Ray Tracing & Ray Casting

Realistic Graphics Inpsired by Nature

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Motivation

Why Learn This?

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Elsa's Castle in Frozen

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Cyberpunk 2077 with RTX

- **Realistic graphics** of your favourite animated movies are the result of ground-breaking work in Ray Tracing by studios like Disney, Pixar, and DreamWorks. Do you know these films take years to render? 30 hours per frame!

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- Lately, **RTX** is all the rage in gaming. New titles boast ray-tracing effects in real-time, not 30 hours per frame!

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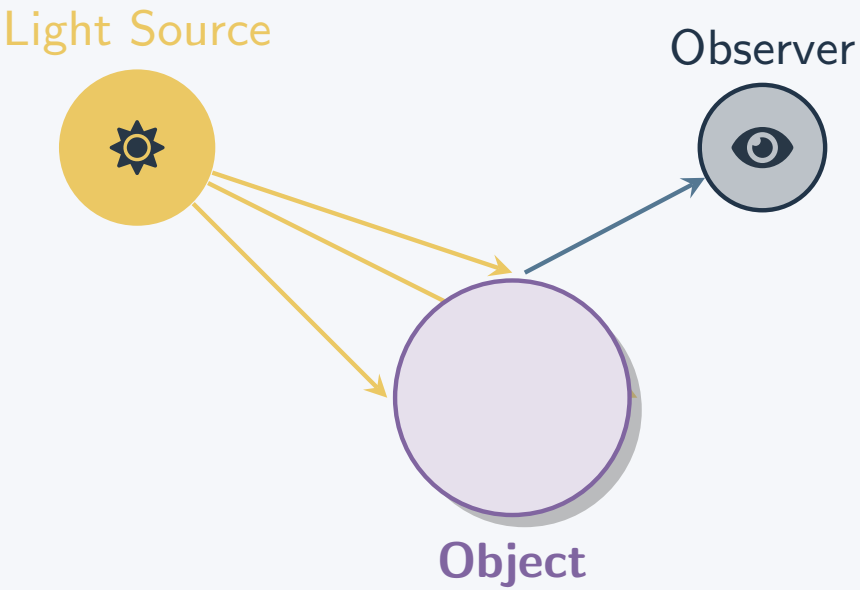
Cyberpunk 2077 with RTX

- **Realistic graphics** of your favourite animated movies are the result of ground-breaking work in Ray Tracing by studios like Disney, Pixar, and DreamWorks. Do you know these films take years to render? 30 hours per frame!
- Lately, **RTX** is all the rage in gaming. New titles boast ray-tracing effects in real-time, not 30 hours per frame!
- It's fun! You will know when you create your first ray-traced image!

The Story of Light

How Do We See?



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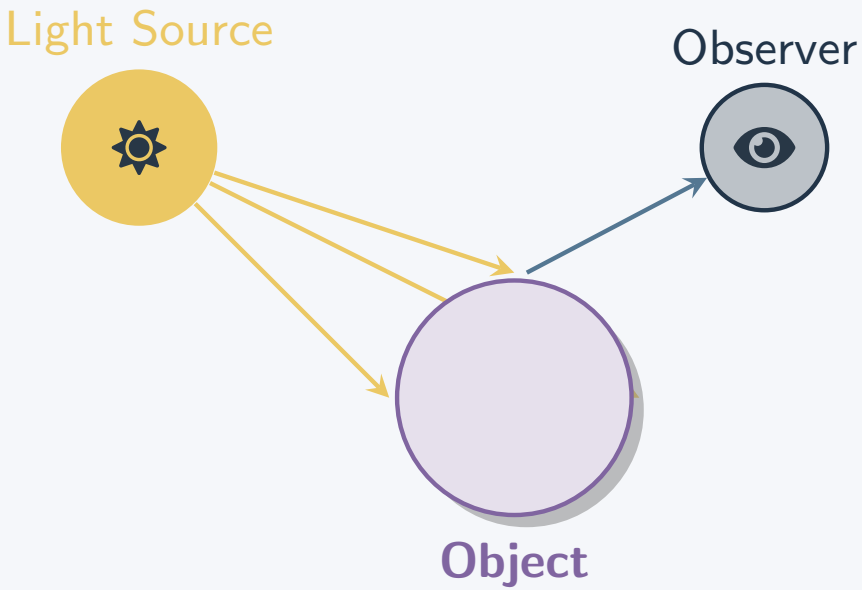
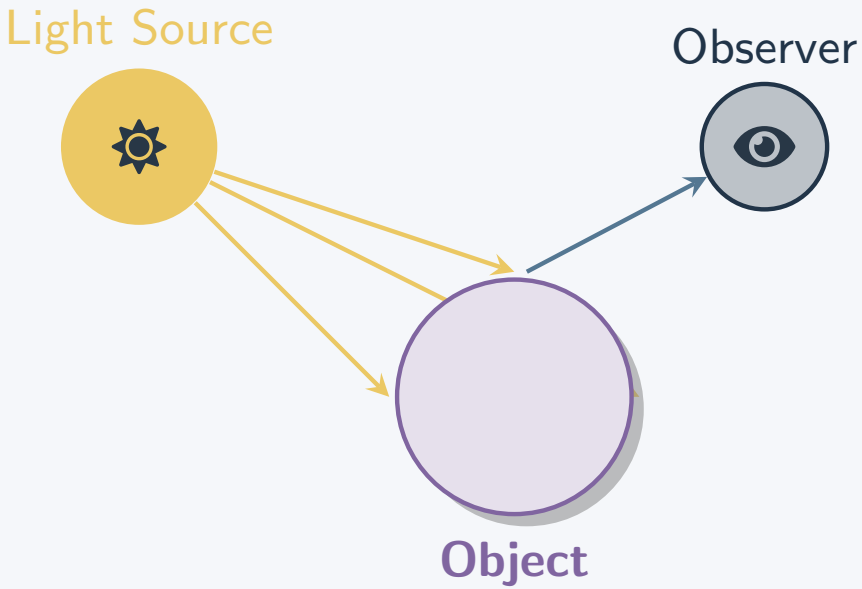


Natural Process

1. Light travels from source
2. Light hits objects
3. Light bounces to our eyes
4. Our brain interprets the signal

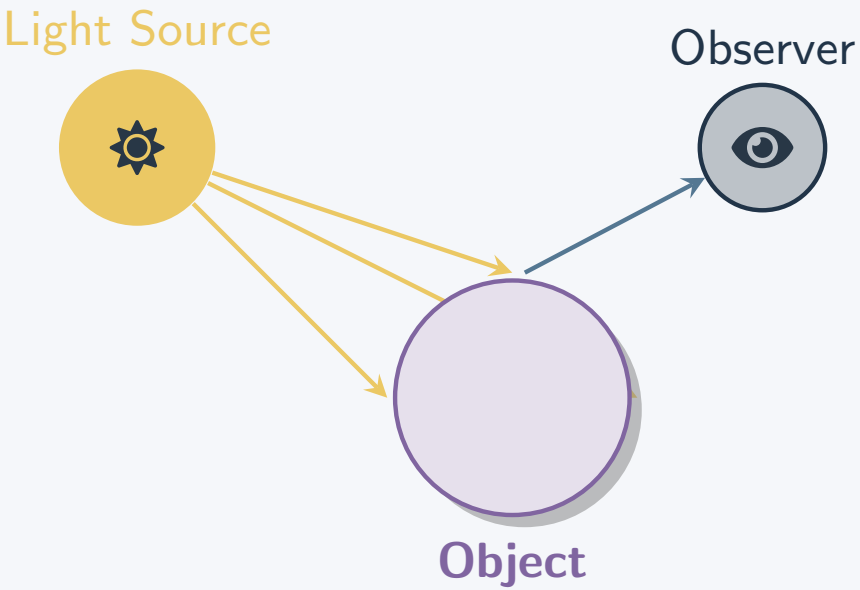
How Do We See?



Physical Process

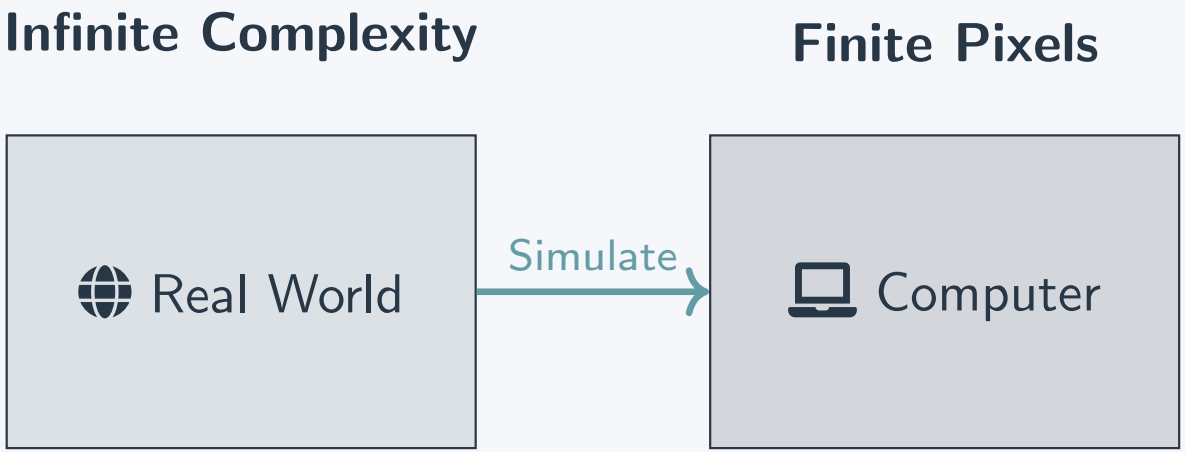
1. Photon is emitted from source
2. Photon hits objects
3. Part of the photon is reflected or absorbed
4. The reflected photons reach our eyes
5. The rods and cones in our retina detect the photons
6. Our brain interprets the signal
7. **Colour**: The wavelength of the photons
8. **Brightness**: The number of photons

How Do We See?



Question: How do we simulate this?

