

# Conditional Kernel Imitation Learning for Continuous State Environments

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## Abstract

Imitation Learning (IL) is an important paradigm within the broader reinforcement learning (RL) methodology. Unlike most of RL, it does not assume availability of reward-feedback. Classical methods such as behavioral cloning and inverse reinforcement learning are highly sensitive to estimation errors, especially in continuous state space problems. Meanwhile, state-of-the-art (SOTA) IL algorithms often require additional online interaction data to be effective. In this paper, we consider the problem of imitation learning in continuous state space environments based solely on observed behavior, without access to transition dynamics information, reward structure, or, most importantly, any additional interactions with the environment. Our approach is based on the Markov balance equation and introduces a novel conditional kernel density estimation-based imitation learning framework. It uses conditional kernel density estimators for transition dynamics and seeks to satisfy a balance equation for the environment. We establish that our estimators satisfy asymptotic consistency and present a detailed analysis of their sample complexity. Through a series of numerical experiments on continuous state benchmark environments, we show consistently superior empirical performance over many SOTA IL algorithms. The full paper with the appendix is available at: <https://github.com/rishabh-1086/CKIL>.

**Keywords:** Imitation Learning, Reinforcement Learning, Offline learning.

## 1. Introduction

Reinforcement Learning (RL) has produced a series of breakthroughs over the last decade from exceeding human proficiency at playing simple games such as in the Atari suite (Mnih et al., 2015) to Go (Silver et al., 2016) and StarCraft (Vinyals et al., 2019), and to protein structure prediction systems (Jumper et al., 2021), etc. A fundamental premise that the ‘reward is enough’ (Silver et al., 2021) underlies all of such RL methodology. And yet, in most problems, a natural reward function is not available. Nor may it be possible to engineer one from intuition. Thus, a lot of effort is spent on reward shaping (Ng et al., 1999) to make RL algorithms work, often without success.

This problem is particularly acute with humans in the loop, either as demonstrators, or as evaluators. Often, demonstration data comes from human experts and it is impossible to infer precisely what reward function human experts really have in mind while taking actions. To be fair, several inverse RL (IRL) algorithms such as MaxEntropy-IRL (Ziebart et al., 2008) use a methodology wherein a reward function is first inferred from the demonstration data, and then used in conjunction with RL algorithms to design near-optimal policies. This has two lacunae. First, the performance of the RL algorithms can be very sensitive to errors in the reward function estimate. And second, the expert may not be using a policy that is optimal with respect to any reward objective at all! There is

thus a need to develop imitation learning (IL) algorithms that do not depend on reward inference as a first step (Arora and Doshi, 2021).

Behavioral Cloning (BC) is a straightforward approach to imitation learning (Pomerleau, 1988), using supervised learning to map states to actions. In fact, it largely ignores the inherent sequential nature of reinforcement learning problems. Unfortunately, it suffers from severe covariate shift issues as it fails to generalize to less-visited parts of the state space and ignores the sequential decision-making aspect. This results in propagation of errors in the agent’s performance (Ross and Bagnell, 2010) resulting in a limited practical ability to generalize effectively.

To address the issue of compounding errors that afflicts methods like BC, IRL algorithms (Abbeel and Ng, 2004; Ng and Russell, 2000; Syed and Schapire, 2007; Ziebart et al., 2008; Levine et al., 2011) first learn a reward function, and then use reinforcement learning to find a policy, which makes them expensive to run, and sensitive to reward function estimation errors.

Adversarial Imitation Learning (AIL) on the other hand, is based on distribution matching through adversarial learning (Ho and Ermon, 2016; Fu et al., 2018; Ke et al., 2021) between the target state-action distribution and that generated by the behavioral policy during its interaction with the environment. Unfortunately, this is also its drawback: it needs new samples from the behavioral policy in every iteration, and is thus not suitable when only offline data is available. It is also challenging for continuous control problems since each visited state is visited at most once. There is broader IL literature (Ross et al., 2011) that also assumes access to a generative model to generate more data on the fly.

In this paper, we propose an imitation learning algorithm designed to address several key challenges. Our approach does not require *reward feedback*, avoids *distribution matching* through on-policy samples, and moves beyond *behavioral cloning* by leveraging the Markovian structure of the dynamics. It does not rely on access to a *generative model*, supports continuous state spaces, and enables batch processing of offline datasets. This formulation is particularly relevant for real-world decision-making applications, such as healthcare, robotics, and autonomous vehicles (Le Mero et al., 2022), where experimentation is costly or unsafe.

