University of Maryland at College Park

Dept. of Aerospace Engineering

ENAE 788K: Estimation and Control of Stochastic Systems

Problem Set #5

Issued: 28 Nov. 2022
Due: 6 Dec. 2022

Question 1:

The file `F16s.mat` gives a linear perturbation model for an F16 aircraft in level flight at 10,000 ft. The outputs of this model are the aircraft's speed and climb rate, both in ft/sec. The two inputs to the model are stochastic processes, $d_1(t)$ and $d_2(t)$ representing wind gust effects on the vertical component of the aircraft velocity, and on the pitch rate of the aircraft, respectively.

Using the Dryden model for vertical wind turbulence, we can model $d_1(t)$ and $d_2(t)$ as zero mean WSS processes with spectra

$$S_{d_1 d_1}(\omega) = \frac{K(1 + 3(a\omega)^2)}{(1 + (a\omega)^2)^2}$$

and

$$S_{d_2 d_2}(\omega) = \left[\frac{\omega^2}{1 + (b\omega)^2} \right] S_{d_1 d_1}(\omega)$$

Moreover, these disturbances are assumed to be correlated, which can be modeled by using a single white noise source to drive the shaping filters for both disturbances. (Hence d_2 can be modeled by passing d_1 through an additional shaping filter, given the structure of its spectrum.) For the flight conditions above, the values $a = 3$, $b = 0.1$, are appropriate.

a.) The constant K controls the RMS (root-mean-square) magnitude of the vertical gust component. Choose this constant so that $\sqrt{E[d_1(t)^2]} = 7$.

b.) With the value of K found in a.), compute the RMS magnitude of the variations in speed and climb rate that result from the disturbance inputs.

c.) Compute the components F and Q for the stochastic discretization of the complete system. Assume a sample rate of 20Hz.

d.) Show that the discretization computed in c.) is consistent with the calculations in b.). That is, show that the discrete output sequence $\mathbf{y}_k$ has the same steady-state variance as found in b.)

Question 2:

Suppose in Question #1 that a pitot tube can measure the speed of the aircraft at a rate of 20Hz. These measurements are corrupted by noise, modeled as white and WSS with a variance of 8 ft/sec. The measurements are uncorrelated with the disturbances at all times.

Assume that a Kalman filter is implemented to estimate the states based on these measurements. Letting $\mathbf{x}(t)$ be the states of the complete physical model, assume that $E[\mathbf{x}(0)] = 0$ and $E[\mathbf{x}(0)\mathbf{x}(0)^T] = \frac{1}{2}\boldsymbol{I}$. Plot the evolution in RMS uncertainty of the true values of the two outputs (i.e. $E[\tilde{y}_1(t)^2]$ and $E[\tilde{y}_2(t)^2]$). What is the steady-state RMS uncertainty in each output?

HINT: Note that the measurements (airspeed) used by the filter in this problem are different from the outputs we are interested in estimating! This poses no conceptual difficulties; just use a $\boldsymbol{C}$ matrix in the design of the estimator that reflects the actual measurement, then use the original $\boldsymbol{C}$ matrix to assess the output estimation error variance.

Question 3:

On landing approach, proper knowledge (and control) of the descent rate is crucial. Suppose that the RMS uncertainty in the climb rate must below ± 0.5 ft/sec to ensure safe landing. Can this be assured with the given measurements? If not, how small would the variance in each speed measurement need to be to ensure this level of uncertainty in the climb rate? Conversely, if the given measurements can assure this level, how "bad" could the pitot tube measurements to be (i.e. how large can the variance of the measurement noise be) and still provide the required RMS uncertainty level?

NOTE: The aircraft speed, and the disturbance spectra, would generally be different at landing as opposed to the cruise condition considered above. We'll ignore these effects, rather than recalculate everything for a new flight condition!