

Lean4Less: Translating Lean to Smaller Theories via an Extensional-to-Intensional Translation

Rish Vaishnav

Presented at ICTAC 2025 in Marrakesh, Morocco

December 4, 2025

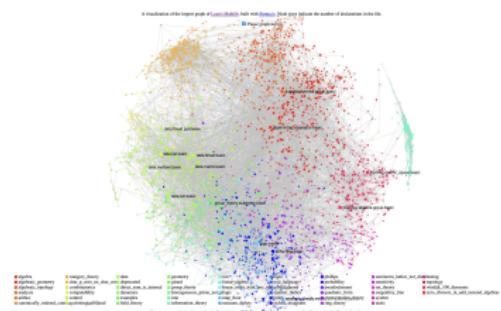
DEDUC
H
EAM



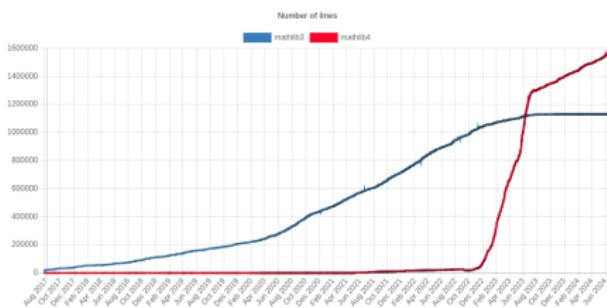
école
normale
supérieure
paris-saclay

Introduction: Lean

- Lean (<https://lean-lang.org/>): proof assistant developed by the Lean FRO (<https://lean-fro.org/>)
- Type theory: calculus of inductive constructions with an infinite universe hierarchy and impredicative `Prop`
- mathlib4: large library of mathematics formalized in Lean 4



mathlib's import graph



mathlib's growth

Lean's Type Theory: Basics

Lean's type theory is based on the Calculus of Inductive Constructions (CIC), w/ an infinite hierarchy of “type collections”, i.e. **Sorts**, in particular:

Lean's Type Theory: Basics

Lean's type theory is based on the Calculus of Inductive Constructions (CIC), w/ an infinite hierarchy of “type collections”, i.e. **Sorts**, in particular:

- **Type** contains constructed mathematical concepts:
 - `(Nat : Type)`: type of natural numbers
 - `(Bool : Type)`: type of booleans
 - `(Real : Type)`: type of real numbers
 - `(List : Type → Type)`: type of lists of a specified type

Lean's Type Theory: Basics

Lean's type theory is based on the Calculus of Inductive Constructions (CIC), w/ an infinite hierarchy of “type collections”, i.e. **Sorts**, in particular:

- **Type** contains constructed mathematical concepts:
 - `(Nat : Type)`: type of natural numbers
 - `(Bool : Type)`: type of booleans
 - `(Real : Type)`: type of real numbers
 - `(List : Type → Type)`: type of lists of a specified type
- **Prop** contains logical statements:
 - `0 < 1`
 - `False → True`
 - `(l : List Nat).rev.rev = l`

Lean's Type Theory: Basics

Lean's type theory is based on the Calculus of Inductive Constructions (CIC), w/ an infinite hierarchy of “type collections”, i.e. **Sorts**, in particular:

- **Type** contains constructed mathematical concepts:
 - `(Nat : Type)`: type of natural numbers
 - `(Bool : Type)`: type of booleans
 - `(Real : Type)`: type of real numbers
 - `(List : Type → Type)`: type of lists of a specified type
- **Prop** contains logical statements:
 - `0 < 1`
 - `False → True`
 - `(l : List Nat).rev.rev = l`

Key to Lean's proof capabilities is the **propositions-as-types** principle:

```
-- any proposition implies itself
theorem ex1 (P : Prop) : P → P :=
fun (p : P) => p -- a function is a proof
```

Lean's Type Theory: Inductive Types

Lean's rich expressivity comes from the use of **inductive types**:

```
inductive Nat where -- the natural numbers
| zero : Nat           -- zero, the smallest natural number
| succ (n : Nat) : Nat -- the successor of a natural number `n`
```

Lean's Type Theory: Inductive Types

Lean's rich expressivity comes from the use of **inductive types**:

```
inductive Nat where -- the natural numbers
| zero : Nat           -- zero, the smallest natural number
| succ (n : Nat) : Nat -- the successor of a natural number `n`
```

Inductive types allow functions to be defined via **elimination**:

-- the addition operation; `Nat.add a b` is abbreviated `a + b`

```
def Nat.add : Nat → Nat → Nat
| a, Nat.zero    => a
| a, Nat.succ b => Nat.succ (Nat.add a b)
```

Lean's Type Theory: Definitional Equalities

To aid in formalization, Lean also features certain **definitional equalities**:

```
inductive Vec : (n : Nat) → Type where ... -- vector of length `n`  
def ex2 (v : Vec n) : Vec (n + 0) := v
```

Lean's Type Theory: Definitional Equalities

To aid in formalization, Lean also features certain **definitional equalities**:

```
inductive Vec : (n : Nat) → Type where ... -- vector of length `n`  
def ex2 (v : Vec n) : Vec (n + 0) := v
```

- Type of `v` inferred as `Vec n`, which Lean identifies w/ `Vec (n + 0)`

Lean's Type Theory: Definitional Equalities

To aid in formalization, Lean also features certain **definitional equalities**:

```
inductive Vec : (n : Nat) → Type where ... -- vector of length `n`  
def ex2 (v : Vec n) : Vec (n + 0) := v
```

- Type of `v` inferred as `Vec n`, which Lean identifies w/ `Vec (n + 0)`

Definitional equality is utilized in Lean's “conversion” typing rule:

$$\frac{\Delta \vdash A, B : \text{Sort } u \quad \Delta \vdash A \equiv B \quad \Delta \vdash t : A}{\Delta \vdash t : B} \text{ [CONV]}$$

- $\Delta \vdash t : T$ is Lean's typing judgment
- $\Delta \vdash a \equiv b$ is Lean's definitional equality judgment

Lean's Type Theory: Definitional Equalities

To aid in formalization, Lean also features certain **definitional equalities**:

```
inductive Vec : (n : Nat) → Type where ... -- vector of length `n`  
def ex2 (v : Vec n) : Vec (n + 0) := v
```

- Type of v inferred as $\text{Vec } n$, which Lean identifies w/ $\text{Vec } (n + 0)$

Definitional equality is utilized in Lean's “conversion” typing rule:

$$\frac{\Delta \vdash A, B : \text{Sort } u \quad \Delta \vdash A \equiv B \quad \Delta \vdash t : A}{\Delta \vdash t : B} \text{ [CONV]}$$

- $\Delta \vdash t : T$ is Lean's typing judgment
- $\Delta \vdash a \equiv b$ is Lean's definitional equality judgment

Above, $\Delta \vdash \text{Vec } n \equiv \text{Vec } (n + 0)$, so $\Delta \vdash t : \text{Vec } (n + 0)$ by [CONV].

Propositional Equality

It is possible to formulate an equality inductive type in Lean:

```
-- equality inductive type; `Eq a b` is abbreviated `a = b`
inductive Eq {A : Type} : A → A → Prop where
-- Eq.refl : {A : Type} → (a : A) → Eq a a
| refl (a : A) : Eq a a
```

Propositional Equality

It is possible to formulate an equality inductive type in Lean:

```
-- equality inductive type; `Eq a b` is abbreviated `a = b`
inductive Eq {A : Type} : A → A → Prop where
-- Eq.refl : {A : Type} → (a : A) → Eq a a
| refl (a : A) : Eq a a
```

By [CONV], we have `Eq.refl a : a = b` for any defeq `a` and `b`, e.g.:

```
theorem ex3 (n : Nat) : n + 0 = n := Eq.refl n
```

Propositional Equality

It is possible to formulate an equality inductive type in Lean:

```
-- equality inductive type; `Eq a b` is abbreviated `a = b`
inductive Eq {A : Type} : A → A → Prop where
-- Eq.refl : {A : Type} → (a : A) → Eq a a
| refl (a : A) : Eq a a
```

By [CONV], we have `Eq.refl a : a = b` for any defeq `a` and `b`, e.g.:

```
theorem ex3 (n : Nat) : n + 0 = n := Eq.refl n
```

- Therefore, any definitional equality corresponds to a provable propositional equality

Explicit Type Conversion

The cast operation (a.k.a. type transport) makes conversion explicit:

```
def cast {A B : Sort u} (h : A = B) (a : A) : B := h.rec a
```

Explicit Type Conversion

The cast operation (a.k.a. type transport) makes conversion explicit:

```
def cast {A B : Sort u} (h : A = B) (a : A) : B := h.rec a
```

- cast allows you to type a term under a provably equal type

Explicit Type Conversion

The cast operation (a.k.a. type transport) makes conversion explicit:

```
def cast {A B : Sort u} (h : A = B) (a : A) : B := h.rec a
```

- cast allows you to type a term under a provably equal type

The following definition is ill-typed in Lean:

```
def ex4 (n : Nat) (v : Vec n) : Vec (0 + n) :=
v -- ERROR! expected type `Vec n`
```

- Problem: $0 + n$ is not defeq to n (addition matches on second arg)

Explicit Type Conversion

The cast operation (a.k.a. type transport) makes conversion explicit:

```
def cast {A B : Sort u} (h : A = B) (a : A) : B := h.rec a
```

- cast allows you to type a term under a provably equal type

The following definition is ill-typed in Lean:

```
def ex4 (n : Nat) (v : Vec n) : Vec (0 + n) :=
v -- ERROR! expected type `Vec n`
```

- Problem: $0 + n$ is not defeq to n (addition matches on second arg)

To get Lean to accept it, we wrap it with `cast` and a proof:

```
def ex4' (n : Nat) (v : Vec n) : Vec (0 + n) :=
cast
-- proof that `Vec n = Vec (0 + n)`
(congr rfl (zero_add n).symm)
v
```

Explicit Type Conversion

The cast operation (a.k.a. type transport) makes conversion explicit:

```
def cast {A B : Sort u} (h : A = B) (a : A) : B := h.rec a
```

- cast allows you to type a term under a provably equal type

The following definition is ill-typed in Lean:

```
def ex4 (n : Nat) (v : Vec n) : Vec (0 + n) :=
v -- ERROR! expected type `Vec n`
```

- Problem: $0 + n$ is not defeq to n (addition matches on second arg)

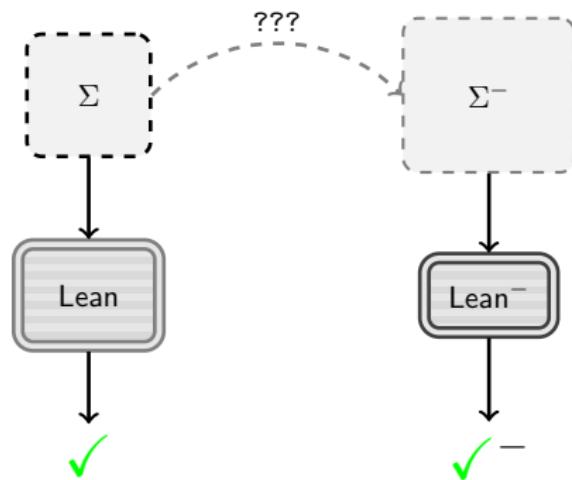
To get Lean to accept it, we wrap it with `cast` and a proof:

```
def ex4' (n : Nat) (v : Vec n) : Vec (0 + n) :=
cast
-- proof that `Vec n = Vec (0 + n)`
(congr rfl (zero_add n).symm)
v
```

- The cast around `v` is rather unsightly, but it works!

