ECE 223: Computer Systems Engineering **Spring 2014**
Machine Problem 6 **Due: 11:59 pm, Sunday, April 6**

The goal of this machine problem is to implement a module for Dijkstra's algorithm, and to apply the module to solve a variety of shortest path graph problems.

Use a layered design to develop a clean interface to a module that implements Dijkstra's algorithm. You must design and document your interface, clearly defining the required inputs, data structures, and output for the module. You must also design test drivers and implement the user interface as described below. In addition to submitting all source code you must also submit a `makefile` that compiles all code and creates an executable with the name "lab6". Your code must have no compiler warnings and no memory leaks or other `valgrind` errors.

Two additional documents must be submitted. One is a **test plan** that describes details of your implementation and demonstrates, with a test script, how you verified that the code works correctly. The verification should include detailed prints from your program to show that you program operates correctly. The second document describes your **performance evaluation**, and the details are described below.

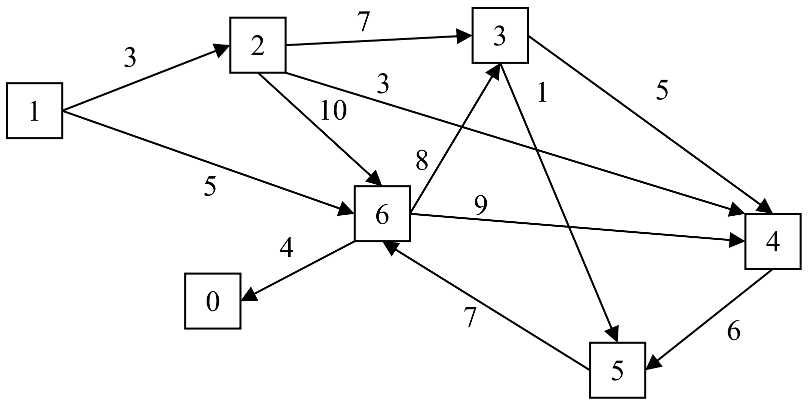
Interface specifications

Command line arguments must be used to set run-time parameters. Use the `getopt` function to implement processing of the command line arguments. See MP4 and MP5 for examples using `getopt`.

-g [1 | 2 | 3]	Graph type
-n *N*	Number of vertices in graph
-a *A*	Approximate number of adjacent vertices $0 < A < N$
-h [1 | 2]	Graph operation
-s *S*	Number of the source vertex $0 \leq S < N$
-d *D*	Number of the destination vertex $0 \leq D < N$
-v	enable verbose output
-r 1234567	seed for the uniform random number generator

Graphs Types (-g [1 | 2 | 3])

1. Use the **weakly-connected directed graph** shown in the figure below. The –n and –a options are not used.



2. **Strongly-connected directed graph with *N* vertices**. The vertices are numbered 0 through *N* – 1, where *N* is set by the command line with the –n option. The weight of the edge from vertex *i* to vertex *j* is

$$w_{i,j} = \begin{cases} |i-j| + (i-j+2)^2 + 3j & i \neq j \\ 0 & i = j \end{cases}.$$

Notice that the edges in this graph are not symmetric, and there is an edge between every pair of vertices.

3. **Random graphs**, with *N* vertices and approximate number of adjacent neighbors *A* (–n *N* –a *A*). The vertices are numbered 0 through *N* – 1.
   - Vertex 0 is at location (0.0, 0.0)
   - Vertex *N* – 1 is at location (1.0, 1.0)
   - All other vertices have a random (*x*, *y*) location, uniformly located in a square of size 1. That is, for each vertex *v* set *v.x* = `drand48()` and *v.y* = `drand48()`.
   - The distance between vertex i and vertex j is $D(i,j) = \sqrt{(i.x - j.x)^2 + (i.y - j.y)^2}$
   - For a vertex *v*, the average number of adjacent vertices in a large random graph is approximately $A = N\pi C^2$, where C is the radius of a circle centered at *v* and a vertex must be located in the circle to be a neighbor. Since we are specifying *A*, the radius (or normalized communication range), *C*, is

$$C = \sqrt{\frac{A}{N\pi}}$$

   - The weight of the edge from vertex *i* to vertex *j* is

$$w_{i,j} = \begin{cases} 0, & i = j \\ 1, & D(i,j) \leq \frac{C}{2} \\ 2, & \frac{C}{2} < D(i,j) \leq \frac{3C}{4} \\ 4, & \frac{3C}{4} < D(i,j) \leq C \\ \infty, & \text{otherwise} \end{cases}$$

Notice this graph is undirected ($w_{i,j} = w_{j,i}$), but the graph may not be connected. Generally, for moderately large graphs there is a chance that a path between each pair of vertices cannot be found if the average number of neighbors, *A*, is smaller than approximately 10. With high probability the graph is connected if *A* is larger than approximately 20.

For random graphs you **must** print the following information after the graph is built: The average, maximum, and minimum number of adjacent vertices taken over all *N* vertices.

Graph Operations (-h [1 | 2])

1. **Shortest path**. Given input parameters -s *S* and -d *D*, find and print the cost of the shortest path from *S* to *D* and the vertices in the path in order from the source to the destination. If there does not exist a path from the source to the destination, print a message instead of the path.

For example, if the parameters at –g 1 –h 1 –s 1 –d 5, your program should print that the cost of the path is 11 and the path is **`1 -- 2 -- 3 -- 5`**. If the parameters are –g 1 –h 1 –s 5 –d 1, your program should print that a path does not exist between these vertices. For –g 2 –n 100 –h 1 –s 0 –d 99 the cost is 2,172 and the path you print contains 11 vertices (there are multiple paths with the same cost, and the path you find depends on how ties are broken). Your implementation of Dijkstra's algorithm must achieve performance that is no worse than $O(n^2)$ where *n* is the number of vertices. If you choose, you can make an implementation that can achieve $O(n \log n)$ for sparse graphs if you use adjacency lists and a priority queue with a heap representation, however, the more efficient operation is not required.

