Module 4 | Topic 1 | Video 1

Introduction to Inferential Statistics and Hypothesis Testing

Descriptive Vs Inferential Statistics

Descriptive Statistics	Vs	Inferential Statistics
Objective is to organize, summarize and describe the given data		Objective is to make inference from the sample and make generalization about the population
Common tools used are 1. Visualization such as bar charts, line charts, box plots etc. 2. Statistical summary measures such as mean, median, mode, standard deviation, variance, etc.		Common tools used are 1. Probability distribution 2. Hypothesis testing, ANOVA etc.

Statistical Hypothesis

- A statement about the population parameter. It may be true or may not be true.
- To verify the truthfulness of this statement a hypothesis testing is done.
- A statistical hypothesis is an assertion concerning one or more populations
- To prove that a hypothesis is true, or false, with absolute certainty, we need absolute knowledge. i.e. we need to know fully about the population.
- Instead hypothesis testing concerns on how to use a random sample to judge if it is evidence that supports or not the hypothesis.

Hypothesis Testing

Null and Alternative hypothesis:

- Null hypothesis is status quo. It is based on the known old information.
- Alternative hypothesis is what is being ascertained. It is the assertion.

Ho : Earth is the center of the universe

Ha : Sun is the center of the universe

In a hypothesis test, we do not prove the alternative hypothesis, but instead we strive to reject the null hypothesis.

When we fail to reject the null hypothesis, then the status quo prevails.

You have been focusing a lot on your fitness for the last 5 years and have run a few marathons in the past.

You are getting trained to run a marathon in a year and you note that you have averaged 4 hour 30 minutes with a standard deviation of 24 minutes.

You want to improve your marathon performance by at least half hour. So you have enrolled a personal fitness program to improve your stamina to run a marathon.

What are the null and alternative hypotheses?

Ho : Finish the marathon in 4 hour 30 minutes

Ha : Finish the marathon in less than 4 hours

Implication: gather data to reject the null hypothesis

FreshCo process analyst asserts that after running a pilot, the average order to delivery time is now 9 hours.

What are the null and alternative hypotheses?

Ho : mean order to delivery time is same as the status quo = 10 hours

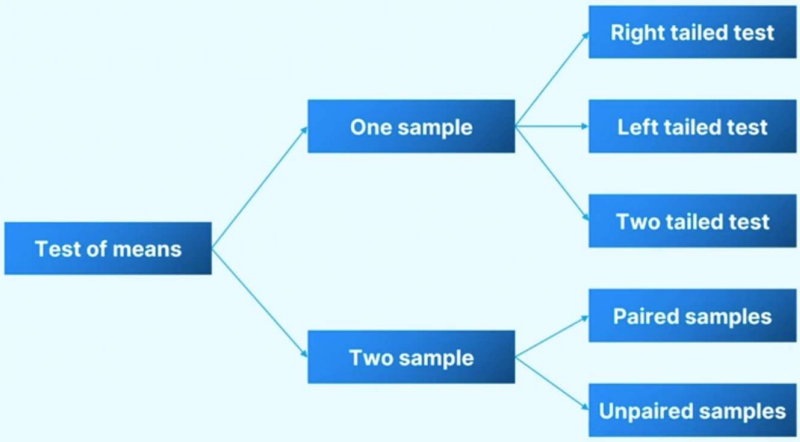
Ha : mean order to delivery time has reduced to 9 hours

Implication: gather data to reject the null hypothesis

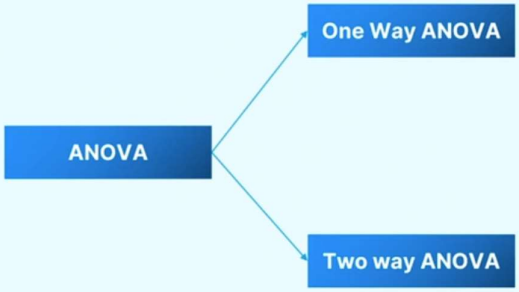
Types of Hypothesis Testing

- Test of Means
- Analysis of Variance
- Test of Independence
- Test of median
- Test of Normality
- Test of Stationarity

Test of Means



Analysis of Variance



Module 4 | Topic 1 | Video 2

Introduction to Test of Means

Learning Objective

- Test of Means
  - Test of Means Intuition
  - Defining hypothesis
  - Significance and Acceptance Criteria
- Types of Test of Means
  - One sample test: Left, Right, Two tail tests
  - Two sample test: Paired Vs Unpaired tests

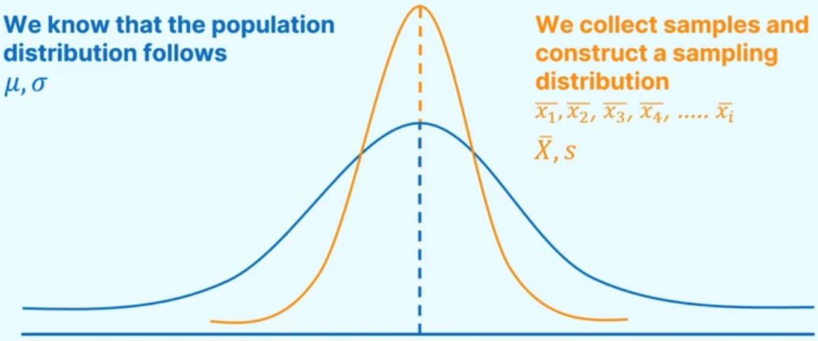
In Test of means, we observe the samples and infer something about the population mean

The true population mean is unknown

Central Limit Theorem

We know that the population distribution follows
μ, σ

We collect samples and construct a sampling distribution
$\overline{x_1}, \overline{x_2}, \overline{x_3}, \overline{x_4}, \ldots\ldots \overline{x_i}$
$\bar{X}, s$

