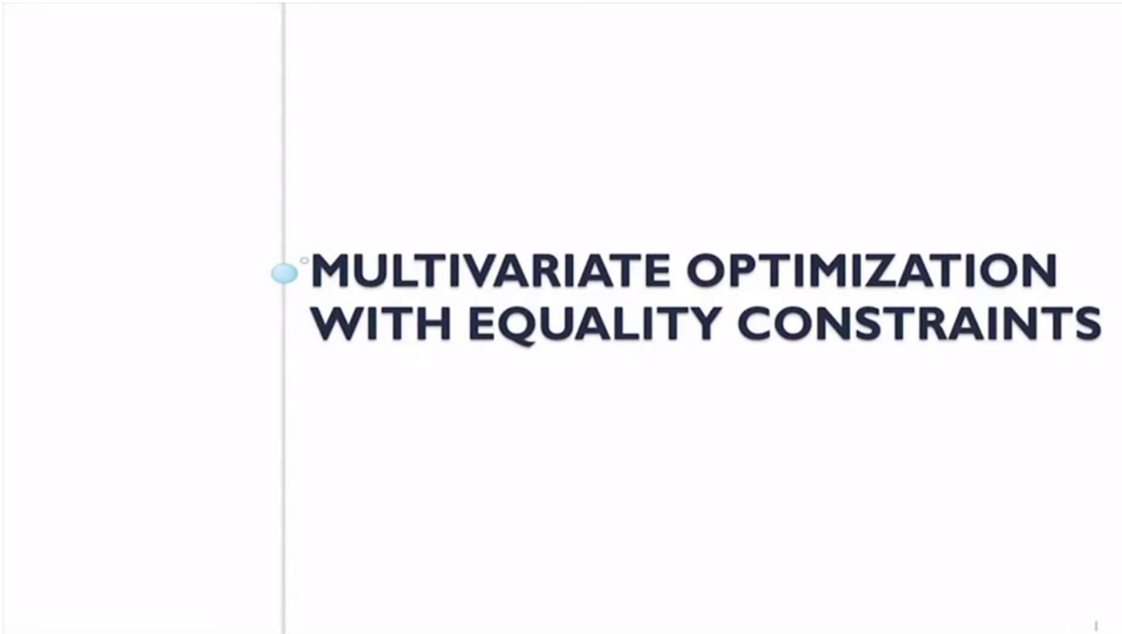
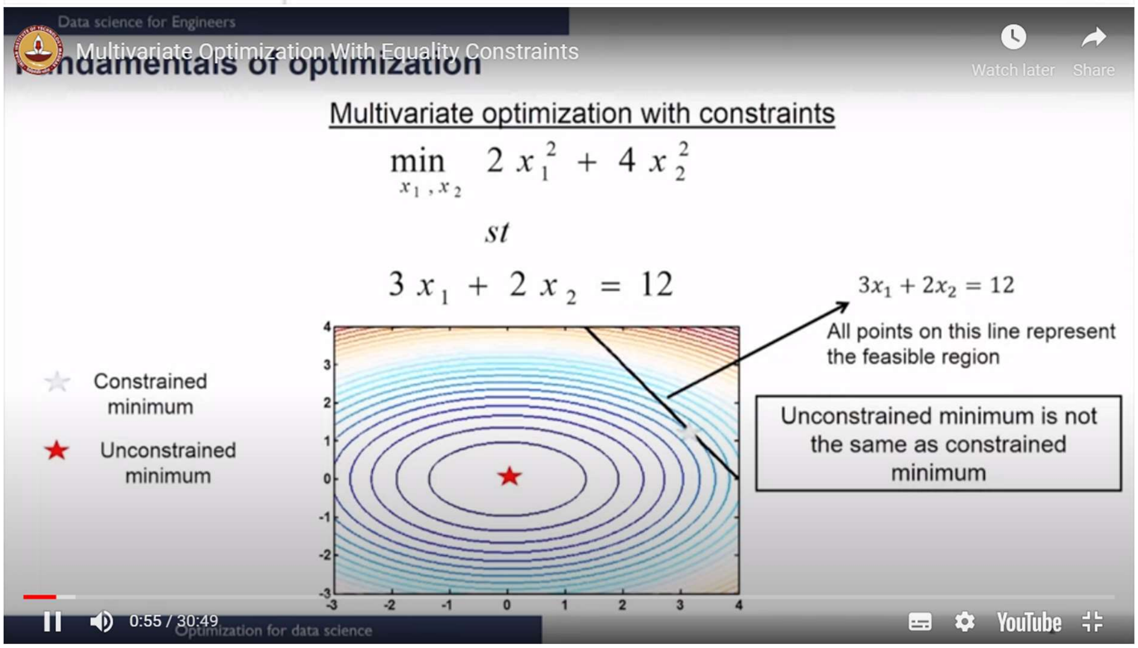

MULTIVARIATE OPTIMIZATION
WITH EQUALITY CONSTRAINTS


Data science for Engineers
Multivariate Optimization With Equality Constraints
Fundamentals of optimization
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Multivariate optimization with constraints
$\min_{x_1, x_2} \; 2x_1^2 + 4x_2^2$
st
$3x_1 + 2x_2 = 12$
$3x_1 + 2x_2 = 12$
All points on this line represent the feasible region
Unconstrained minimum is not the same as constrained minimum
Constrained minimum
Unconstrained minimum
0:55 / 30:49
Optimization for data science

