CADA MID-II Assignment

1) Explain how disjoint sets can be represented?

>> The operations we wish to perform on these sets are:

1) Disjoint set union:

If S_i and S_j are two disjoint sets then their union $S_i \cup S_j$

= all elements x such that x is in S_i or S_j

2) Find(i):

Given element i, find set containing i.

>> In presenting union and find algorithms, we ignore set names and identify sets just by roots of trees representing sets.

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2) Discuss single source shortest paths algorithm with suitable example.

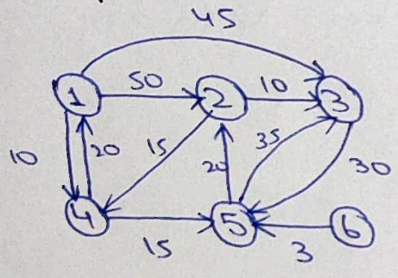
```
Algorithm ShortestPaths(v,cost,dist,n)
{
  for i:=1 to n do
  {
     S[i]:=false;
     dist[i]:=cost[v,i];
  }
  S[v]:= true;
  dist[v]:=0.0;
  for num:=2 to n do
  {
     S[u]:=true;
     for(each w adjacent to u with S[w]=false) do
        if(dist[w]>dist[u]+cost[u,w])) then
```

dist[w] := dist[u] + cost[u,w];

}

}

Example:



Graph

Path	Length
1) 1,4	10
2) 1,4,5	25
3) 1,4,5,2	45
4) 1,3	45

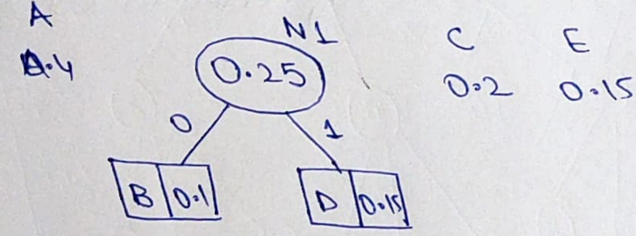
Shortest Paths from 1

Iteration	Path	Selected vertex	Distance 1	2	3	4	5	6
Initial	-	-	0	50	45	10	∞	∞
1	{1}	4	0	50	45	10	25	∞
2	{1,4}	5	0	45	45	10	25	∞
3	{1,4,5}	3	0	45	45	10	25	∞
4	{1,4,5,3}	2	0	45	45	10	25	∞
5	{1,4,5,3,2}	-	-					

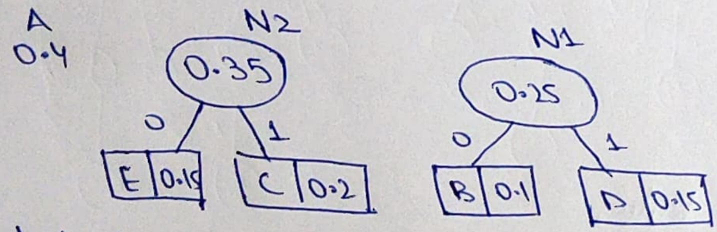
3) Explain Huffman coding and construct Huffman Code for the following.

char :	A	B	C	D	E
prob :	0.4	0.1	0.2	0.15	0.15

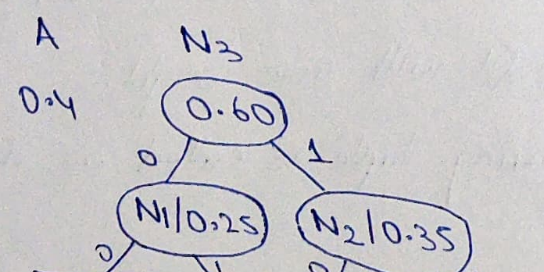
Step 1 : Select two less frequently used characters and constuct tree, with total of their frequency as root node. The less frequently used characters are B and D. Mark left away as 0 and right away as 1. The left child should be shortest one. Update data with new node.



Step 2 : Next select characters with less frequencies. So E & C are selected.

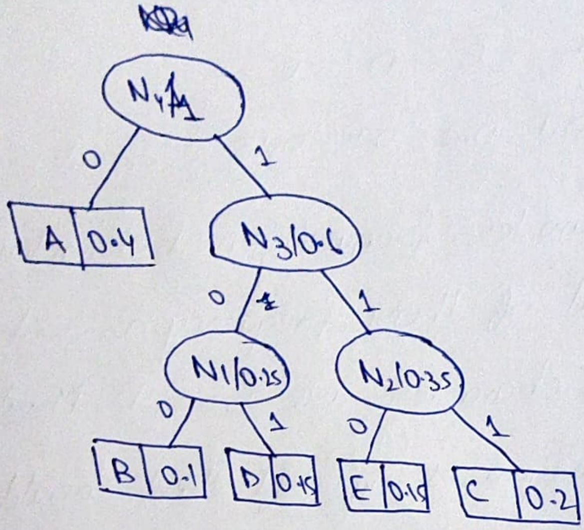


Step 3 : Select characters with less frequencies from updated table, so N1 & N2 are selected



Step 4: Select characters with less frequency from updated table,

so A & N_3 are selected.



