PATTERN RECOGNITION ASSIGNMENT

BY

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ICE – B

110116070

```
X = [-1 -0.9 -0.8 -0.7 -0.6 -0.5 -0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4
 0.5 0.6 0.7 0.8 0.9 1];
t = [5.12 4.97 4.92 4.83 4.90 5.06 5.29 5.34 5.36 5.76 5.99 6.30 6.66
 6.70 7.49 7.92 8.48 9.09 9.70 10.30 10.98];
x_out = [-0.95 -0.85 -0.75 -0.65 -0.55 -0.45 -0.35 -0.25 -0.15 -0.05
 0.05 0.15 0.25 0.35 0.45 0.55 0.65 0.75 0.85 0.95];
g = [5 4.92 4.88 4.88 4.92 5 5.12 5.28 5.48 5.72 6 6.32 6.68 7.08 7.52
 8 8.52 9.08 9.68 10.32 11];


A = [];
t_a = [];
a_out = [];
B = [];
t_b = [];
b_out = [];
C = [];
t_c = [];
c_out = [];
t_exp = [];

w_exp_a = [0 0 0];
w_exp_b = [0 0 0];
w_exp_c = [0 0 0];

disp(length(X))
for i = 1:3:length(X)
 A = [A, X(i)];
 t_a = [t_a, t(i)];
end

for i = 2:3:length(X)
 B = [B, X(i)];
 t_b = [t_b, t(i)];
end

for i = 3:3:length(X)
 C = [C, X(i)];
 t_c = [t_c, t(i)];
end

for i =1:3:length(x_out)
 a_out = [a_out, x_out(i)];
end

for i =2:3:length(x_out)
 b_out = [b_out, x_out(i)];
end

for i =3:3:length(x_out)
 c_out = [c_out, x_out(i)];
end
```

```matlab
phix_a = A;
phix_a = [phix_a ; exp((-(A-0.5).^2)/0.1)];
phix_a = [phix_a ; exp((-(A+0.5).^2)/0.1)];

phix_b = B;
phix_b = [phix_b ; exp((-(B-0.5).^2)/0.1)];
phix_b = [phix_b ; exp((-(B+0.5).^2)/0.1)];

phix_c = C;
phix_c = [phix_c ; exp((-(C-0.5).^2)/0.1)];
phix_c = [phix_c ; exp((-(C+0.5).^2)/0.1)];

phi_xout_a = a_out;
phi_xout_a = [phi_xout_a ; exp((-(a_out-0.5).^2)/0.1)];
phi_xout_a = [phi_xout_a ; exp((-(a_out+0.5).^2)/0.1)];

phi_xout_b = b_out;
phi_xout_b = [phi_xout_b ; exp((-(b_out-0.5).^2)/0.1)];
phi_xout_b = [phi_xout_b ; exp((-(b_out+0.5).^2)/0.1)];

phi_xout_c = c_out;
phi_xout_c = [phi_xout_c ; exp((-(c_out-0.5).^2)/0.1)];
phi_xout_c = [phi_xout_c ; exp((-(c_out+0.5).^2)/0.1)];

figure(10)
scatter(A,t_a,'filled','b');
title('data1');

figure(11)
scatter(B,t_b,'filled','g');
title('data2');

figure(12)
scatter(C,t_c,'filled','r');
title('data3');

var_reg = [];
bias_reg = [];
for lambda = [0,2,4,5,6,10]

 disp("LAMBDA=");
 disp(lambda);
 %disp("w_exp_a = ");
 w_exp_a = inv(phix_a*transpose(phix_a) +
 lambda*eye(length(w_exp_a)))*phix_a*transpose(t_a);

 %disp("w_exp_b = ");
 w_exp_b = inv(phix_b*transpose(phix_b) +
 lambda*eye(length(w_exp_b)))*phix_b*transpose(t_b);

 %disp("w_exp_c = ");
```

```matlab
w_exp_c = inv(phix_c*transpose(phix_c) +
lambda*eye(length(w_exp_c)))*phix_c*transpose(t_c);

hold on
figure(1)
plot(A,transpose(w_exp_a)*phix_a);
title('x train1 vs test 1');
legend('reg = 0','reg = 2','reg = 4','reg = 5','reg = 6','reg = 10')
grid
hold off

hold on
figure(2)
plot(a_out,transpose(w_exp_a)*phi_xout_a);
title('x out vs y out1');
legend('reg = 0','reg = 2','reg = 4','reg = 5','reg = 6','reg = 10')
grid
hold off

hold on
figure(3)
plot(B,transpose(w_exp_b)*phix_b);
title('x train2 vs test 2');
legend('reg = 0','reg = 2','reg = 4','reg = 5','reg = 6','reg = 10')
grid
hold off

hold on
figure(4)
plot(b_out,transpose(w_exp_b)*phi_xout_b);
title('x out vs y out2');
legend('reg = 0','reg = 2','reg = 4','reg = 5','reg = 6','reg = 10')
grid
hold off

hold on
figure(5)
plot(C,transpose(w_exp_c)*phix_c);
title('x train3 vs test 3');
legend('reg = 0','reg = 2','reg = 4','reg = 5','reg = 6','reg = 10')
grid
hold off

hold on
figure(6)
plot(c_out,transpose(w_exp_c)*phi_xout_c);
title('x out vs y out3');
legend('reg = 0','reg = 2','reg = 4','reg = 5','reg = 6','reg = 10')
grid
hold off


t_1 = [];
t_2 = [];
```

```matlab
    t_3 = [];
    t_exp = [];
    c1 = 0;
    c2 = 0;
    c3 = 0;

    t_1 = transpose(w_exp_a)*phix_a;
    t_2 = transpose(w_exp_b)*phix_b;
    t_3 = transpose(w_exp_c)*phix_c;


    for i = 1:length(t)
     if mod(i+2,3)==0
      c1 = c1+1;
      t_exp = [t_exp,t_1(c1)];
     elseif mod(i+1,3)==0
      c2 = c2+1;
      t_exp = [t_exp,t_2(c2)];
     else
      c3 = c3+1;
      t_exp = [t_exp, t_3(c3)];
     end
    end


    mean = 0;
    for i = 1:length(t_exp)
     mean = mean+t_exp(i);
    end
    mean = mean/length(t_exp);

    var = 0;
    bias = 0;
    for i = 1:length(t_exp)
     var = var + (t_exp(i)-mean)^2;
     bias = bias + (g(i)-mean)^2;
    end
    var = var/length(t_exp);
    bias = bias/length(g);

    var_reg = [var_reg, var];
    bias_reg = [bias_reg, bias];
    dis1 = ["VAR with lambda", lambda, " = "];
    dis2 = ["BIAS with lambda", lambda, " = "];
    disp(dis1);
    disp(var);
    disp(dis2);
    disp(bias);


end

hold on
figure(15)
```

```
plot(var_reg,[0,2,4,5,6,10]);
title('variance vs regularization');
grid
hold off

hold on
figure(16)
plot(bias_reg,[0,2,4,5,6,10]);
title('bias vs regularization');
grid
hold off
```