We present a novel framework grounded in the premise that demonstration trajectory data satisfies the balance equation linking the demonstration policy, the Markov decision process (MDP) transition density, and the induced Markov chain. We then estimate the MDP and Markov chain transition densities using conditional kernel density estimators. Starting with the discrete state and action setting, we extend the framework to continuous state spaces, proving estimators’ asymptotic consistency and analyzing their sample complexities. Validation on continuous state problems demonstrates strong numerical performance. The *novel* combination of these two ideas together enables its excellent numerical performance. While extending to continuous action spaces is conceptually straightforward, additional effort is needed for numerical robustness.

**Other Related Work.** As already mentioned, behavioral cloning often has poor performance due to failure to account for Markovian dynamics and discarding distributional insights from the demonstrations (Ross and Bagnell, 2010; Piot et al., 2014). This is sought to be addressed (Ross et al., 2011; Piot et al., 2016) by either further online interactions with the environment, or the demonstrator, or using insights into model dynamics or the sparsity of rewards, all of which are in general impractical. The recent work (Xu et al., 2022) aims to overcome these by using additional data from non-expert policies without needing additional online interactions but such offline data may not be available. The EDM approach (Jarrett et al., 2020a) captures the expert’s state occupancy measure

by training an explicit energy-based model but has many limitations as discussed in (Swamy et al., 2021).

There also have been efforts to further develop IRL approaches to overcome the limitations of earlier algorithms. (Klein et al., 2011, 2012) introduce *LSTD- $\mu$* , a temporal difference technique that shares the weaknesses of least squares estimators, and is highly sensitive to basis feature selection and training data distribution. (Lee et al., 2019b) propose *DSFN* which uses a transition-regularized imitation network that produces an initial policy close to expert behavior and an efficient feature representation but assumes complete knowledge about reward features which is unrealistic (Arora and Doshi, 2021).

(Piot et al., 2014) introduced *RCAL*, a non-parametric algorithm that employs a boosting method to minimize a large margin objective augmented with a regularization term taking into account the MDP structure, avoids feature selection steps and can help tackle some of the above issues. (Chan and van der Schaar, 2021b) propose *AVRIL*, which jointly learns an approximate posterior distribution over reward and policy. (Garg et al., 2021a) introduce *IQ-Learn*, an off-policy IRL method that implicitly represents reward and policy using a learned soft Q-function. Unfortunately, both suffer from significant covariate shift problems, and thus encounter significant reward extrapolation errors, leading to misguided outcomes in novel environments. To address this, the *CLARE* (Yue et al., 2023) model-based offline Inverse Reinforcement Learning (IRL) approach introduces conservatism to its estimated reward but assumes access to an additional diverse dataset. It employs an IRL algorithm within an estimated dynamics model to learn the reward. However, it has been shown (Zeng et al., 2023) to have poor performance when there are predominantly low-quality transition samples from a behavior policy.

Adversarial Imitation Learning (AIL) approaches (Ho and Ermon, 2016) were a breakthrough when they were introduced a few years ago (Blondé and Kalousis, 2019; Kostrikov et al., 2019a). However, these approaches require online interactions with the environment, and thus are not applicable when we must work only with offline data. (Kostrikov et al., 2019b) introduces *ValueDICE* that employs a distribution matching strategy between the imitator and expert policies, and undertakes a complex alternating maximization-minimization optimization procedure. Still, it has difficulties in estimating the expectation of an exponential that introduces bias when approximating gradients using mini-batches (Jarrett et al., 2020a). The algorithm we present is quite distinct from the above, and demonstrates promising preliminary empirical results.

## 2. Preliminaries

**The Imitation Learning Problem.** An infinite horizon discounted Markov decision process (MDP)  $M$  is defined by the tuple  $(S, A, T, r, \gamma)$  with states  $s \in S$ , actions  $a \in A$  and successor states  $s' \in S$  drawn from the transition function  $T(s'|s, a)$ . The reward function  $r : S \times A \rightarrow \mathbb{R}$  maps state-action pairs to scalar rewards, and  $\gamma$  is the discount factor. Policy  $\pi$  is a probability distribution over actions conditioned on state and is given by  $\pi(a_t|s_t) = P_\pi(A_t = a_t|S_t = s_t)$ , where  $a_t \in A$ ,  $s_t \in S$ ,  $\forall t = 0, 1, 2, \dots$ . The induced occupancy measure of a policy is given as  $\rho_\pi(s, a) := \mathbb{E}_\pi[\sum_{t=0}^{\infty} \gamma^t \mathbf{1}_{s_t=s, a_t=a}]$ , where the expectation is taken over  $a_t \sim \pi(\cdot|s_t)$ ,  $s_{t+1} \sim T(\cdot|s_t, a_t)$  for all  $t$ , and the initial state  $s_0$ . The corresponding state-only occupancy measure is given as  $\rho_\pi(s) = \sum_a \rho_\pi(s, a)$ . In the offline imitation learning (IL) framework, the agent is provided with trajectories generated by a demonstration policy  $\pi_D$ , collected as  $D = \{(s_0, a_0), (s_1, a_1), (s_2, a_2), \dots\}$ ; and is not allowed any further interaction with the environment. The data  $D$  does *not* include any reward