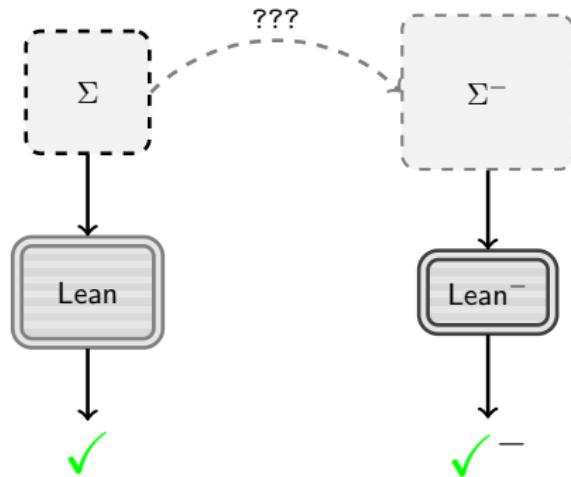
Speculation: Translation to a Weaker Theory?

Above, we have compensated for a *lack of expressivity* in Lean by using Eq and cast. What if Lean was even *less expressive* (with fewer defeqs)? Can we generalize this process to translate to this smaller theory?



Speculation: Translation to a Weaker Theory?

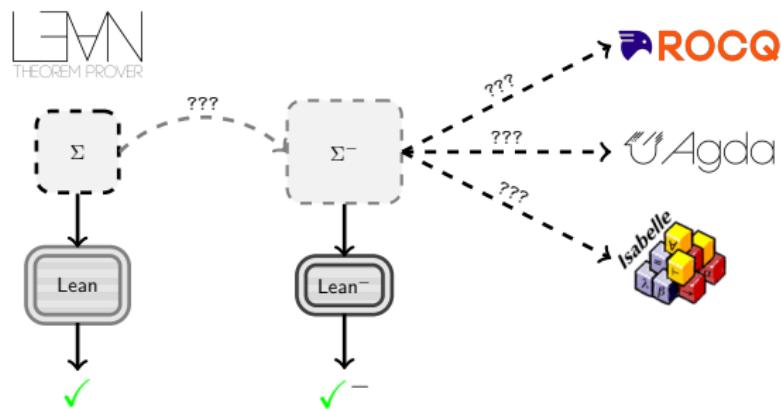
Above, we have compensated for a *lack of expressivity* in Lean by using `Eq` and `cast`. What if Lean was even *less* expressive (with fewer defeqs)? Can we generalize this process to translate to this smaller theory?



- Benefit: smaller kernels are easier to implement and verify, and are therefore more trustworthy

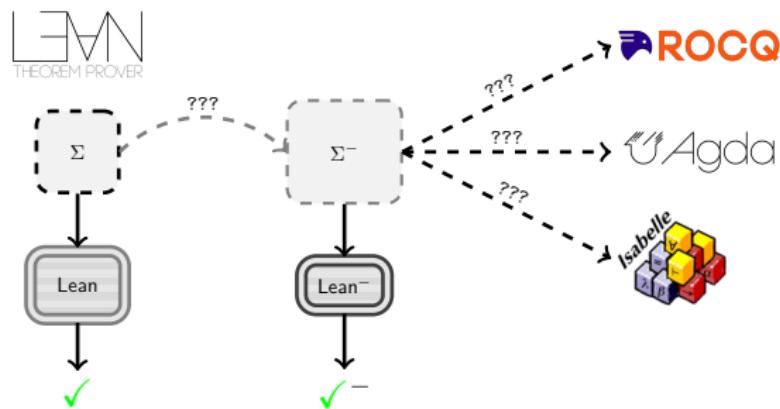
Speculation: Translation to a Weaker Theory?

Another benefit of translation to a smaller theory: easier **proof translation**



Speculation: Translation to a Weaker Theory?

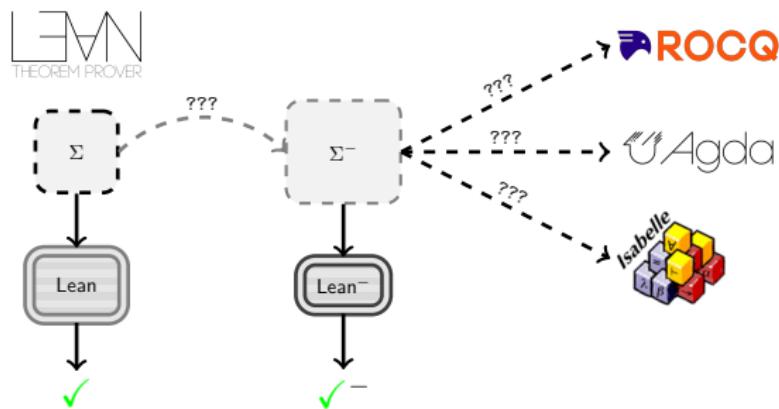
Another benefit of translation to a smaller theory: easier **proof translation**



- Fewer assumptions need to be made on target theory

Speculation: Translation to a Weaker Theory?

Another benefit of translation to a smaller theory: easier **proof translation**



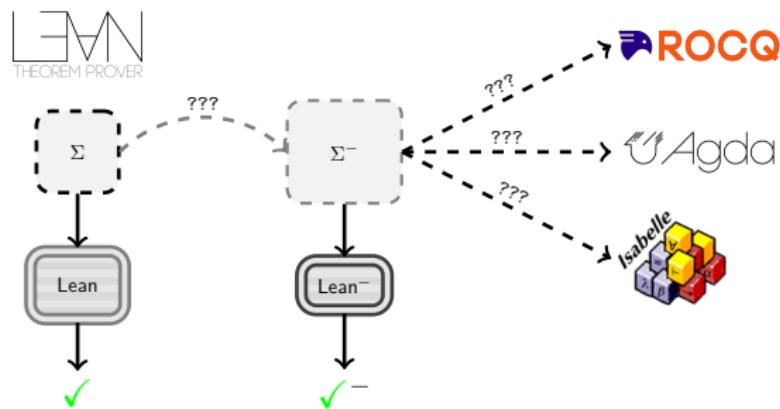
- Fewer assumptions need to be made on target theory

Some benefits of translation:

- Make Lean's formalizations available to them to extend/adapt

Speculation: Translation to a Weaker Theory?

Another benefit of translation to a smaller theory: easier **proof translation**



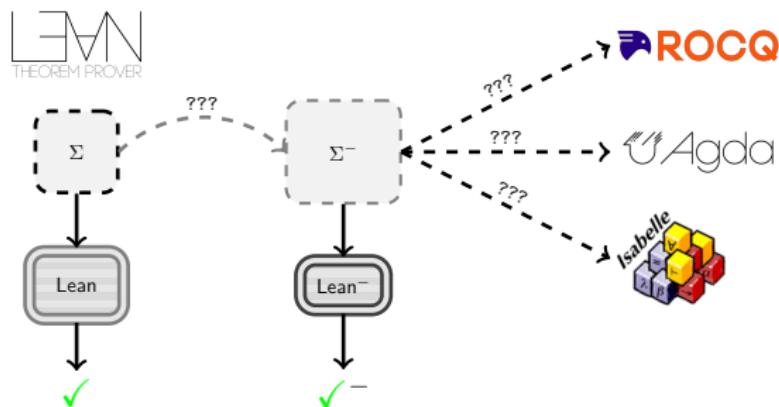
- Fewer assumptions need to be made on target theory

Some benefits of translation:

- Make Lean's formalizations available to them to extend/adapt
- Improve confidence in Lean's proof libraries through cross-checking

Speculation: Translation to a Weaker Theory?

Another benefit of translation to a smaller theory: easier **proof translation**



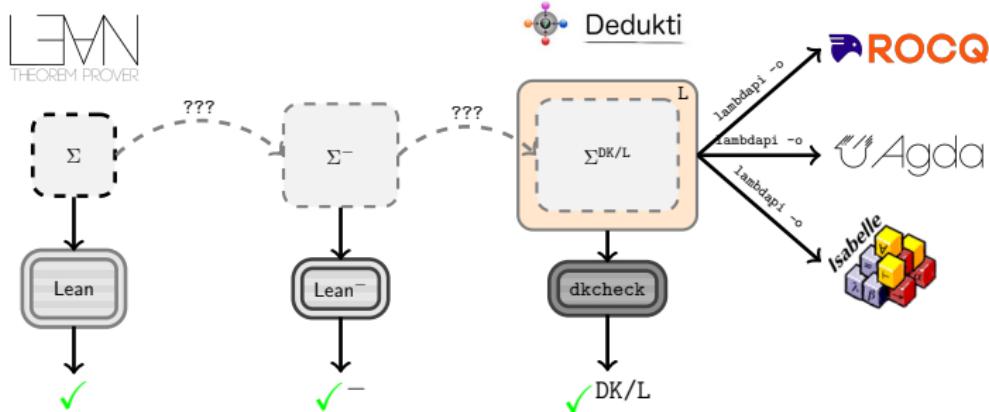
- Fewer assumptions need to be made on target theory

Some benefits of translation:

- Make Lean's formalizations available to them to extend/adapt
- Improve confidence in Lean's proof libraries through cross-checking
- Prevent duplication of work in writing libraries, tooling, etc.