21

LAMBDA=

    0

    "VAR with lambda"   *"0"*   *" = "*

  11.0762

    "BIAS with lambda"   *"0"*   *" = "*

    6.3912

LAMBDA=

    2

    "VAR with lambda"   *"2"*   *" = "*

    2.5073

    "BIAS with lambda"   *"2"*   *" = "*

  25.8510

LAMBDA=

    4

    "VAR with lambda"   *"4"*   *" = "*

    1.1718

    "BIAS with lambda"   *"4"*   *" = "*

  33.5532

LAMBDA=

    5

    "VAR with lambda"   *"5"*   *" = "*

    0.8796

```
"BIAS with lambda"      "5"      " = "

35.7748

LAMBDA=
    6

"VAR with lambda"      "6"      " = "

0.6859

"BIAS with lambda"      "6"      " = "

37.4425

LAMBDA=
    10

"VAR with lambda"      "10"      " = "

0.3204

"BIAS with lambda"      "10"      " = "

41.3318
```
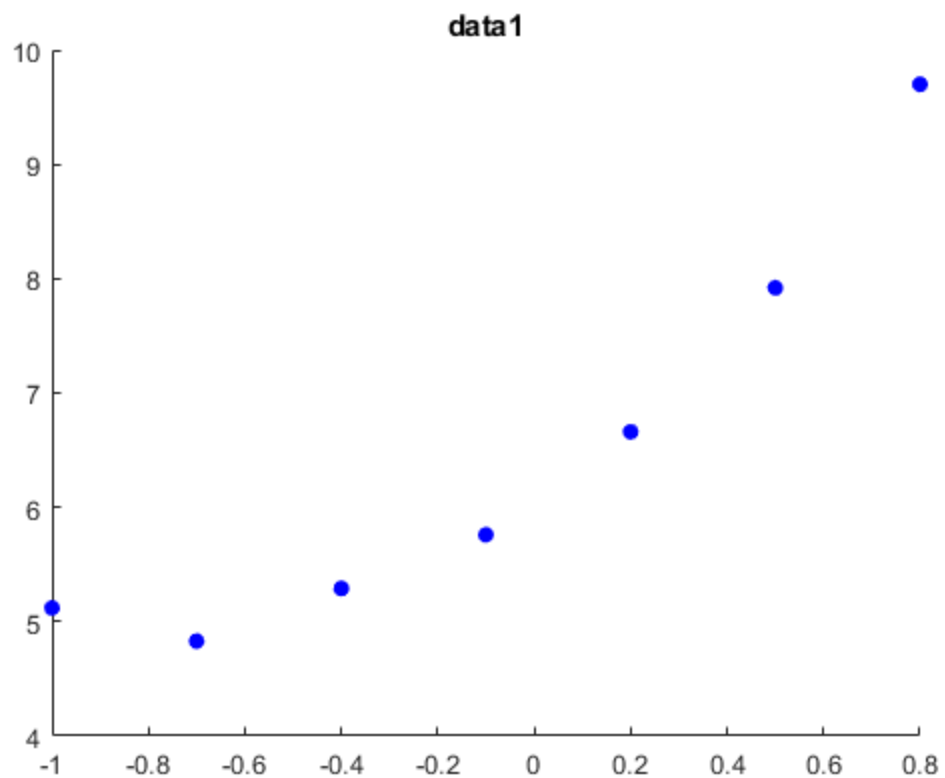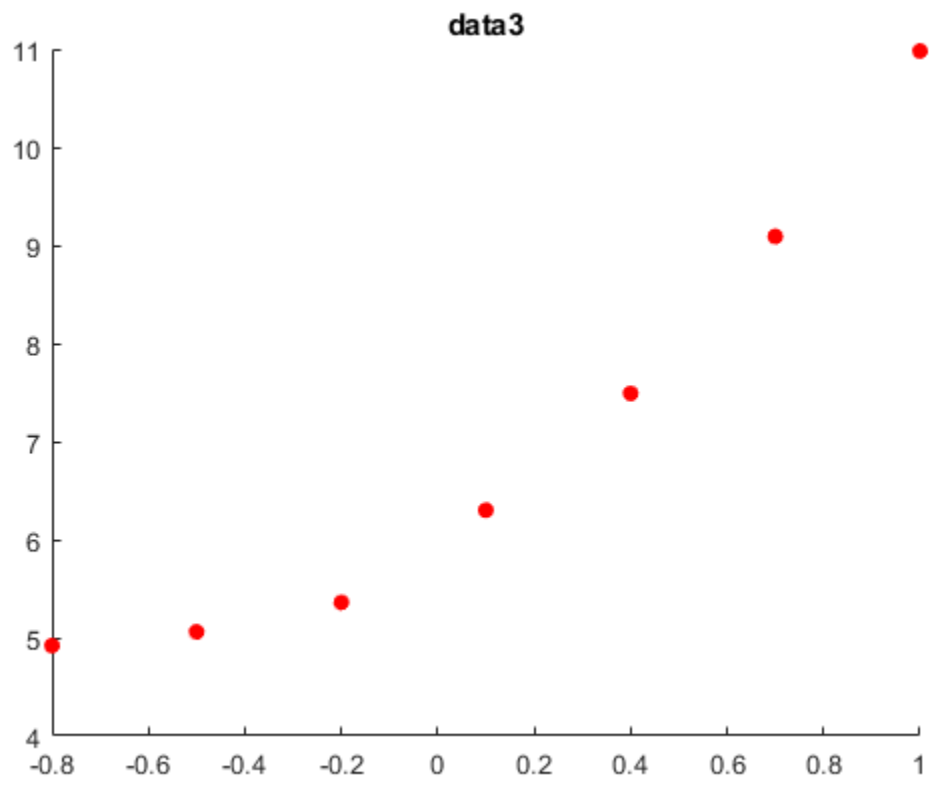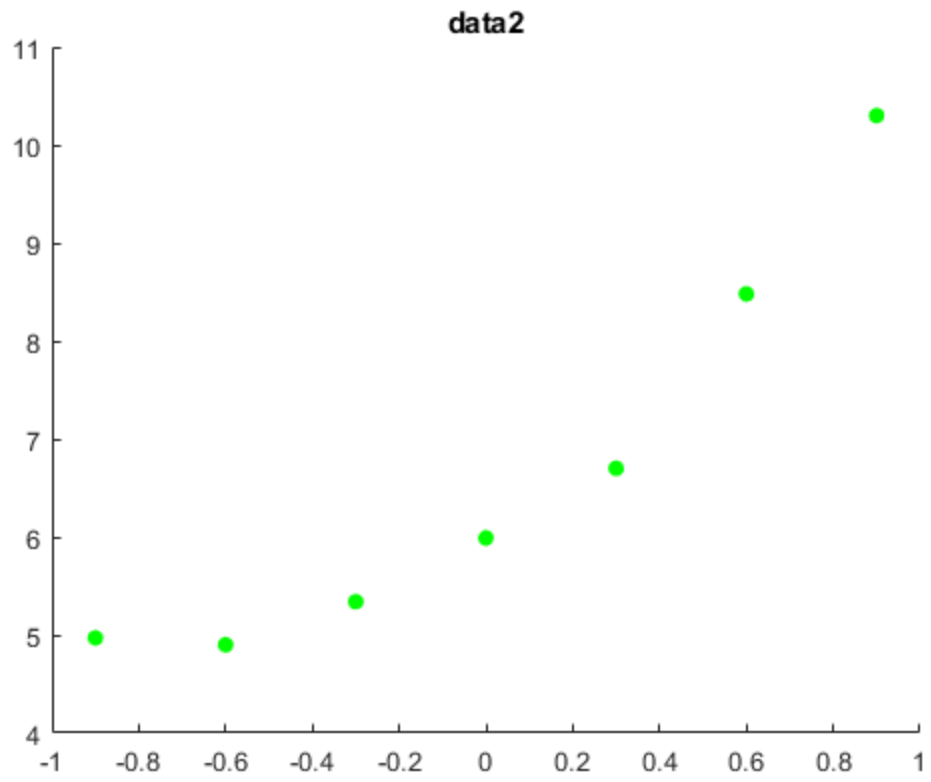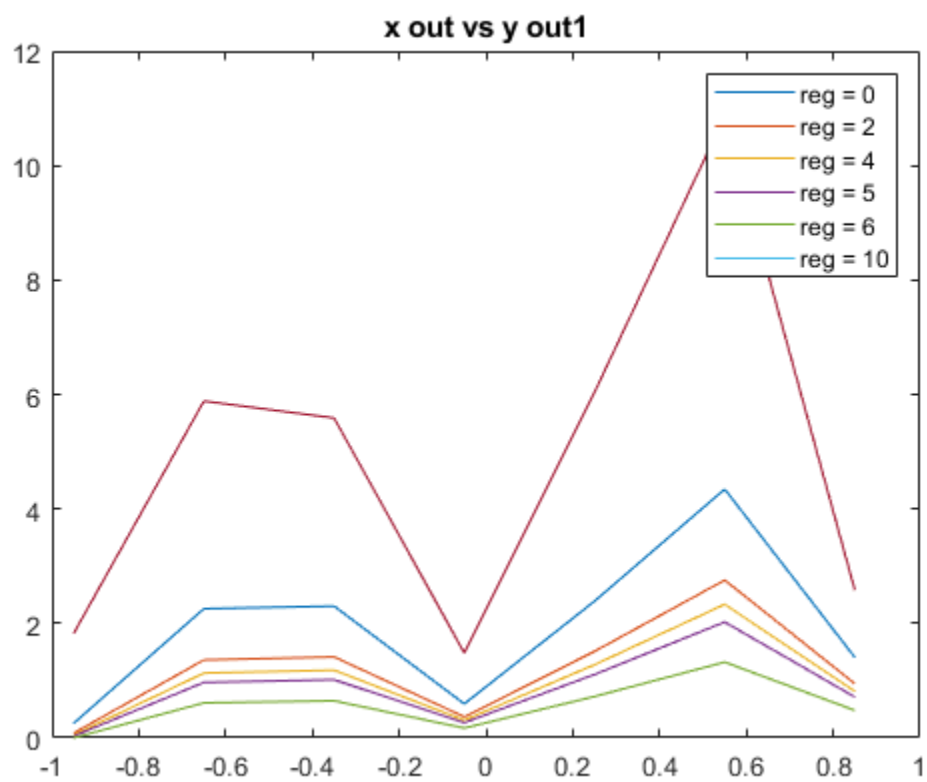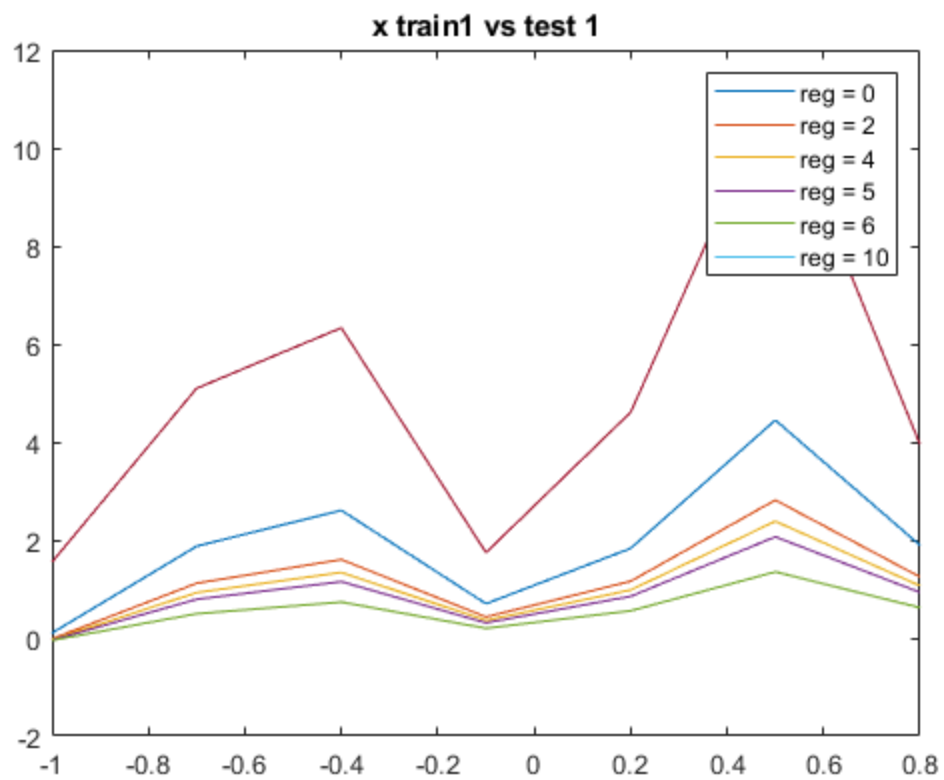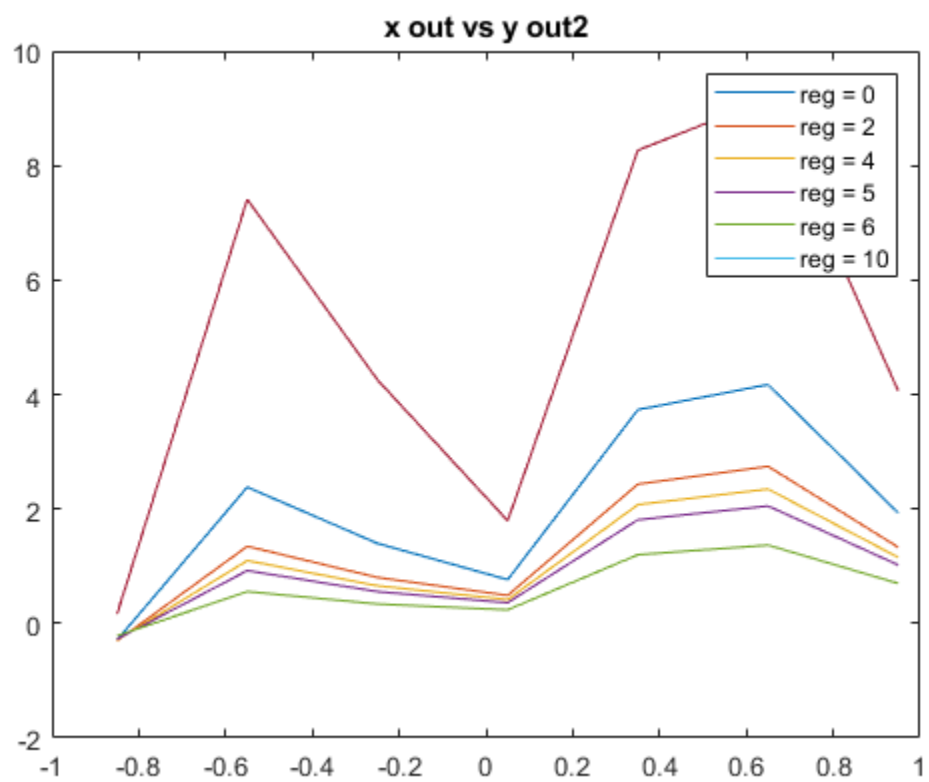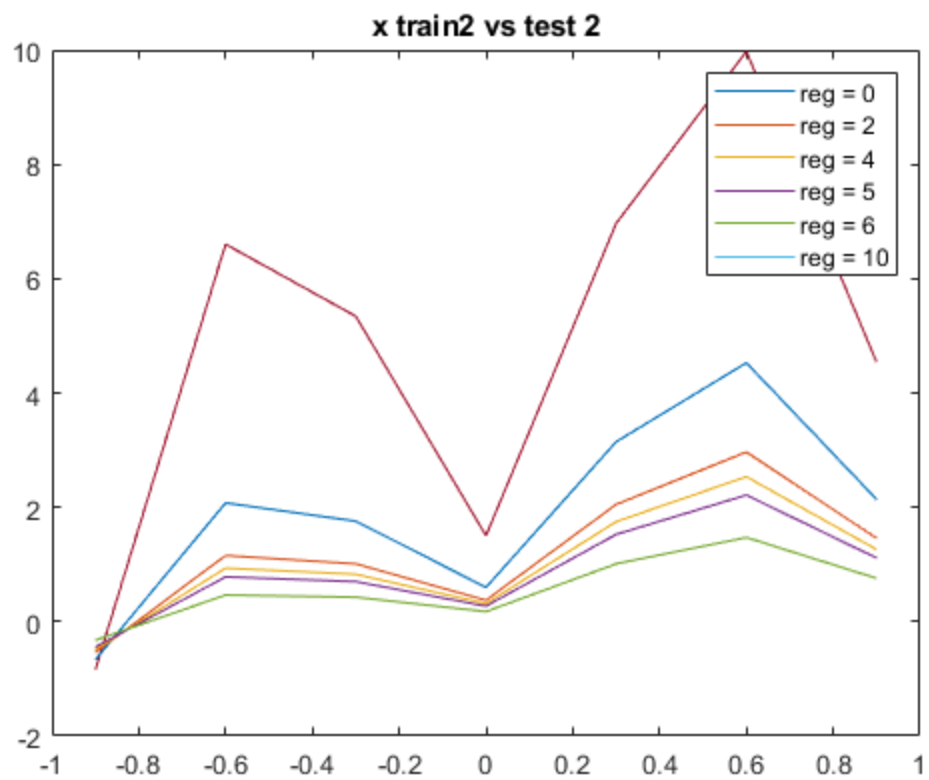