The Computer Graphics Challenge



Challenges:

- Infinite light rays/photons
- Complex physics
- High computational cost

Ray Casting: Foundation

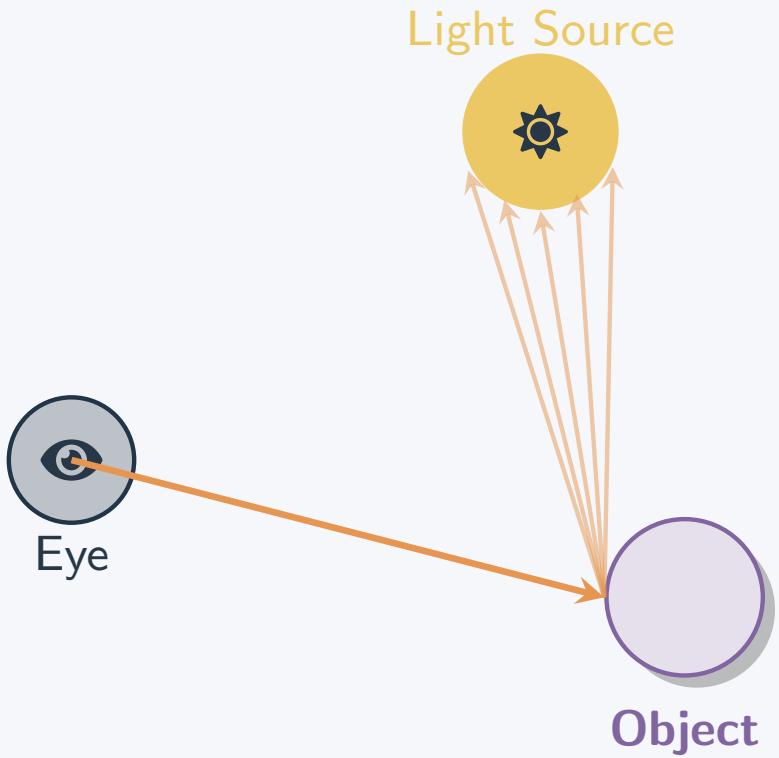
The Key Insight

1. Reverse Engineering

Instead of following light rays from light sources —

Let's trace backwards!

Shoot rays from the eye, find where it hits and find out how much light reaches there.



This is the opposite of what happens in reality. **Why does this work?**

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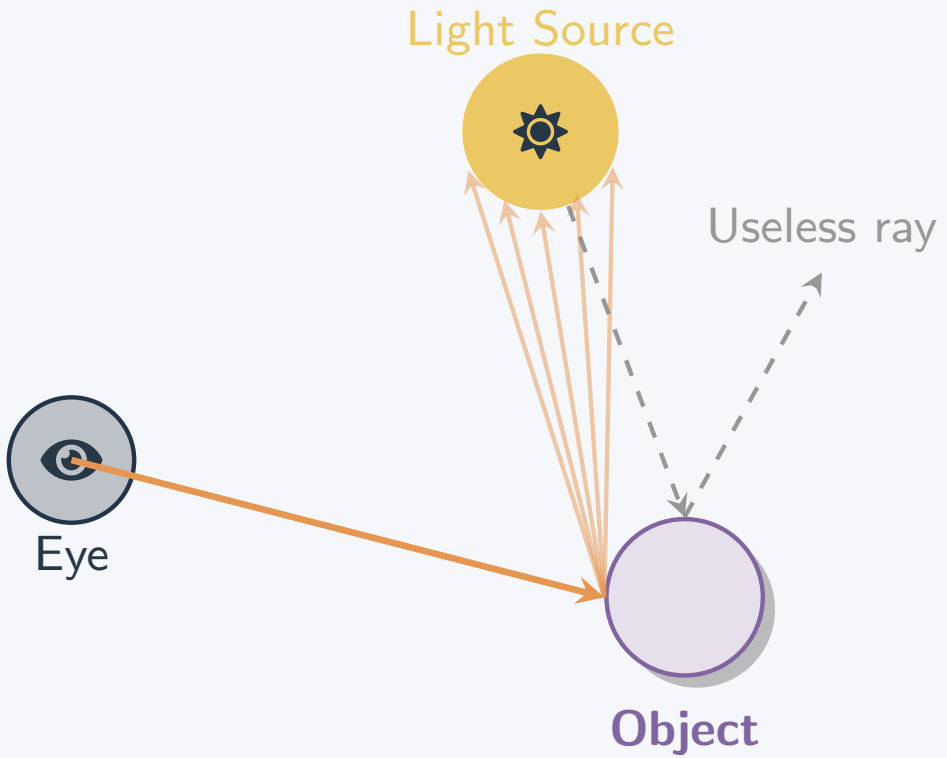

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- Most light never reaches our eyes

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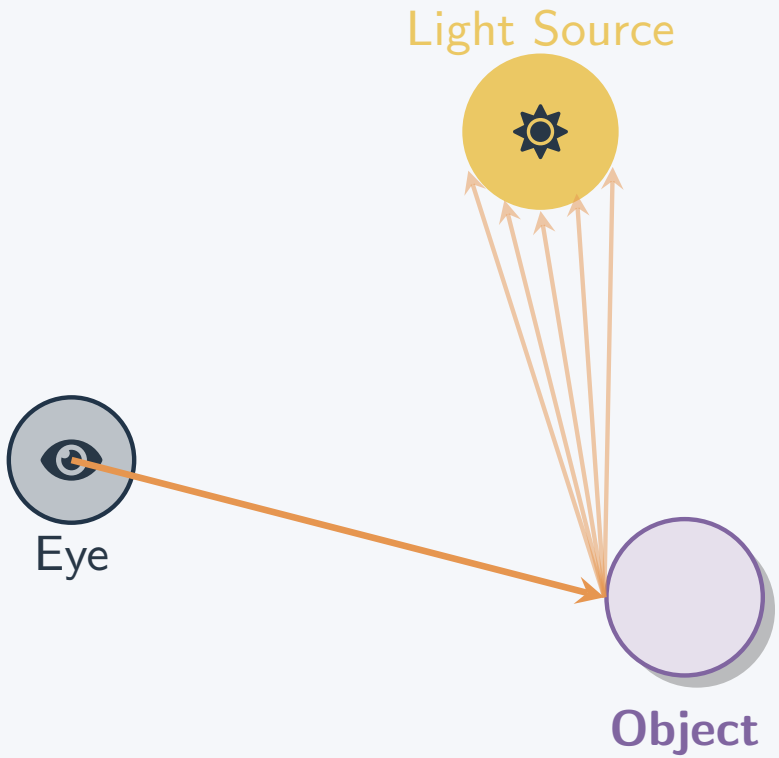

This is the opposite of what happens in reality. **Why does this work?**

- Most light never reaches our eyes
- Only trace rays that matter
- Much more efficient!

From Infinite Rays to Finite Pixels

2. Cutting Costs

Instead of tracing infinite rays —

Trace one ray per pixel.

This comes with little tradeoff, because:

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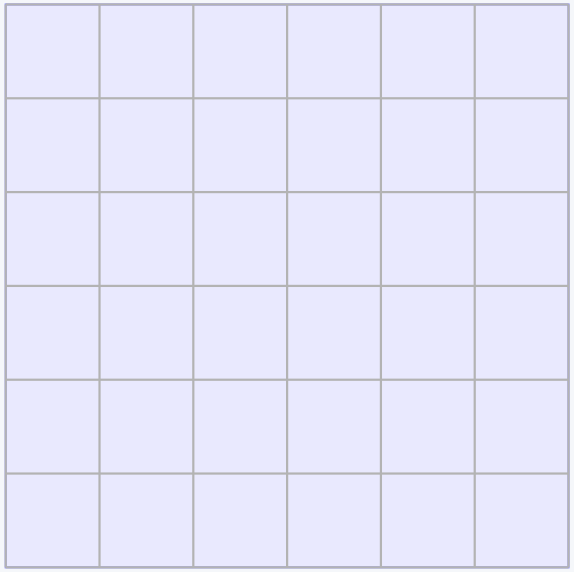

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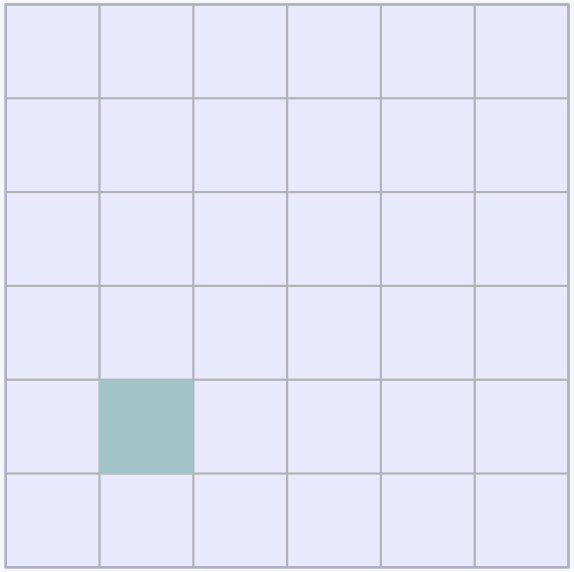

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- Each pixel can only be of one color

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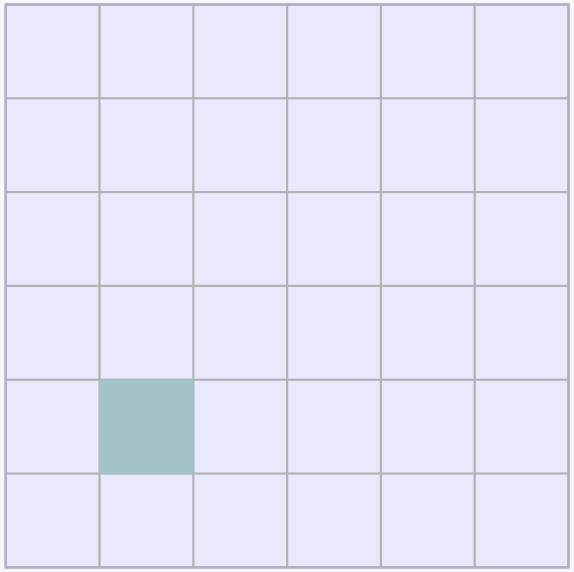