$r_t$  at each time step. Indeed, rather than long-term reward maximization, the IL objective is to learn a policy  $\pi^*$  that is close to  $\pi_D$  in the following sense (Yue and Le, 2018):

$$\pi^* \in \arg \min_{\pi \in \Pi} \mathbb{E}_{s \sim \rho_\pi} [\mathcal{L}(\pi(\cdot|s), \pi_D(\cdot|s))], \quad (1)$$

where  $\Pi$  is the set of all randomized (Markovian) stationary policies, and  $\mathcal{L}$  is a chosen loss function. In practice, (1) can only be solved approximately since  $\pi_D$  is unknown and only transitions are observed in the dataset  $D$ .

**Conditional Kernel Density Estimation (CKDE).** The imitation learning approach we introduce depends on transition density estimation. Though statistical theory exists for it, conditional density estimation is a difficult problem due to a lack of clarity on what parametric families of density functions are good candidates. Thus, we adopt kernel density estimation (KDE), a nonparametric framework for the estimation of general continuous distributions (Wand and Jones, 1994).

We next outline the method for two continuous random variables,  $X$  and  $Y$  for the sake of simplicity. Let  $f$  and  $g$  denote the joint density of  $(X, Y)$  and the marginal density of  $X$ , respectively. The conditional distribution of  $Y$ , given  $X$ , is denoted as  $h_{Y|X}(y|x) = f_{X,Y}(x, y)/g_X(x)$ . Selecting a pair of kernel functions  $K : \mathbb{R} \rightarrow \mathbb{R}$  and  $K' : \mathbb{R} \rightarrow \mathbb{R}$  with respective scalar bandwidth parameters  $h > 0$  and  $h' > 0$  and given a set of  $n$  samples  $\{(x_i, y_i)\}_{i=1}^n$ , the KDE approximations  $\hat{f}$  and  $\hat{g}$  for the joint and marginal distributions, respectively, are obtained as follows:

$$\begin{aligned} \hat{f}_{X,Y}(x, y) &= \frac{1}{n} \sum_{i=1}^n \frac{1}{h} K\left(\frac{x - x_i}{h}\right) \frac{1}{h'} K'\left(\frac{y - y_i}{h'}\right), \\ \hat{g}_X(x) &= \frac{1}{n} \sum_{i=1}^n \frac{1}{h} K\left(\frac{x - x_i}{h}\right). \end{aligned} \quad (2)$$

Using the approximations in (2), the approximate conditional density  $\hat{h}_{Y|X}$  can be computed as

$$\hat{h}_{Y|X}(y|x) = \frac{\hat{f}_{X,Y}(x, y)}{\hat{g}_X(x)}. \quad (3)$$

In more general cases involving random vectors, analogous estimates to those in (2) and (3) may be obtained using kernel functions defined according to

$$K_H(x) = |H|^{-\frac{1}{2}} K(H^{-\frac{1}{2}} x), \quad (4)$$

where  $H$  is a symmetric positive definite *bandwidth matrix* of appropriate dimension,  $m$ , with determinant  $|H|$ , and  $K$  is a real-valued function satisfying  $\int_{\mathbb{R}^m} K(x) dx = 1$ . For example, the KDE estimate for the marginal distribution of random vector  $X$  is defined as

$$\hat{g}_X(x; H) = \frac{1}{n} \sum_{i=1}^n K_H(x - x_i). \quad (5)$$

An example of such a multivariate kernel function is the standard  $m$ -variate normal density function

$$K(x) := (2\pi)^{-\frac{m}{2}} \exp\left(-\frac{x^T x}{2}\right).$$

We note that conditional density estimation is quite difficult numerically, and conditional kernel density estimation (CKDE), the adaptation of KDE to conditional density estimation, is amongst the most effective methods available (see (Chacón and Duong, 2018) for more details).