data1

data2

data3

x train1 vs test 1

x out vs y out1

x train2 vs test 2

x out vs y out2

x train3 vs test 3

x out vs y out3

variance vs regularization
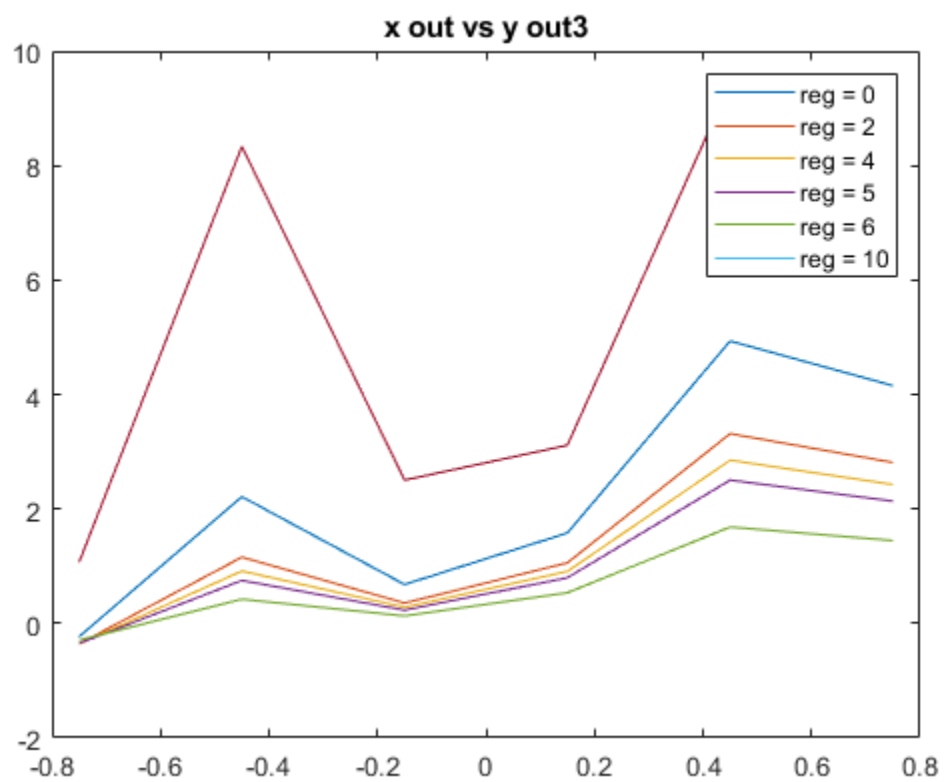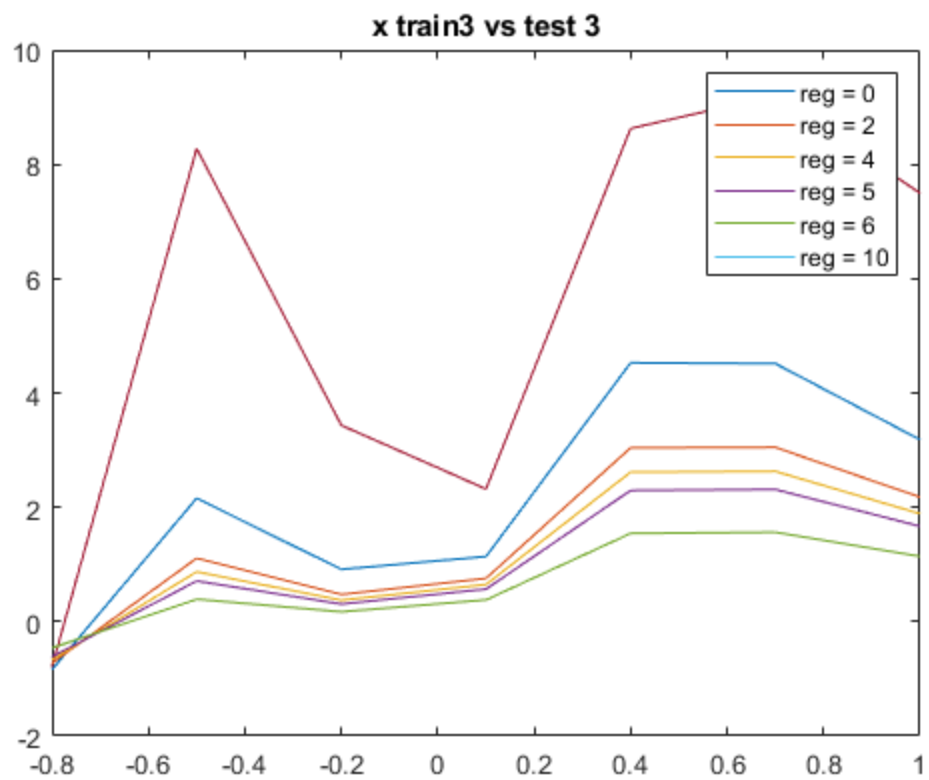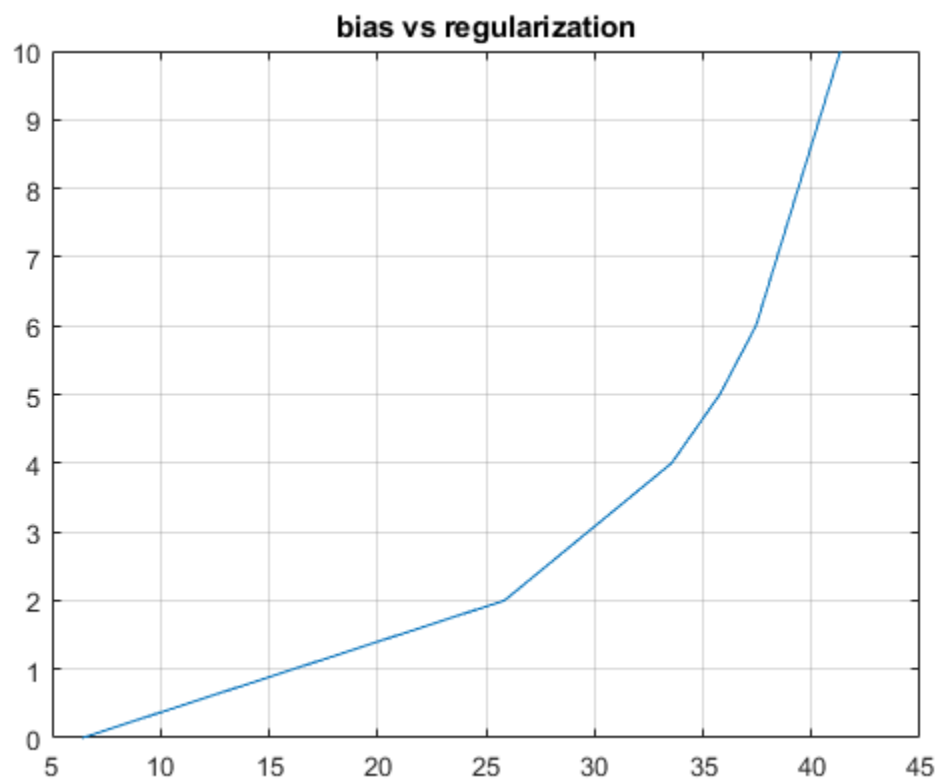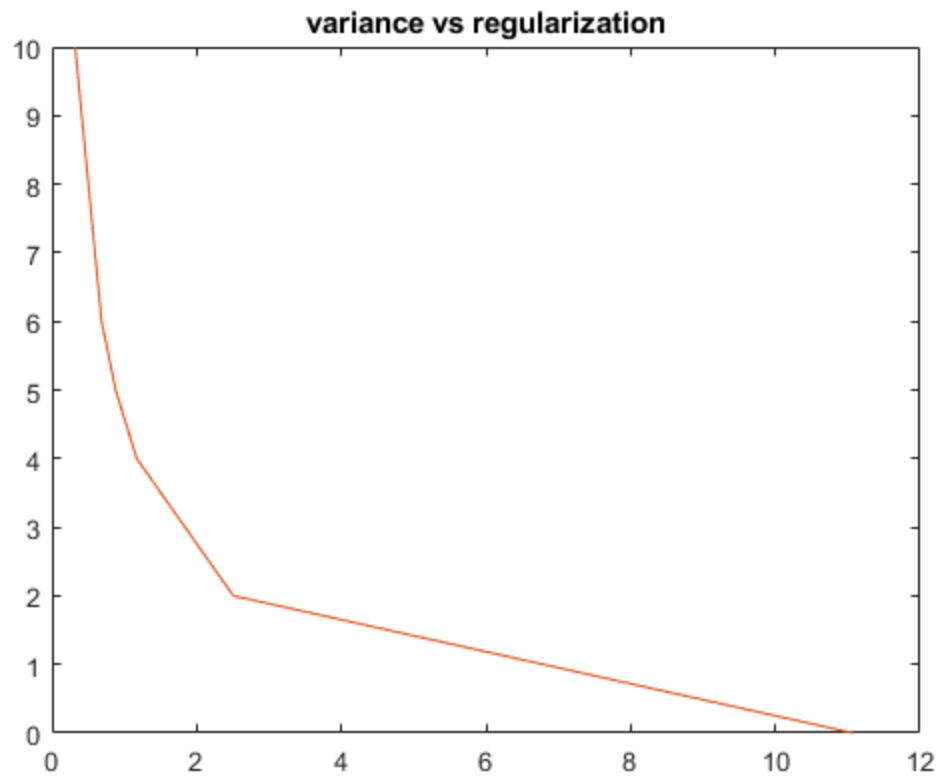