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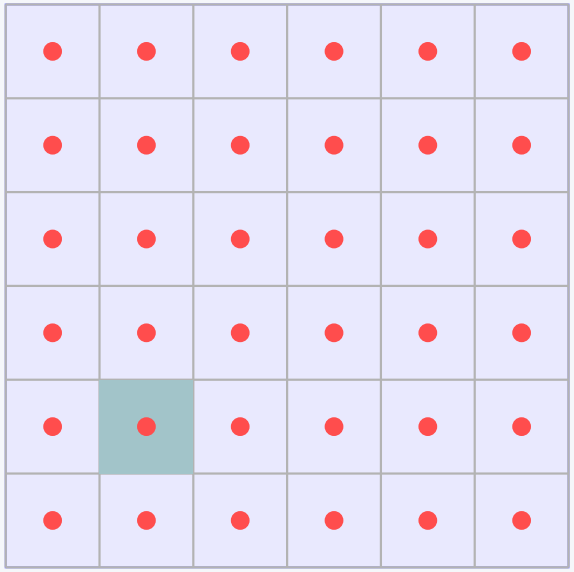
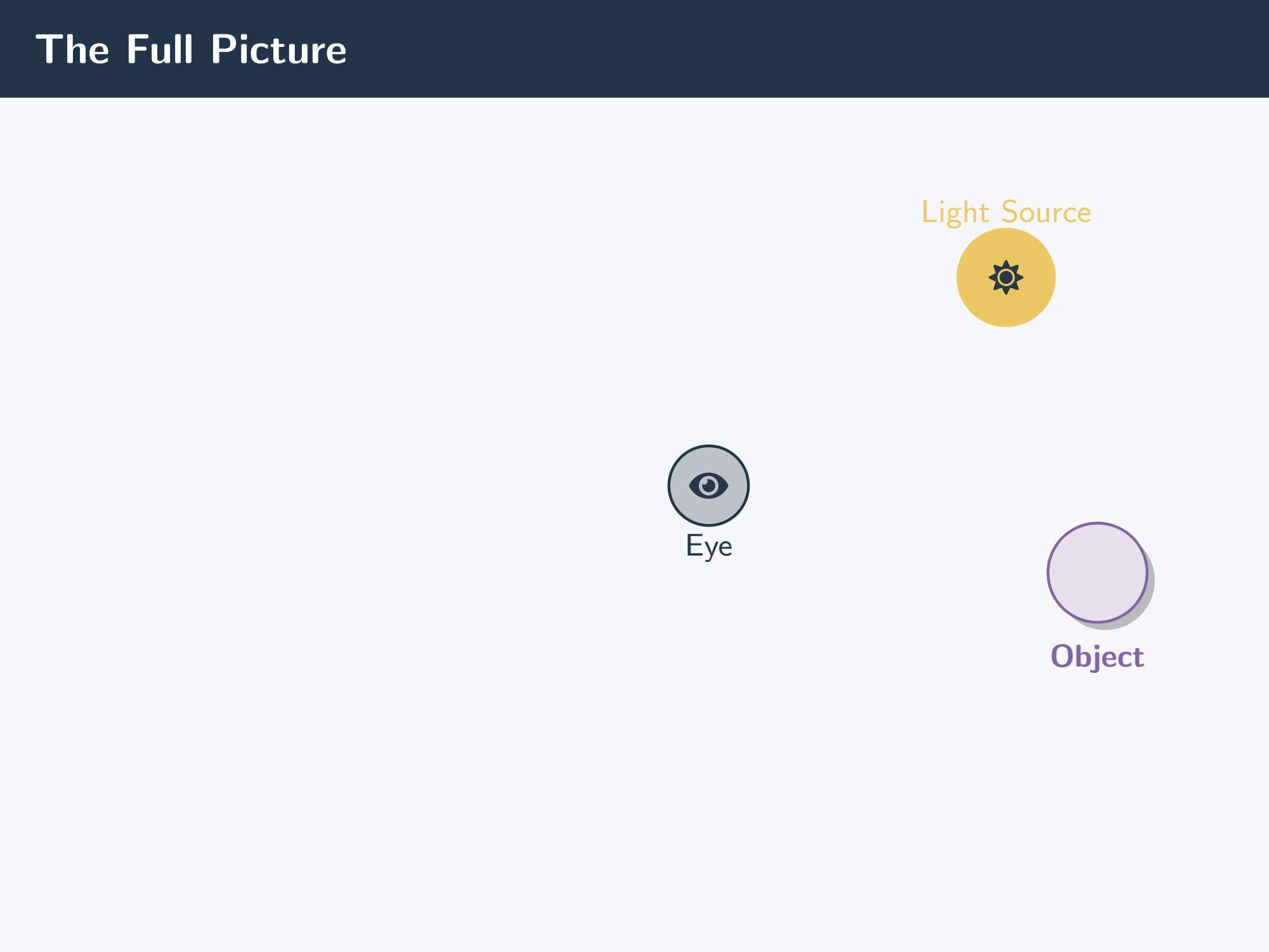


This comes with little tradeoff, because:

- An image is just a grid of pixels
- Each pixel can only be of one color
- In the end, we just need to know the *color of each pixel*
- Hence, one ray from the mid-point of each pixel should be a good approximation*

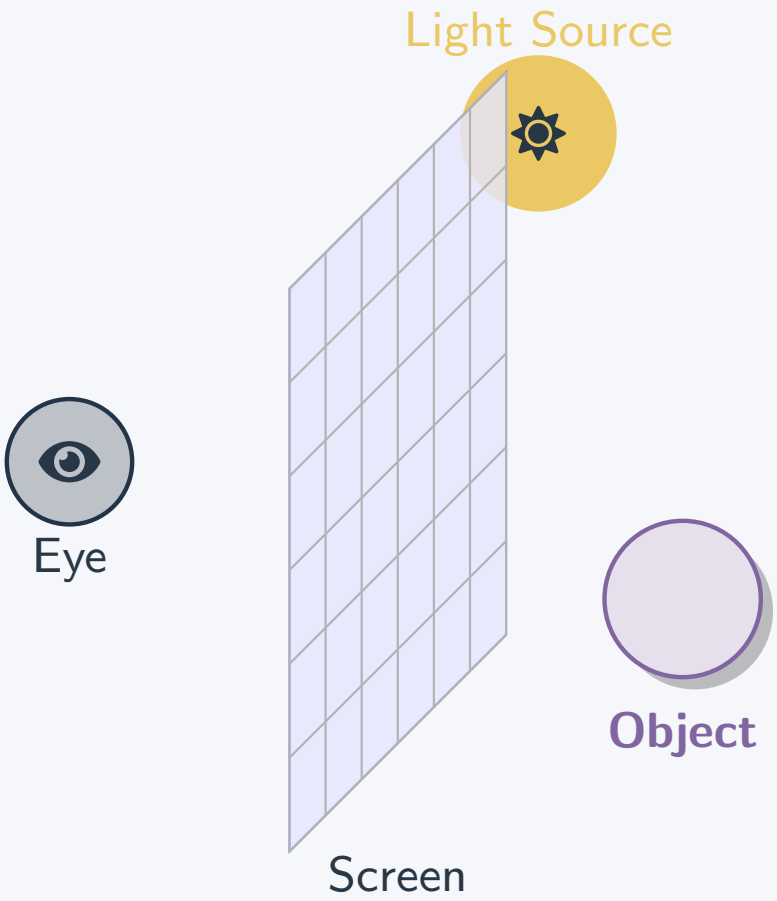
* We will discuss more advanced techniques later that improve quality

The Full Picture



The Full Picture

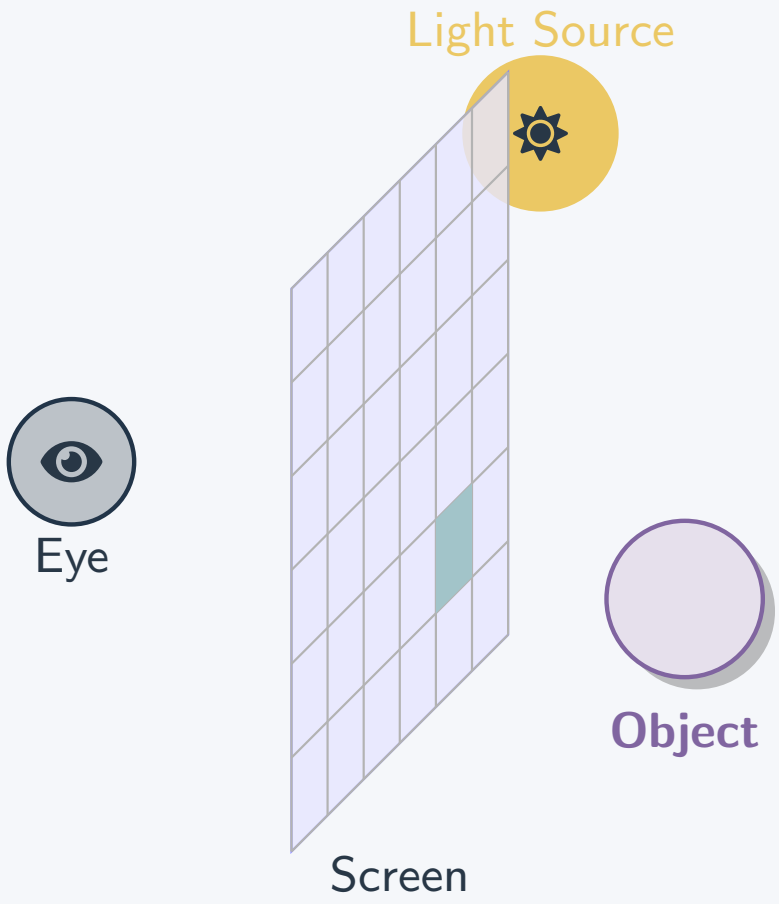
Place screen in front of eye



The Full Picture

Place screen in front of eye

For each pixel



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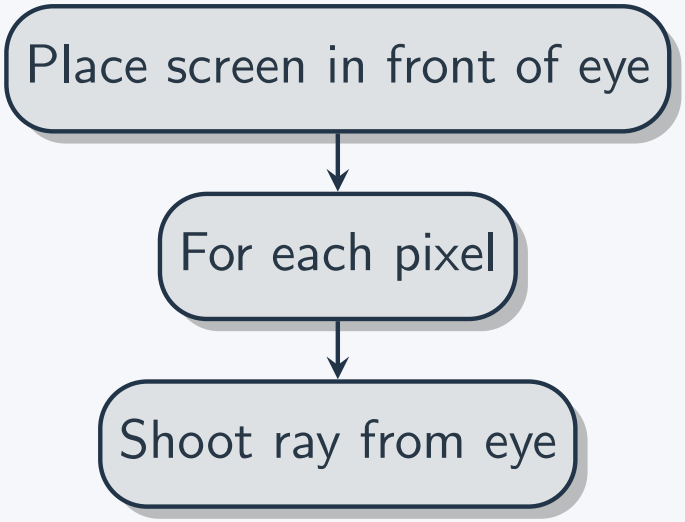