» Traverse from root node to leaf nodes to construct Huffman codes.

A – 0

B – 100

C – 111

D – 101

E – 110

4) Define Dominance Rule.

» Dominance Rule is a technique used to optimize solution by eliminating certain items from consideration. Removing the tuple which gives less profit with more weight.

» The process of removing tuple is called as dominance rule/ purging.

6) Explain application of dynamic programming with an example of matrix chain pa multiplication.

Sol: Applications of dynamic programming:

1) Knapsack Problem
2) Shortest Path Problem
3) Matrix chain multiplication
4) Coin change Problem.

» Consider $A_1 = 5 \times 4$, $A_2 = 4 \times 6$, $A_3 = 6 \times 2$, $A_4 = 2 \times 7$

$A_1 = 5 \times 4$
$A_2 = 4 \times 6$
$A_3 = 6 \times 2$
$A_4 = 2 \times 7$

	1	2	3	4
1	$M_{11}=0$	$M_{12}=120$ $k=1$	$M_{13}=88$ $k=1$	$M_{14}=158$ $k=3$
2		$M_{22}=0$	$M_{23}=48$ $k=2$	$M_{24}=104$ $k=3$
3			$M_{33}=0$	$M_{34}=84$ $k=3$
4				$M_{44}=0$

$P_1 = 5, P_2 = 4, P_3 = 6, P_4 = 2, P_5 = 7$

$$M_{12} = M_{11} + M_{22} + P_1 P_2 P_3$$
$$= 0 + 0 + (5 \times 4 \times 6) = 120 \ (k=1)$$

$$M_{23} = M_{22} + M_{33} + P_2 P_3 P_4$$
$$= 0 + 0 + (4 \times 6 \times 2) = 48 \ (k=2)$$

$$M_{34} = M_{33} + M_{44} + P_3 P_4 P_5$$
$$= 0 + 0 + (6 \times 2 \times 7) = 84 \ (k=3)$$

$$M_{13} = \min\{[M_{11} + M_{23} + P_1 P_2 P_4], [M_{12} + M_{33} + P_1 P_3 P_4]\}$$
$$= \min\{0 + 48 + (5 \times 4 \times 2), 120 + 0 + (5 \times 6 \times 2)\}$$
$$= \min\{88, 180\} = 88 \ (k=1)$$

$$M_{24} = \min\{[M_{22}+M_{34}+P_2P_3P_5],[M_{23}+M_{44}+P_2P_4P_5]\}$$

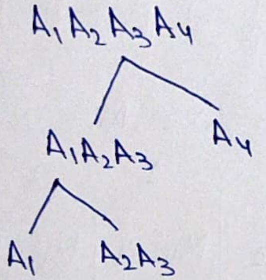
$$= \min\{0+84+(4\times6\times7),\ 48+0+(4\times2\times7)\}$$

$$= \min\{252,104\} = 104\ (k=3)$$

$$M_{14} = \min\{[M_{11}+M_{24}+P_1P_2P_5],[M_{12}+M_{34}+P_1P_3P_5],[M_{13}+M_{44}+P_1P_4P_5]\}$$

$$= \min\{0+104+(5\times4\times7),\ 120+84+(5\times6\times7),\ 88+0+(5\times2\times7)\}$$

$$= \min\{244,414,158\} = 158\ (k=3)$$



$((A_1(A_2A_3))A_4)$

7) Define Branch and Bound.

>> It is one of techniques used for problem solving.

>> It is similar to Backtracking.

>> It is used for solving optimization problems.

>> The technique explores solution space while efficiently eliminating certain branches based on bounds, reducing search space and improving overall efficiency.

5) Explain features of dynamic programming.

» Dynamic programming is a technique used in ~~a~~DAA to solve optimization problems by breaking them down in smaller overlapping subproblems and efficiently solving each only once.

» Some of features are:

i) Optimal Substructure

ii) Overlapping subproblem

iii) Recursion and Iteration

iv) Memoization

v) Tabulation

vi) Time and Space complexity

vii) Applicability

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9) Explain non-deterministic algorithms and write non-deterministic algorithm for sorting.

» Algorithms with property (result of every operation is uniquely defined) are termed as non-deterministic algorithms.

» To specify such algorithms, we introduce 3 new functions:

i) Choice(S) arbitrarily chooses one of elements of set S.

ii) Failure() signals an unsuccessful completion.