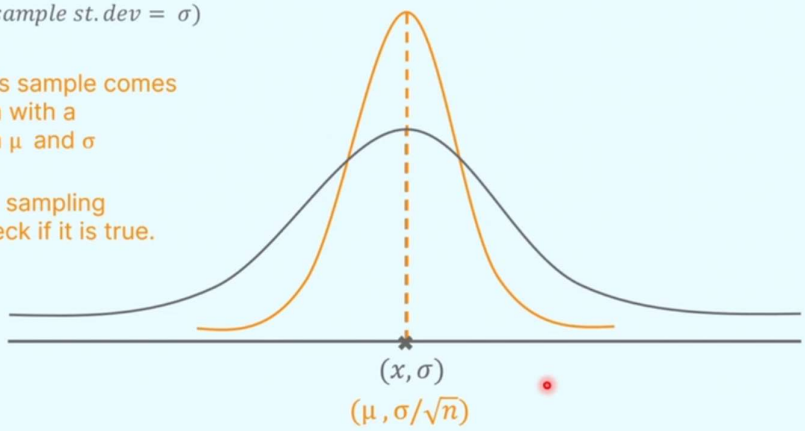


And we observe that the population and sampling distribution are related
$\boldsymbol{\mu = \bar{X}, s = \sigma/\sqrt{n}}$

Since we want to know about the population we collect n samples following a distribution,
$(sample\ mean = x, sample\ st.dev = \sigma)$

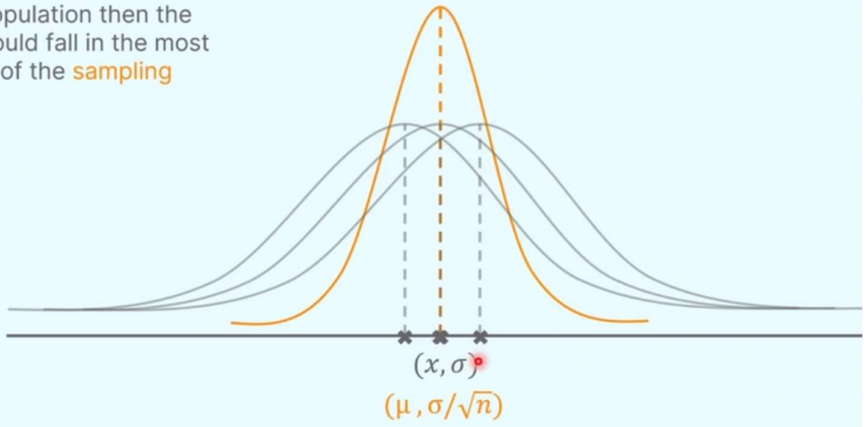
We assume that this sample comes from the population with a hypothesized mean μ and σ

We can construct a sampling distribution and check if it is true.



Due the randomness in sampling, several copies of samples is possible

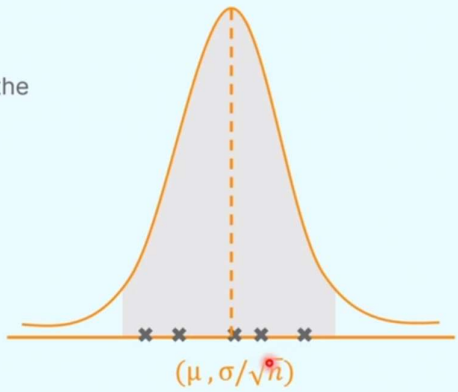
If the sample truly comes from the hypothesized population then the sample mean would fall in the most probable region of the sampling distribution.



If the sample mean falls in the most probable region of the hypothesized sampling distribution

Then the sample mean and the population means are same

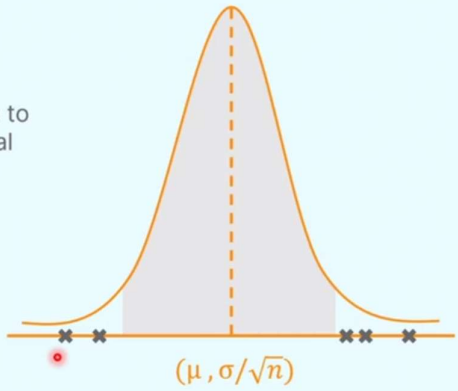
The observed difference is due to the randomness in sampling



However, if the sample mean falls in the least probable regions of the hypothesized sampling distribution i.e. the tails

Then the sample mean and the population means are different

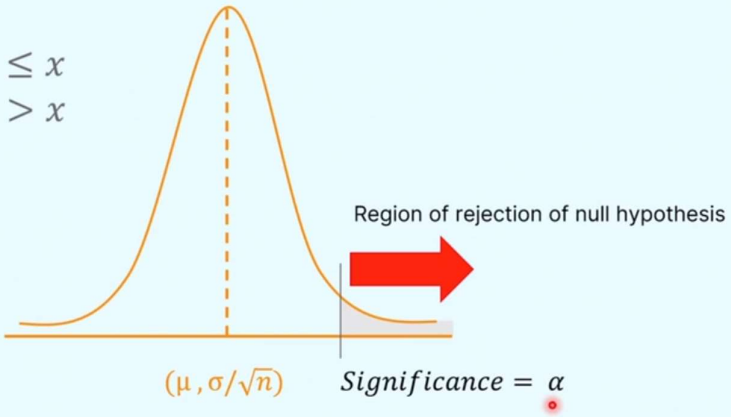
The observed difference is not due to the randomness in sampling but real