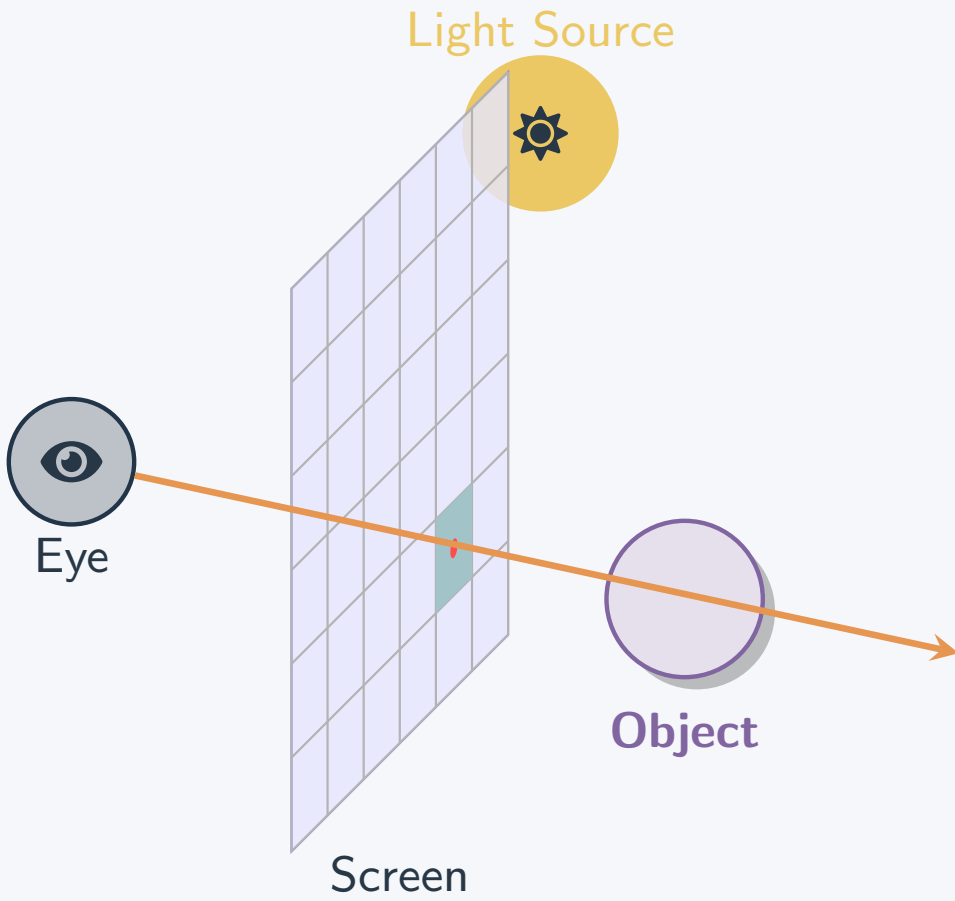

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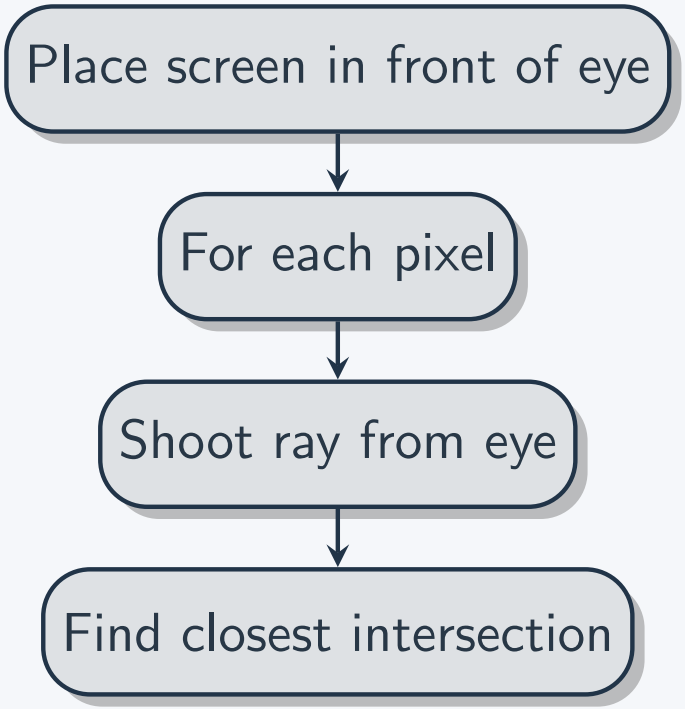

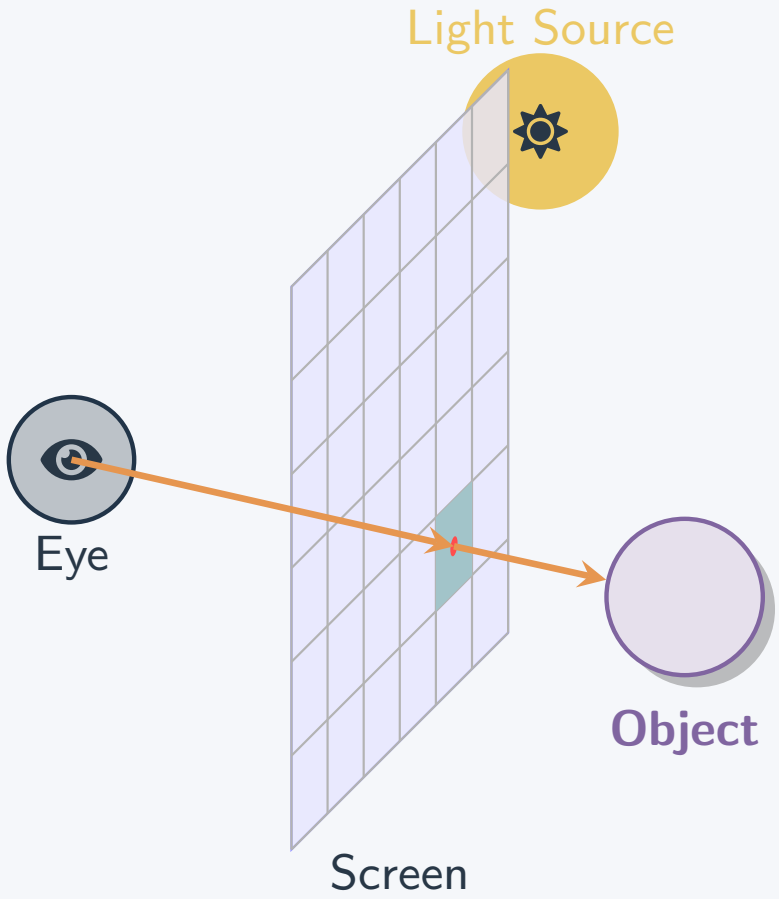

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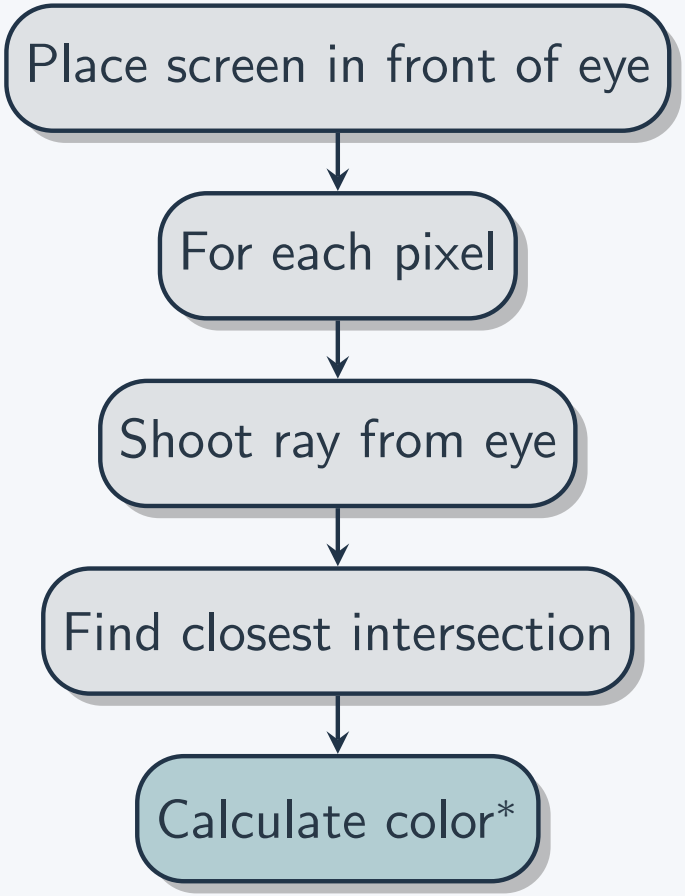