bias vs regularization

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```matlab
X = [-1 -0.9 -0.8 -0.7 -0.6 -0.5 -0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4
 0.5 0.6 0.7 0.8 0.9 1];
t = [5.12 4.97 4.92 4.83 4.90 5.06 5.29 5.34 5.36 5.76 5.99 6.30 6.66
 6.70 7.49 7.92 8.48 9.09 9.70 10.30 10.98];
x_out = [-0.95 -0.85 -0.75 -0.65 -0.55 -0.45 -0.35 -0.25 -0.15 -0.05
 0.05 0.15 0.25 0.35 0.45 0.55 0.65 0.75 0.85 0.95];
g = [5 4.92 4.88 4.88 4.92 5 5.12 5.28 5.48 5.72 6 6.32 6.68 7.08 7.52
 8 8.52 9.08 9.68 10.32 11];

w = [0;0;0];

X_t = X(1:14);
Y = X(15:21);

phix = X_t;
phix = [phix ; exp((-(X_t-0.5).^2)/0.1)];
phix = [phix ; exp((-(X_t+0.5).^2)/0.1)];

phiy = Y;
phiy = [phiy ; exp((-(Y-0.5).^2)/0.1)];
phiy = [phiy ; exp((-(Y+0.5).^2)/0.1)];

lr = 0.1;

sse_ep = [];
for k = 1:10
 error = 0;
 sse = 0;
 for i =1:length(X_t)
  error = t(i)-transpose(w)*phix(:,i);
  %disp(error);
  w = w + lr*error*phix(:,i);
 end

 for j = 1:length(Y)
  sse = sse + (t(j)-transpose(w)*phiy(:,j))^2;
 end
 sse_ep = [sse_ep, sse];
end


plot([1,2,3,4,5,6,7,8,9,10], sse_ep);
title('epoch vs SSE with lr = 0.1');
grid


disp("ERROR=");
disp(error);

disp("SSE=");
disp(sse);
```

```matlab
disp("WEIGHTS=");
disp(w);
%disp(length(error));
```

```
ERROR=
    2.8302


SSE=
  121.0279


WEIGHTS=
   -2.9658
    7.3157
    5.0216
```


epoch vs SSE with lr = 0.1
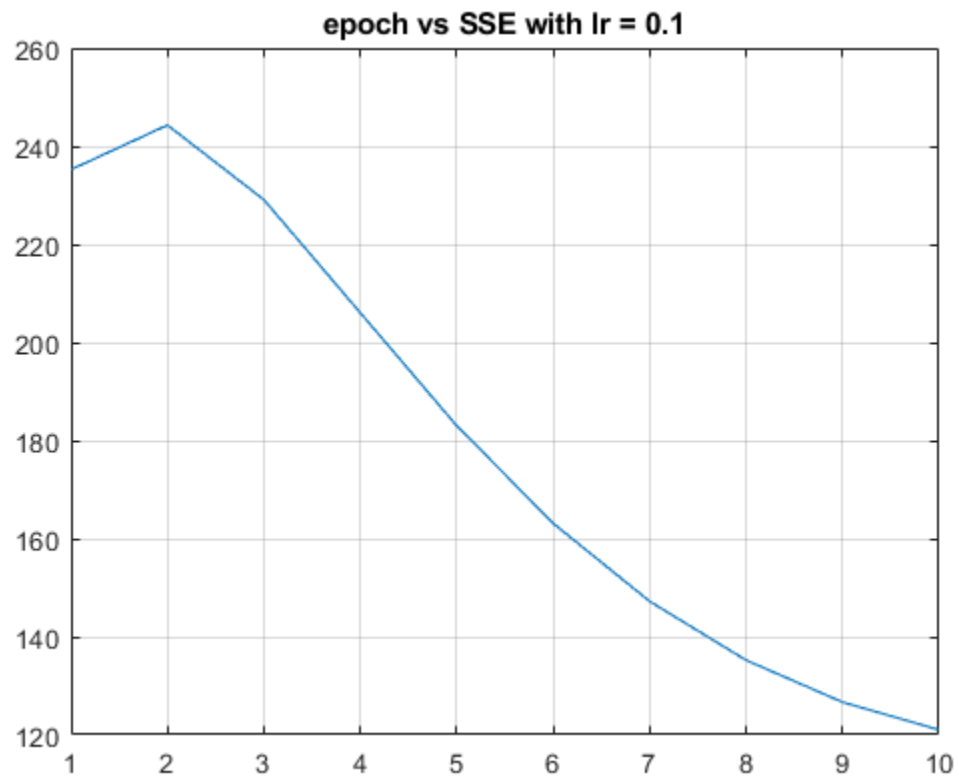
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```matlab
X_i = [-1 -0.7 -0.4 -0.1 0.2 0.5 0.8];
V_i = [-0.9 -0.6 -0.3 0 0.3 0.6 0.9];
T_i = [-0.8 -0.5 -0.2 0.1 0.4 0.7 1];
X_o = [5.12 4.83 5.29 5.76 6.66 7.92 9.70];
V_o = [4.97 4.90 5.34 5.99 6.70 8.48 10.30];
T_o = [4.92 5.06 5.36 6.30 7.49 9.09 10.98];

M = [];
V_ohat = [];
T_ohat = [];
sse = 0;

sse_v = [];
for sig = [0.1,0.2,0.3,0.4,0.5]
 V_ohat = [];
 T_ohat = [];
 for i = 1:length(V_i)
  M = [];
   for j =1:length(X_i)
    M = [M; exp(-(X_i(j)-V_i(i))^2/(2*sig^2))];
    %disp(length(M));
   end
  M = M/sum(M);
   %disp(length(M));
   %disp(length(X_o));
   V_ohat = [V_ohat X_o*M];
   T_ohat = [T_ohat T_i*M];
   %disp(V_ohat);
  end

  %disp(V_ohat);
  sse = (V_o - V_ohat)*transpose(V_o - V_ohat);
  disp(length(sse));
  disp1 = ["SSE for sig^2 = ",sig];
  disp(disp1);
  disp(sse);

 sse_v = [sse_v sse];
end

figure(1)
plot([0.1 0.2 0.3 0.4 0.5], sse_v);
title('Sigma^2 vs SSE');
grid

sse2 = (T_o - T_ohat)*transpose(T_o - T_ohat);
disp("SSE2=");
disp(sse2);

     1

    "SSE for sig^2 = "    "0.1"
```

```
0.4680

 1

"SSE for sig^2 = "    "0.2"

0.9392

 1

"SSE for sig^2 = "    "0.3"

2.0292

 1

"SSE for sig^2 = "    "0.4"

3.5089

 1

"SSE for sig^2 = "    "0.5"

5.3094
```