### 3. Conditional Kernel Imitation Learning

We next describe our imitation learning algorithm. A key premise of our algorithm is that the demonstration trajectories from an expert must satisfy the Markov balance equation under the demonstrator’s policy  $\pi_D$ , and thus we must use that to guide the agent’s learning. The use of the balance equation then requires estimation of certain transition (conditional probability) density functions which we obtain via conditional kernel density estimation methods. Then, the problem reduces to identifying policies that best fit the balance equation. We elucidate this procedure in Algorithm 1, prove its theoretical properties and then present numerical evidence of its efficacy on a number of benchmark environments.

**The Markov Balance Equation.** Consider a demonstration policy  $\pi_D$  that is used to take actions starting from an initial state  $s_0$ . Let  $T(s'|s, a)$  denote the transition density function of the MDP. Note that  $\pi_D(a|s)$  is a randomized Markovian, stationary policy that denotes the probability of taking action  $a$  in state  $s$ . This will induce a Markov chain on the state space  $S$ . Let its transition density be denoted  $P(s'|s)$ . Then, the Markov balance equation is given by

$$P(s'|s) = \sum_a \pi_D(a|s)T(s'|s, a).$$

Unfortunately, this involves a sum, and hence is difficult to use. Therefore, we use the following alternate form which is a transition density of the induced Markov chain on the state-action space,

$$P_{\pi_D}(s', a'|s, a) = \pi_D(a'|s')T(s'|s, a). \quad (6)$$

The above balance equation is the basis of our IL approach. If we can estimate  $P_{\pi_D}$  and  $T$  in (6) (estimates denoted by  $\hat{P}$  and  $\hat{T}$  respectively), we can then infer a policy  $\pi_D$  that satisfies it. Unfortunately, the problem is ill-conditioned, and we will need to impose additional criterion such as a regularization term.

We consider a class of policies parametrized by  $\theta$  and formulate the following optimization problem:

$$\begin{aligned} \min_{\theta \in \Theta} \int_{(s', a')} \int_{(s, a)} [\hat{P}(s', a'|s, a) - \pi_\theta(a'|s')\hat{T}(s'|s, a)]^2 d\mu(s, a) d\mu(s', a') \\ - \lambda \int_{s'} H(\pi_\theta(\cdot|s')) d\nu(s'). \end{aligned} \quad (7)$$

In (7), the squared loss first term ensures that the balance equation is satisfied approximately. It is a simple but novel loss function rarely used in imitation learning in conjunction with a balance equation. The second term,  $H(\pi_\theta(\cdot|s'))$  is the entropy of the probability distribution  $\pi_\theta(\cdot|s')$  on actions when the state is  $s'$ . It penalizes less randomized policies in favor of more randomized policies.  $\lambda \geq 0$  is a regularization parameter. Here,  $\mu$  and  $\nu$  denote reference probability measures on state-action pairs and states respectively. For example, they can be the counting measures obtained from the dataset.  $\Theta$  is a given parameter set. The parameters could be weights of a neural network, for example.

**Transition Density Estimation.** We now discuss how to use kernel density estimation methods for estimating the two conditional densities  $P_{\pi_D}$  and  $T$ , first for the discrete state and action space setting, where the form of estimates is intuitive, and then for the continuous setting.

**Discrete Spaces.** When both the state and action spaces are discrete, the estimates  $\hat{P}$  and  $\hat{T}$  can be calculated as:

$$\hat{T}(s'|s, a) := \frac{\eta(s, a, s')}{\eta(s, a)}, \text{ and } \hat{P}(s', a'|s, a) := \frac{\eta(s, a, s', a')}{\eta(s, a)} \quad (8)$$

where  $\eta$  represents the counting measure, i.e., the frequency of a tuple or sequence in dataset  $D$ . If the denominator is zero, the conditional density is assumed uniform.

**Continuous Spaces.** Estimation of transition densities in this setting is more challenging since no visited state would appear twice, and most of the states would never be visited in any given dataset. This, thus calls for conditional density estimation using more sophisticated methods.

These include a range of techniques such as parametric approaches like mixture density network (Bishop, 1994), normalizing flows (Trippe and Turner, 2018); non-parametric methods like Gaussian process conditional density estimation (Dutordoir et al., 2018), CKDE (Li and Racine, 2006); and semi-parametric methods like least squares conditional density estimation (Sugiyama et al., 2010). In this study, we opt for using CKDE since it is a closed-form, non-parametric method that can be easily implemented and adapted to different data types. Further, CKDE provides a consistent estimator under appropriate conditions (Chacón and Duong, 2018).