2. **Network diameter**. If the graph is connected, find the source-destination pair (*s*, *d*) for which the cost of the shortest path from *s* to *d* has the largest value among all possible values for *s* and *d*. If the graph is not connected, then the output should print the value for the source-destination pair that is connected and has the largest cost, and must print that the graph is not connected. The –s and –d command line parameters are not used. For example, for –g 1 –h 2, the diameter is 21 for the path `4 -- 5 -- 6 -- 3`, and the graph is not connected. You must design an algorithm that is no worse than $O(n^3)$ where *n* is the number of vertices.

Performance Evaluation

1. Computational complexity verification for Dijkstra'a algorithm. As we discussed in class, Dijkstra's algorithm requires time that is $O(n^2)$ in general, where *n* is the number of vertices. However, if a graph is sparse (i.e., the average number of neighbors is bounded and does not depend on the size of the graph), then an implementation of Dijkstra's algorithm using adjacency lists and a priority queue with a heap representation can achieve $O(n \log n)$.
   a. For graph type two (strongly-connected and directed) show that your implementation is $O(n^2)$. This graph is dense and represents a worst-case scenario because there is an edge between each pair of vertices. One way to demonstrate this is find a value for the number of vertices, *N*, that is large enough so that the run time is greater than one second for the options -g 2 -h 1 -n *N* -s 0 -d *N*-1. Measure the time to run Dijkstra's algorithm using the `clock()` function (only measure the time Dijkstra's algorithm requires and do not include the time to build the graph). Run the same experiment but with the number of vertices as *N*/2. Show that the ratio of the two run times is approximately 4. (Why does this verify that the performance is $O(n^2)$?)
   b. Repeat the experiment for graph type three (random) with the input for the number of adjacent vertices equal to 20 (i.e., -a 20). Demonstrate if your implementation is $O(n^2)$ or $O(n \log n)$, and explain why.
2. Computational complexity verification for finding the network diameter. Using graph type three (random) with -a 20, demonstrate that your implementation is either is $O(n^3)$ or $O(n^2 \log n)$, and explain why.
3. Node density for random graphs. For the types of random graphs generated with the –g 3 option, there is a relatively narrow range for the value of A (the approximate number of adjacent vertices) such that the graph changes from a low probability of being connected to a high probability. This range does not depend on the number of vertices. The graph operation –h 2 (network diameter) is one way to determine if a graph is connected. Consider random graphs with -a 7 and show that with the -g 3 -h 2 -n 100 -r seed -a 7 that the graph is usually not connected (try with some different seeds for the random number generator). In a second experiment, show that if you take -a 20, then it is very rare that the graph is not connected (again show the output for some different trials with different values for the seed).

Notes

1. The system file #include <limits.h> has definitions for maximum and minimum numbers. Be very careful if you use INFINITY. This symbol is defined in <math.h> and if used in a calculation can "overflow at translation time". Be very careful if you use a negative number as infinity. Searches for the largest or smallest value become very confusing.
2. You must choose the approach for representing a graph, using either an adjacency matrix or adjacency list. If you use a matrix, see http://c-faq.com/aryptr/dynmuldimary.html to review how to allocate two-dimensional arrays. For example:

   ```
   int **array = (int **) malloc(NumVertices * sizeof(int *));
   for(i = 0; i < NumVertices; i++)
       array[i] = (int *) malloc(NumVertices * sizeof(int));
   ```

   Gives `array[x][y]` for `0 <= x < NumVertices` and the same limits for `y`
3. Begin with a top-down design. Once you have a design then you start with bottom-up programming (never write more than ~10 lines of code before testing!). For example, you might decide to make a graph module that includes:

   ```
   graph_t * graph_construct(NumVertices)
   graph_destruct(G)
   ```

   At least two operations seems likely:
   ```
   graph_add_edge(G, link_src, link_dest, link_weight)
   graph_shortest_path(G, path_src)
   ```

   Every program we have done has had a debugging function to print internal details. So add
   ```
   graph_debug_print(G)
   ```

   Once you decide on a design for your module you might have something like:

   ```
   G = graph_construct(NumVertices)

   if GraphType is 1
       graph_add_edge(G, 1, 2, 3);
       graph_add_edge(G, 1, 6, 5);
       graph_add_edge(G, 2, 3, 7);
       graph_add_edge(G, 2, 4, 3);
       graph_add_edge(G, 2, 6, 10);
       graph_add_edge(G, 3, 4, 5);
       graph_add_edge(G, 3, 5, 1);
       graph_add_edge(G, 4, 5, 6);
       graph_add_edge(G, 5, 6, 7);
       graph_add_edge(G, 6, 0, 4);
       graph_add_edge(G, 6, 3, 8);
       graph_add_edge(G, 6, 4, 9);

   else if GraphTYpe is 2
     for(link_src = 0 ; link_src < NumVertices; link_src++)
       for(link_dest = 0 ; link_dest < NumVertices; link_dest++) {
         if (link_src == link_dest) continue;
         graph_add_edge(G, link_src, link_dest, weight(link_src, link_dest));
       }
   ```

See the ECE 223 Programming Guide for additional requirements that apply to all programming assignments. All code and documentation files must be turned in by email to assign@assign.ece.clemson.edu. Use as subject header ECE223-1,#6 . **Work must be completed by each individual student, and see the course syllabus for additional policies**.