Proof Translation: Motivations

We target Dedukti¹, a proof system and logical framework designed for interoperability, as an intermediate theory from which we export proofs

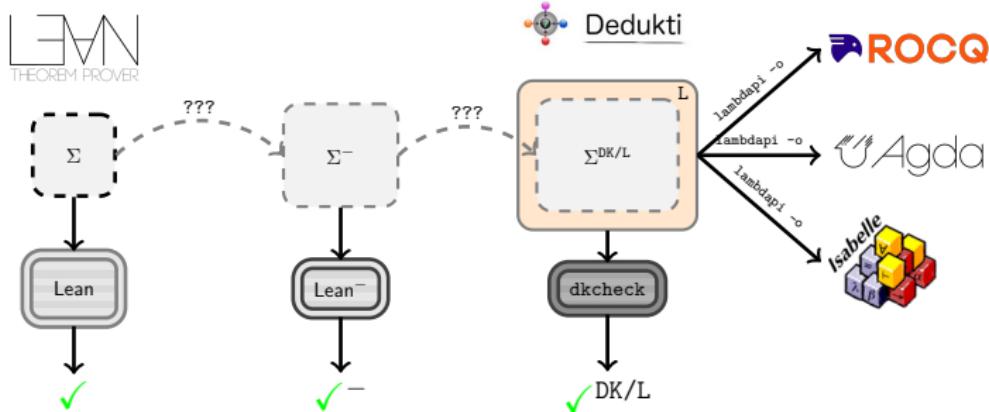


- Dedukti encoding L accounts for defeqs specific to Lean⁻

¹<https://deducteam.github.io/>

Proof Translation: Motivations

We target Dedukti¹, a proof system and logical framework designed for interoperability, as an intermediate theory from which we export proofs

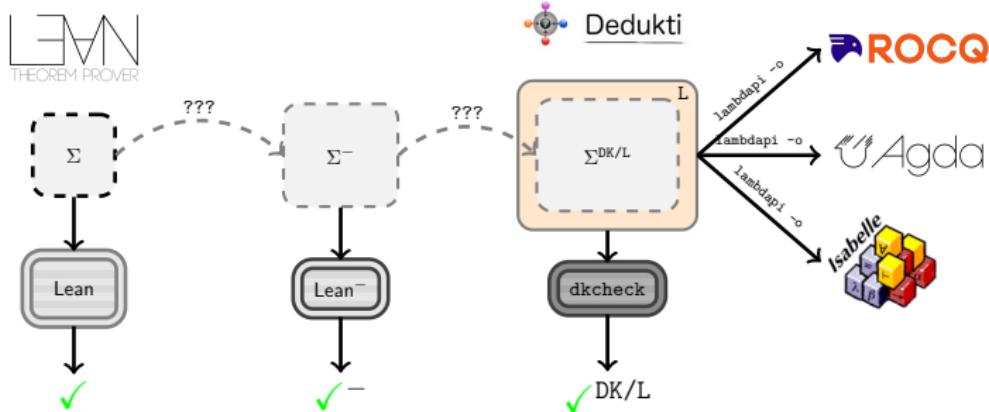


- Dedukti encoding L accounts for defeqs specific to Lean⁻
So: what defeqs should we eliminate in the initial translation?

¹<https://deducteam.github.io/>

Proof Translation: Motivations

We target Dedukti¹, a proof system and logical framework designed for interoperability, as an intermediate theory from which we export proofs

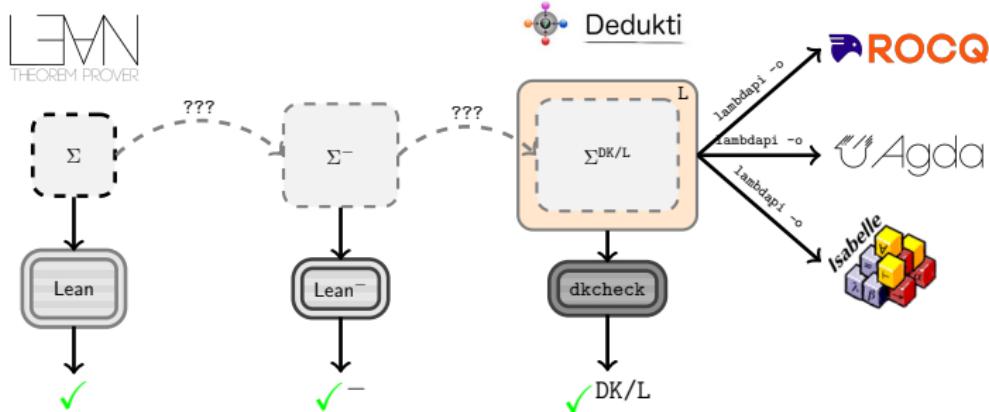


- Dedukti encoding L accounts for defeqs specific to Lean⁻
So: what defeqs should we eliminate in the initial translation?
- Whichever ones are *not directly encodable* within Dedukti

¹<https://deducteam.github.io/>

Proof Translation: Motivations

We target Dedukti¹, a proof system and logical framework designed for interoperability, as an intermediate theory from which we export proofs



- Dedukti encoding L accounts for defeqs specific to Lean⁻

So: what defeqs should we eliminate in the initial translation?

- Whichever ones are *not directly encodable* within Dedukti
- It turns out that Lean's particular rules of “proof irrelevance” and “K-like reduction” do not give way to an encoding

¹<https://deducteam.github.io/>

Lean's Type Theory: Proof Irrelevance

Lean features a special defeq rule known as **proof irrelevance**:

$$\frac{\Delta \vdash P : \text{Prop} \quad \Delta \vdash p, q : P}{\Delta \vdash p \equiv q} [\text{PI}]$$

- Any two proofs of the same proposition are identified

Lean's Type Theory: Proof Irrelevance

Lean features a special defeq rule known as **proof irrelevance**:

$$\frac{\Delta \vdash P : \text{Prop} \quad \Delta \vdash p, q : P}{\Delta \vdash p \equiv q} [\text{PI}]$$

- Any two proofs of the same proposition are identified

This is useful, for, instance, in subtyping:

```
-- type of `Nat`'s less than 5
inductive LT5 : Type where
| mk : (n : Nat) → (p : n < 5) → LT5
theorem ex5 (n : Nat) (p1 p2 : n < 5) :
  LT5.mk n p1 = LT5.mk n p2 :=
-- `p1` and `p2` are defeq by proof irrelevance, which gives us
-- that `LT5.mk n p1` and `LT5.mk n p2` are defeq as well
refl
```

Lean's Type Theory: Proof Irrelevance

Lean features a special defeq rule known as **proof irrelevance**:

$$\frac{\Delta \vdash P : \text{Prop} \quad \Delta \vdash p, q : P}{\Delta \vdash p \equiv q} [\text{PI}]$$

- Any two proofs of the same proposition are identified

This is useful, for, instance, in subtyping:

```
-- type of `Nat`'s less than 5
inductive LT5 : Type where
| mk : (n : Nat) → (p : n < 5) → LT5
theorem ex5 (n : Nat) (p1 p2 : n < 5) :
  LT5.mk n p1 = LT5.mk n p2 :=
-- `p1` and `p2` are defeq by proof irrelevance, which gives us
-- that `LT5.mk n p1` and `LT5.mk n p2` are defeq as well
rfl
```

However, the typing requirement on p and q make this hard to encode.

- Dedukti's rewrite rules cannot "match" based on typing

Lean's Type Theory: K-Like Reduction

Lean features another special rule known as **K-like reduction**:

$$\frac{\Delta \vdash \text{mk } p_1 \dots p_n : K \dots p_n i_1 \dots i_m \quad \Delta \vdash t : K \ p_1 \dots p_n i_1 \dots i_m}{\Delta \vdash t \rightsquigarrow \text{mk } p_1 \dots p_n} \text{ [KLR]}$$

This applies to any “K-like” type K , an inductive proposition with one constructor without (non-parametric) arguments

Lean's Type Theory: K-Like Reduction

Lean features another special rule known as **K-like reduction**:

$$\frac{\Delta \vdash \text{mk } p_1 \dots p_n : K \dots p_n i_1 \dots i_m \quad \Delta \vdash t : K \ p_1 \dots p_n i_1 \dots i_m}{\Delta \vdash t \rightsquigarrow \text{mk } p_1 \dots p_n} \text{ [KLR]}$$

This applies to any “K-like” type K , an inductive proposition with one constructor without (non-parametric) arguments

- This is a **reduction rule**, not a definitional equality – it relates to the reduction subroutine of defeq-checking

Lean's Type Theory: K-Like Reduction

Lean features another special rule known as **K-like reduction**:

$$\frac{\Delta \vdash \text{mk } p_1 \dots p_n : K \dots p_n i_1 \dots i_m \quad \Delta \vdash t : K \ p_1 \dots p_n i_1 \dots i_m}{\Delta \vdash t \rightsquigarrow \text{mk } p_1 \dots p_n} \quad [\text{KLR}]$$

This applies to any “K-like” type K , an inductive proposition with one constructor without (non-parametric) arguments

- This is a **reduction rule**, not a definitional equality – it relates to the reduction subroutine of defeq-checking

As Eq is K-like, [KLR] allows the kernel to eliminate redundant casts:

```
theorem ex6 (n : Nat) (v : Vec n) (h : Vec n = Vec (n + 0)) :  
  v = cast h v :=  
  -- `v` and `cast h v` are defeq thanks to [KLR]  
  rfl
```

Lean's Type Theory: K-Like Reduction

Lean features another special rule known as **K-like reduction**:

$$\frac{\Delta \vdash \text{mk } p_1 \dots p_n : K \dots p_n i_1 \dots i_m \quad \Delta \vdash t : K \ p_1 \dots p_n i_1 \dots i_m}{\Delta \vdash t \rightsquigarrow \text{mk } p_1 \dots p_n} \quad [\text{KLR}]$$

This applies to any “K-like” type K , an inductive proposition with one constructor without (non-parametric) arguments

- This is a **reduction rule**, not a definitional equality – it relates to the reduction subroutine of defeq-checking

As Eq is K-like, [KLR] allows the kernel to eliminate redundant casts:

```
theorem ex6 (n : Nat) (v : Vec n) (h : Vec n = Vec (n + 0)) :  
  v = cast h v :=  
  -- `v` and `cast h v` are defeq thanks to [KLR]  
  rfl
```

[KLR] has a complex typing requirement on t , and is also hard to encode.

Our Target Theory: Lean⁻

In our target theory Lean⁻, we have removed [PI] and [KLR]:

$$\frac{\Delta \vdash P : \text{Prop} \quad \Delta \vdash p, q : P}{\Delta \vdash p \equiv q} \text{[PI]} \quad \frac{\Delta \vdash \text{mk } p_1 \dots p_n : K \dots \quad \Delta \vdash t : K \ p_1 \dots}{\Delta \vdash t \rightsquigarrow \text{mk } p_1 \dots p_n} \text{[KLR]}$$

Let's use $\Delta \vdash^- t : T$ for Lean⁻'s typing judgment.

Our Target Theory: Lean⁻

In our target theory Lean⁻, we have removed [PI] and [KLR]:

$$\frac{\Delta \vdash P : \text{Prop} \quad \Delta \vdash p, q : P}{\Delta \vdash p \equiv q} \text{[PI]} \quad \frac{\Delta \vdash \text{mk } p_1 \dots p_n : K \dots \quad \Delta \vdash t : K \ p_1 \dots}{\Delta \vdash t \rightsquigarrow \text{mk } p_1 \dots p_n} \text{[KLR]}$$

Let's use $\Delta \vdash^- t : T$ for Lean⁻'s typing judgment.

- We will need to add proof irrelevance as an axiom:

```
axiom prfIrrel {P : Prop} (p q : P) : p = q
```

as otherwise, there would be proofs we can no longer express.

Our Target Theory: Lean⁻

In our target theory Lean⁻, we have removed [PI] and [KLR]:

$$\frac{\Delta \vdash P : \text{Prop} \quad \Delta \vdash p, q : P}{\Delta \vdash p \equiv q} \text{[PI]} \quad \frac{\Delta \vdash \text{mk } p_1 \dots p_n : K \dots \Delta \vdash t : K \ p_1 \dots}{\Delta \vdash t \rightsquigarrow \text{mk } p_1 \dots p_n} \text{[KLR]}$$

Let's use $\Delta \vdash^- t : T$ for Lean⁻'s typing judgment.