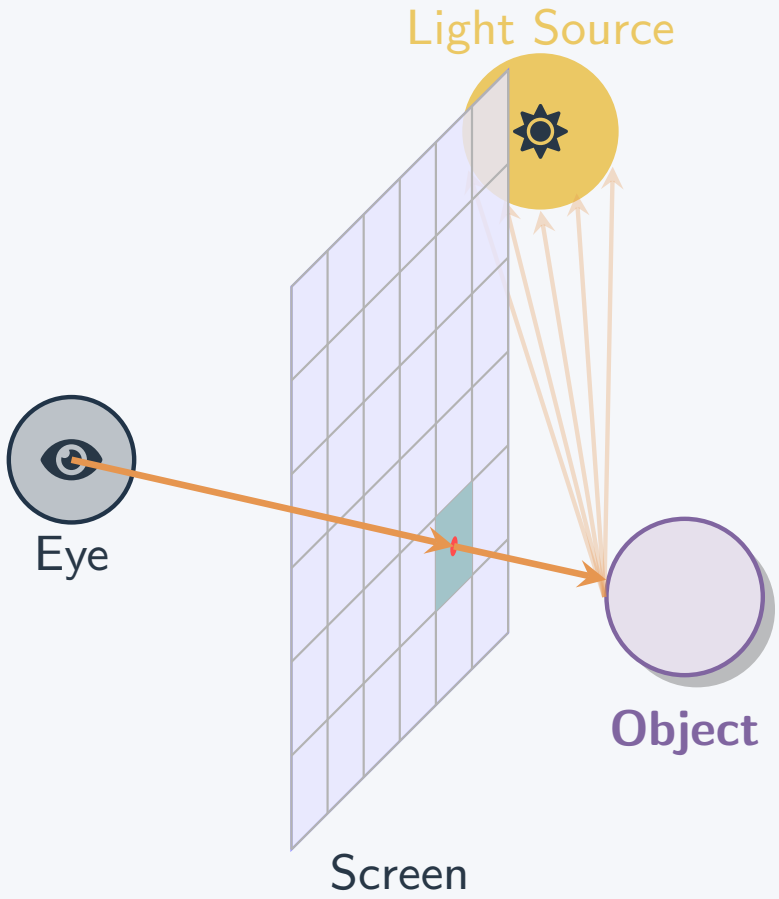

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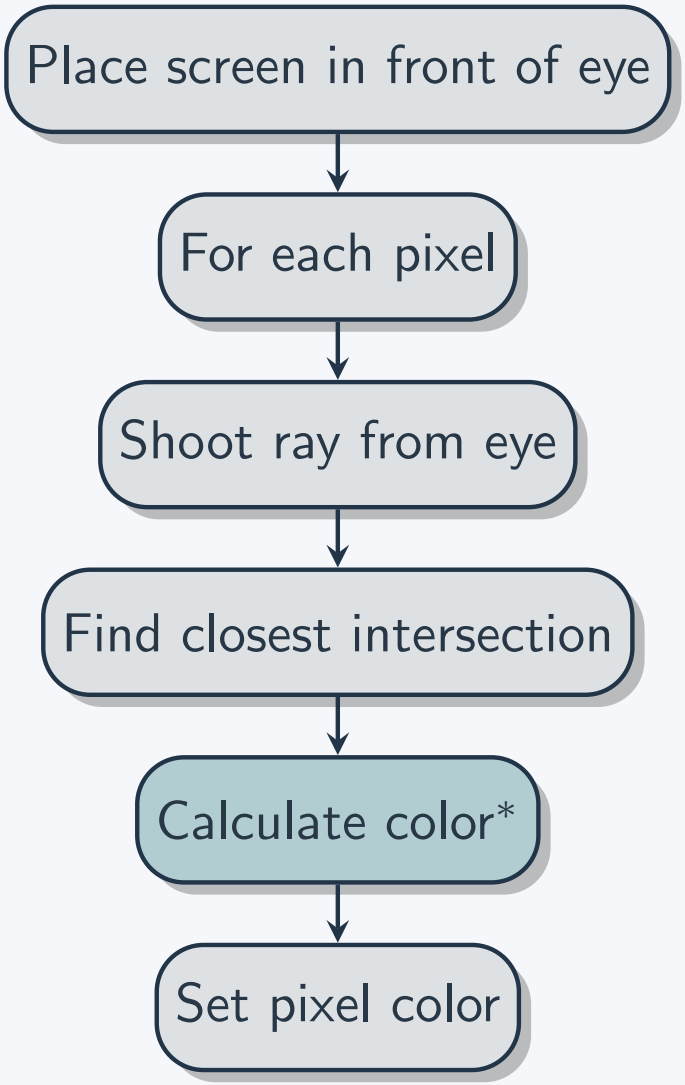

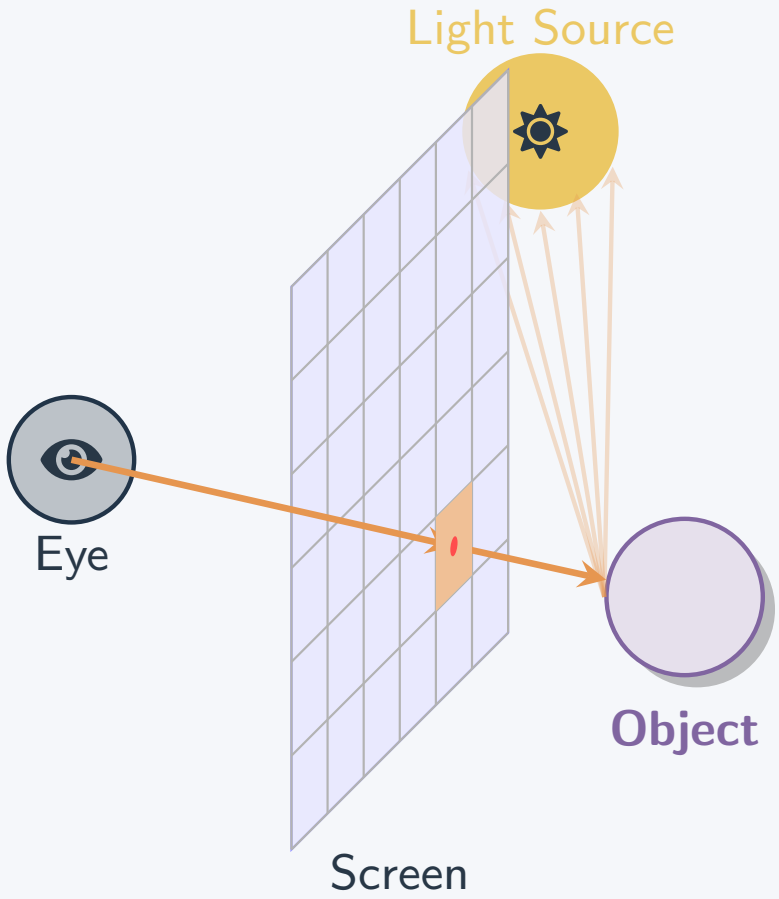

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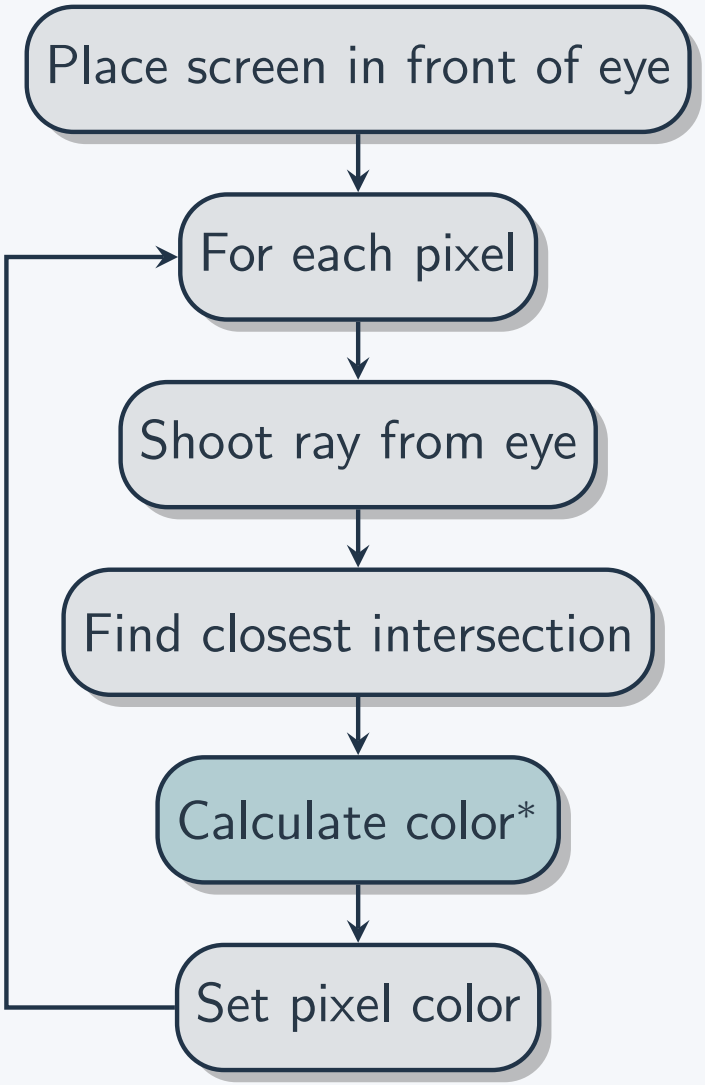
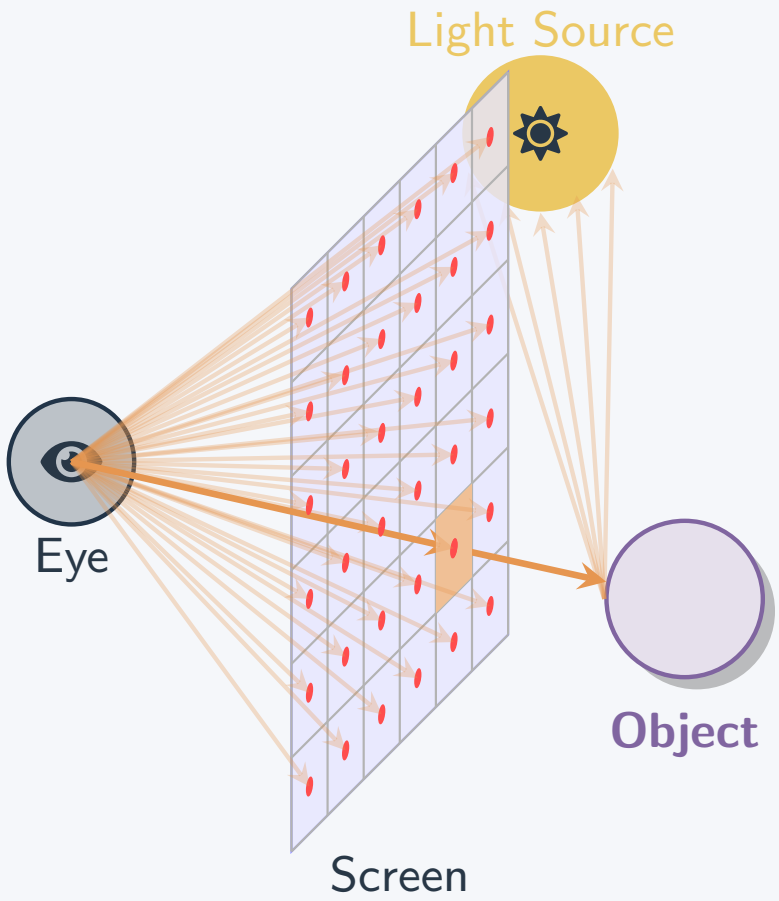




The Mathematics of Rays

What is a Ray?

