On equality of solid angles ratio and corresponding volumes ratio

This work goes on proving the next equality:

$$\frac{\Omega_t}{\Omega_{us}} = \frac{V_t}{V_{us}}$$

,where *us* means unit sphere and $\Omega_{us} = 4\,\pi$. Ω_t is arbitrary solid angle on the sphere and V_t is the volume of corresponding quasi segment of the sphere. Since solid angle is not necessary represented as pure sphere segment surface, we first show that the volumes of arbitrary figure quasi-segments are equal if corresponding sold angles are equal. Using the Gauss – Ostrogradsky theorem, we get the formula:

$$V = \frac{1}{3}\iint_S \bar{a}\,\bar{n}\;dS$$

, where $\bar{n}$ is surface normal in at $\bar{a}$ coordinate. Which gives us

$$V = \frac{1}{3}\iint_S R\,dS = \frac{1}{3}R\iint_S dS$$

for the sphere case because $\bar{a}$ and $\bar{n}$ are facing the same direction(and $\bar{n}$ is unit vector). For the unit sphere it goes as simple as that

$$V = \frac{1}{3}\,\Omega$$

since the area of some unit sphere surface is the direct definition for the term "solid angle". At this point we proven that volume quantity for sphere quasi segment is agnostic to a figure of the unit sphere surface defining the segment(since it relies only on term R and S).

Then we have

$\frac{\Omega_t}{\Omega_{us}} = \frac{V_t}{V_{us}} \Leftrightarrow$

$\frac{\Omega_t}{\Omega_{us}} = \frac{\left(\frac{1}{3}\right)\Omega_t}{\left(\frac{1}{3}\right)\Omega_{us}}$,

so the initial statement is proven.

For non unit sphere, since $\Omega = \frac{A}{r^2}$

$$V = \frac{1}{3}R\iint_S dS = \frac{1}{3}R\,A = \frac{1}{3}\Omega\,r^3$$

So the ratio equation keeps being correct for arbitrary sphere.

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