- We will need to add proof irrelevance as an axiom:

`axiom prfIrrel {P : Prop} (p q : P) : p = q`

as otherwise, there would be proofs we can no longer express.

- Our goal: define a translation $|\cdot|^-$ such that:

If $\Delta \vdash t : T$, then `prfIrrel :: |Δ|^- ⊢^- |t|^- : |T|^-`.

That is, typeability should be preserved by our translation
(assuming `prfIrrel` in the Lean⁻ typing context)

Designing a Translation: Existing Work?

So, how can we design and implement a translation eliminating these judgments? Is there any existing work that we can take advantage of?

Designing a Translation: Existing Work?

So, how can we design and implement a translation eliminating these judgments? Is there any existing work that we can take advantage of?

- Relative to Lean⁻, Lean is a theory where the axiom `prfIrrel` is “promoted” to a definitional equality

Designing a Translation: Existing Work?

So, how can we design and implement a translation eliminating these judgments? Is there any existing work that we can take advantage of?

- Relative to Lean⁻, Lean is a theory where the axiom `prfIrrel` is “promoted” to a definitional equality
- Recall our difficulties around how `0 + n` is not defeq to `n`: what if we promoted every propositional equality to a definitional one?

Designing a Translation: Existing Work?

So, how can we design and implement a translation eliminating these judgments? Is there any existing work that we can take advantage of?

- Relative to Lean⁻, Lean is a theory where the axiom `prfIrrel` is “promoted” to a definitional equality
- Recall our difficulties around how `0 + n` is not defeq to `n`: what if we promoted every propositional equality to a definitional one?

This is the well-studied topic of extensional type theory (ETT), characterized by the “equality reflection rule”:

$$\frac{\Delta \vdash_e A : \text{Sort } u \quad \Delta \vdash_e t, s : A \quad \Delta \vdash_e _ : t = s}{\Delta \vdash_e t \equiv s}$$

Designing a Translation: Existing Work?

So, how can we design and implement a translation eliminating these judgments? Is there any existing work that we can take advantage of?

- Relative to Lean⁻, Lean is a theory where the axiom `prfIrrel` is “promoted” to a definitional equality
- Recall our difficulties around how `0 + n` is not defeq to `n`: what if we promoted every propositional equality to a definitional one?

This is the well-studied topic of extensional type theory (ETT), characterized by the “equality reflection rule”:

$$\frac{\Delta \vdash_e A : \text{Sort } u \quad \Delta \vdash_e t, s : A \quad \Delta \vdash_e _ : t = s}{\Delta \vdash_e t \equiv s}$$

- Lean and Lean⁻ are examples of “intensional” type theories (ITT)

Designing a Translation: Existing Work?

So, how can we design and implement a translation eliminating these judgments? Is there any existing work that we can take advantage of?

- Relative to Lean⁻, Lean is a theory where the axiom `prfIrrel` is “promoted” to a definitional equality
- Recall our difficulties around how `0 + n` is not defeq to `n`: what if we promoted every propositional equality to a definitional one?

This is the well-studied topic of extensional type theory (ETT), characterized by the “equality reflection rule”:

$$\frac{\Delta \vdash_e A : \text{Sort } u \quad \Delta \vdash_e t, s : A \quad \Delta \vdash_e _ : t = s}{\Delta \vdash_e t \equiv s}$$

- Lean and Lean⁻ are examples of “intensional” type theories (ITT)
- There is a good amount of existing work regarding translation from ETT to ITT – can we adapt this for our purposes?

Lean_e^- : an Extensional Theory

Suppose we add [REFL] to Lean^- to obtain an extensional theory Lean_e^- :

$$\frac{\Delta \vdash_e^- A : \text{Sort } u \quad \Delta \vdash_e^- t, s : A \quad \Delta \vdash_e^- _ : t = s}{\Delta \vdash_e^- t \equiv s} \text{ [REFL]}$$

Lean_e^- : an Extensional Theory

Suppose we add [REFL] to Lean^- to obtain an extensional theory Lean_e^- :

$$\frac{\Delta \vdash_e^- A : \text{Sort } u \quad \Delta \vdash_e^- t, s : A \quad \Delta \vdash_e^- _ : t = s}{\Delta \vdash_e^- t \equiv s} \text{ [REFL]}$$

- Lean_e^- is strictly more expressive than Lean, since we can recover definitional proof irrelevance via [REFL]:

$$\frac{\Delta \vdash_e^- P : \text{Prop} \quad \Delta \vdash_e^- p, q : P \quad \Delta \vdash_e^- \text{prfIrrel } p \ q : p = q}{\Delta \vdash_e^- p \equiv q}$$

(we can also show that [KLR] is recovered)

Theories Overview

To summarize, we have the following theories:

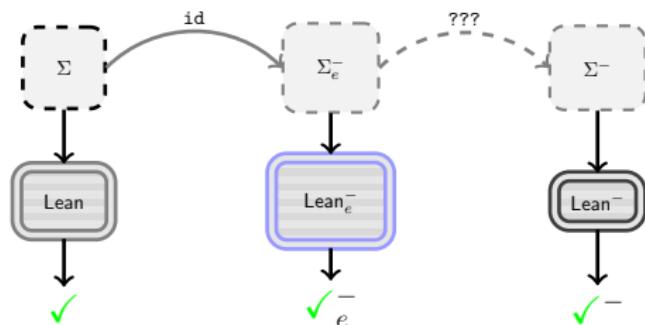
Theory	Rules	Axioms	\subsetneq
Lean ⁻ (\vdash^-)		prfIrrel	Lean
Lean (\vdash)	[PI], [KLR]		Lean _e ⁻
Lean _e ⁻ (\vdash_e^-)	[REFL]	prfIrrel	

Theories Overview

To summarize, we have the following theories:

Theory	Rules	Axioms	\subsetneq
Lean ⁻ (\vdash^-)		prfIrrel	Lean
Lean (\vdash)	[PI], [KLR]		Lean ⁻ _e
Lean ⁻ _e (\vdash_e^-)	[REFL]	prfIrrel	

Can we translate from Lean to Lean⁻ using Lean⁻_e as a “middle ground”?

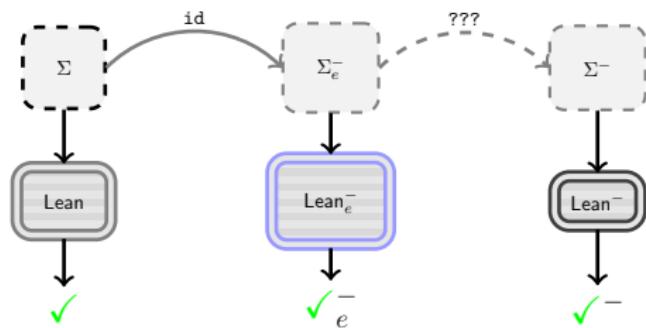


Theories Overview

To summarize, we have the following theories:

Theory	Rules	Axioms	\subsetneq
Lean ⁻ (\vdash^-)		prfIrrel	Lean
Lean (\vdash)	[PI], [KLR]		Lean ⁻ _e
Lean ⁻ _e (\vdash_e^-)	[REFL]	prfIrrel	

Can we translate from Lean to Lean⁻ using Lean⁻_e as a “middle ground”?



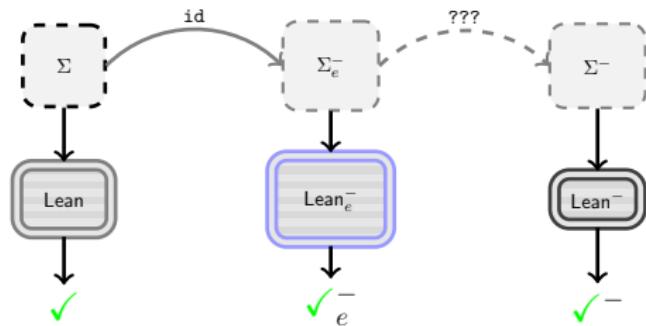
- Lean \rightarrow Lean⁻_e is the identity function

Theories Overview

To summarize, we have the following theories:

Theory	Rules	Axioms	\subsetneq
Lean ⁻ (\vdash^-)		prfIrrel	Lean
Lean (\vdash)	[PI], [KLR]		Lean ⁻ _e
Lean ⁻ _e (\vdash_e^-)	[REFL]	prfIrrel	

Can we translate from Lean to Lean⁻ using Lean⁻_e as a “middle ground”?



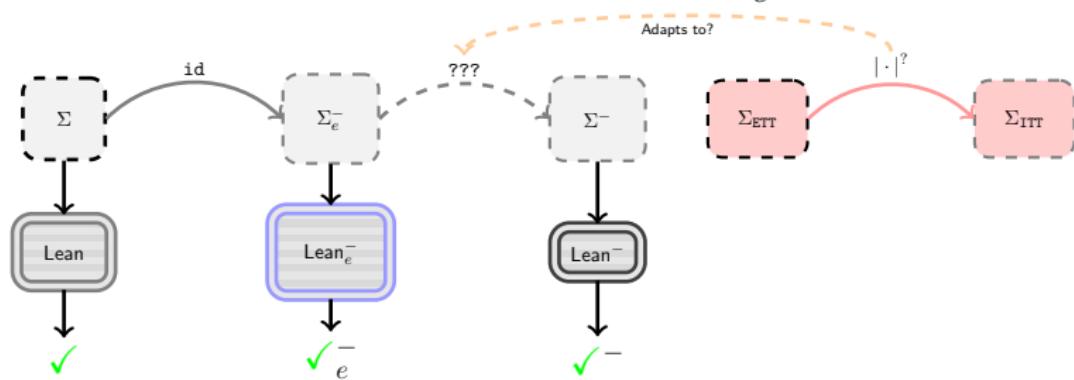
- Lean \rightarrow Lean⁻_e is the identity function
- Lean⁻_e \rightarrow Lean⁻ requires “eliminating” [REFL]: can this be done?

Theories Overview

To summarize, we have the following theories:

Theory	Rules	Axioms	\subsetneq
Lean ⁻ (\vdash^-)		prfIrrel	Lean
Lean (\vdash)	[PI], [KLR]		Lean ⁻ _e
Lean ⁻ _e (\vdash_e^-)	[REFL]	prfIrrel	

Can we translate from Lean to Lean⁻ using Lean⁻_e as a “middle ground”?



- Lean \rightarrow Lean⁻_e is the identity function
- Lean⁻_e \rightarrow Lean⁻ requires “eliminating” [REFL]: can this be done?
 - Existing work on the translation from ETT to ITT may help

From ETT to ITT

What existing work on translating from ETT to ITT can we make use of?

From ETT to ITT

What existing work on translating from ETT to ITT can we make use of?

- First conservativity result of ETT over ITT shown by Hofmann [3]

From ETT to ITT

What existing work on translating from ETT to ITT can we make use of?