Population mean μ > hypothesized value

>> This is a right tail test

$$Ho: \mu \leq x$$
$$Ha: \mu > x$$

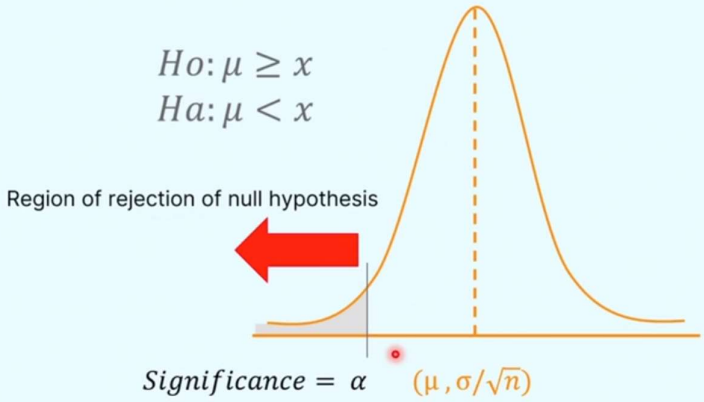


Population mean μ < hypothesized value

>> This is a left tail test

$Ho: \mu \geq x$

$Ha: \mu < x$

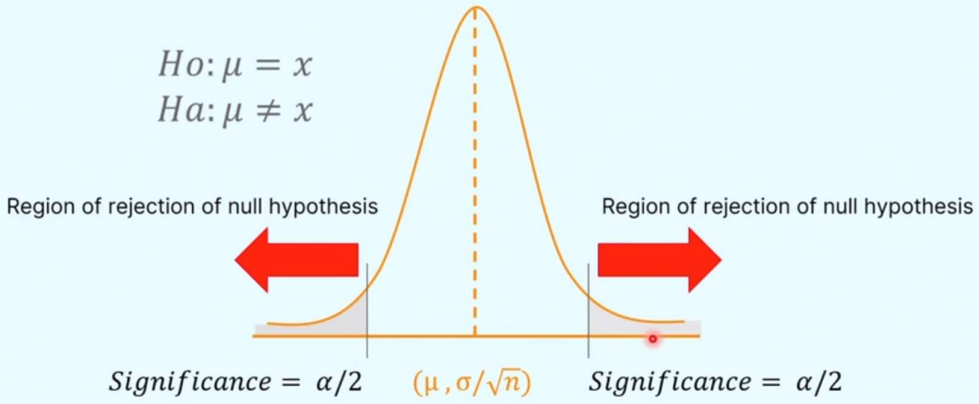


Population mean μ ≠ hypothesized value

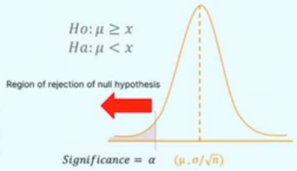
>> This is a two tail test

$Ho: \mu = x$

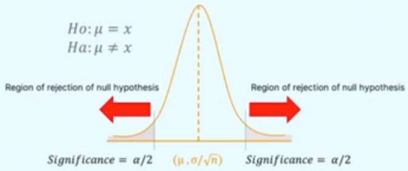
$Ha: \mu \neq x$



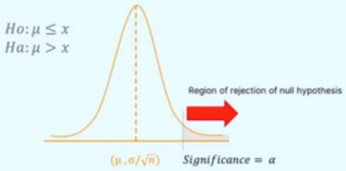
Population mean μ < hypothesized value
>> This is a left tail test



Population mean μ = hypothesized value
>> This is a two tail test



Population mean μ > hypothesized value
>> This is a right tail test

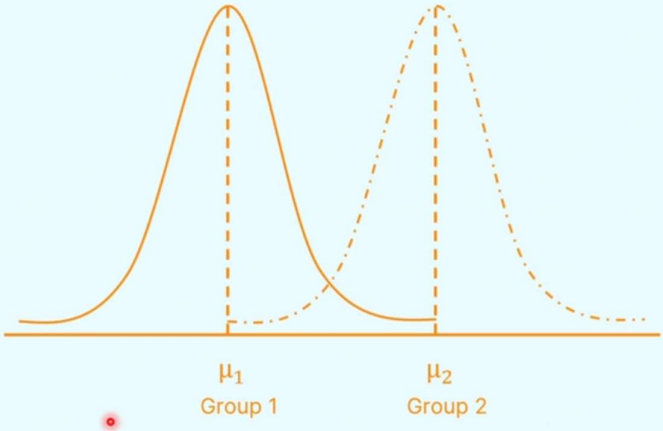


All of these are one sample tests because we collect only one sample and compare the population mean against a fixed hypothesized value

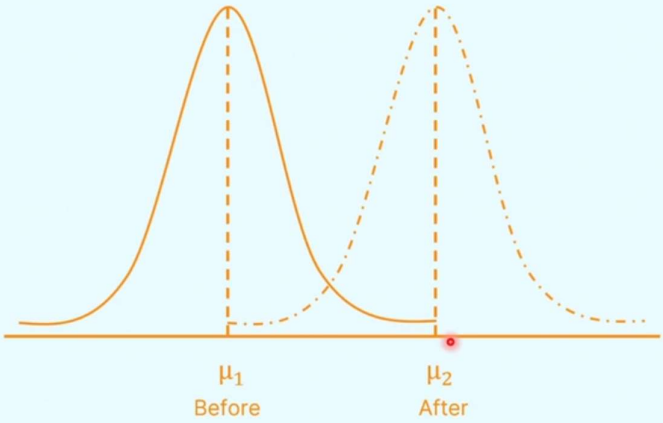
Population mean μ_1 > Population μ_2
Population mean μ_1 < Population μ_2
Population mean μ_1 = Population μ_2
>> These are two tail tests

All of these are two sample tests because we collect two sets of samples and compare the population mean of one with the other

Independent or unpaired two samples



Paired two samples



Module 4 | Topic 1 | Video 3

Steps for Conducting Test of Means

Launching a Learning Portal

Learning and Development Manager of a leading firm intends to launch an organization wide skill development portal.

By conducting a pilot with 25 employees she found that they spent on an average 62 minutes per week on the portal with a standard deviation of 10 minutes.

The criteria for launching the skill portal organization wide is that the pilot employees must spend at least 60 minutes a week on an average as it would make significant improvements on the employee skills.

Should the project go for a launch?