As described in Section 2, the kernel functions use a difference between two samples/values (e.g.,  $x - x_i$ ) as their argument (5). This difference can be alternatively replaced by a suitable distance metric, as indicated in prior work (Haasdonk and Bahlmann, 2004). We define three distinct distance metrics: one to assess the dissimilarity between (next state, next action) pairs, another for (state, action) pairs, and a final one for next states. These metrics are denoted as  $d_1 : (S \times A) \times (S \times A) \rightarrow \mathbb{R}_+$ ,  $d_2 : (S \times A) \times (S \times A) \rightarrow \mathbb{R}_+$ , and  $d_3 : S \times S \rightarrow \mathbb{R}_+$  respectively. Similarly, we define  $H_1$ ,  $H_2$ , and  $H_3$  as bandwidth matrices for the kernels  $K_{H_1}$ ,  $K_{H_2}$ , and  $K_{H_3}$ , respectively.  $H_1$ ,  $H_2$ , and  $H_3$  are square matrices with dimensions matching those of the  $(s', a')$  pair,  $(s, a)$  pair, and  $s'$ , respectively. The CKDE approximations  $\hat{P}$  and  $\hat{T}$  are then computed as

$$\begin{aligned} \hat{P}(s', a'|s, a) &= \frac{\sum_{l=1}^n K_{H_1}(d_1((s', a'), (s'_l, a'_l))) K_{H_2}(d_2((s, a), (s_l, a_l)))}{\sum_{l=1}^n K_{H_2}(d_2((s, a), (s_l, a_l)))}, \\ \text{and } \hat{T}(s'|s, a) &= \frac{\sum_{l=1}^n K_{H_3}(d_3(s', s'_l)) K_{H_2}(d_2((s, a), (s_l, a_l)))}{\sum_{l=1}^n K_{H_2}(d_2((s, a), (s_l, a_l)))}. \end{aligned} \quad (9)$$

We combine the transition estimation procedures of (8) and (9) with the balance equation based optimization problem in (7) in our conditional kernel imitation learning (CKIL) algorithm whose pseudo-code is presented in Algorithm 1.

**Remarks.** 1. Aside from the regularization parameter  $\lambda$  in (11), the only other hyperparameters in our algorithm are the three kernel bandwidth parameters, which are not particularly sensitive. 2. While scalability can be a concern, conditional density estimation is a difficult problem, and kernel methods are among the best for them in terms of numerical performance while also coming with theoretical consistency guarantees which we provide next. 3. CKIL allows for batch learning, wherein it uses only  $N_{batch}$  (the batch size) tuples at each gradient step, thus allowing it to scale since it does not need to grow with the dataset size.

**Theoretical Guarantees.** Given standard assumptions, we demonstrate that as the training dataset size  $n$  approaches infinity, the CKDE estimates in (9) converge in probability to the true conditional distributions in Theorem 1. We then provide sample complexity result to show how fast these estimates concentrate around the true distribution in 2.



**Theorem 1** Let  $\hat{P}_n$  and  $\hat{T}_n$  be the CKDE estimates constructed using (9) and a buffer  $B$  with  $n$  tuples. Then, under appropriate conditions, for each  $(s, a, s', a')$ , as  $n \rightarrow \infty$ ,

$$\hat{P}_n(s', a' | s, a) \xrightarrow{P} P_{\pi_D}(s', a' | s, a), \text{ and } \hat{T}_n(s' | s, a) \xrightarrow{P} T(s' | s, a). \quad (10)$$

**Theorem 2** Let  $\hat{P}_n$  and  $\hat{T}_n$  be the CKDE estimates constructed using (9). Then, under appropriate conditions, for each  $(s, a, s', a')$ , the following holds:

1. For  $\hat{T}_n(s' | s, a)$ ,

$$\sup_{T \in \Sigma(\beta, L)} \mathbb{P}(|\hat{T}_n(s' | s, a) - T(s' | s, a)| > \epsilon) < \delta$$

$$, \forall n > \log\left(\frac{2}{\delta}\right) \max \left[ \frac{C_1}{h_3^{m_3} h_2^{m_2}} \max \left( \frac{1}{\epsilon^2}, \frac{1}{(\epsilon - c_1(h_3^2 + h_2^2)^{\frac{\beta}{2}})} \right), \frac{C_2}{h_2^{m_2}} \max \left( \frac{1}{\epsilon^2}, \frac{1}{(\epsilon - c_2 h_2^\beta)} \right) \right]$$