- First conservativity result of ETT over ITT shown by Hofmann [3]
- A *constructive* proof first shown by Winterhalter et. al. [1]
 - Formalized in Rocq in the repository `ett-to-itt` [4]

From ETT to ITT

What existing work on translating from ETT to ITT can we make use of?

- First conservativity result of ETT over ITT shown by Hofmann [3]
- A *constructive* proof first shown by Winterhalter et. al. [1]
 - Formalized in Rocq in the repository `ett-to-itt` [4]

The translation by Winterhalter et. al. takes *typing derivations* as input, and works with the more general **heterogeneous equality** type:

```
inductive HEq : {A : Sort u} → A → {B : Sort u} → B → Prop where
| refl (a : A) : HEq a a
```

- Allows for non-defeq LHS and RHS types

From ETT to ITT

What existing work on translating from ETT to ITT can we make use of?

- First conservativity result of ETT over ITT shown by Hofmann [3]
- A *constructive* proof first shown by Winterhalter et. al. [1]
 - Formalized in Rocq in the repository `ett-to-itt` [4]

The translation by Winterhalter et. al. takes *typing derivations* as input, and works with the more general **heterogeneous equality** type:

```
inductive HEq : {A : Sort u} → A → {B : Sort u} → B → Prop where
| refl (a : A) : HEq a a
```

- Allows for non-defeq LHS and RHS types

So, can we use `ett-to-itt` to translate $\text{Lean} \rightarrow \text{Lean}^-$? Some problems:

- `ett-to-itt` is defined w.r.t. very specific ETT and ITT theories
- We have no way to access typing derivations in Lean!

The Lean4Lean kernel implementation

How can we get at the typing derivations needed for our translation?

The Lean4Lean kernel implementation

How can we get at the typing derivations needed for our translation?

Idea: repurpose a kernel typechecker.

- Typecheckers implement a “search” for valid typing derivations
- Steps can be correlated with uses of typing rules – our “input”!

The Lean4Lean kernel implementation

How can we get at the typing derivations needed for our translation?

Idea: repurpose a kernel typechecker.

- Typecheckers implement a “search” for valid typing derivations
- Steps can be correlated with uses of typing rules – our “input”!

Lean4Lean [2]: project to verify Lean’s kernel & formalize its meta-theory

- Contains a Lean typechecker kernel that is *implemented in Lean*:

The screenshot shows a GitHub repository page for 'digama0/lean4lean'. The page has a light gray header with the repository name and a public status. Below the header is a navigation bar with links for Code, Issues, Pull requests, Actions, Projects, Security, and Insights. The main content area shows a code editor with a file named 'c22fe58' containing two branches and no tags. A search bar and a 'Code' button are also present. To the right of the code editor is an 'About' section. The 'About' section includes a brief description of the project: 'Lean 4 kernel / "external checker" written in Lean 4', a 'Readme' link, an 'Activity' link, and statistics: 67 stars, 6 watching, 3 forks, and a 'Report repository' link. Below the 'About' section is a 'Releases' section which states 'No releases published'.

The Lean4Lean kernel implementation

How can we get at the typing derivations needed for our translation?

Idea: repurpose a kernel typechecker.

- Typecheckers implement a “search” for valid typing derivations
- Steps can be correlated with uses of typing rules – our “input”!

Lean4Lean [2]: project to verify Lean’s kernel & formalize its meta-theory

- Contains a Lean typechecker kernel that is *implemented in Lean*:
 - Allows us to use Lean’s existing utilities to build our translation output

The screenshot shows the GitHub repository page for `digama0/lean4lean`. The page has a "Public" status, a "Code" tab selected, and 2 branches. The main content area displays the `README` file, which contains the following text:

```
This is an implementation of the Lean 4 kernel written in (mostly) pure Lean 4. It is derived directly from the C++ kernel implementation, and as such likely shares some implementation bugs with it (it's not really an independent implementation), although it also benefits from the same algorithmic performance improvements existing in the C++ Lean kernel.

The project also houses some metatheory regarding the Lean system, in the same general direction as the MetaCog project.
```

The right sidebar provides an "About" summary:

- Lean 4 kernel / 'external checker' written in Lean 4
- Readme
- Activity
- 67 stars
- 6 watching
- 3 forks
- Report repository

Below the "About" section is a "Releases" section stating "No releases published".

The Lean4Lean kernel implementation

How can we get at the typing derivations needed for our translation?

Idea: repurpose a kernel typechecker.

- Typecheckers implement a “search” for valid typing derivations
- Steps can be correlated with uses of typing rules – our “input”!

Lean4Lean [2]: project to verify Lean’s kernel & formalize its meta-theory

- Contains a Lean typechecker kernel that is *implemented in Lean*:
 - Allows us to use Lean’s existing utilities to build our translation output
 - Leaves the door open for a formally verified translation

The screenshot shows a GitHub repository page for 'digama0/lean4lean'. The page has a 'Code' tab selected. At the top, there's a search bar and navigation links for 'Notifications', 'Fork 3', and 'Star 67'. Below the header, there are sections for 'Code' (with a dropdown showing 'c22fe58'), 'Issues', 'Pull requests', 'Actions', 'Projects', 'Security', and 'Insights'. A 'README' file is shown, which contains the text 'Lean-for-Lean'. The README describes the project as an implementation of the Lean 4 kernel written in (mostly) pure Lean 4, derived from the C++ kernel implementation. It also mentions that the project houses some metatheory regarding the Lean system. To the right of the README, there's an 'About' section with details like 'Readme', 'Activity', '6 stars', '6 watching', '3 forks', and a note about the repository being 'Report repository'. Below the README, there's a 'Releases' section stating 'No releases published'.

Lean4Less implementation: The main functions

Our translation, “Lean4Less”, has been adapted from Lean4Lean.
We repurpose Lean4Lean’s main typechecking functions as follows:

```
def inferType (e : Expr) : RecM (Expr × Option Expr) := ...
def isDefEq (t s : Expr) : RecM (Bool × Option Expr) := ...
```

Both functions have a new second return value of type Option Expr:

Lean4Less implementation: The main functions

Our translation, “Lean4Less”, has been adapted from Lean4Lean.
We repurpose Lean4Lean’s main typechecking functions as follows:

```
def inferType (e : Expr) : RecM (Expr × Option Expr) := ...
def isDefEq (t s : Expr) : RecM (Bool × Option Expr) := ...
```

Both functions have a new second return value of type Option Expr:

- `inferType` produces a translation in parallel to type inference
 - Explicit type conversions via `cast` are applied to subterms as necessary

Lean4Less implementation: The main functions

Our translation, “Lean4Less”, has been adapted from Lean4Lean.
We repurpose Lean4Lean’s main typechecking functions as follows:

```
def inferType (e : Expr) : RecM (Expr × Option Expr) := ...
def isDefEq (t s : Expr) : RecM (Bool × Option Expr) := ...
```

Both functions have a new second return value of type Option Expr:

- `inferType` produces a translation in parallel to type inference
 - Explicit type conversions via `cast` are applied to subterms as necessary
- `isDefEq` produces a proof of equality between the LHS and RHS when they are judged `defeq`
 - Needed by the `casts` inserted by `inferType`
 - A sort of “trace” on the `defeq` derivation

Lean4Less implementation: The main functions

Our translation, “Lean4Less”, has been adapted from Lean4Lean.
We repurpose Lean4Lean’s main typechecking functions as follows:

```
def inferType (e : Expr) : RecM (Expr × Option Expr) := ...
def isDefEq (t s : Expr) : RecM (Bool × Option Expr) := ...
```

Both functions have a new second return value of type Option Expr:

- `inferType` produces a translation in parallel to type inference
 - Explicit type conversions via `cast` are applied to subterms as necessary
- `isDefEq` produces a proof of equality between the LHS and RHS when they are judged `defeq`
 - Needed by the `casts` inserted by `inferType`
 - A sort of “trace” on the `defeq` derivation
- We return an `Option` because of an important optimization: we only produce translated terms/proofs *when necessary*

Results: Some example translations

For instance, the following proof requires [PI] to be well-typed:

```
variable (P : Prop) (p : P) (q : P) (T : P → Prop)
-- `T p` is defeq to `T q` (due to proof irrelevance)
def ex7 (t : T p) : T q := t
```

Results: Some example translations

For instance, the following proof requires [PI] to be well-typed:

```
variable (P : Prop) (p : P) (q : P) (T : P → Prop)
-- `T p` is defeq to `T q` (due to proof irrelevance)
def ex7 (t : T p) : T q := t
```

- Our translation wraps a cast around `t`, translating it to:

```
def ex7' (t : T p) : T q := cast (congrArg T (prfIrrel p q)) t
```

Results: Some example translations

For instance, the following proof requires [PI] to be well-typed:

```
variable (P : Prop) (p : P) (q : P) (T : P → Prop)
-- `T p` is defeq to `T q` (due to proof irrelevance)
def ex7 (t : T p) : T q := t
```

- Our translation wraps a cast around t , translating it to:
`def ex7' (t : T p) : T q := cast (congrArg T (prfIrrel p q)) t`
- Need to use `prfIrrel` axiom to prove this (not possible otherwise)

Results: Some example translations

For instance, the following proof requires [PI] to be well-typed:

```
variable (P : Prop) (p : P) (q : P) (T : P → Prop)
-- `T p` is defeq to `T q` (due to proof irrelevance)
def ex7 (t : T p) : T q := t
```

- Our translation wraps a cast around t , translating it to:
`def ex7' (t : T p) : T q := cast (congrArg T (prfIrrel p q)) t`
- Need to use `prfIrrel` axiom to prove this (not possible otherwise)

Translations can get quite complex, esp. involving dependent types:

```
variable (P : Prop) (Q : P → Prop) (p q : P) (Qp : Q p) (Qq : Q q)
variable (U : (p : P) → Q p → Prop)
def ex8 (t : U p Qp) : U q Qq := t
```

Results: Some example translations

For instance, the following proof requires [PI] to be well-typed:

```
variable (P : Prop) (p : P) (q : P) (T : P → Prop)
-- `T p` is defeq to `T q` (due to proof irrelevance)
def ex7 (t : T p) : T q := t
```

- Our translation wraps a cast around t , translating it to:
`def ex7' (t : T p) : T q := cast (congrArg T (prfIrrel p q)) t`
- Need to use `prfIrrel` axiom to prove this (not possible otherwise)

Translations can get quite complex, esp. involving dependent types:

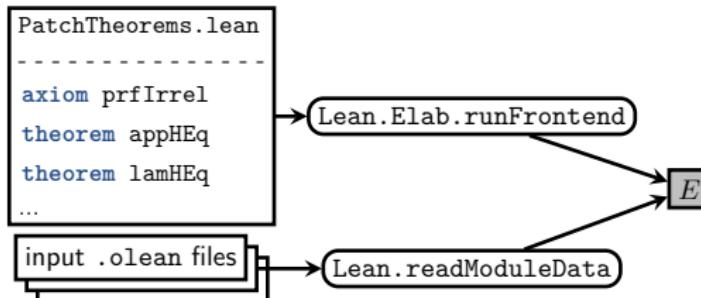
```
variable (P : Prop) (Q : P → Prop) (p q : P) (Qp : Q p) (Qq : Q q)
variable (U : (p : P) → Q p → Prop)
def ex8 (t : U p Qp) : U q Qq := t
```