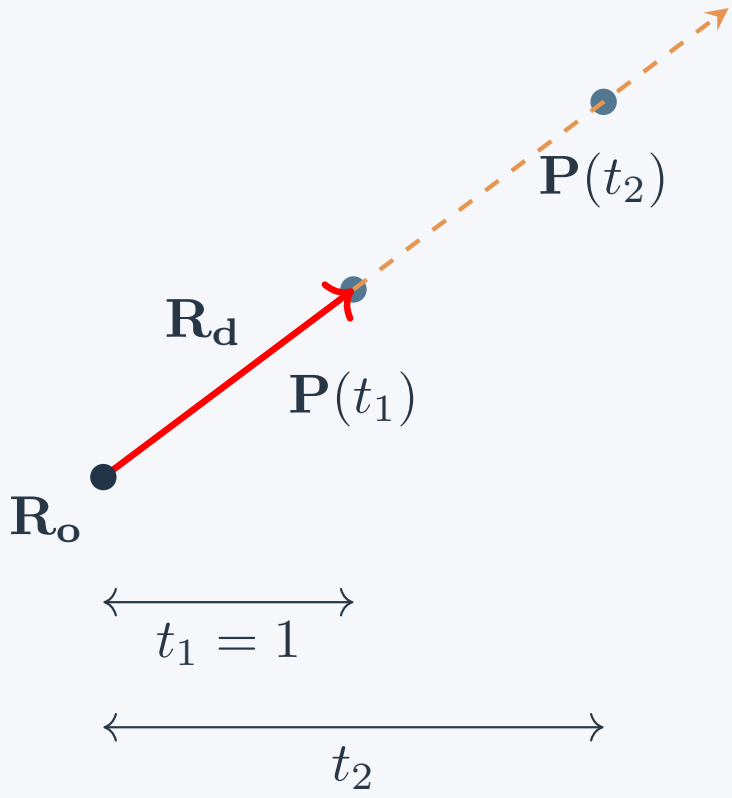
Ray Representation

A ray is defined by:

$$\mathbf{P}(t) = \mathbf{R_o} + t \cdot \mathbf{R_d} \quad (1)$$

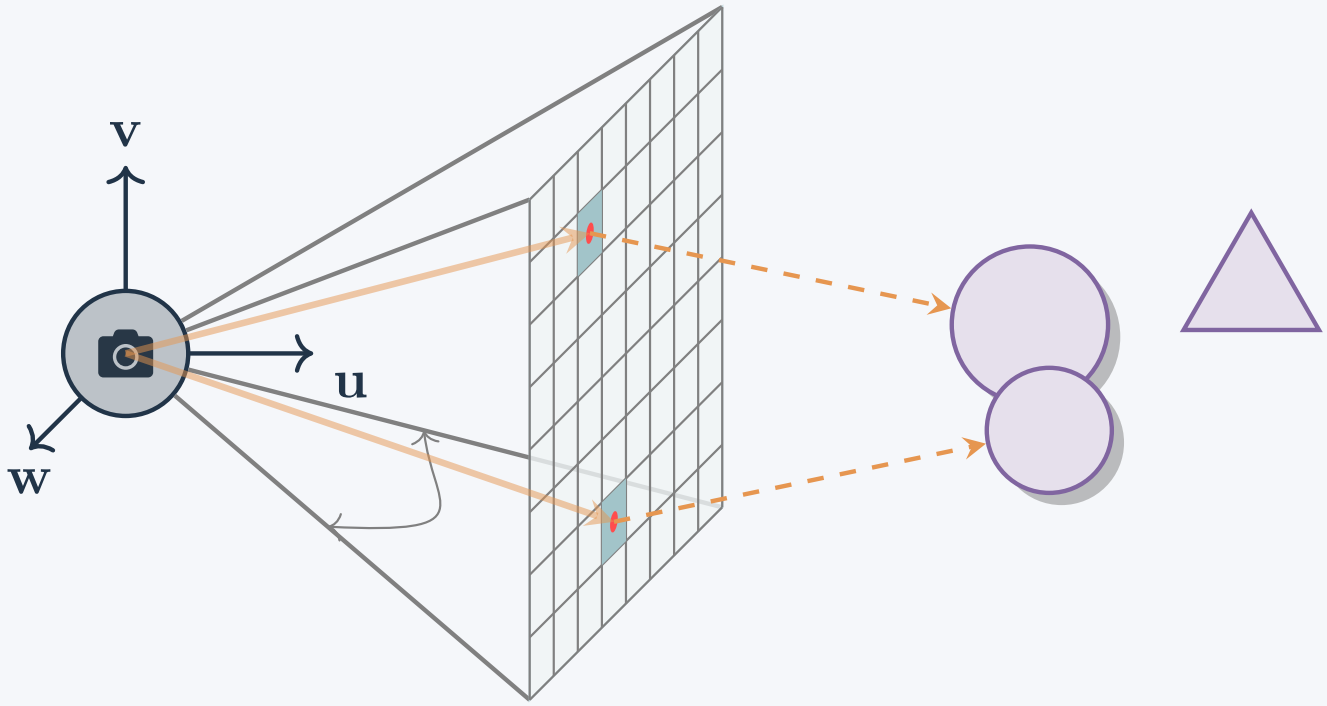
where:

- $\mathbf{R_o}$ = Origin point
- $\mathbf{R_d}$ = Direction vector
- t = Parameter ($t \geq 0$)



Check out here on desmos.

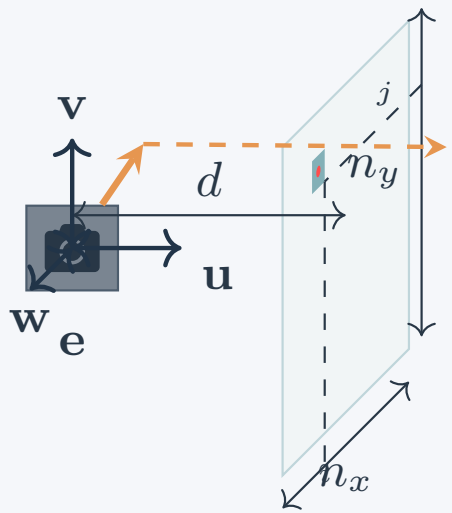
Camera and Ray Generation



Camera Parameters

Camera Definition: Eye point $\mathbf{e}$, orthobasis $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$, field of view, image dimensions

Ray Generation Mathematics



Ray Equation

For pixel (i, j):

$$s = \frac{i + 0.5}{n_x} \tag{2}$$

$$t = \frac{j + 0.5}{n_y} \tag{3}$$

Ray direction:

$$\mathbf{d} = (s - 0.5) \cdot \text{FOV} \cdot \mathbf{u} \tag{4}$$

$$+ (t - 0.5) \cdot \text{FOV} \cdot \mathbf{v} \tag{5}$$

$$+ d \cdot \mathbf{w} \tag{6}$$

Parametric ray:

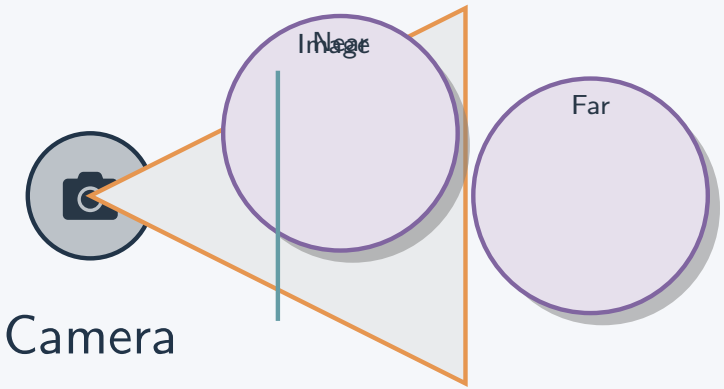
Camera Models in Ray Tracing

Why Camera Models Matter

The Camera's Role

The camera determines:

- **Field of view** - What we see
- **Perspective** - How objects appear
- **Ray generation** - Where rays start
- **Image formation** - Final rendering



Different camera models = Different visual effects!