Step 1: Define Hypothesis

$\mu = Population\ Mean = 60$

$S = 10$

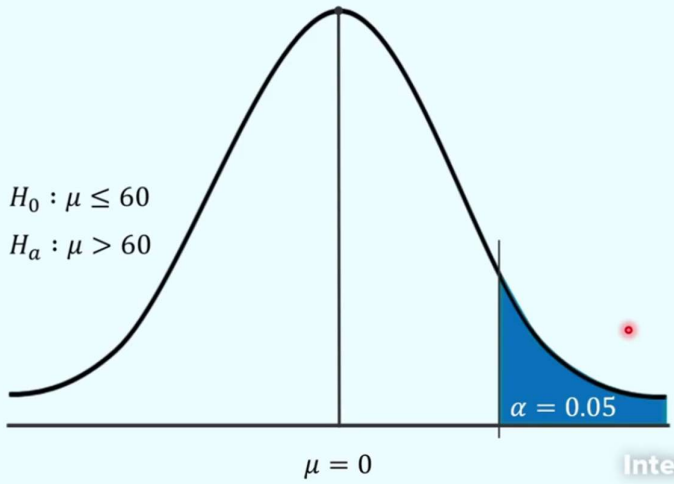
$\bar{X} = 62$

$H_0 : \mu \leq 60$

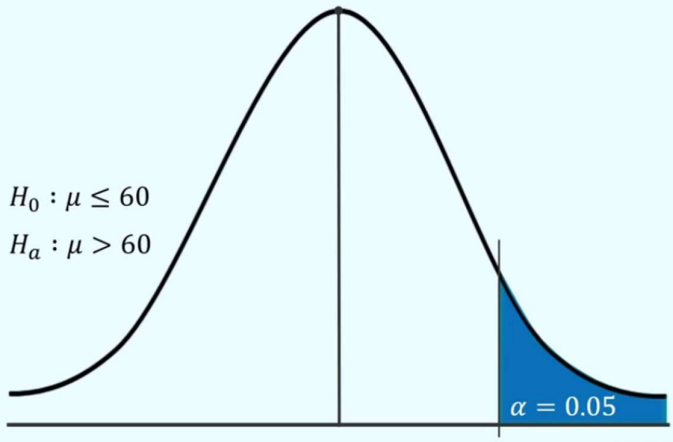
$H_a : \mu > 60$

Right Tail Test

Step 2: Define Acceptance Criteria



Step 2: Define Acceptance Criteria



Step 3a: Calculate Tstat

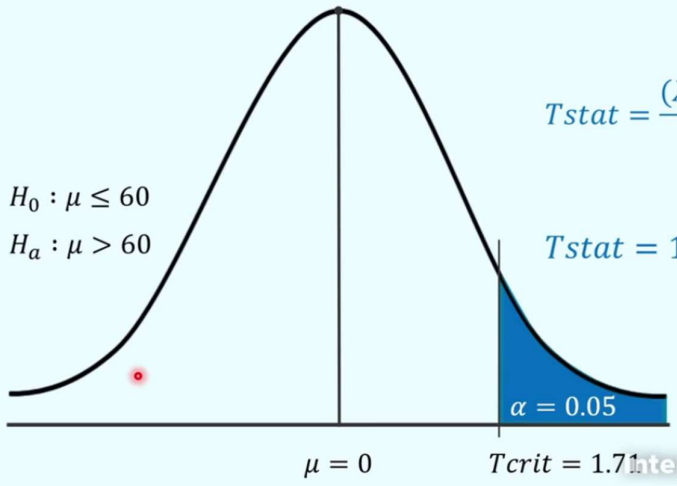
$$Tstat = \frac{(\bar{X} - \mu)}{\frac{S}{\sqrt{n}}}$$

- If n < 30
- If population is unknown

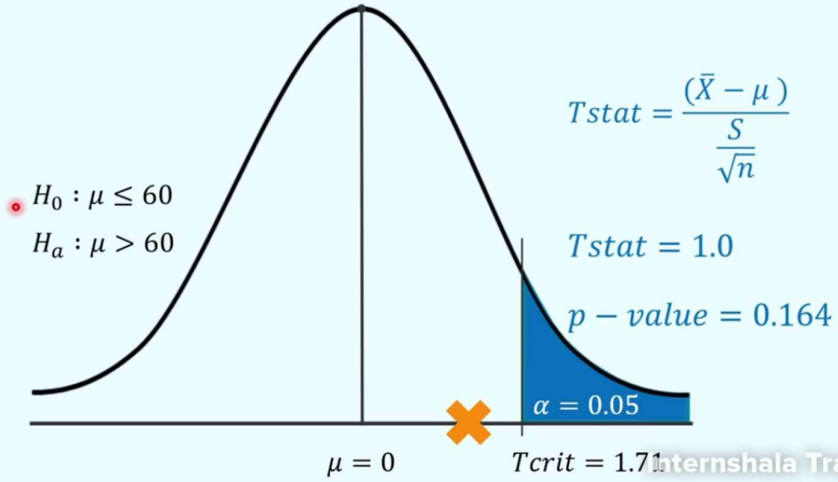
$$Zstat = \frac{(\bar{X} - \mu)}{\frac{\sigma}{\sqrt{n}}}$$