- This uses [PI] in a nested manner, leading to a larger translation:

```
def ex8' (t : T p Qp) : T q Qq := cast (eq_of_heq
  (appHEq (congrArg Q (eq_of_heq (prfIrrel rfl p q)))
    (fun _ _ _ => HEq.rfl))
  (appHEq rfl ... HEq.rfl (prfIrrel rfl p q))
  (prfIrrel (congrArg Q (eq_of_heq (prfIrrel rfl p q)))
    Qp Qq))) t
```

Translation and verification workflow

Lean4Less generates and verifies its output as follows:

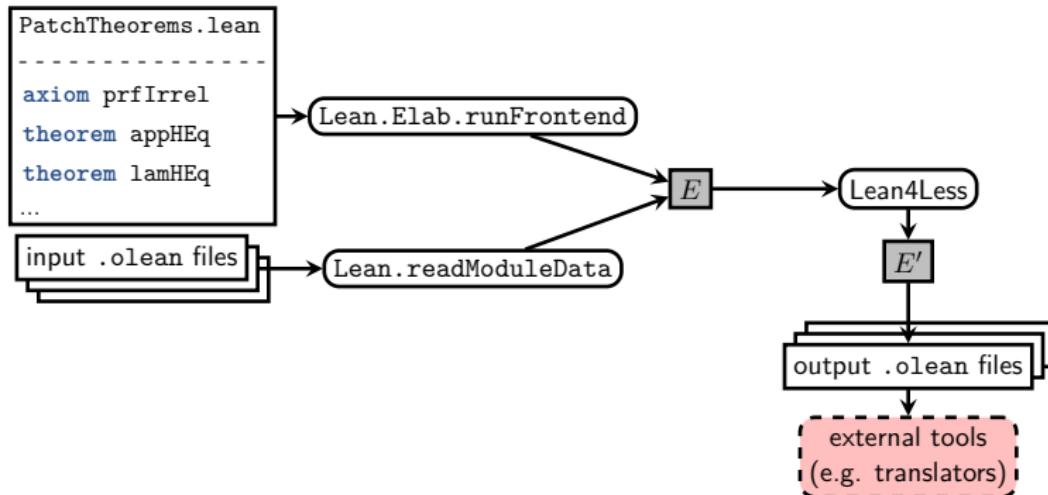


- Input environment E :

- set of preliminary translation defs from file `PatchTheorems.lean`
- constants from pre-elaborated source `.olean` files

Translation and verification workflow

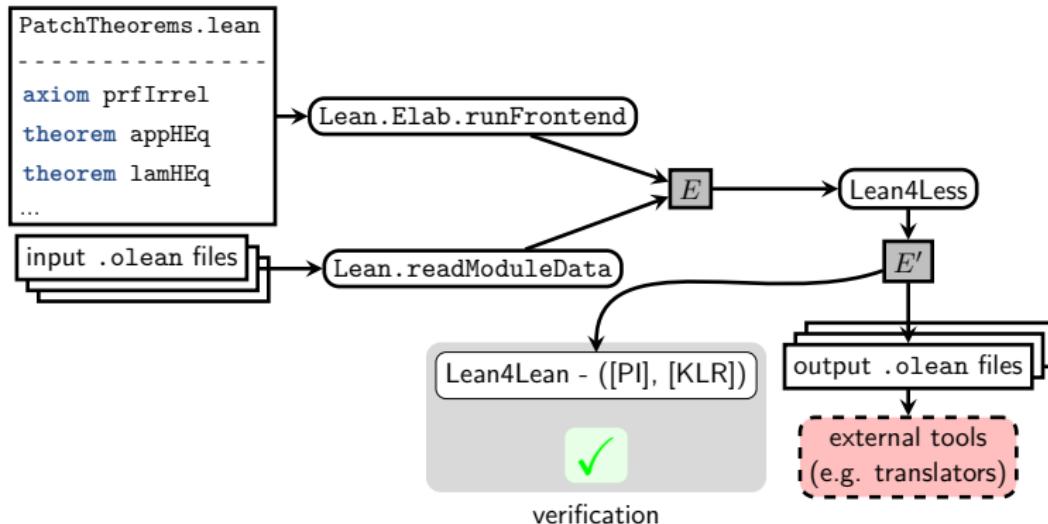
Lean4Less generates and verifies its output as follows:



- Input environment E :
 - set of preliminary translation defs from file `PatchTheorems.lean`
 - constants from pre-elaborated source `.olean` files
- Output env. E' : translated environment for export as `.olean` files

Translation and verification workflow

Lean4Less generates and verifies its output as follows:



- Input environment E :
 - set of preliminary translation defs from file `PatchTheorems.lean`
 - constants from pre-elaborated source `.olean` files
- Output env. E' : translated environment for export as `.olean` files
- Verification via a modified Lean⁻ kernel (lacking `[PI]` and `[KLR]`)

Results: Library translations

We have been able to translate (and verify) translations of the Lean standard library, as well as some smaller mathlib modules. Some numbers:

Module	Total Constants	Constants Using [PI]/[KLR] (% of total)	Input/Output Environment Size (Overhead)	Translation Runtime	Input/Output Typechecking Runtime (Overhead) ²
Std	29859	1736/134 (6.3%)	226MB/261MB (15.5%)	18m02s	2m19s/3m9s (36.0%)
Algebra.Order.Field.Rat	113899	2965/237 (2.8%)	1485MB/1501MB (1.1%)	32m16s	5m11s/5m44s (10.6%)

Results: Library translations

We have been able to translate (and verify) translations of the Lean standard library, as well as some smaller mathlib modules. Some numbers:

Module	Total Constants	Constants Using [PI]/[KLR] (% of total)	Input/Output Environment Size (Overhead)	Translation Runtime	Input/Output Typechecking Runtime (Overhead) ²
Std	29859	1736/134 (6.3%)	226MB/261MB (15.5%)	18m02s	2m19s/3m9s (36.0%)
Algebra.Order.Field.Rat	113899	2965/237 (2.8%)	1485MB/1501MB (1.1%)	32m16s	5m11s/5m44s (10.6%)

- Translation output size overheads are reasonable

Results: Library translations

We have been able to translate (and verify) translations of the Lean standard library, as well as some smaller mathlib modules. Some numbers:

Module	Total Constants	Constants Using [PI]/[KLR] (% of total)	Input/Output Environment Size (Overhead)	Translation Runtime	Input/Output Typechecking Runtime (Overhead) ²
Std	29859	1736/134 (6.3%)	226MB/261MB (15.5%)	18m02s	2m19s/3m9s (36.0%)
Algebra.Order.Field.Rat	113899	2965/237 (2.8%)	1485MB/1501MB (1.1%)	32m16s	5m11s/5m44s (10.6%)

- Translation output size overheads are reasonable
- Translation runtime is far from ideal, and comes coupled with high memory requirements that prevent us from effectively scaling

Results: Library translations

We have been able to translate (and verify) translations of the Lean standard library, as well as some smaller mathlib modules. Some numbers:

Module	Total Constants	Constants Using [PI]/[KLR] (% of total)	Input/Output Environment Size (Overhead)	Translation Runtime	Input/Output Typechecking Runtime (Overhead) ²
Std	29859	1736/134 (6.3%)	226MB/261MB (15.5%)	18m02s	2m19s/3m9s (36.0%)
Algebra.Order.Field.Rat	113899	2965/237 (2.8%)	1485MB/1501MB (1.1%)	32m16s	5m11s/5m44s (10.6%)

- Translation output size overheads are reasonable
- Translation runtime is far from ideal, and comes coupled with high memory requirements that prevent us from effectively scaling
 - While the implementation is well-optimized in many respects, more investigation/work must be done to address this

Prospects: extensionality in Lean

Lean4Less's translation framework should be consistent with the general ETT to ITT translation (Winterhalter et al.)

Prospects: extensionality in Lean

Lean4Less's translation framework should be consistent with the general ETT to ITT translation (Winterhalter et al.)

- So, should be possible to extend to eliminate other definitional equalities (w/ new axioms/lemmas for each of them).

Prospects: extensionality in Lean

Lean4Less's translation framework should be consistent with the general ETT to ITT translation (Winterhalter et al.)

- So, should be possible to extend to eliminate other definitional equalities (w/ new axioms/lemmas for each of them).
- This could include *new, user-defined* definitional equalities.

Prospects: extensionality in Lean

Lean4Less's translation framework should be consistent with the general ETT to ITT translation (Winterhalter et al.)

- So, should be possible to extend to eliminate other definitional equalities (w/ new axioms/lemmas for each of them).
- This could include *new, user-defined* definitional equalities.
- While full ETT is undecidable, could add *partial* extensionality via a mechanism for registering/deriving new definitional equalities.

Prospects: extensionality in Lean

Lean4Less's translation framework should be consistent with the general ETT to ITT translation (Winterhalter et al.)

- So, should be possible to extend to eliminate other definitional equalities (w/ new axioms/lemmas for each of them).
- This could include *new, user-defined* definitional equalities.
- While full ETT is undecidable, could add *partial* extensionality via a mechanism for registering/deriving new definitional equalities.

Could add a rule for “algorithmic reflection” to Lean:

$$\frac{\Delta \vdash_{e^*}^- A : \text{Sort } u \quad \Delta \vdash_{e^*}^- t, u : A \quad \Delta \vdash_{e^*}^- _ : t = u \text{ computable}}{\Delta \vdash_{e^*}^- t \equiv u}$$

and extend Lean4Less to translate from this theory “Lean_{e*}”.

Prospects: extensionality in Lean

Lean4Less's translation framework should be consistent with the general ETT to ITT translation (Winterhalter et al.)

- So, should be possible to extend to eliminate other definitional equalities (w/ new axioms/lemmas for each of them).
- This could include *new, user-defined* definitional equalities.
- While full ETT is undecidable, could add *partial* extensionality via a mechanism for registering/deriving new definitional equalities.

Could add a rule for “algorithmic reflection” to Lean:

$$\frac{\Delta \vdash_{e^*} A : \text{Sort } u \quad \Delta \vdash_{e^*} t, u : A \quad \Delta \vdash_{e^*} _ : t = u \text{ computable}}{\Delta \vdash_{e^*} t \equiv u}$$

and extend Lean4Less to translate from this theory “Lean_{e*}”.

Lean4Less could then be integrated with Lean’s elaborator, allowing for reasoning modulo a extensible set of computable definitional equalities.