2. For  $\hat{P}_n(s', a' | s, a)$ ,

$$\sup_{P \in \Sigma(\beta, L)} \mathbb{P}(|\hat{P}_n(s', a' | s, a) - P(s', a' | s, a)| > \epsilon) < \delta$$

$$, \forall n > \log\left(\frac{2}{\delta}\right) \max \left[ \frac{C_3}{h_1^{m_1} h_2^{m_2}} \max \left( \frac{1}{\epsilon^2}, \frac{1}{(\epsilon - c_3(h_1^2 + h_2^2)^{\frac{\beta}{2}})} \right), \frac{C_4}{h_2^{m_2}} \max \left( \frac{1}{\epsilon^2}, \frac{1}{(\epsilon - c_4 h_2^\beta)} \right) \right]$$

, where  $m_i$  is the order of diagonal matrix  $H_i, \forall i \in [3]$ .  $C_1, C_2, C_3, C_4 > 0$  and  $c_1, c_2, c_3, c_4$  are some constants.

We prove the Theorems 1 and 2 and outline the assumptions made therein in the Appendix A.

## 4. Experimental Results

**Experimental Setup.** We evaluate our algorithm’s performance by assessing its effectiveness across diverse benchmark environments sourced from OpenAI Gym (Brockman et al., 2016). These environments represent a wide spectrum of complexities commonly encountered in reinforcement learning. These include the MountainCar environment (Moore, 1990), CartPole (Barto et al., 1983), Acrobot (Sutton, 1995), and LunarLander (Klimov, 2019). To generate demonstration datasets  $D$ , we leverage pre-trained and hyperparameter-optimized agents available in the RL Baselines Zoo (Raffin, 2020). Specifically, we employ a PPO agent for LunarLander-v2, a DQN agent for CartPole-v1, and an A2C agent for Acrobot-v1.

**Baseline Algorithms.** We compare the performance of our CKIL algorithm (Algorithm 1), with a range of offline IRL/IL/AIL baselines, including several recent SOTA algorithms. These include Behavioral Cloning (BC), ValueDICE (VDICE) (Kostrikov et al., 2019b), reward-regularized classification (RCAL) (Piot et al., 2014), Energy-based Distribution Matching (EDM) (Jarrett et al., 2020a), AVRIL (Chan and van der Schaar, 2021b), and Deep Successor Feature Network (DSFN) (Lee et al., 2019b). Furthermore, we also compare against IQ-Learn (Garg et al., 2021a), a state-of-the-art model-free offline IRL algorithm. Please note that recent offline IRL/IL methods, like

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**Algorithm 1** Conditional Kernel Imitation Learning (CKIL)
 

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**Input:** Expert dataset of trajectories  $D = \{(s_i, a_i)\}_{i=1}^n$ 
**Output:**  $\theta^*$ 

- 1: Initialize policy parameter  $\theta$
- 2: Transform dataset  $D$  into  $(s, a, s', a')$  tuples, then store them in buffer  $B$ .
- 3: **for**  $iter = 0, 1, \dots$  **do**
- 4:   Sample a batch  $b_{iter}$  of  $(s, a, s', a')$  tuples from  $B$
- 5:   Obtain  $\hat{P}, \hat{T}$  in (9) via CKDE on  $b_{iter}$
- 6:   Calculate empirical estimate of the objective function in (7) using all  $(s, a, s', a') \in b_{iter}$  as:

$$\sum_{(s', a')} \sum_{(s, a)} [\hat{P}(s', a' | s, a) - \pi_{\theta}(a' | s') \hat{T}(s' | s, a)]^2 + \lambda \sum_{s'} \sum_{a'} \pi_{\theta}(a' | s') \log(\pi_{\theta}(a' | s')) \quad (11)$$

- 7:   Update the policy parameter  $\theta$  using gradient update to minimize the calculated empirical estimate of the objective function
  - 8: **end for**
  - 9: **return**  $\theta^*$
- 

CLARE (Yue et al., 2023), have emerged in the literature. Some of these methods require additional diverse data alongside expert data, while others permit interaction with the environment, with "offline" meaning that these methods cannot query the expert. However, our work strictly assumes access to only expert data without further interactions or access to additional diverse data. Hence, we exclude these algorithms from comparison to maintain fairness. More detailed discussion can be found in Appendix C.