SSE2=
  351.5135

Sigma^2 vs SSE
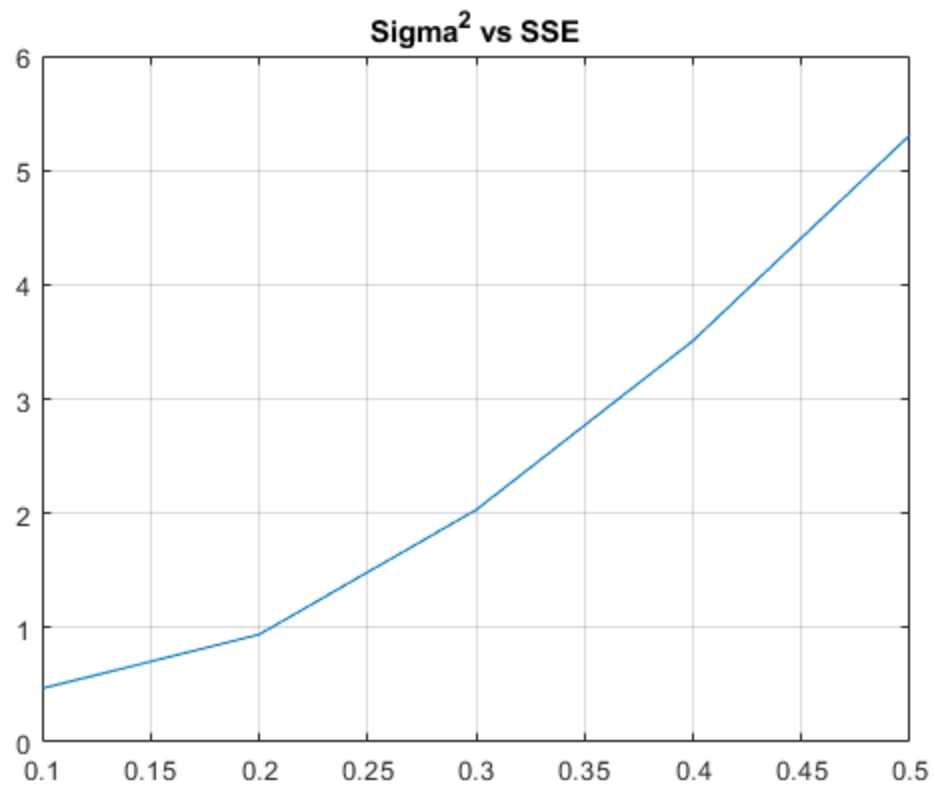
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```
x = [0.67 0.71 0.48 0.72 0.29 0.89 -1.07 -2.94 0.33 1.38; -1.20 1.63
 1.03 -0.3 -0.78 -1.15 -0.81 1.44 -0.75 -1.71]
m = [0.5 0.5]
c = [0.9 0.1; 0.1 0.9]
[e,d] = eig(c)
d = d^0.5
y = e*(d)*x
y = transpose(y);
mu1 = 0;
mu2 = 0;
for i = 1:10
 mu1 = mu1+y(i,1);
 mu2 = mu2+y(i,2);
end
mu1 = mu1/10
mu2 = mu2/10


sum1 = 0;
sum2 = 0;
sum3 = 0;
for i = 1:10
 sum1 = sum1 + (y(i,1)-mu1)^2;
 sum2 = sum2 + (y(i,1)-mu1)*(y(i,2)-mu2);
 sum3 = sum3 + (y(i,2)-mu2)^2;
end

sum1 = sum1/10
sum2 = sum2/10
sum3 = sum3/10

exp = [sum1 sum2; sum2 sum3]
cov(y);


x = transpose(x);
mux1 = 0;
mux2 = 0;
for i = 1:10
 mux1 = mux1+x(i,1);
 mux2 = mux2+x(i,2);
end
mux1 = mux1/10
mux2 = mux2/10


sumx1 = 0;
sumx2 = 0;
sumx3 = 0;
for i = 1:10
 sumx1 = sumx1 + (x(i,1)-mux1)^2;
```

```matlab
  sumx2 = sumx2 + (x(i,1)-mux1)*(x(i,2)-mux2);
  sumx3 = sumx3 + (x(i,2)-mux2)^2;
end

sumx1 = sumx1/10
sumx2 = sumx2/10
sumx3 = sumx3/10

ex = [sumx1 sumx2; sumx2 sumx3]
cov(x)

[e,lam] = eig(ex)

x = transpose(x);
y_p = transpose(e)*x


x_1d_1 = transpose(e(:,1))*x
x_1d_2 = transpose(e(:,2))*x


x1_hat = e(:,1)*y_p(1,:) + e(:,2)*y_p(2,:)
x2_hat = e(:,1)*y_p(1,:)

subplot(2,1,1)
plot(x(1,:),x(2,:),'ro')
title('Data before projection')
xlabel('x-coordinate')
ylabel('y-coordinate')
subplot(2,1,2)
plot(x2_hat(1,:),0,'g^')
title('Data after projection')
```

```
x =

  Columns 1 through 7

    0.6700    0.7100    0.4800    0.7200    0.2900    0.8900   -1.0700
   -1.2000    1.6300    1.0300   -0.3000   -0.7800   -1.1500   -0.8100

  Columns 8 through 10

   -2.9400    0.3300    1.3800
    1.4400   -0.7500   -1.7100


m =

    0.5000    0.5000


c =
```

```
    0.9000    0.1000
    0.1000    0.9000


e =

   -0.7071    0.7071
    0.7071    0.7071


d =

    0.8000         0
         0    1.0000


d =

    0.8944         0
         0    1.0000


y =

  Columns 1 through 7

   -1.2723    0.7035    0.4247   -0.6675   -0.7350   -1.3761    0.1040
   -0.4248    1.6016    1.0319    0.2432   -0.3681   -0.2503   -1.2495

  Columns 8 through 10

    2.8777   -0.7390   -2.0819
   -0.8412   -0.3216   -0.3364


mu1 =

   -0.2762


mu2 =

   -0.0915


sum1 =

    1.7583


sum2 =

    0.0693
```

```
sum3 =

    0.6469


exp =

    1.7583    0.0693
    0.0693    0.6469


mux1 =

    0.1460


mux2 =

   -0.2600


sumx1 =

    1.4167


sumx2 =

   -0.6213


sumx3 =

    1.2719


ex =

    1.4167   -0.6213
   -0.6213    1.2719


ans =

    1.5741   -0.6903
   -0.6903    1.4132


e =

   -0.6649   -0.7469
   -0.7469    0.6649
```

```
lam =

    0.7188         0
         0    1.9697


y_p =

  Columns 1 through 7

    0.4508   -1.6896   -1.0885   -0.2547    0.3898    0.2671    1.3165
   -1.2983    0.5535    0.3264   -0.7372   -0.7353   -1.4294    0.2606

  Columns 8 through 10

    0.8794    0.3407    0.3596
    3.1534   -0.7452   -2.1678


x_1d_1 =

  Columns 1 through 7

    0.4508   -1.6896   -1.0885   -0.2547    0.3898    0.2671    1.3165

  Columns 8 through 10

    0.8794    0.3407    0.3596


x_1d_2 =

  Columns 1 through 7

   -1.2983    0.5535    0.3264   -0.7372   -0.7353   -1.4294    0.2606

  Columns 8 through 10

    3.1534   -0.7452   -2.1678


x1_hat =

  Columns 1 through 7

    0.6700    0.7100    0.4800    0.7200    0.2900    0.8900   -1.0700
   -1.2000    1.6300    1.0300   -0.3000   -0.7800   -1.1500   -0.8100

  Columns 8 through 10

   -2.9400    0.3300    1.3800
    1.4400   -0.7500   -1.7100
```

```
x2_hat =

  Columns 1 through 7

   -0.2997    1.1234    0.7238    0.1693   -0.2592   -0.1776   -0.8754
   -0.3367    1.2619    0.8130    0.1902   -0.2911   -0.1995   -0.9833