The Pinhole Camera Model

Pinhole Camera

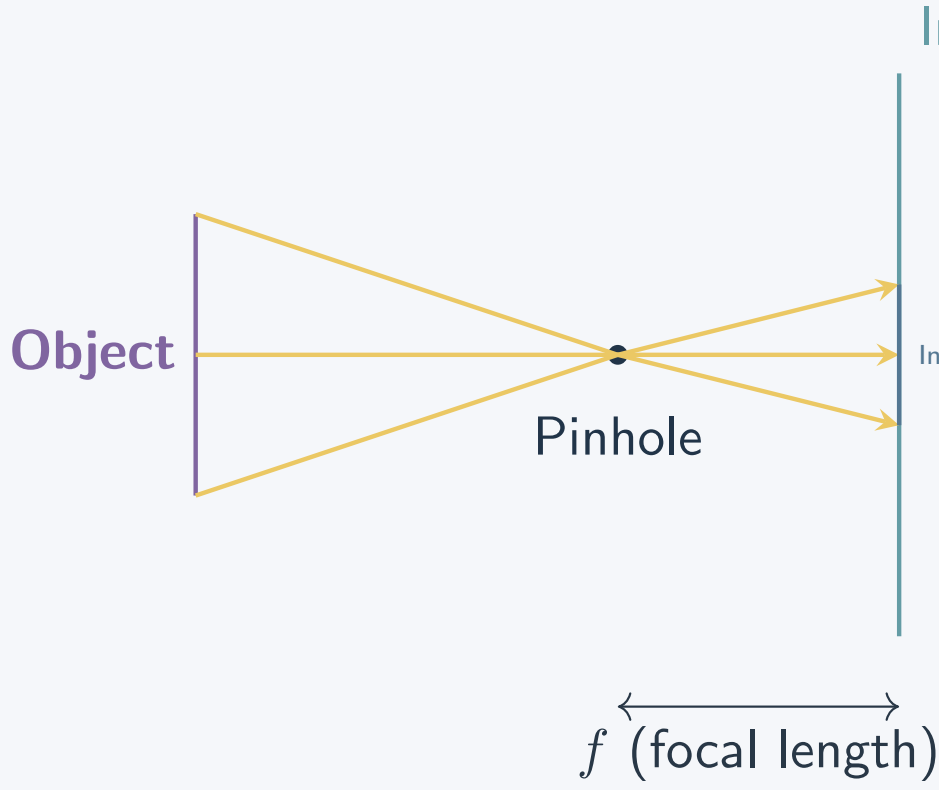
Key Properties:

- Point aperture (no lens)
- Perfect focus everywhere
- Linear perspective
- No depth of field

Ray Generation:

$$\mathbf{R_o} = \mathbf{eye} \tag{8}$$

$$\mathbf{R_d} = \mathbf{pixel} - \mathbf{eye} \tag{9}$$



Physical Reality

Real pinhole cameras exist! They create sharp images but require

Simplified Pinhole Camera

Simplification

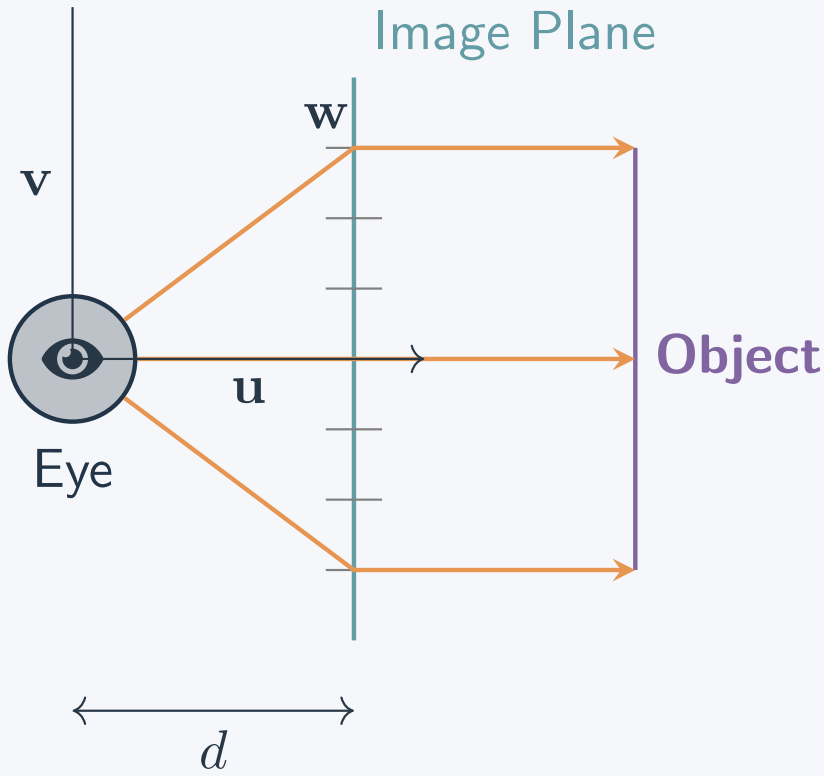
Problem: Real pinhole creates inverted image

Solution: Place image plane in front!

$$\mathbf{pixel} = \mathbf{eye} + d \cdot \mathbf{w} + u \cdot \mathbf{u} + v \cdot \mathbf{v} \tag{10}$$

where:

- d = distance to image plane
- u, v = pixel coordinates
- $\mathbf{u}, \mathbf{v}, \mathbf{w}$ = camera basis



Advantage

View Frustum

Viewing Frustum

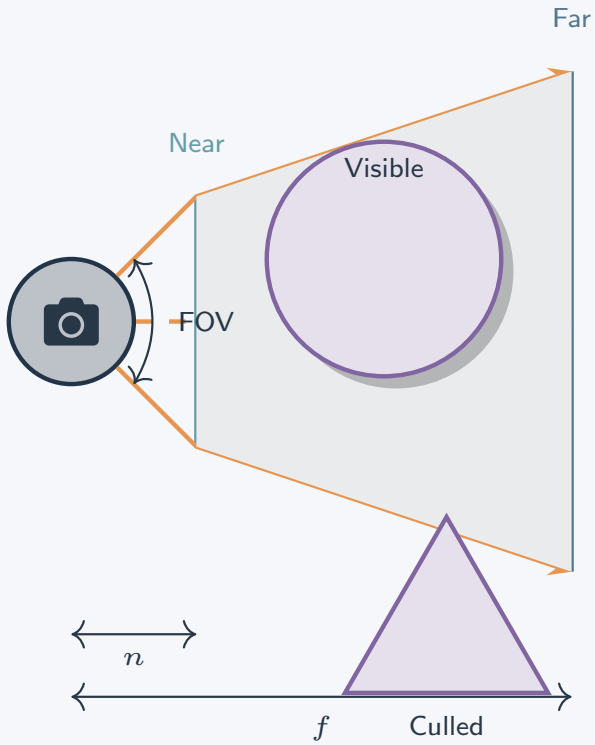
Definition: The 3D region visible to the camera

Boundaries:

- **Near plane** - Closest visible distance
- **Far plane** - Farthest visible distance
- **Left/Right** - Horizontal field of view
- **Top/Bottom** - Vertical field of view

Field of View:

$$\text{FOV} = 2 \arctan\left(\frac{h}{2d}\right)$$



Orthographic Camera

Orthographic Projection

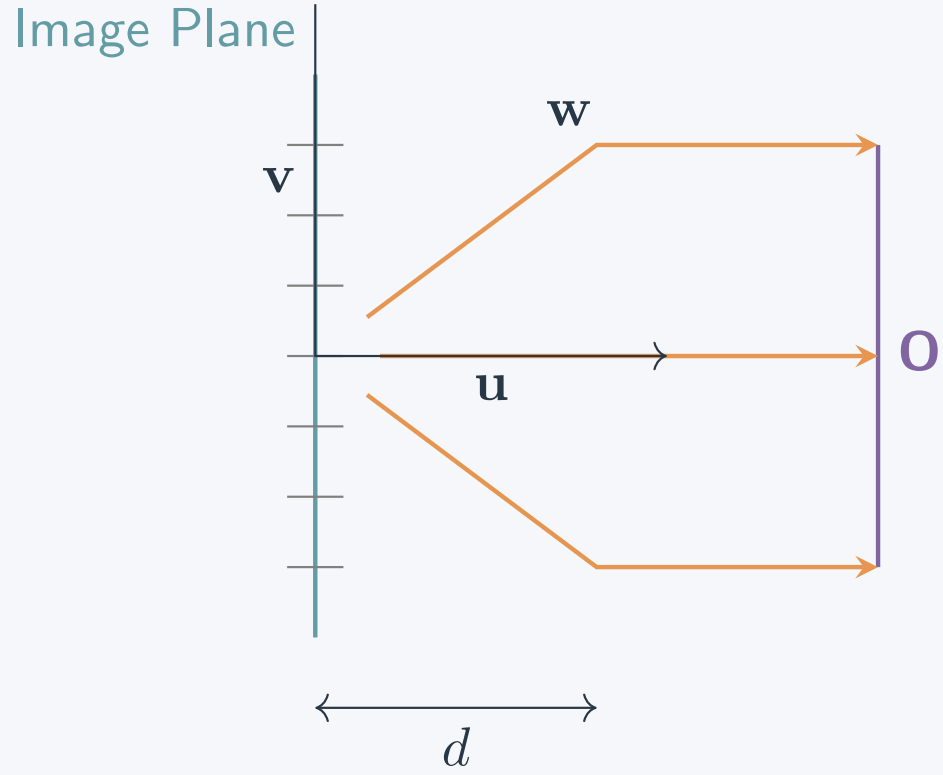
Key Properties:

- No perspective distortion
- Parallel projection rays
- Objects same size regardless of distance
- Infinite focal length

