Classical definition of Probability

If a random experiment (E) or trial results in 'n' exhaustive, mutually exclusive and equally likely outcomes, out of which 'm' are favourable to the occurrence of an event A, the the probability 'p' of the occurrence of A, usually denoted by P(A) is given by

$$p = P(A) = \frac{\text{no. of outcomes favourable to 'A'}}{\text{Total no. of outcomes}}$$

$$= \frac{n(A)}{n(E)}$$

$$= \frac{m}{n}.$$

Note! -

(i) $n(E) = n$ must be finite in number

(ii) $0 \leq P(A) \leq 1$.

(iii) $P(E) = 1$

(iv) $P(\bar{A}) = 1 - P(A)$.

Criticisms of the Classical definition of Probability

If we look into the classical definition of probability a little more closely we find that there is a logical drawback in the definition. We note that the definition can be used only if it is possible to ascertain that all the simple events are equally likely. In many problems considerations of symmetry and similarity enable us to decide whether, in the problem, simple events are equally likely. For example, if a die be symmetric, then the simple events connected to the random experiment of throwing the die may be considered to be equally likely. But it is very difficult to explain the nature of 'symmetry' and 'similarity' as stated above. It was found after many serious investigations that the phrase 'equally likely' cannot be explained without the prior idea of probability.

Moreover, the definition is restricted to the event spaces which are finite and where all the simple events are equally likely.

Hence, the classical definition of probability cannot be applied where the simple events are not equally likely or where the event space is infinite. Hence, using this definition it is impossible to treat the case of a

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biased (loaded) die, since here intutively we can ~~except~~ expect that a face can turn up in preference to others and concequently simple events <u>are not necessarily equally likely</u>. the case of predicting the number of telephone calls in a given interval of time is also consisting of infinite number of simple events.

In order to avoid the limitations of the classical definition of probability and to make the definition more widely applicable, we now define another definition, called the <u>frequency definition of probability</u>:

Frequency definition of Probability :-

Let A be an event of a given random experiment E. Let the event A occur $N(A)$ times when the random experiment E is repeated N times under identical conditions. Then on the basis of statistical regularity we can assume that $\lim_{N\to\infty} \frac{N(A)}{N}$ exists finitely and the value of this limit is called the probability of the event A, denoted by $P(A)$.

i.e., $$P(A) = \lim_{N\to\infty} \frac{N(A)}{N} = \lim_{N\to\infty} f(A).$$

where $f(A) = \frac{N(A)}{N}$ is the frequency ratio of the event A in N repetitions of the corresponding random experiment under identical conditions.

Deductions :-

(a) $0 \leq P(A) \leq 1$, for any event A

(b) $P(S) = 1$, S is the sample space related to the random experiment E.

(c) $P(\Phi) = 0$

(d) $P(\bar{A}) = 1 - P(A)$.

Proof :- (a) We have $0 \leq N(A) \leq N$

where N and N(A) are given in frequency definition of probability.

$$\therefore \quad 0 \leq \frac{N(A)}{N} \leq 1$$

$$\Rightarrow \quad 0 \leq \lim_{N \to \infty} \frac{N(A)}{N} \leq 1.$$

Hence, $0 \leq P(A) \leq 1$.

(b) $$P(S) = \lim_{N \to \infty} \frac{N(S)}{N} = \lim_{N \to \infty} \frac{N}{N} = 1.$$

(c) $$P(\Phi) = \lim_{N \to \infty} \frac{N(\Phi)}{N} = \lim_{N \to \infty} \frac{0}{N} = 0.$$

(d) $$P(\bar{A}) = \lim_{N \to \infty} \frac{N(\bar{A})}{N} = \lim_{N \to \infty} \left[\frac{N - N(A)}{N}\right]$$

$$= \lim_{N \to \infty} \left[1 - \frac{N(A)}{N}\right] = 1 - P(A).$$

Hence, proved.

Axiomatic Definition of probability

Let E be a given random experiment and S be the corresponding event space. Also let Δ be the class of subsets of S forming the class of events of E.

A mapping $P : \Delta \to R$ is called a probability function defined on Δ and the unique real number $P(A)$ determined by P is called the probability of the event A where $A \in \Delta$ if the following axioms, known as axiom of probability, are satisfied.

Axiom (a) : $P(A) \geq 0$ for every event $A \in \Delta$.

Axiom (b) : $P(S) = 1$

Axiom (c) : If $A_1, A_2, \ldots, A_n, \ldots$ be countably infinite number of pairwise mutually exclusive events, i.e., if $A_i \cap A_j = \Phi$ whenever $i \neq j$ and $A_i, A_j \in \Delta$.

then $P(A_1 + A_2 + \cdots + A_n + \cdots) = P(A_1) + P(A_2) + \cdots + P(A_n) + \cdots$

$$\Rightarrow P\left(\sum_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i).$$

The ordered 3-tuple (S, Δ, P) is called a probability space.