Prospects: Meta-theoretical simplifications

Lean^- is a smaller theory, making it more feasible to prove **consistency**:

Prospects: Meta-theoretical simplifications

Lean⁻ is a smaller theory, making it more feasible to prove **consistency**:

- Consistency property: there is no (axiom-free) proof of False

Prospects: Meta-theoretical simplifications

Lean⁻ is a smaller theory, making it more feasible to prove **consistency**:

- Consistency property: there is no (axiom-free) proof of False
 - Important for ensuring that **proofs can be trusted**, i.e. kernel is “safe”

Prospects: Meta-theoretical simplifications

Lean⁻ is a smaller theory, making it more feasible to prove **consistency**:

- Consistency property: there is no (axiom-free) proof of False
 - Important for ensuring that **proofs can be trusted**, i.e. kernel is “safe”

If Lean⁻ is proven consistent, it becomes an ideal translation target:

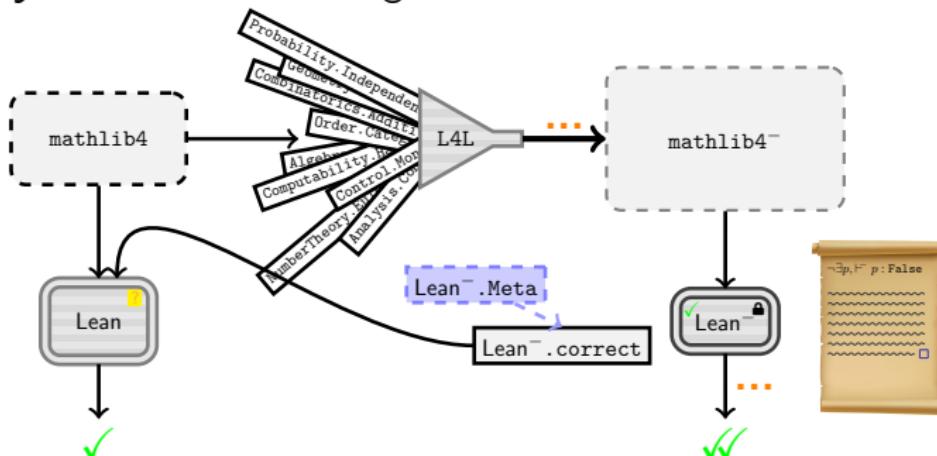
Prospects: Meta-theoretical simplifications

Lean⁻ is a smaller theory, making it more feasible to prove **consistency**:

- Consistency property: there is no (axiom-free) proof of False
 - Important for ensuring that **proofs can be trusted**, i.e. kernel is “safe”

If Lean⁻ is proven consistent, it becomes an ideal translation target:

- Can possibly use Lean4Less to translate proofs to be verified with a **provably safe kernel** deciding Lean⁻:



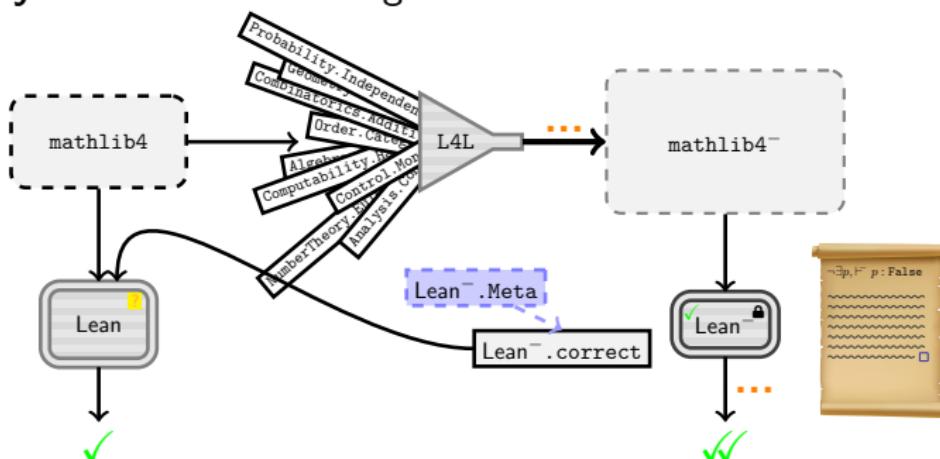
Prospects: Meta-theoretical simplifications

Lean⁻ is a smaller theory, making it more feasible to prove **consistency**:

- Consistency property: there is no (axiom-free) proof of False
 - Important for ensuring that **proofs can be trusted**, i.e. kernel is “safe”

If Lean⁻ is proven consistent, it becomes an ideal translation target:

- Can possibly use Lean4Less to translate proofs to be verified with a **provably safe kernel** deciding Lean⁻:



- Issue: translation does not currently scale very well

Prospects: Meta-theoretical simplifications

Alternatively, we could formally verify the following properties:

Prospects: Meta-theoretical simplifications

Alternatively, we could formally verify the following properties:

- The correctness of the translation implemented by Lean4Less

Prospects: Meta-theoretical simplifications

Alternatively, we could formally verify the following properties:

- The correctness of the translation implemented by Lean4Less
- The correctness of the Lean kernel w.r.t. the Lean theory

Prospects: Meta-theoretical simplifications

Alternatively, we could formally verify the following properties:

- The correctness of the translation implemented by Lean4Less
- The correctness of the Lean kernel w.r.t. the Lean theory

Thus, if Lean^- is consistent (no proof of `False`), then so is Lean:

- Any proof of `False` in Lean translates to a proof of `False` in Lean^-

Prospects: Meta-theoretical simplifications

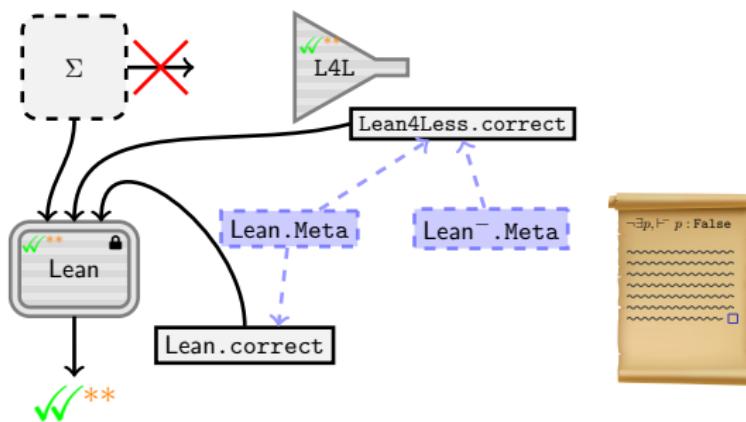
Alternatively, we could formally verify the following properties:

- The correctness of the translation implemented by Lean4Less
- The correctness of the Lean kernel w.r.t. the Lean theory

Thus, if Lean^- is consistent (no proof of False), then so is Lean:

- Any proof of False in Lean translates to a proof of False in Lean^-

This justifies using a verified Lean kernel w/o translating entire libraries:



Prospects: Meta-theoretical simplifications

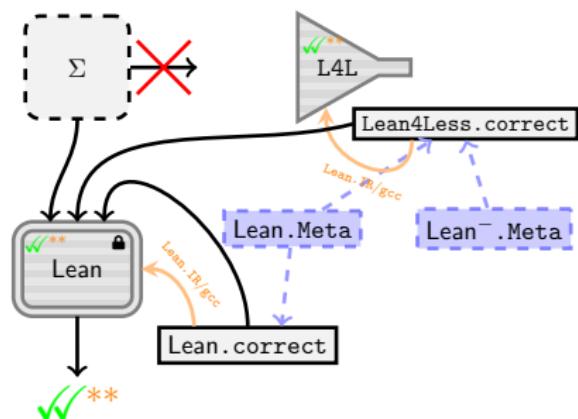
Alternatively, we could formally verify the following properties:

- The correctness of the translation implemented by Lean4Less
- The correctness of the Lean kernel w.r.t. the Lean theory

Thus, if Lean^- is consistent (no proof of False), then so is Lean:

- Any proof of False in Lean translates to a proof of False in Lean^-

This justifies using a verified Lean kernel w/o translating entire libraries:



However, some caveats:

- Requires extra trust in code generators and compilers



Prospects: Meta-theoretical simplifications

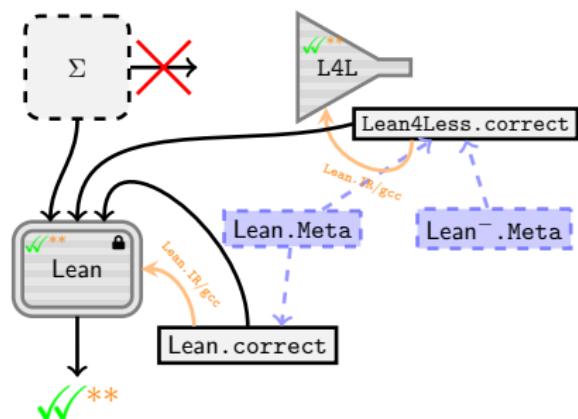
Alternatively, we could formally verify the following properties:

- The correctness of the translation implemented by Lean4Less
- The correctness of the Lean kernel w.r.t. the Lean theory

Thus, if Lean^- is consistent (no proof of False), then so is Lean:

- Any proof of False in Lean translates to a proof of False in Lean^-

This justifies using a verified Lean kernel w/o translating entire libraries:



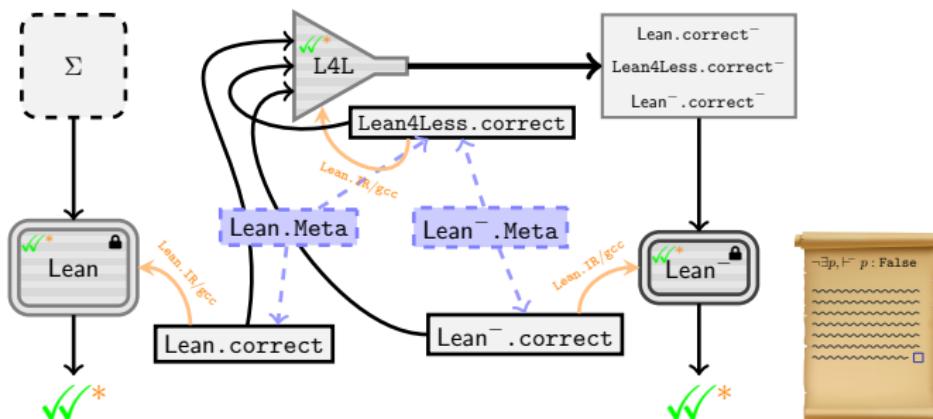
However, some caveats:

- Requires extra trust in code generators and compilers
- Lean kernel's consistency proof is partly checked by the *same* kernel



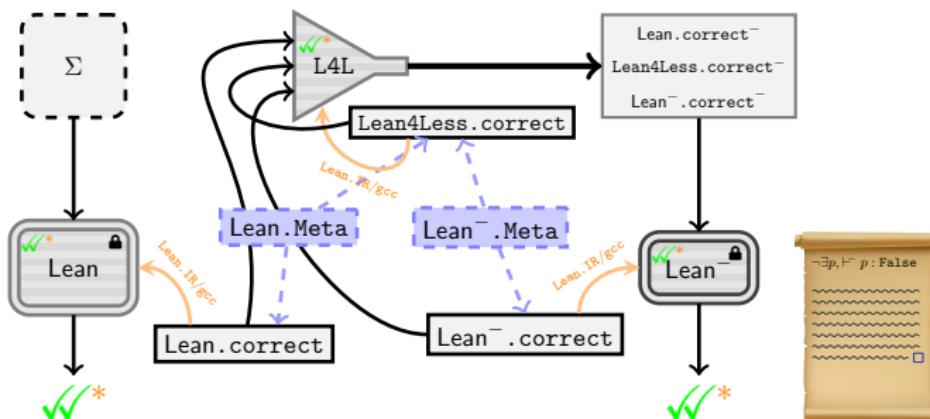
Prospects: Meta-theoretical simplifications