  Columns 8 through 10

   -0.5847   -0.2266   -0.2391
   -0.6568   -0.2545   -0.2686
```
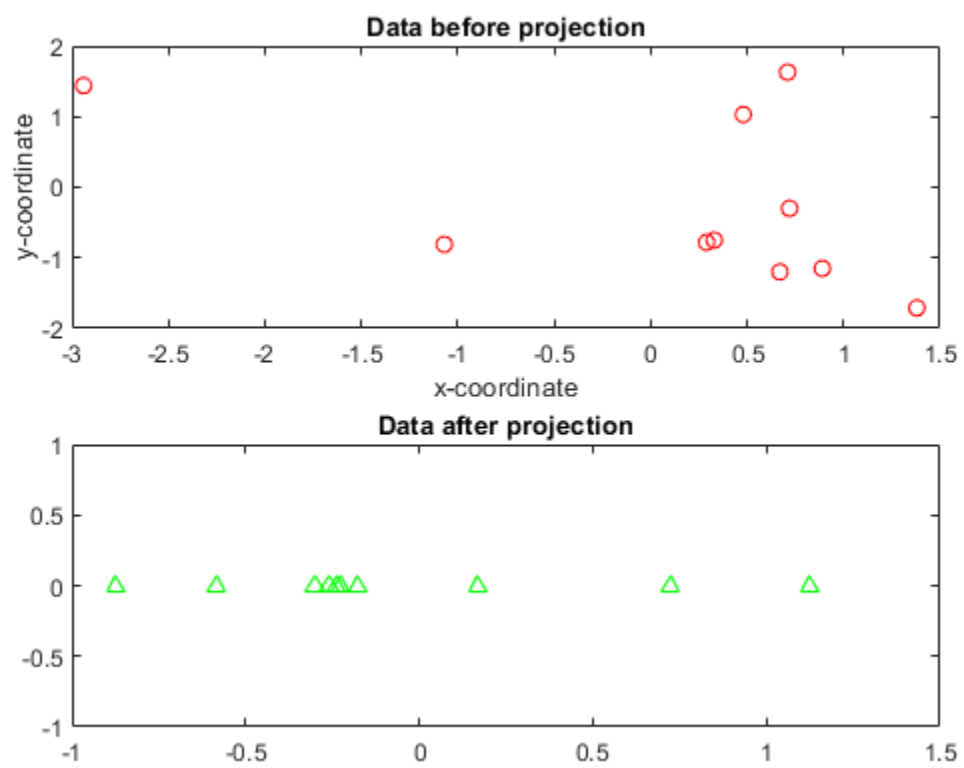

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```matlab
x = [0.59 1.67 1.42 1.11 1.64 0.39 1.42 1.43 1.45 1.45 0.70 1.21; 0.59
 1.73 1.10 1.09 1.79 0.66 1.52 1.52 1.54 1.54 0.81 1.31]

x = transpose(x);
mux1 = 0;
mux2 = 0;
for i = 1:12
 mux1 = mux1+x(i,1);
 mux2 = mux2+x(i,2);
end
mux1 = mux1/12
mux2 = mux2/12


sumx1 = 0;
sumx2 = 0;
sumx3 = 0;
for i = 1:12
 sumx1 = sumx1 + (x(i,1)-mux1)^2;
 sumx2 = sumx2 + (x(i,1)-mux1)*(x(i,2)-mux2);
 sumx3 = sumx3 + (x(i,2)-mux2)^2;
end

sumx1 = sumx1/12;
sumx2 = sumx2/12;
sumx3 = sumx3/12;

ex = [sumx1 sumx2; sumx2 sumx3]
cov(x)

[e,lam] = eig(ex)

temp1 = e(:,1);
e(:,1) = e(:,2);
e(:,2) = temp1;

x = transpose(x);
y_p = transpose(e)*x

x1_hat = e(:,1)*y_p(1,:) + e(:,2)*y_p(2,:)
x2_hat = e(:,1)*y_p(1,:)

subplot(2,1,1)
plot(x(1,:),x(2,:),'ro')
title('Data before projection')
xlabel('x-coordinate')
ylabel('y-coordinate')
subplot(2,1,2)
plot(x2_hat(1,:),0,'g^')
title('Data after projection')


x =
```

```
    Columns 1 through 7

      0.5900     1.6700     1.4200     1.1100     1.6400     0.3900     1.4200
      0.5900     1.7300     1.1000     1.0900     1.7900     0.6600     1.5200

    Columns 8 through 12

      1.4300     1.4500     1.4500     0.7000     1.2100
      1.5200     1.5400     1.5400     0.8100     1.3100


mux1 =

    1.2067


mux2 =

    1.2667


ex =

    0.1646     0.1509
    0.1509     0.1551


ans =

    0.1796     0.1646
    0.1646     0.1692


e =

    0.6960    -0.7181
   -0.7181    -0.6960


lam =

    0.0089          0
         0     0.3108


y_p =

  Columns 1 through 7

   -0.8343    -2.4032    -1.7852    -1.5557    -2.4234    -0.7394    -2.0775
   -0.0131    -0.0800     0.1984    -0.0102    -0.1440    -0.2025    -0.1032

  Columns 8 through 12
```

```
   -2.0847    -2.1130    -2.1130    -1.0664    -1.7806
   -0.0963    -0.0967    -0.0967    -0.0945    -0.0986


x1_hat =

  Columns 1 through 7

    0.5900     1.6700     1.4200     1.1100     1.6400     0.3900     1.4200
    0.5900     1.7300     1.1000     1.0900     1.7900     0.6600     1.5200

  Columns 8 through 12

    1.4300     1.4500     1.4500     0.7000     1.2100
    1.5200     1.5400     1.5400     0.8100     1.3100


x2_hat =

  Columns 1 through 7

    0.5991     1.7257     1.2820     1.1171     1.7402     0.5309     1.4918
    0.5806     1.6725     1.2424     1.0827     1.6866     0.5146     1.4459

  Columns 8 through 12

    1.4970     1.5173     1.5173     0.7658     1.2786
    1.4509     1.4705     1.4705     0.7422     1.2392
```
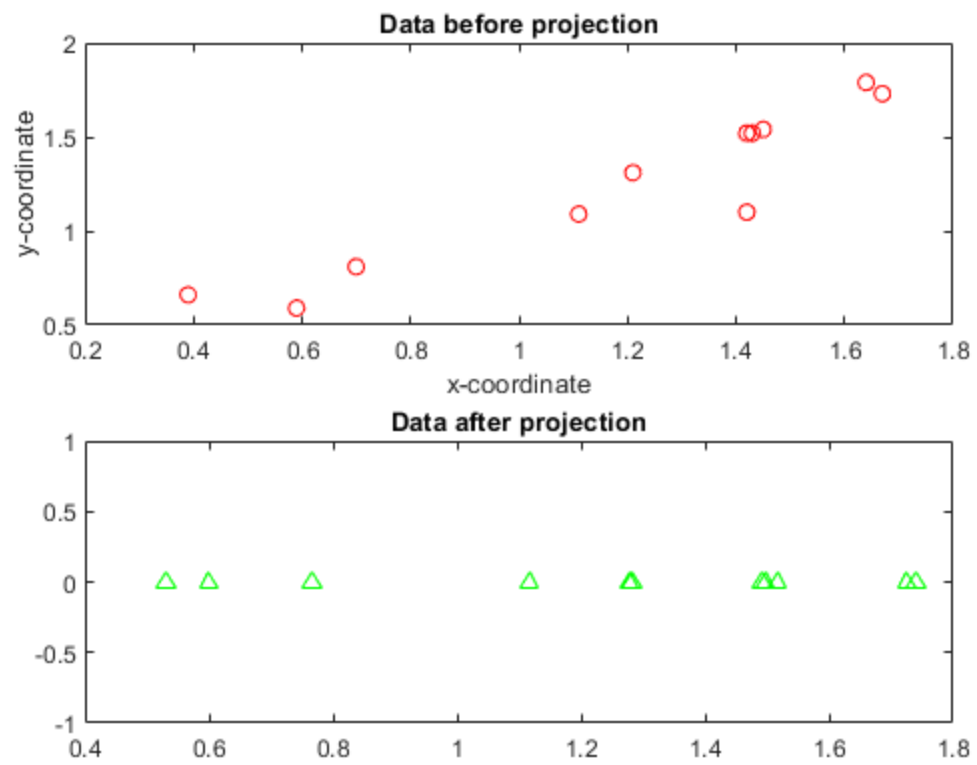
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```matlab
A1 = [1.08 0.75 0.85 0.94 0.40 1.25 1.19 0.99 0.69 1.32; 0.08 -0.19
 -0.11 0.00 -0.09 -0.21 0.07 0.04 -0.02 0.02]
A2 = [0.01 -0.01 0.09 -0.05 -0.45 0.07 -0.33 -0.06 -0.33 -0.24; 0.85
 1.05 0.93 1.41 1.45 1.20 0.88 1.08 1.10 1.01]

c1 = [0;0];
c2 = [0;0];
c = [0;0];
for i=1:10
    c1 = c1+A1(:,i);
    c2 = c2+A2(:,i);
    c = c+A1(:,i)+A2(:,i);
end

c1 = c1/10
c2 = c2/10
c = c/10

sb = (c-c1)*transpose(c-c1) + (c-c2)*transpose(c-c2);
sb = sb/2

A1 = transpose(A1);
A2 = transpose(A2);

cov1 = cov(A1)
cov2 = cov(A2)

A1 = transpose(A1);
A2 = transpose(A2);

sw = 0.05*(10*cov1 + 10*cov2)

new = inv(sw)*sb
[e, lam] = eig(new)

temp = e(:,1) ;
e(:,1) = e(:,2);
e(:,2) = temp;

temp = lam(1,1);
lam(1,1) = lam(2,2);
lam(2,2) = temp;

A = [A1, A2]
y_lda = transpose(e)*A

x1_hat = e(:,1)*y_lda(1,:) + e(:,2)*y_lda(2,:)
x2_hat = e(:,1)*y_lda(1,:)

subplot(2,1,1)
plot(A(1,:),A(2,:),'ro')
title('Data before projection')
```

```
xlabel('x-coordinate')
ylabel('y-coordinate')
subplot(2,1,2)
plot(x2_hat(1,:),0,'g^')
title('Data after projection')
```

A1 =

  Columns 1 through 7

| 1.0800 | 0.7500 | 0.8500 | 0.9400 | 0.4000 | 1.2500 | 1.1900 |
| 0.0800 | -0.1900 | -0.1100 | 0 | -0.0900 | -0.2100 | 0.0700 |

  Columns 8 through 10

| 0.9900 | 0.6900 | 1.3200 |
| 0.0400 | -0.0200 | 0.0200 |

A2 =

  Columns 1 through 7

| 0.0100 | -0.0100 | 0.0900 | -0.0500 | -0.4500 | 0.0700 | -0.3300 |
| 0.8500 | 1.0500 | 0.9300 | 1.4100 | 1.4500 | 1.2000 | 0.8800 |

  Columns 8 through 10

| -0.0600 | -0.3300 | -0.2400 |
| 1.0800 | 1.1000 | 1.0100 |

c1 =

   0.9460
   -0.0410

c2 =

   -0.1300
   1.0960

c =

   0.8160
   1.0550

sb =

   0.4559   -0.0906

```
   -0.0906    0.6014


cov1 =

    0.0803    0.0084
    0.0084    0.0108


cov2 =

    0.0365   -0.0103
   -0.0103    0.0421


sw =

    0.0584   -0.0010
   -0.0010    0.0265


new =

    7.7532   -1.1671
   -3.1332   22.6749


e =

   -0.9793    0.0767
   -0.2024   -0.9971


lam =

    7.5120         0
         0   22.9160


A =

  Columns 1 through 7

    1.0800    0.7500    0.8500    0.9400    0.4000    1.2500    1.1900
    0.0800   -0.1900   -0.1100         0   -0.0900   -0.2100    0.0700

  Columns 8 through 14

    0.9900    0.6900    1.3200    0.0100   -0.0100    0.0900   -0.0500
    0.0400   -0.0200    0.0200    0.8500    1.0500    0.9300    1.4100

