

Given: k blocks of data/info (to store) in fault tolerant way using error code (e) size

such that total no. of block (n)

$$(k+e) \leq n < k+2e$$

For this we can, assume c_1, c_2, \dots, c_k (k blocks)
then

A polynomial:
$$P = \sum_{i=1}^k c_i x^{k-i}$$

$$\Rightarrow P = c_1 x^{k-1} + c_2 x^{k-2} + \dots + \underbrace{c_k x^0}_{c_k}$$

Generate n points randomly using this polynomial i.e. $P(x_i) = x_i$

or $(x_i, P(x_i))$ pairs for $1 \leq i \leq n$

lets say e blocks are ~~err~~ corrupted out of these n blocks,

and using Lagrange interpolation, we ~~need~~ ^{can unique construct} polynomial of degree n with $(n+1)$ points

So, here k points are needed to construct back the original (P) .

$$\therefore n - e \geq k \Rightarrow (k+e) \leq n$$

also from info theory $n < k+2e$

hence, using this method $\Rightarrow (k+e) \leq n < k+2e$

We can use digital signature to detect which of these are corrupted.

So, the exact steps :-

- (*) For given k blocks, generate k degree polynomial
- (*) Generate random $(k+e)$ points (n) where e is maximum (no. of blocks which can be corrupted)
- (*) Sign all block points & send.
- (*) if ~~any~~ for any pair of $(m, \text{Sign}(m))$ is not correct (we can discard it)
- (*) And ~~for~~ from remaining k uncorrupted blocks can generate the polynomial back & hence will get k blocks as its coefficients.