Ideally, we could also translate these correctness proofs to Lean^- :



Prospects: Meta-theoretical simplifications

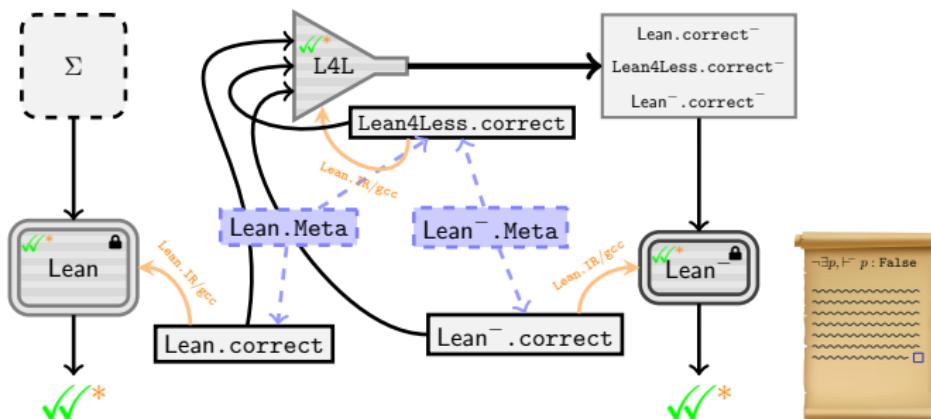
Ideally, we could also translate these correctness proofs to Lean^- :



- Can also show correctness of the Lean^- kernel w.r.t. the Lean^- theory

Prospects: Meta-theoretical simplifications

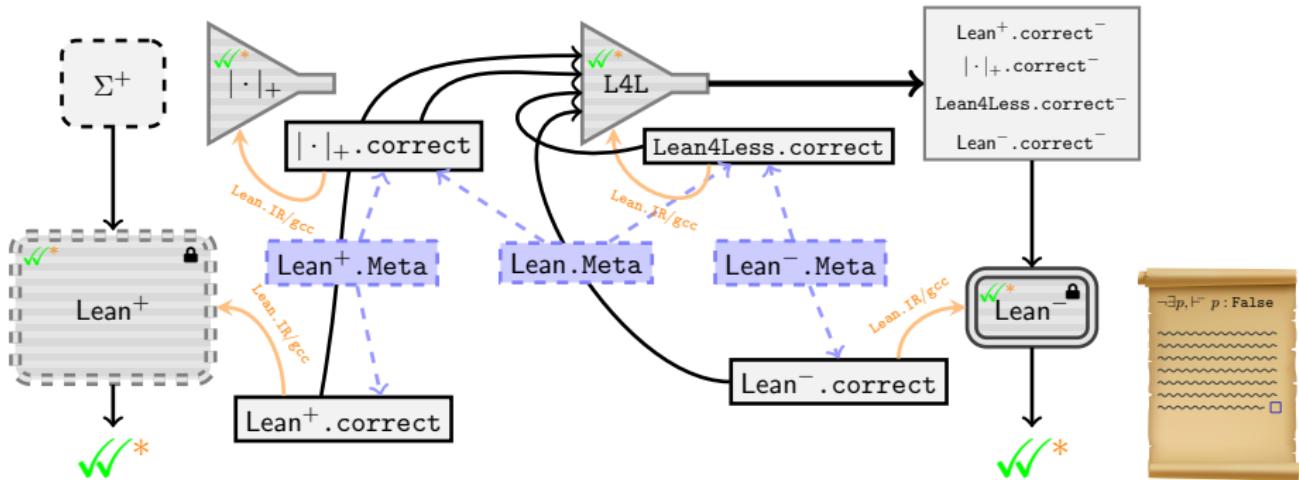
Ideally, we could also translate these correctness proofs to Lean^- :



- Can also show correctness of the Lean^- kernel w.r.t. the Lean^- theory
- Translation should be practical, but only enough to translate these select few proofs

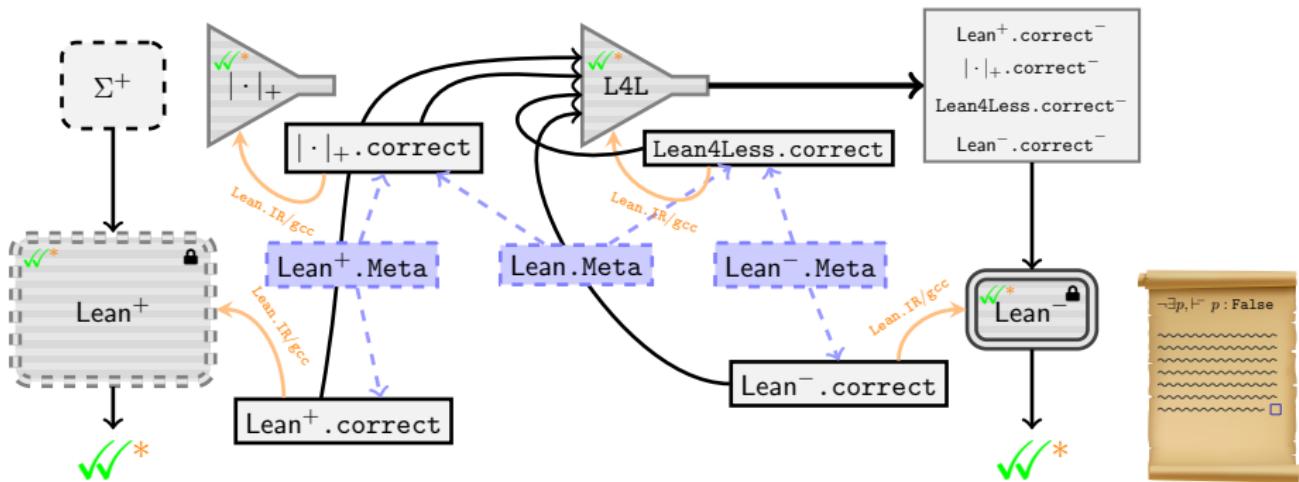
Prospects: Meta-theoretical simplifications

Can also expand this approach to certain extensions of Lean:



Prospects: Meta-theoretical simplifications

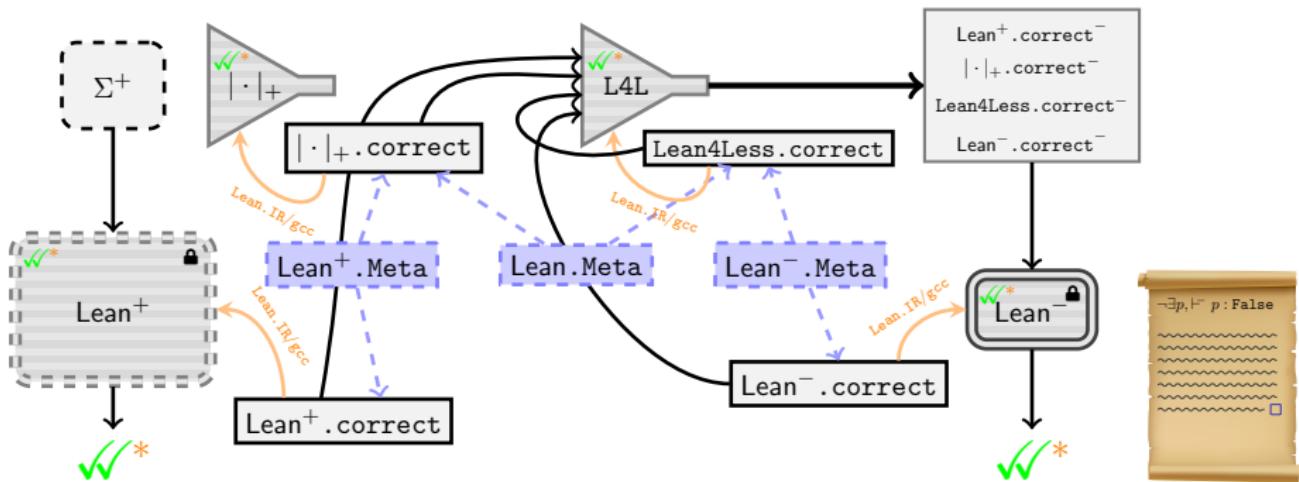
Can also expand this approach to certain extensions of Lean:



- Can define a translation from some Lean⁺ theory back to Lean

Prospects: Meta-theoretical simplifications

Can also expand this approach to certain extensions of Lean:



- Can define a translation from some Lean⁺ theory back to Lean
- If translation and Lean⁺ kernel are verified, Lean⁺ kernel is consistent

Conclusion

To conclude:

Conclusion

To conclude:

- Translating from Lean to smaller subtheories can be interpreted as a special case of a translation extensional to intensional type theory

Conclusion

To conclude:

- Translating from Lean to smaller subtheories can be interpreted as a special case of a translation extensional to intensional type theory
- Such a translation *is possible* in practice, by modifying a kernel typechecker to construct translated terms

Conclusion

To conclude:

- Translating from Lean to smaller subtheories can be interpreted as a special case of a translation extensional to intensional type theory
- Such a translation *is possible* in practice, by modifying a kernel typechecker to construct translated terms
- Our translation, Lean4Less, implements the framework of a first (somewhat) practical translation from ETT to ITT that could possibly be extended to enable real-time extensional reasoning in Lean

Conclusion

To conclude:

- Translating from Lean to smaller subtheories can be interpreted as a special case of a translation extensional to intensional type theory
- Such a translation *is possible* in practice, by modifying a kernel typechecker to construct translated terms
- Our translation, Lean4Less, implements the framework of a first (somewhat) practical translation from ETT to ITT that could possibly be extended to enable real-time extensional reasoning in Lean
- Verifying translation correctness could simplify meta-theoretical analyses of Lean relating to the consistency property, facilitating trust in future possible extensions of Lean's kernel

References

- [1] Théo Winterhalter et. al. “Eliminating reflection from type theory”. In: *ACM SIGPLAN International Conference on Certified Programs and Proofs* (2019).
- [2] Mario Carneiro. *Lean4Lean: Towards a formalized metatheory for the Lean theorem prover*. 2024. arXiv: 2403.14064 [cs.PL].
- [3] Martin Hofmann. “Conservativity of Equality Reflection over Intensional Type Theory”. In: *International Workshop on Types for Proofs and Programs*. 1995.
- [4] Théo Winterhalter and Nicolas Tabareau. *ett-to-itt (Github)*.

The End

Thanks for listening!