  Columns 15 through 20

   -0.4500    0.0700   -0.3300   -0.0600   -0.3300   -0.2400
```

```
     1.4500     1.2000     0.8800     1.0800     1.1000     1.0100


  y_lda =

    Columns 1 through 7

      0.0031     0.2470     0.1749     0.0721     0.1204     0.3053     0.0215
     -1.0738    -0.6960    -0.8102    -0.9206    -0.3735    -1.1816    -1.1795

    Columns 8 through 14

      0.0361     0.0729     0.0814    -0.8467    -1.0477    -0.9204    -1.4097
     -0.9776    -0.6717    -1.2967    -0.1818    -0.2027    -0.2763    -0.2364

    Columns 15 through 20

     -1.4803    -1.1911    -0.9027    -1.0814    -1.1221    -1.0254
      0.1473    -0.3114     0.1451    -0.1598     0.1006     0.0306


  x1_hat =

    Columns 1 through 7

      1.0519     0.7006     0.8068     0.9070     0.3750     1.1806     1.1568
      0.2142    -0.1054    -0.0104     0.1144    -0.0445    -0.0653     0.2172

    Columns 8 through 14

      0.9602     0.6634     1.2762     0.1131     0.1181     0.2000     0.1233
      0.1618     0.0632     0.1813     0.8810     1.0856     0.9736     1.4534

    Columns 15 through 20

     -0.2578     0.2135    -0.2114     0.0735    -0.1846    -0.1087
      1.4461     1.2506     0.8707     1.1106     1.0984     1.0162


  x2_hat =

    Columns 1 through 7

      0.0002     0.0190     0.0134     0.0055     0.0092     0.0234     0.0017
     -0.0031    -0.2463    -0.1744    -0.0719    -0.1201    -0.3044    -0.0215

    Columns 8 through 14

      0.0028     0.0056     0.0062    -0.0650    -0.0804    -0.0706    -0.1082
     -0.0360    -0.0727    -0.0811     0.8442     1.0446     0.9176     1.4055

    Columns 15 through 20

     -0.1136    -0.0914    -0.0693    -0.0830    -0.0861    -0.0787
```
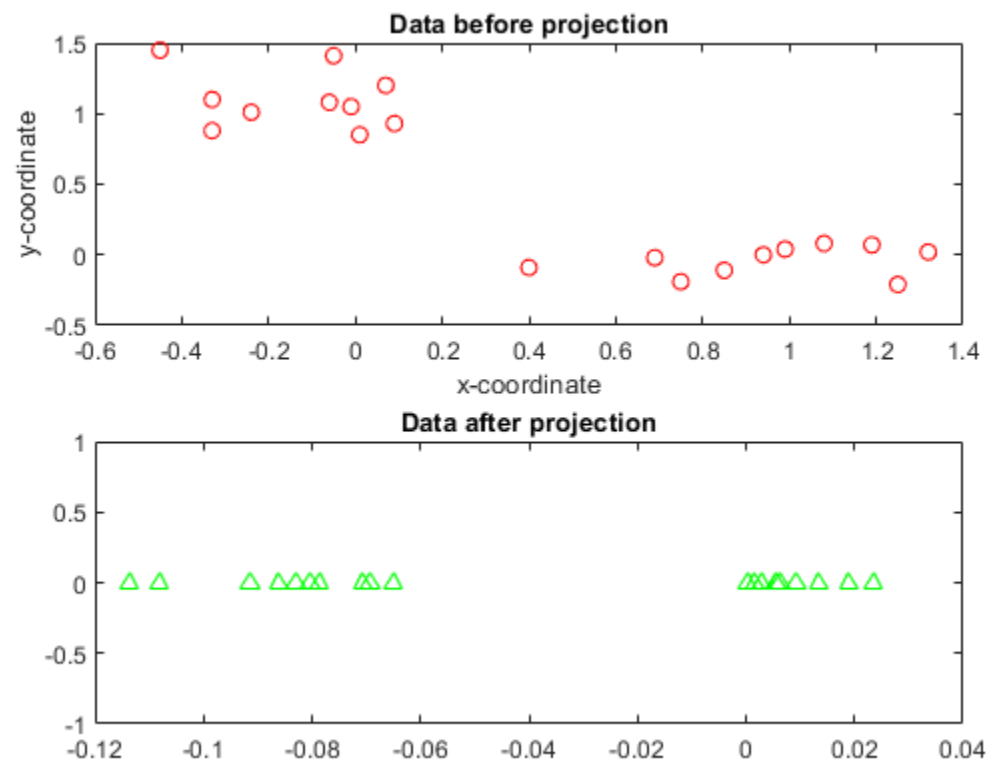
```
1.4759    1.1876    0.9001    1.0782    1.1188    1.0224
```

Data before projection

Data after projection

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```matlab
A1 = [1.08 0.75; 0.08 -0.19]
A2 = [0.01 0.09; 0.85 0.93]
A = [A1 A2]

G = [];
for i = 1:4
    for j = 1:4
        G(i,j) = kernel(A(:,i),A(:,j));
    end
end

A1 = [];
A2 = [];

for i = 1:2
    A1 = [A1 G(:,i)];
    A2 = [A2 G(:,2+i)];
end

A1
A2
A = [A1 A2]

c1 = zeros(4,1);
c2 = zeros(4,1);
c = zeros(4,1);

for i = 1:2
    c1 = c1 + A1(:,i);
    c2 = c2 + A2(:,i);
end

c1 = c1/2
c2 = c2/2

c = (c1+c2)/2

sb = 0.5*((c-c1)*transpose(c-c1) + (c-c2)*transpose(c-c2))

A1 = transpose(A1);
A2 = transpose(A2);

cov1 = cov(A1)
cov2 = cov(A2)

A1 = transpose(A1);
A2 = transpose(A2);

sw = (2*cov1 + 2*cov2)/4

new = inv(sw)*sb
[e, lam] = eigs(new,4)
```

```matlab
y = transpose(e)*A


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A1 = [1.08 0.75; 0.08 -0.19]
A2 = [0.01 0.09; 0.85 0.93]
A = [A1 A2]

B1 = [1.08 0.75 0.85 0.94 0.40 1.25 1.19 0.99 0.69 1.32; 0.08 -0.19
 -0.11 0.01 -0.09 -0.21 0.07 0.04 -0.02 0.02]
B2 = [0.01 -0.01 0.09 -0.05 -0.45 0.07 -0.33 -0.06 -0.33 -0.24; 0.85
 1.05 0.93 1.41 1.45 1.20 0.88 1.08 1.10 1.01]
B = [B1 B2]

G1 = [];
proj = [];

for i = 1:20
    for j = 1:4
        G1(j,i) = kernel(A(:,j),B(:,i));
    end
end

for i = 1:10
    B12 = G1(:,i);
    B22 = G1(:,10+i);
end

for i = 1:20
    proj(:,i) = e(:,1)'*G1(:,i);
end

y1 = [];
y2 = [];
for i = 1:10
    y1 = [y1 proj(:,i)];
    y2 = [y2 proj(:,10+i)];
end

y1
y2

subplot(2,1,1)
plot(B1(1,:),B1(2,:),'ro',B2(1,:),B2(2,:),'g^')
title('Data before projection')
xlabel('x-coordinate')
ylabel('y-coordinate')
legend('class1','class2')
subplot(2,1,2)
plot(y1(1,:),0,'ro',y2(1,:),0,'g^')
title('Data after projection')
```

```
function k = kernel(p,q)
sig = 1;
k = exp(-(transpose(p-q)*(p-q))/(2*sig^2));
end
```

A1 =

*   1.0800   0.7500*
*   0.0800   -0.1900*

A2 =

*   0.0100   0.0900*
*   0.8500   0.9300*

A =

*   1.0800   0.7500   0.0100   0.0900*
*   0.0800   -0.1900   0.8500   0.9300*

A1 =

*   1.0000   0.9131*
*   0.9131   1.0000*
*   0.4194   0.4428*
*   0.4269   0.4296*

A2 =

*   0.4194   0.4269*
*   0.4428   0.4296*
*   1.0000   0.9936*
*   0.9936   1.0000*

A =

*   1.0000   0.9131   0.4194   0.4269*
*   0.9131   1.0000   0.4428   0.4296*
*   0.4194   0.4428   1.0000   0.9936*
*   0.4269   0.4296   0.9936   1.0000*

c1 =

*   0.9566*
*   0.9566*
```

```
    0.4311
    0.4282


c2 =

    0.4231
    0.4362
    0.9968
    0.9968


c =

    0.6898
    0.6964
    0.7140
    0.7125


sb =

    0.0711    0.0694   -0.0754   -0.0758
    0.0694    0.0677   -0.0736   -0.0740
   -0.0754   -0.0736    0.0800    0.0804
   -0.0758   -0.0740    0.0804    0.0808


cov1 =

    0.0038   -0.0038   -0.0010   -0.0001
   -0.0038    0.0038    0.0010    0.0001
   -0.0010    0.0010    0.0003    0.0000
   -0.0001    0.0001    0.0000    0.0000


cov2 =

   1.0e-04 *

    0.2773   -0.4937   -0.2375    0.2375
   -0.4937    0.8790    0.4229   -0.4229
   -0.2375    0.4229    0.2035   -0.2035
    0.2375   -0.4229   -0.2035    0.2035


sw =

    0.0019   -0.0019   -0.0005   -0.0000
   -0.0019    0.0019    0.0005    0.0000
   -0.0005    0.0005    0.0001    0.0000
   -0.0000    0.0000    0.0000    0.0000
```

```
Warning: Matrix is close to singular or badly scaled. Results may be
 inaccurate.
RCOND =  2.739728e-19.

new =

   1.0e+19 *

    1.4693    1.4333   -1.5582   -1.5662
    0.4899    0.4779   -0.5195   -0.5222
    3.3323    3.2508   -3.5340   -3.5521
    2.6354    2.5709   -2.7949   -2.8092


e =

   -0.3249    0.5162    0.8310    0.7216
   -0.1083   -0.3173   -0.0228    0.1363
   -0.7369   -0.4585    0.2798    0.1376
   -0.5828    0.6501    0.4803    0.6647


lam =

   1.0e+19 *

   -4.3960         0         0         0
         0    0.0000         0         0
         0         0   -0.0000         0
         0         0         0   -0.0000


y =

   -0.9817   -0.9817   -1.5002   -1.5002
    0.3117    0.2302    0.2634    0.2785
    1.1325    1.0662    1.0955    1.1032
    1.1874    1.1416    1.1611    1.1680


