Q2

06 December 2022 15:40

Question 2:

Suppose in Question #1 that a pitot tube can measure the speed of the aircraft at a rate of 20Hz. These measurements are corrupted by noise, modeled as white and WSS with a variance of 8 ft/sec. The measurements are uncorrelated with the disturbances at all times.

Assume that a Kalman filter is implemented to estimate the states based on these measurements. Letting $\mathbf{x}(t)$ be the states of the complete physical model, assume that $E[\mathbf{x}(0)] = 0$ and $E[\mathbf{x}(0)\mathbf{x}(0)^T] = \frac{1}{2}\boldsymbol{I}$. Plot the evolution in RMS uncertainty of the true values of the two outputs (i.e. $E[\tilde{y}_1(t)^2]$ and $E[\tilde{y}_2(t)^2]$). What is the steady-state RMS uncertainty in each output?

HINT: Note that the measurements (airspeed) used by the filter in this problem are different from the outputs we are interested in estimating! This poses no conceptual difficulties; just use a $\boldsymbol{C}$ matrix in the design of the estimator that reflects the actual measurement, then use the original $\boldsymbol{C}$ matrix to assess the output estimation error variance.

Given:-

$$y_k = H_k x_k + v$$

$$R_k = 8 \text{ ft/sec} = \underset{\sim}{\text{var}}[v]$$

$\because$ only one measurement

$$E[y_k v_k] = 0$$

$$\underset{\sim}{x}(t) = \begin{bmatrix} 9 \times 1 \end{bmatrix}, \text{ from the augmented model in Q1,}$$

$$E[\underset{\sim}{x}(0)] = 0, \quad P_0 = E[\underset{\sim}{x}(0)\underset{\sim}{x}(0)^T] = \frac{1}{2}\mathbb{I}$$

$$\Downarrow$$

$$\hat{\underset{\sim}{x}}_0^- = 0$$

$$P_k = E[\tilde{\underset{\sim}{x}}_k \tilde{\underset{\sim}{x}}_k^T], \quad \text{where, } \tilde{\underset{\sim}{x}}_k = \underset{\sim}{x}_k - \hat{\underset{\sim}{x}}_k$$

To find,

$$E[\tilde{y}_1(t)^2], \quad E[\tilde{y}_2(t)^2],$$

$$\Rightarrow \quad C_a P_k C_a'$$

augmented C_a from Q1

using the Kalman Filter algorithm,

H_k = 1^{st} row of C_a of augmented matrix. ($\because$ one measurement available)

Coding the algorithm in matlab

```
H_k = Ca(1,:);
R_k = [8];
P_ko = 1/2*eye(9,9);
S_o = inv(H_k*P_ko*H_k' + R_k);
K_ko = P_ko*H_k'*S_o;
P_kc = (eye(9,9) - K_ko*H_k)*P_ko;
y_o = Ca*P_kc*Ca';
y11 = [sqrt((y_o(1,1)))];
y22 = [sqrt((y_o(2,2)))];
```

```
for i = 1:3500
    %%prediction
    P_kp = F*P_kc*F' + Q_bar;
    S = inv(H_k*P_kp*H_k' + R_k);
    K_k = P_kp*H_k'*S;
    %%correction
    P_kc = (eye(9,9) - K_k*H_k)*P_kp;
    x(i) = i;
    y_o = Ca*P_kc*Ca';
    y11(i) = sqrt((y_o(1,1)));
    y22(i) = sqrt(y_o(2,2));
end
figure(1)
plot(x, y11)
xlabel('iterations')
ylabel('RMS Uncertainty in speed')
plot(x, y22)
xlabel('iterations')
ylabel('RMS Uncertainty in climb rate')
```

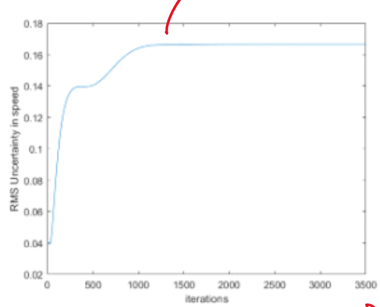
measurement Ca(1,:);


```
H_k = Ca(1,:);
R_k = [8];
P_ko = 1/2*eye(9,9);
S_o = inv(H_k*P_ko*H_k' + R_k);
K_ko = P_ko*H_k'*S_o;
P_kc = (eye(9,9) - K_ko*H_k)*P_ko;
y_o = Ca*P_kc*Ca';
y11 = [sqrt((y_o(1,1)))];
y22 = [sqrt((y_o(2,2)))];
```