Ray Generation:

$$\mathbf{R_o} = \mathbf{pixel} \quad (12)$$

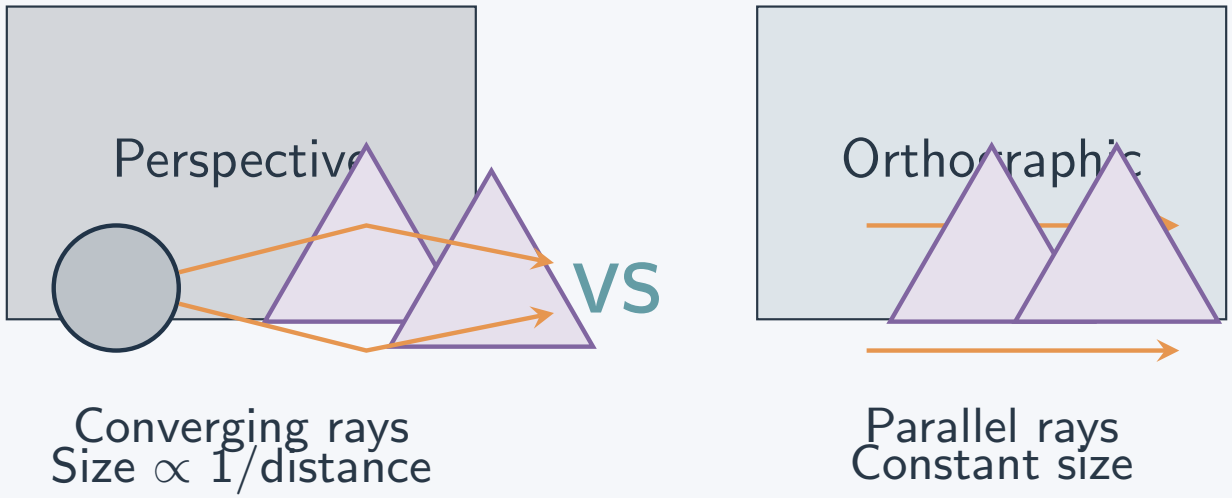
$$\mathbf{R_d} = \mathbf{w} \text{ (constant)} \quad (13)$$



Applications

Technical drawings, CAD software, 2D games, architectural

Perspective vs Orthographic



When to use Perspective

- Natural/realistic scenes
- Human vision simulation
- Games and films
- Depth perception important

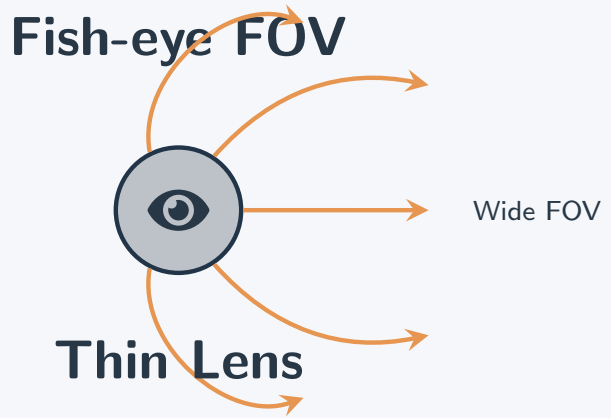
When to use Orthographic

- Technical illustrations
- CAD/Engineering
- UI elements overlay
- Precise measurements

Other Camera Types

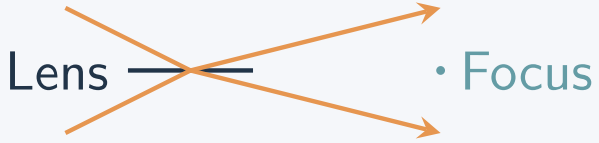
Fish-eye Camera

- Very wide field of view (¿180°)
- Non-linear distortion
- Curved ray paths
- Surveillance, VR applications



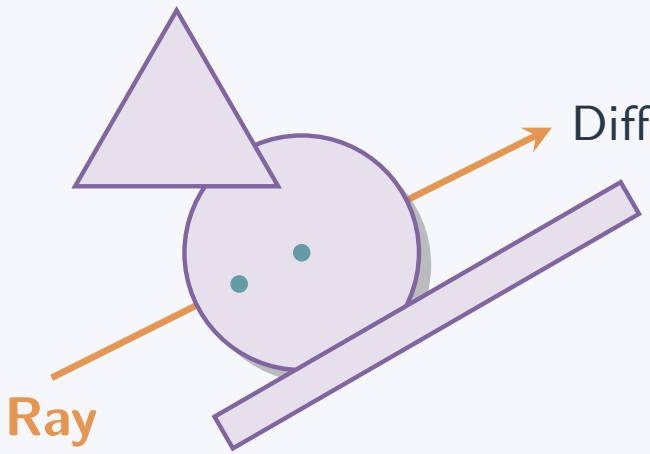
Thin Lens Camera

- Simulates real camera lens
- Depth of field effects
- Focal blur



Ray-Object Intersections

Finding Intersections



Different objects, different math!

Key Objects:

- **Planes** - Linear equations
- **Spheres** - Quadratic equations
- **Triangles** - Barycentric coordinates
- **General Quadrics** - Polynomial solving

Challenge: Find the **closest** intersection efficiently!

Ray-Plane Intersection

Plane Equation

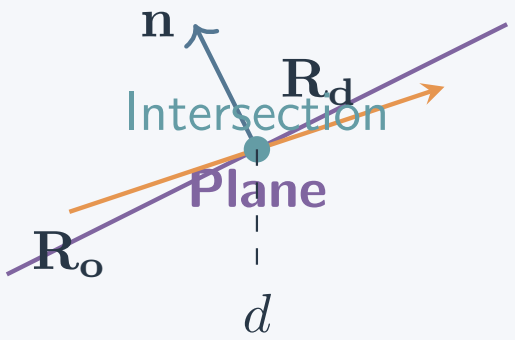
Implicit form:

$$\mathbf{n} \cdot \mathbf{P} + D = 0 \tag{14}$$

Substituting ray equation:

$$\mathbf{n} \cdot (\mathbf{R_o} + t\mathbf{R_d}) + D = 0 \tag{15}$$

$$t = -\frac{D + \mathbf{n} \cdot \mathbf{R_o}}{\mathbf{n} \cdot \mathbf{R_d}} \tag{16}$$



Key Insight

Explicit ray equation meets **implicit** plane equation = Clean intersection formula!

Ray-Sphere Intersection

Sphere Equation

Implicit form (centered at origin):

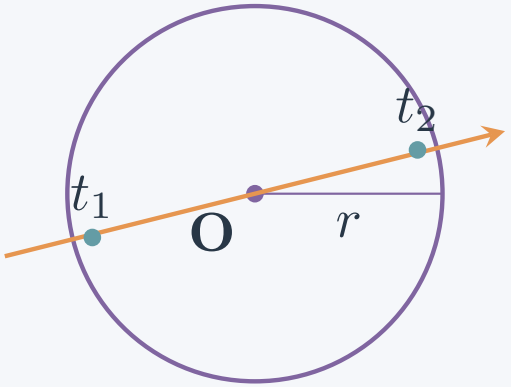
$$\mathbf{P} \cdot \mathbf{P} - r^2 = 0 \qquad (17)$$

Substituting ray equation:

$$(\mathbf{R_o} + t\mathbf{R_d}) \cdot (\mathbf{R_o} + t\mathbf{R_d}) - r^2 = 0 \qquad (18)$$

$$t^2 + 2(\mathbf{R_d} \cdot \mathbf{R_o})t + (\mathbf{R_o} \cdot \mathbf{R_o} - r^2) = 0 \qquad (19)$$

Quadratic formula: $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$



Sphere

$\Delta > 0$: 2 roots
$\Delta = 0$: 1 root
$\Delta < 0$: no roots

Ray-Triangle Intersection

Barycentric Approach

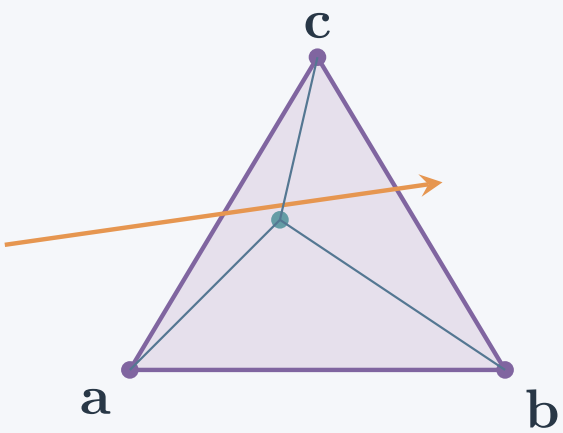
Triangle defined by vertices $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$:

$$\mathbf{P}(\beta, \gamma) = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a}) \tag{20}$$

Set equal to ray equation:

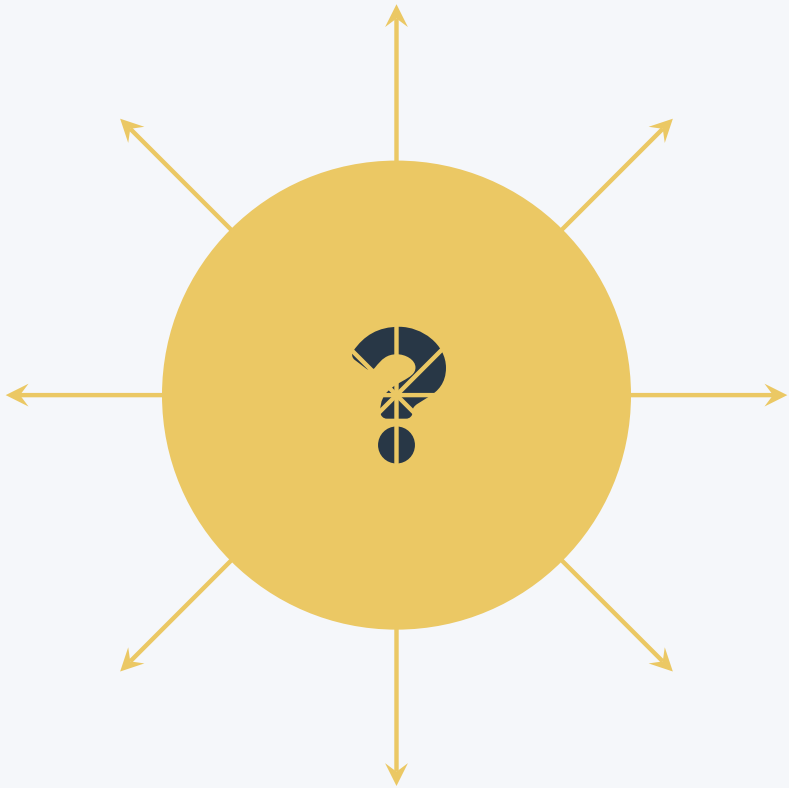
$$\mathbf{R_o} + t\mathbf{R_d} = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a}) \tag{21}$$

Solve 3×3 system for t, β, γ



Inside if:
$\beta \geq 0$
$\gamma \geq 0$
$\beta + \gamma \leq 1$

Questions?



References & Further Reading

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