential heuristic



Video Game Maps

- 168–6,240 states
- Octile connectivity: diagonals cost 1.5
- Maps are like corridors to which differential heuristics are well suited; a competing W^{+}_{diff} is introduced:

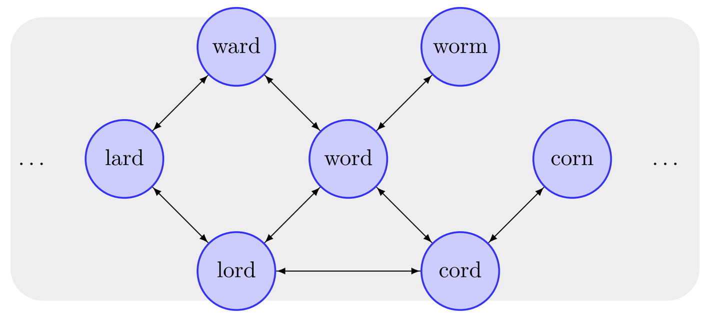
$$W^{+}_{\text{diff}} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & \epsilon & \epsilon \\ 1 & \epsilon & 0 & \epsilon \\ 1 & \epsilon & \epsilon & 0 \end{bmatrix} \quad (\epsilon = 10^{-3})$$



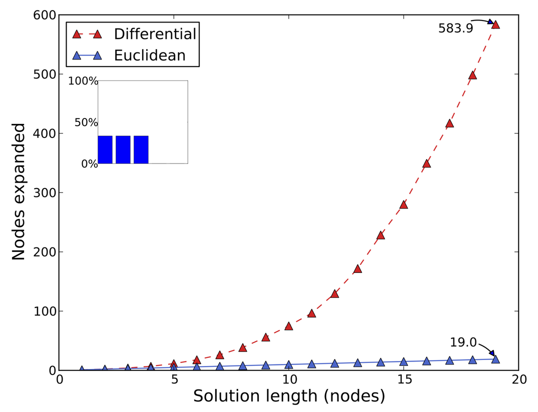
Dragon Age: Origins (BioWare)

Four-Letter Word Graph

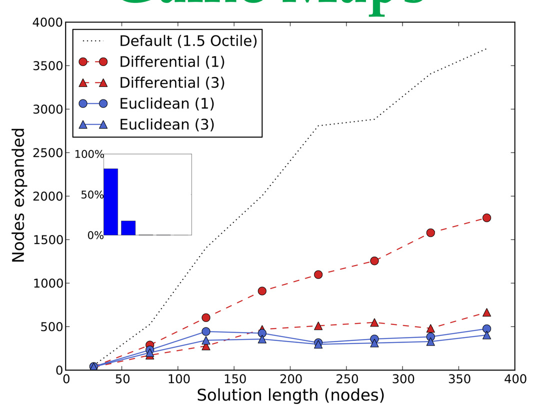
- Connected graph of 4,820 words
- Agent changes 1 letter per step
- High dimensional domain



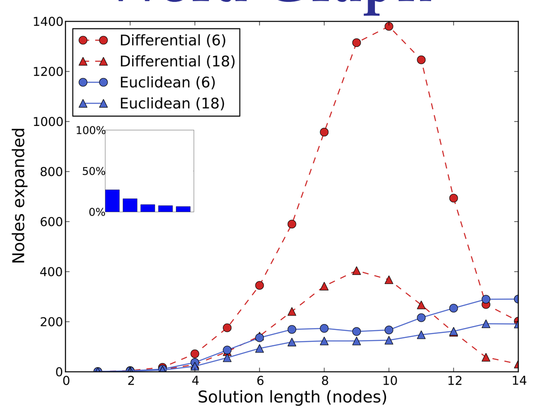
Cube World



Game Maps



Word Graph



- Bar plots: variance in each dimension of the uniformly weighted embedding
- Optimal Euclidean heuristics show promise, storing more in less memory

Thank you

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Anonymous reviewers
NSERC and iCore

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