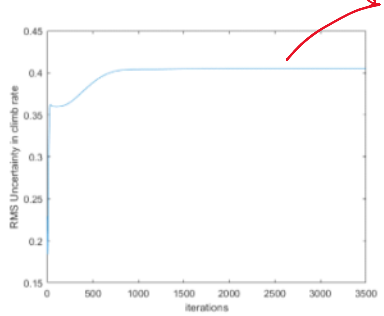

Output covariance using original Ca


```
for i = 1:3500
    %%prediction
    P_kp = F*P_kc*F' + Q_bar;
    S = inv(H_k*P_kp*H_k' + R_k);
    K_k = P_kp*H_k'*S;
    %%correction
    P_kc = (eye(9,9) - K_k*H_k)*P_kp;
    x(i) = i;
    y_o = Ca*P_kc*Ca';
    y11(i) = sqrt((y_o(1,1)));
    y22(i) = sqrt(y_o(2,2));
end
figure(1)
plot(x, y11)
xlabel('iterations')
ylabel('RMS Uncertainty in speed')
plot(x, y22)
xlabel('iterations')
ylabel('RMS Uncertainty in climb rate')
```


~0.1667



~0.4051



using "dare" function to check the steady state using "dare" function to check the steady state RMS uncertainty in each output,

$$P_{\infty}^{-} = \text{dare}(F', H', Q, R)$$

using Discrete algebraic Ricatti equation.

$$K_{\infty} = P_{\infty}^{-} \times H_k \times (H_k \times P_{\infty}^{-} \times H_k^T + R_k)$$

$$P_{\infty} = (\mathbb{I} - K_{\infty} H_k) P_{\infty}^{-}$$

```
%%%dare to
[Ppinf,L_,G_] = dare(F',H_k',Q_bar,R_k)
Sinf = inv(H_k*Ppinf*H_k' + R_k);
K_inf = Ppinf*H_k'*S
P_cinf = (eye(9,9) - K_inf*H_k)*Ppinf;
var = Ca*P_cinf*Ca'
```

```
iterations

Ppinf = 9×9
    9.8095   -0.0580   -0.2206   -0.3488   -5.9368   59.3944 ...
   -0.0580    8.7122    0.1480    0.3525    0.1298  -29.9727
   -0.2206    0.1480    1.1309    0.0365    9.0615    0.3169
   -0.3488    0.3525    0.0365    0.8616   -1.9959  -15.1075
   -5.9368    0.1298    9.0615   -1.9959  107.9913   12.2599
   59.3944  -29.9727    0.3169  -15.1075   12.2599  922.2464
   -0.3180    0.4250    0.0308    0.8156    7.0108  -10.3520
   -0.5619   -0.0295    0.9031   -0.2811   10.0981    2.2612
    5.9956   -2.9943   -0.0586   -1.4826    0.2162   91.9985

L_ = 9×1 complex
   0.9979 + 0.0030i
   0.9979 - 0.0030i
   0.9835 + 0.0000i
   0.9835 + 0.0000i
   0.9834 + 0.0028i
   0.9834 - 0.0028i
   0.9409 + 0.0691i
   0.9409 - 0.0691i
   0.6065 + 0.0000i

G_ = 1×9
    0.0446   -0.0446   -0.0029   -0.0014   -0.0427    0.3897 ...

K_inf = 9×1
    0.0444
   -0.0447
   -0.0029
   -0.0016
   -0.0419
    0.3918
   -0.0019
   -0.0040
    0.0396

var = 2×2
    0.0278   -0.0023
   -0.0023    0.1641
```

$$\boxed{\therefore \; E[\tilde{y}_1(t)^2] = \sqrt{0.0278} \simeq 0.16673}$$

(speed at steady state)

$$\boxed{E[\tilde{y}_2(t)^2] = \sqrt{0.1641} \simeq 0.40509}$$

(climb rate at steady state)