**Implementation.** The policy  $\pi_{\theta}$  in (11) is embodied by a neural network (NN) architecture. This NN comprises two hidden layers featuring the Rectified Linear Unit (ReLU) activation function. The final layer employs a softmax function to produce a probability distribution over actions when given a state as an input. To facilitate comparison, all benchmarks adopt a common neural network architecture consisting of two hidden layers comprising 64 units each, with Exponential Linear Unit (ELU) activation functions. Training is carried out using the Adam optimizer (Kingma and Ba, 2015) with individually tuned learning rates. The implementation details, including hyperparameters of CKIL and benchmark algorithms, can be found in Appendix B and Appendix C, respectively.

**Choice of Kernel.** We use the Gaussian kernel due to its ability to uniformly approximate any continuous target function on a compact subset (Micchelli et al., 2006) which is particularly helpful in density estimation. We consider a Euclidean distance metric for  $d_1$ ,  $d_2$ , and  $d_3$  and utilize a diagonal bandwidth matrix with the same values across its diagonal elements. These matrices can then be denoted as  $H_i = h_i I_{m_i}$ , where  $m_i$  is the corresponding appropriate dimension for  $i = 1, 2, 3$ . Each  $h_i$  was tuned slightly but the conclusions of our experimental results are not very sensitive to this choice. We would like to emphasize that in prior research (Mammen et al., 2011; Schölkopf and Smola, 2002; Silverman, 1986; Ahmad and Ran, 2004), approaches for systematic selection of bandwidth parameters, which should decrease as the dataset size grows, have been developed. These methods can be applied to more intricate problems where manual tuning is impractical. Specific values of  $h_i$  employed for various experiments are detailed in Appendix B.



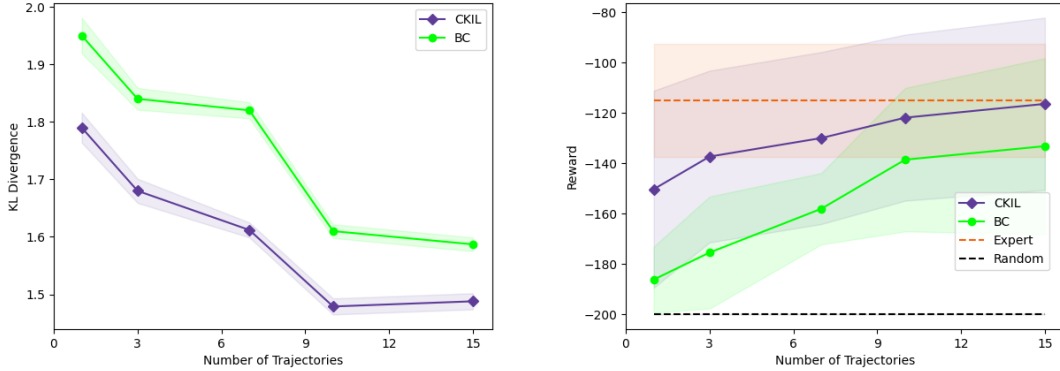


Figure 1: Plots for CKIL agent in a discretized MountainCar environment for a varying number of trajectories (a) Empirical KL divergence (lower values indicate better performance) (b) Average rewards attained (higher values indicate better performance).

## Results

**Discrete States and Actions.** While our focus is primarily on continuous state-space settings requiring conditional density estimation, we first illustrate the effectiveness of our general approach in a discrete state-space problem. We compare CKIL’s performance against Behavior Cloning and a random action agent using the discretized MountainCar environment. The 2-dimensional state space is discretized into a  $15 \times 15$  grid, and  $\hat{P}$  and  $\hat{T}$  are estimated using equation (8). The dataset  $D$  is generated with an  $\epsilon$ -perturbation of the policy in (Xiao, 2019), with  $\epsilon = 0.05$ . Starting with one trajectory in  $D$  and increasing to 15, Figure 1 summarizes the results. The KL-divergence plot shows increasing alignment between agent and expert policies as dataset size grows. With 15 trajectories, CKIL achieves expert-level proficiency, consistently outperforming Behavioral Cloning.

**Continuous States.** We next address our main goal, namely imitation learning for continuous state environments. When provided ample demonstration data, all benchmarks exhibit the capability to attain performance comparable to the optimized demonstration agents in each environment. Thus, we evaluate the algorithms’ capacity to manage sample complexity in scenarios with limited data. In particular, we assess the change in performance of each algorithm as the size of the expert dataset  $D$  ranges from a single trajectory to a collection of 15 trajectories. This setup mirrors the configuration described in (Jarrett et al., 2020a). The average trajectory lengths in the expert data are 80 for Acrobot, 500 for CartPole, and 300 for LunarLander. The algorithms were trained to convergence on datasets of 1, 3, 7, 10, or 15 trajectories, each uniformly drawn from 1000 expert trajectories. The trained policies were deployed in each environment, and average scores over 300 episodes, serving as standard benchmarks for IL comparisons, were recorded for each algorithm. This process was repeated 10 times with diverse initialization and observed trajectories in each iteration.