Fundamentals of optimization

Multivariate optimization with constraints

$2x_1^2 + 4x_2^2 = k$

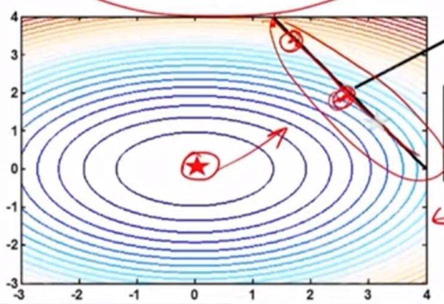
$$\min_{x_1, x_2} \; 2x_1^2 + 4x_2^2$$

$$st$$

$$3x_1 + 2x_2 = 12$$

Constrained minimum

Unconstrained minimum



$3x_1 + 2x_2 = 12$

All points on this line represent the feasible region

Unconstrained minimum is not the same as constrained minimum

Fundamentals of optimization

Multivariate optimization with equality constraints

Min $f(x_1, x_2, \ldots x_n)$

$h(x_1, x_2, \ldots x_n) = 0$

$f(x)$

$f(x_1, x_2, \ldots x_n)$

At optimum (one equality constraint case)

$-\nabla f = \lambda \nabla h$

$$-\nabla f(\overline{x^*}) = \lambda^* \nabla h(\overline{x^*})$$

$-\frac{\partial f}{\partial x_1} = \lambda \frac{\partial h}{\partial x_1}$

$-\frac{\partial f}{\partial x_2} = \lambda \frac{\partial h}{\partial x_2}$

$-\frac{\partial f}{\partial x_n} = \lambda \frac{\partial h}{\partial x_n}$

$h(x_1, x_2, \ldots x_n) = 0$

In higher dimensions and when there are more than one equality constraint

$$-\nabla f(\overline{x^*}) = \sum_{i=1}^{l} [\nabla h_i(\overline{x^*})] \lambda_i^*$$

Gradient lies in the space spanned by the normal of the gradients

Fundamentals of optimization

Multivariate optimization with equality constraints

$\min f(x_1, x_2, \ldots x_n)$

$h(x_1, x_2, \ldots x_n) = 0$

At optimum (one equality constraint case)

$$-\nabla f(\overline{x^*}) = \lambda^* \nabla h(\overline{x^*})$$

$-\nabla f = \lambda \nabla h$

In higher dimensions and when there are more than one equality constraint

$$-\nabla f(\overline{x^*}) = \sum_{i=1}^{l} [\nabla h_i(\overline{x^*})]\, \lambda_i^*$$

Gradient lies in the space spanned by the normal of the gradients

$-\nabla f = \lambda \nabla h$

$f(x)$

$h_1(x) = 0$

$h_2(x) = 0$

$-\nabla f = \lambda_1 \nabla h_1 + \lambda_2 \nabla h_2$

$h_1(x) = 0 ;\ h_2(x) = 0$

Fundamentals of optimization

Multivariate optimization with constraints

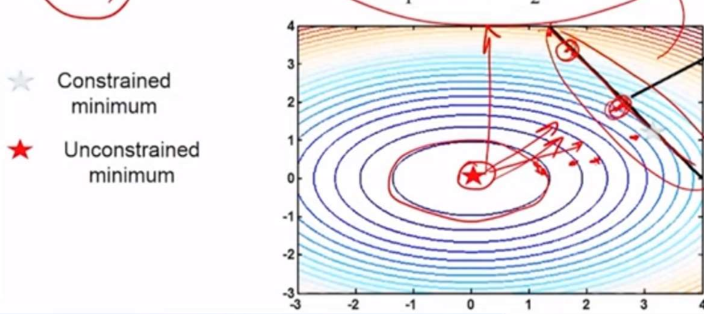
$-\nabla f = \lambda_1 \nabla h_1 + \lambda_2 \nabla h_2$

$$\min_{x_1, x_2} \; 2x_1^2 + 4x_2^2$$

$2x_1^2 + 4x_2^2 = K$

$$st$$

$$3x_1 + 2x_2 = 12$$



$3x_1 + 2x_2 = 12$

All points on this line represent the feasible region

Unconstrained minimum is not the same as constrained minimum

$f(x) = K$

$-\nabla f = \lambda \nabla h$

Fundamentals of optimization

Multivariate optimization

$$\min_{x_1, x_2} \; 2x_1^2 + 4x_2^2$$

$$st$$

$$3x_1 + 2x_2 - 12 = 0$$

$-\nabla f = \lambda \nabla h$

$h(x) = 0$

$-\frac{\partial f}{\partial x_1} = \lambda \frac{\partial h}{\partial x_1}$

$-4x_1 = 3\lambda$

$-8x_2 = 2\lambda$

$-\nabla f = -\begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{pmatrix} = \begin{pmatrix} -4x_1 \\ -8x_2 \end{pmatrix}$

$\begin{pmatrix} \frac{\partial h}{\partial x_1} \\ \frac{\partial h}{\partial x_2} \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

First order condition

$$\begin{aligned} -4x_1 &= 3\lambda \\ -8x_2 &= 2\lambda \\ (3x_1 + 2x_2 - 12) &= 0 \end{aligned} \quad \xrightarrow{\text{solving}} \quad \begin{bmatrix} x_1^* \\ x_2^* \\ \lambda^* \end{bmatrix} = \begin{bmatrix} 3.27 \\ 1.09 \\ -4.36 \end{bmatrix}$$