MTL-106
Probability and Stochastic Processes

Assignment - 1

Deadline : 10th February 2024

You and Alice are managing a team of warriors, Alice 's team consist of warriors with strength $3, 4, 6, 7$ and you have at most 4 warriors with strengths given by **last 4 digits of your entry number**.
For example if your entry number is $2022MT19842$, then you have 4 warriors with strengths $9, 8, 4, 2$, in case of 0 digit you simply omit, i.e. if last 4 digits are 4062 then you only have three warriors with strength $4, 6, 2$.

If a warrior with strength x fights another warrior with strength y, warrior with strength x wins with probability $\frac{x}{x+y}$ and there is no draw (i.e. the other one dies), and the victorious gains confidence and after the match his strength becomes $x+y$.

The tournament is going to consist of You and Alice picking up warriors to fight against one another, one at a time. Alice is going to pick first for each fight (she picks randomly) and winner of the tournament is the person who has at least one gladiator left at the end. Assuming you play optimally, we are interested in the probability of you winning this tournament

To solve this problem lets evaluate some natural strategies

1. *You always pick the best warrior for each fight* : For each fight, pick the most powerful warrior you have and run a Monte Carlo Simulation to estimate probability of winning. [2 pts]
2. *You always pick the worst warrior for each fight* : For each fight, pick the least powerful warrior you have and run a Monte Carlo Simulation to estimate probability of winning. [2 pts]

What do you observe ? After making those simulations we move onto building optimal strategy :

3. If Alice picks a warrior with strength x and you pick a warrior with strength y, what is the expected gain of strength in your team ? Is every match fair(i.e. 0 expected gain) ? [1 pts]
4. Observe TotalStrength = IntialStrength$+\sum_{i\geq 1} \text{gain}_i$, where gain_i is gain of strength in match i, and thus give Monte Carlo Simulation of an optimal strategy and estimate the probability of winning, compare it with the theoretical value. [3 pts]

Remarks

1. Only make up to 10^5 iterations for the simulations at max, and each part shouldn't take more than 5 sec to execute.
2. Clearly show all the calculations done in Part 3 and Part 4.

Submission Format
A single Jupyter Notebook **.ipynb** extension file with format Name_Entry-Number.ipynb, If you don't have Jupyter Notebook installed you can easily start working on it from Google Colab with no installations required on your local machine.