- If n > 30
- If population is known

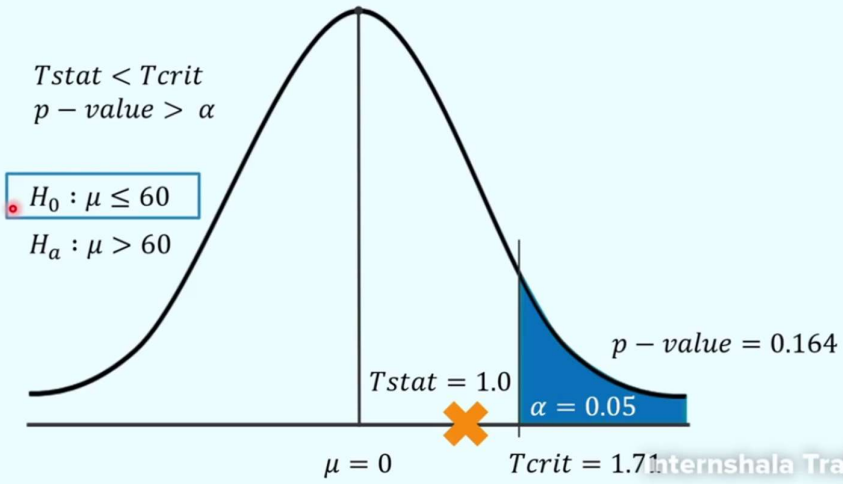
Step 3a: Calculate Tstat



Step 3b: Calculate *p-value*



Step 4: Decision



Step 4: Decision

- **Method 1:** Since Tstat falls in the non-rejection region, we cannot reject the null hypothesis
- **Method 2:** The p-value < α (0.05, in this case), we cannot reject the null hypothesis
- **Decision:** Since p-value < 0.05 and Tstat > Tcrit, we do not reject the null hypothesis and accept it as the truth
- **Conclusion:** There is no enough evidence prove that the mean time spent on the platform is more than 60 minutes. Therefore the project cannot go for a wider launch.

Module 4 | Topic 2 | Video 1

One Sample Right Tail Test

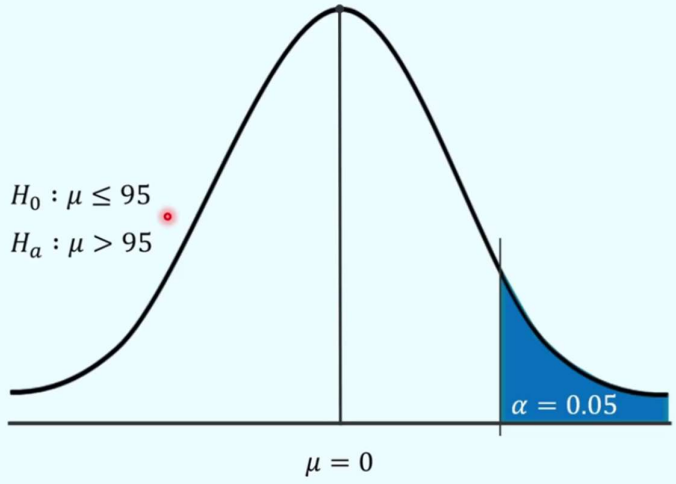
Garment Manufacturer

A garment manufacturer makes 95 shirts a day

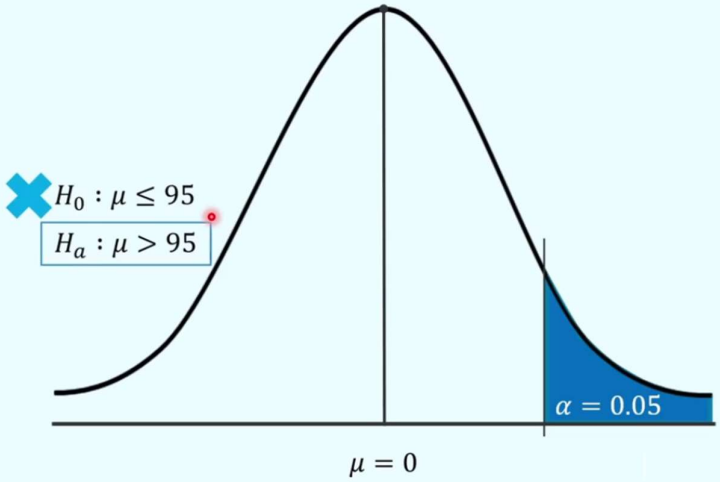
They have made several changes in the process and trained the staffs.

The owner thinks as a result of all of these changes, the daily productivity may have increased to more than 95 shirts a day

Tstat > Tcrit & *p-value* < α



Tstat > Tcrit & *p-value* < α



Module 4 | Topic 2 | Video 2

One Sample Left Tail Test

M4 T2 V2 Lefttailtest-3-55.mp4

Customer Service Center

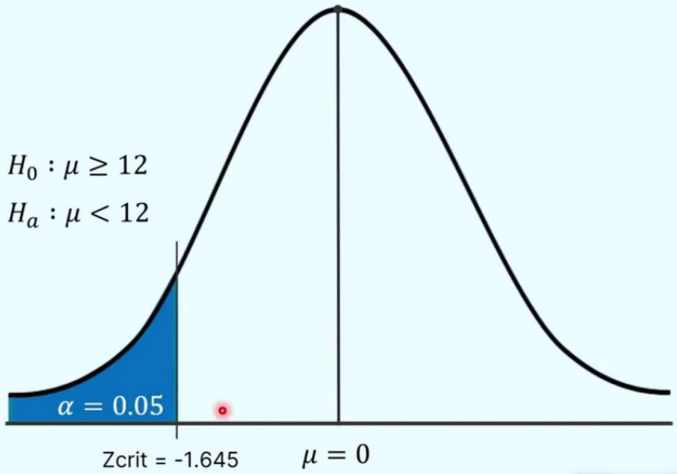
In a customer service center the average call handling time used to be 14 minutes

After making, several changes were made in the procedures, the average time was expected to reduce to less than 12 minutes.

