Q3

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Question 3:

On landing approach, proper knowledge (and control) of the descent rate is crucial. Suppose that the RMS uncertainty in the climb rate must below ±0.5 ft/sec to ensure safe landing. Can this be assured with the given measurements? If not, how small would the variance in each speed measurement need to be to ensure this level of uncertainty in the climb rate? Conversely, if the given measurements can assure this level, how "bad" could the pitot tube measurements to be (i.e. how large can the variance of the measurement noise be) and still provide the required RMS uncertainty level?

NOTE: The aircraft speed, and the disturbance spectra, would generally be different at landing as opposed to the cruise condition considered above. We'll ignore these effects, rather than recalculate everything for a new flight condition!

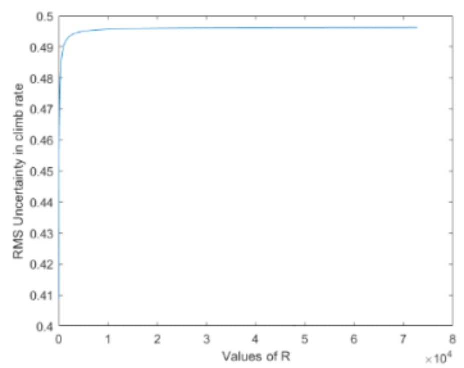
The RMS uncertainty in climb rate is below ±0.5 ft/s, hence safe landing can be ensured with the given measurements, with R = 8 ft/sec for noise.

Varying R to see the output in climb rate,

```
%%%Question3

for j = 1:50
    R_k = R_k*1.2
    [Ppinf,L_,G_] = dare(F',H_k',Q_bar,R_k)
    Sinf = inv(H_k*Ppinf*H_k' + R_k);
    K_inf = Ppinf*H_k'*S
    P_cinf = (eye(9,9) - K_inf*H_k)*Ppinf;
    var = Ca*P_cinf*Ca'
    x_R(j) = R_k
    y(j) = sqrt(var(2,2))

end
figure(1)
plot(x_R,y)
xlabel('Values of R')
ylabel('RMS Uncertainty in climb rate')
```



It can be seen that ↑ing R_k doesn't effect the uncertainty in climb rate.

```
R_k = 1000000;
[Ppinf,L_,G_] = dare(F',H_k',Q_bar,R_k);
Sinf = inv(H_k*Ppinf*H_k' + R_k);
K_inf = Ppinf*H_k'*S;
P_cinf = (eye(9,9) - K_inf*H_k)*Ppinf;
var = Ca*P_cinf*Ca';
sqrt(var(2,2))
```

```
ans = 0.4963
```

Value of RMS uncertainty in climb rate even when variance in measurement noise is 1000000

Hence, even when the variance is infinite the required RMS uncertainty level is attained.

The reason is that system is able to tolerate any amount of noise, since the climb rate RMS uncertainty never crosses 0.5.

" An infinite amount of noise will render the measurements helpful and rely only on propogation." (open loop).

Basically Kalman gain goes to zero as $R_k \to \infty$

$$K_k = \underbrace{P_k^- H_k^T \left[H_k P_k^- H_k^T + R_k \right]^{-1}}_{=0 \text{ when } R_k = \infty}$$

∞

$$\boxed{\text{Hence,} \quad \hat{\underline{x}}_k = \hat{\underline{x}}_k^- + 0 \left[y_k - H_k \hat{\underline{x}}_k^- \right]}$$

$$\Rightarrow \boxed{\hat{\underline{x}}_k = \hat{\underline{x}}_k^-}$$