A1 =

    1.0800    0.7500
    0.0800   -0.1900


A2 =

    0.0100    0.0900
    0.8500    0.9300


A =
```

```
    1.0800    0.7500    0.0100    0.0900
    0.0800   -0.1900    0.8500    0.9300


B1 =

  Columns 1 through 7

    1.0800    0.7500    0.8500    0.9400    0.4000    1.2500    1.1900
    0.0800   -0.1900   -0.1100    0.0100   -0.0900   -0.2100    0.0700

  Columns 8 through 10

    0.9900    0.6900    1.3200
    0.0400   -0.0200    0.0200


B2 =

  Columns 1 through 7

    0.0100   -0.0100    0.0900   -0.0500   -0.4500    0.0700   -0.3300
    0.8500    1.0500    0.9300    1.4100    1.4500    1.2000    0.8800

  Columns 8 through 10

   -0.0600   -0.3300   -0.2400
    1.0800    1.1000    1.0100


B =

  Columns 1 through 7

    1.0800    0.7500    0.8500    0.9400    0.4000    1.2500    1.1900
    0.0800   -0.1900   -0.1100    0.0100   -0.0900   -0.2100    0.0700

  Columns 8 through 14

    0.9900    0.6900    1.3200    0.0100   -0.0100    0.0900   -0.0500
    0.0400   -0.0200    0.0200    0.8500    1.0500    0.9300    1.4100

  Columns 15 through 20

   -0.4500    0.0700   -0.3300   -0.0600   -0.3300   -0.2400
    1.4500    1.2000    0.8800    1.0800    1.1000    1.0100


y1 =

  Columns 1 through 7

   -0.9817   -0.9817   -0.9991   -1.0273   -1.1248   -0.7527   -0.9089
```

```
Columns 8 through 10
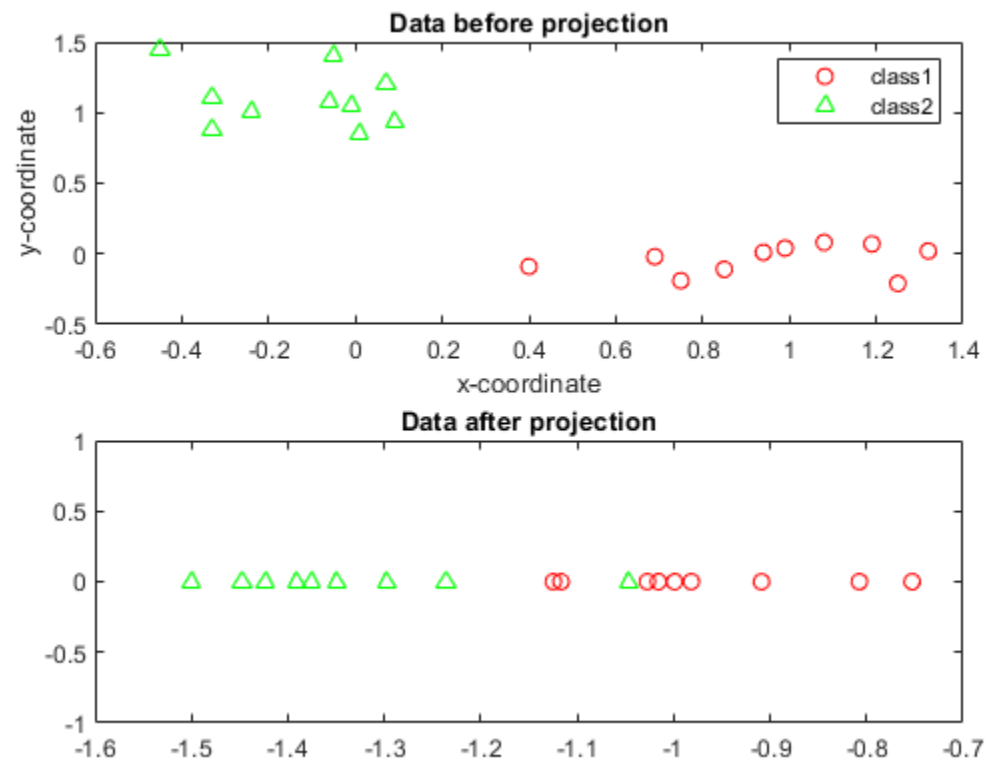
  -1.0158   -1.1168   -0.8074


y2 =

 Columns 1 through 7

  -1.5002   -1.4476   -1.5002   -1.2359   -1.0469   -1.3906   -1.3496

 Columns 8 through 10

  -1.4235   -1.2979   -1.3758
```

```matlab
A1 = [1.08 0.75 0.85 0.94 0.40 1.25 1.19 0.99 0.69 1.32; 0.08 -0.19
 -0.11 0.01 -0.09 -0.21 0.07 0.04 -0.02 0.02]
A2 = [0.01 -0.01 0.09 -0.05 -0.45 0.07 -0.33 -0.06 -0.33 -0.24; 0.85
 1.05 0.93 1.41 1.45 1.20 0.88 1.08 1.10 1.01]
t = [1 1 1 1 1 0 0 0 0 0;0 0 0 0 0 1 1 1 1 1];
train = [];
valid = [];
for i = 1:5
    train = [train A1(:,i)];
    valid = [valid A1(:,5+i)];
end

for i = 1:5
    train = [train A2(:,i)];
    valid = [valid A2(:,5+i)];
end

w1 = [0;0]
w2 = [0;0]
cross = 0

error = [];
for i = 1:10
    cross = 0;
    for j = 1:10
        d = exp(transpose(w1)*train(:,j)) +
 exp(transpose(w2)*train(:,j));
        y1 = exp(transpose(w1)*train(:,j))/d;
        y2 = exp(transpose(w2)*train(:,j))/d;
        cross = cross - (t(1,j)*log(y1) + t(2,j)*log(y2));

        w1 = w1 + 0.1*train(:,j)*(t(1,j)-y1);
        w2 = w2 + 0.1*train(:,j)*(t(2,j)-y2);
    end
    error = [error cross];
end

error
plot(error)
y_out = [];

for i = 1:10
    d = exp(transpose(w1)*valid(:,i)) + exp(transpose(w2)*valid(:,i));
    y_out(1,i) = exp(transpose(w1)*valid(:,i))/d;
    y_out(2,i) = exp(transpose(w2)*valid(:,i))/d;
end

y_out


A1 =
```

```
   Columns 1 through 7

     1.0800     0.7500     0.8500     0.9400     0.4000     1.2500     1.1900
     0.0800    -0.1900    -0.1100     0.0100    -0.0900    -0.2100     0.0700

   Columns 8 through 10

     0.9900     0.6900     1.3200
     0.0400    -0.0200     0.0200


A2 =

   Columns 1 through 7

     0.0100    -0.0100     0.0900    -0.0500    -0.4500     0.0700    -0.3300
     0.8500     1.0500     0.9300     1.4100     1.4500     1.2000     0.8800

   Columns 8 through 10

    -0.0600    -0.3300    -0.2400
     1.0800     1.1000     1.0100


w1 =

     0
     0


w2 =

     0
     0


cross =

     0


error =

   Columns 1 through 7

     5.9878     4.3474     3.3906     2.7740     2.3462     2.0331     1.7945

   Columns 8 through 10

     1.6068     1.4555     1.3310
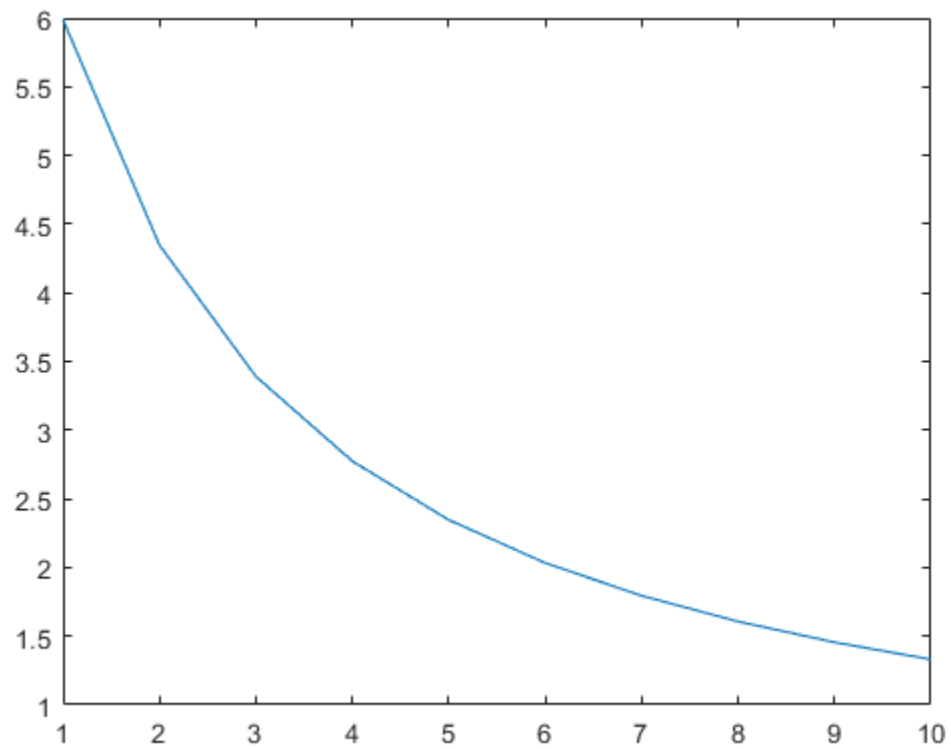

y_out =
```

```
Columns 1 through 7

  0.9558    0.9124    0.8796    0.8160    0.9384    0.0800    0.0698
  0.0442    0.0876    0.1204    0.1840    0.0616    0.9200    0.9302

Columns 8 through 10

  0.0790    0.0446    0.0640
  0.9210    0.9554    0.9360
```


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