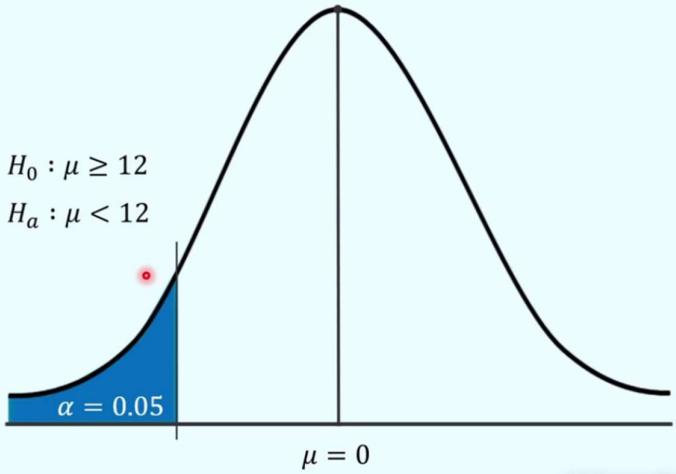
To prove this, 100 samples were studied. The statistics are below:

Mean = 11.5 minutes and standard deviation = 3.5 minutes

Has the average call handling time reduced?



Zstat > Zcrit & *p-value* > α



Module 4 | Topic 2 | Video 3

One Sample Two Tail Test

Internshala Trainings

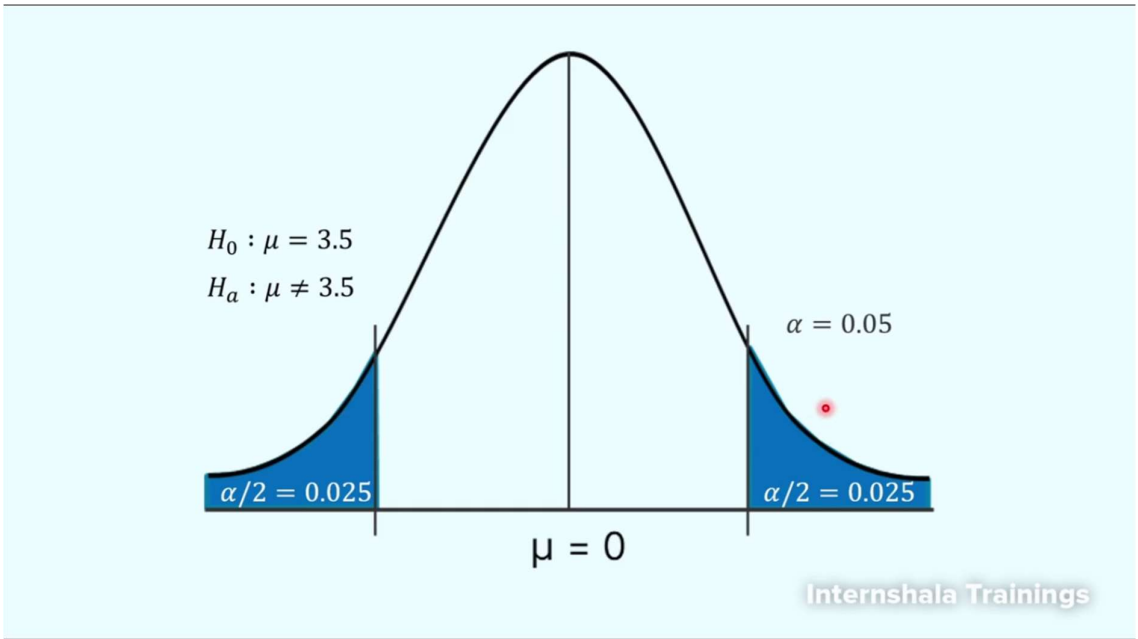
Baby Birth Weight

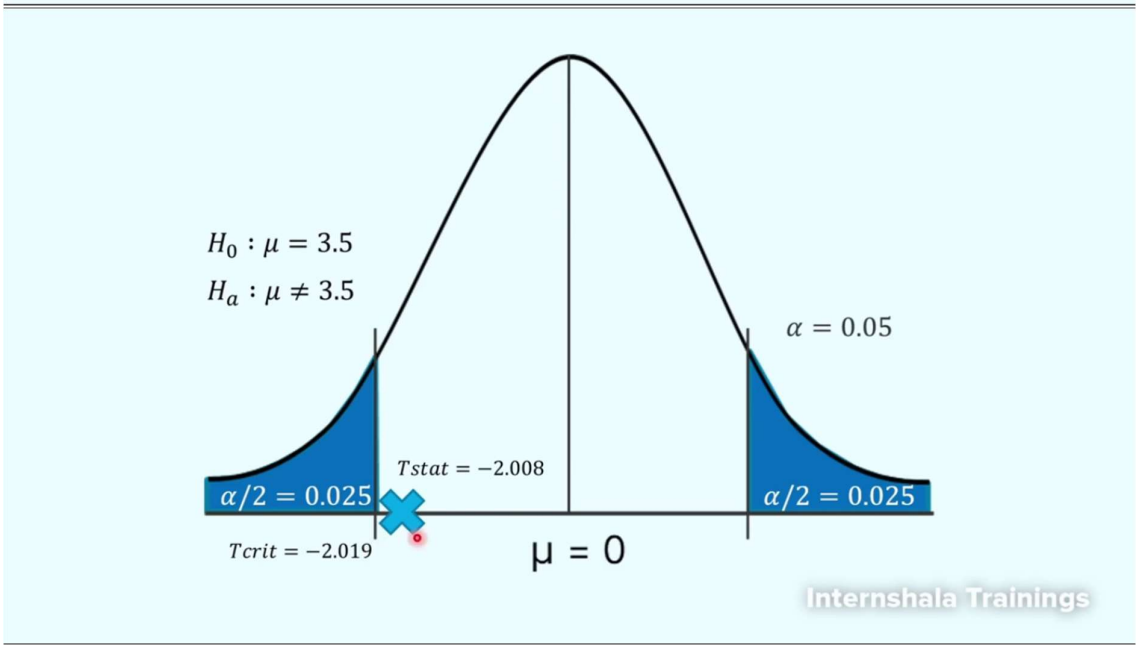
An average new born is expected to weigh 3.5 kgs on birth (or 7.7 lbs)