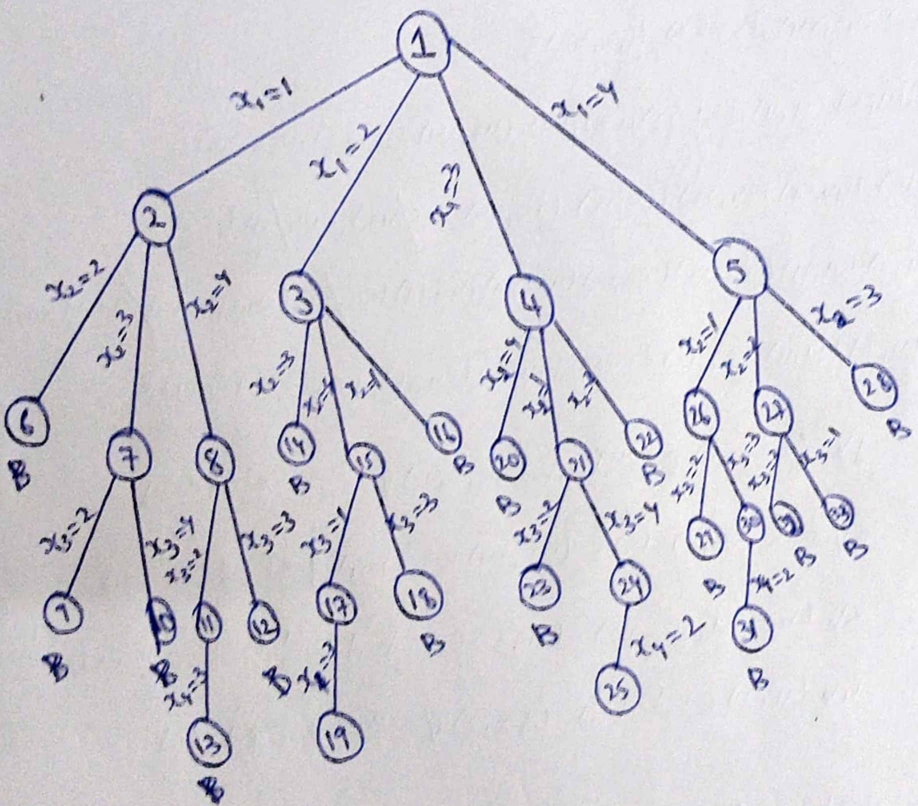
iii) Success() signals a successful completion.

```
Algorithm NSort(A,n)
{
    for i:=1 to n do
        B[i]:=0;
    for i:=1 to n do
    {
        j:=Choice(1,n);
        if B[j] ≠ 0 then Failure();
        B[j]:= A[i];
    }
    for i:=1 to n-1 do
        if B[i] > B[i+1] then Failure();
    write(B[1:n]);
    Success();
}
```

8) Define state space tree. Draw the state space tree for 4-queen problem.

Sol: State space tree:

A state space tree is a tree representing all possible states of problem from root as an initial state to leaf as a terminal state.



» For above state space tree we have 2 solution i.e,

path = 2,4,1,3 and 3,1,4,2

10) Solve 0/1 knapsack problem where $P = (11, 21, 31, 33)$, $W = (2, 11, 22, 15)$

$m = 40$, $n = 4$.

Sol:

$S^0 = \{(0,0)\}$

$S_1^0 = \{(11,2)\}$

$S^1 = \{(0,0),(11,2)\}$

$S_1^1 = \{(21,11),(32,13)\}$

$S^2 = \{(0,0),(11,2),(21,11),(32,13)\}$

$S_1^2 = \{(31,22),(42,24),(52,33),(63,35)\}$

$S^3 = \{(0,0),(11,2),(21,11),\cancel{(31,22)},(32,13),(42,24),(52,33),(63,35)\}$

$S_1^3 = \{(33,15),(44,17),(54,26),(65,28),(75,39),\cancel{(85,48)},\cancel{(99,50)}\}$

$S^4 = \{(0,0)(11,2)(21,11)(32,13)(33,15),\cancel{(42,24)},(44,17),\cancel{(52,33)},(54,26),\cancel{(63,35)},(65,28),(75,39)\}$

$S^4 = \{(0,0),(11,2),(21,11),(32,13),(33,15),(44,17),(54,26),(65,28),(75,39)\}$

1) $(75,39) \in S^4$ but $(75,39) \notin S^3$ $\therefore x^4 = 1$

2) $(75-33, 39-15) = (42,24) \in S^3$ but $\notin S^2$ $\therefore x^3 = 1$

3) $(42-31, 24-22) = (11,2) \in S^2$ but $\in S^1$ $\therefore x^2 = 0$

4) $(11,2) \in S^1$ but $(11,2) \notin S^0$ $\therefore x1 = 1$

$\therefore$ Profit $= P_4 + P_3 + P_1$

$= 33 + 31 + 11$

$= 75$