Figure 2 compares the average rewards achieved by all algorithms as the demonstration dataset size increased in the Acrobot, CartPole, and LunarLander environments. Across all tasks, the results showcase CKIL’s capability to learn effective policies, manifesting robust and consistently superior performance compared to the baselines considered, especially when data is scarce. We observe that CKIL attains expert-level performance in three environments of increasing difficulty, the CartPole, the Acrobot, and the LunarLander, within use of three trajectories. Remarkably, it is

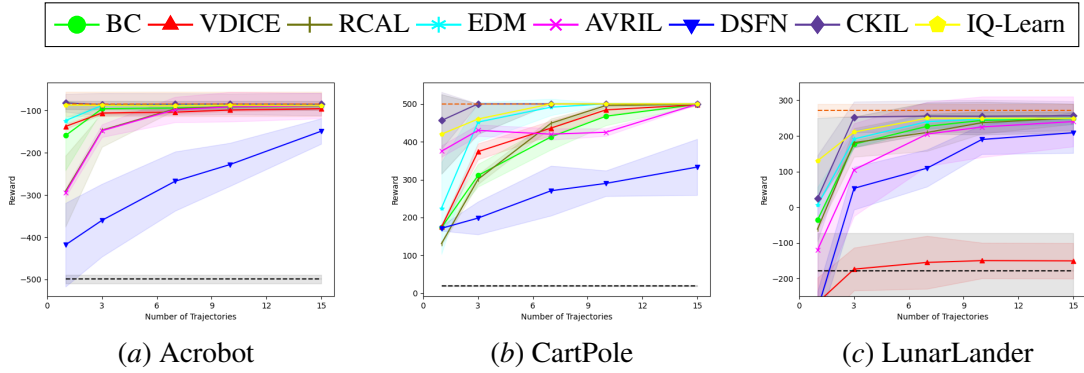


Figure 2: Average rewards of benchmark IRL/IL/AIL and CKIL policies during real-time deployment, plotted against the number of trajectories in dataset  $D$  (higher values indicate better performance). Expert and random agents are represented as --- and ---, respectively.

able to approach expert-level performance on the CartPole environment by use of a single trajectory. Also note that CKIL’s performance is almost uniformly better than that of the various baseline algorithms across the three environments. Among these baselines, the IQ-Learn algorithm demonstrates performance nearly comparable to that of CKIL. However, IQ-Learn, being an IRL algorithm, has significantly higher computational overhead than CKIL. Moreover, instabilities of the IQ-Learn approach is discussed in (Al-Hafez et al., 2023). It is notable that the off-policy adaptations of online algorithms (VDICE, DSFN) do not exhibit the same degree of consistent performance as their inherently offline counterparts. This highlights the inadequacy of solely adopting online algorithms in offline scenarios. The challenges in estimating the expectation of an exponential distribution may contribute to VDICE’s underperformance. Moreover, ablation study is done in Appendix D.

The results suggest that combining a loss function based on the Markov balance equation, conditional kernel density estimation, and an entropy regularizer enables effective imitation learning in continuous state problems. Additionally, CKIL’s design requires minimal hyperparameter tuning, unlike other model-based IL methods. CKIL also demonstrates superior computational efficiency, with training times significantly lower than other baselines. For example, CKIL converges in just a couple of minutes, while EDM and IQ-Learn require about half an hour.

## 5. Conclusions

In this paper, we introduced Conditional Kernel Imitation Learning (CKIL), a simple yet novel approach to imitation learning for continuous state-space problems. CKIL avoids reward modeling by leveraging the Markov balance equation and conditional kernel density estimators for transition densities of both the MDP and induced Markov chain. The algorithm is supported by theoretical consistency and sample complexity analysis, demonstrating strong empirical performance compared to SOTA offline IL, IRL, and AIL methods. Unlike some imitation learning algorithms, CKIL does not require access to a generative model or additional datasets. While imitation learning with limited training data often faces distribution shift challenges, our results suggest CKIL handles this issue more effectively than other strictly batch IL algorithms. Further details on this can be found in Appendix E. Future work may explore using scalable density estimation techniques, such as normalizing flows.

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