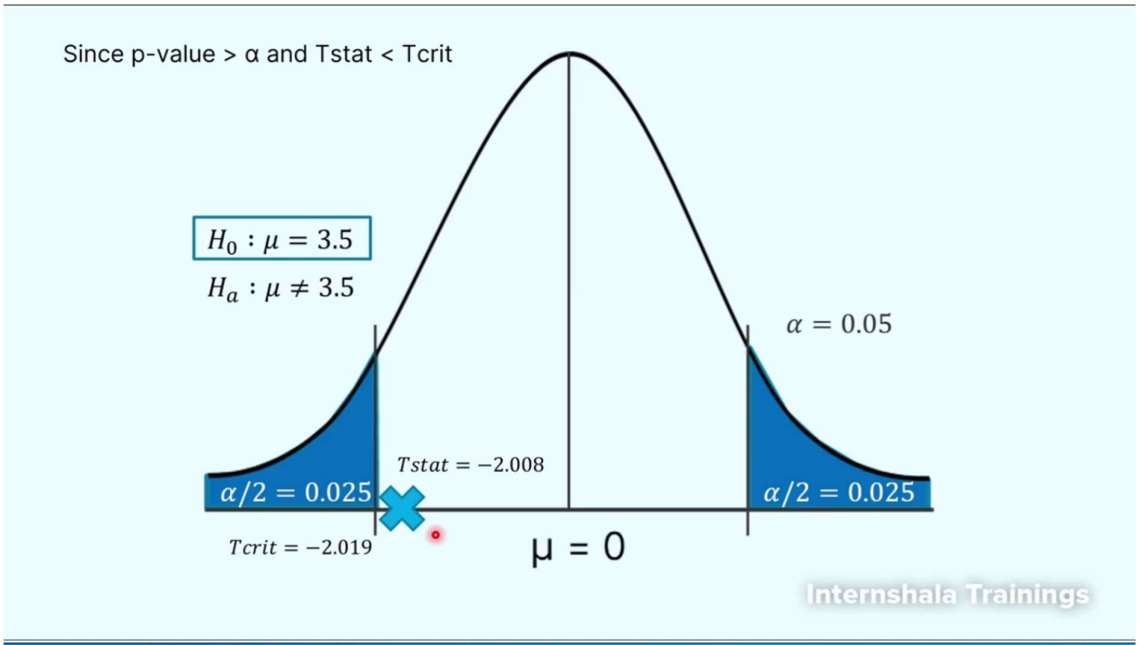
A neonatal center was built as a part of the rural development program. And a sample of 42 babies were studied.

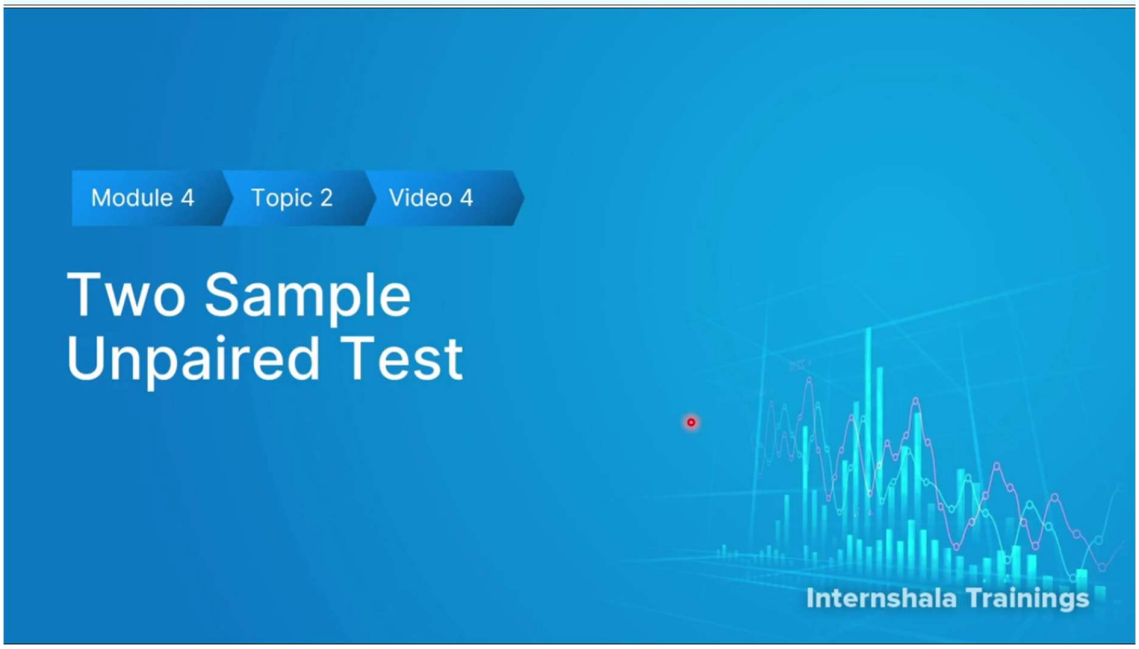
Does the child new born weight observed in the new center same as the population?

Internshala Trainings


$H_0 : \mu = 3.5$
$H_a : \mu \neq 3.5$
$\alpha = 0.05$
$\alpha/2 = 0.025$
$\alpha/2 = 0.025$
μ = 0
Internshala Trainings


$H_0 : \mu = 3.5$
$H_a : \mu \neq 3.5$
$\alpha = 0.05$
$Tstat = -2.008$
$\alpha/2 = 0.025$
$\alpha/2 = 0.025$
$Tcrit = -2.019$
μ = 0
Internshala Trainings


Since p-value > α and Tstat < Tcrit
$H_0 : \mu = 3.5$
$H_a : \mu \neq 3.5$
$\alpha = 0.05$
$Tstat = -2.008$
$\alpha/2 = 0.025$
$\alpha/2 = 0.025$
$Tcrit = -2.019$
μ = 0
Internshala Trainings


Module 4
Topic 2
Video 4
Two Sample
Unpaired Test
Internshala Trainings

Diet Plan

A new diet plan was developed and tested on adults over 6 weeks period.

A group (1) was given the new diet plan in addition with the regular exercise and a control group (2) was not given a diet plan and was asked to only exercise

The results are given.

Is there a significant weight loss due to the diet plan?

Group 1 : Treatment Group - Diet & Exercise
Group 2 : Control Group – Exercise only

$H_0 : \mu_1 = \mu_2$
$H_a : \mu_1 \neq \mu_2$

Module 4 | Topic 2 | Video 5

Two Sample Paired Test

Alpha Coach

Alpha Coach is a coach for standardized MBA entrance test. They specialize is improving the test results of their students within 2 months.

They found that their students improved their scores at least by 30 marks on an average

To prove this they sampled 50 students and recorded their score before joining the program and after joining the program.

Alpha Coach

1) Is their claim valid?
2) Alpha coach wants to run a marketing campaign in which they want to give a promise of 30 marks improvement. Should to they say 30 or something else in the campaign. Assume a significance of 1%

$$H_0 : \mu_2 - \mu_1 \leq 30$$

$$H_a : \mu_2 - \mu_1 > 30$$

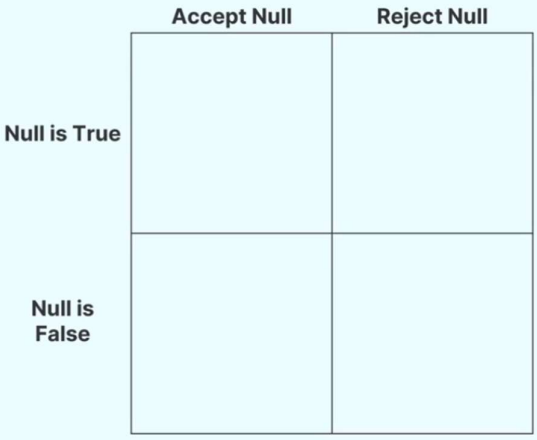
Conclusions

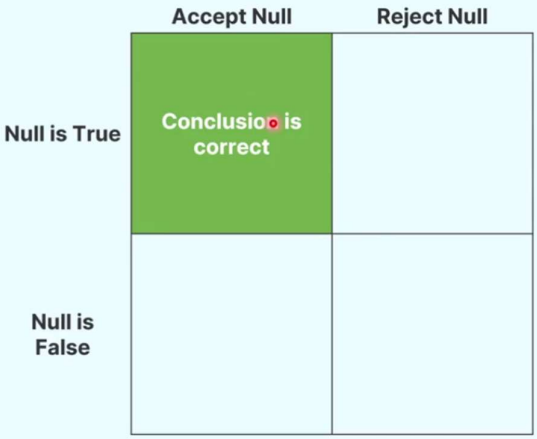
1) The claim made is true: there is enough evidence to say that the score increased by an average of 30

2) Alpha coach can promise an improvement of up to 39 marks in the marketing campaign at a significance of 1%. This means that they would fail to fulfil the promised in only 1% cases.

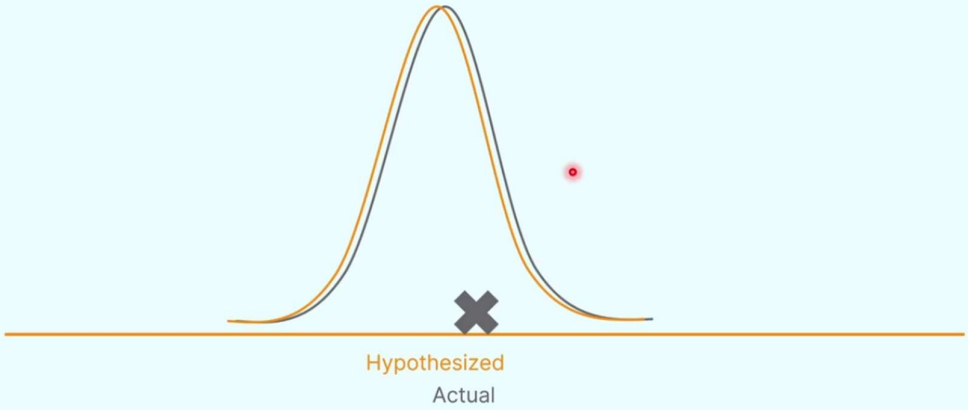
Module 4 | Topic 2 | Video 6

Errors in Test of Means



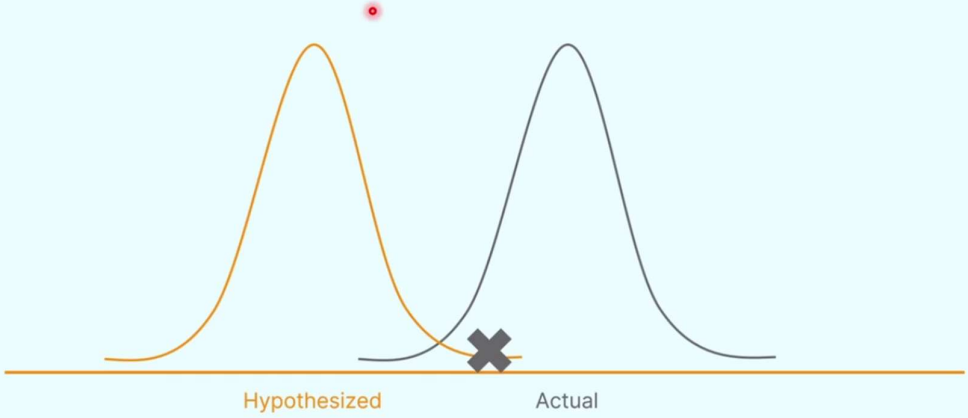


No Error if the decision is to accept null hypothesis



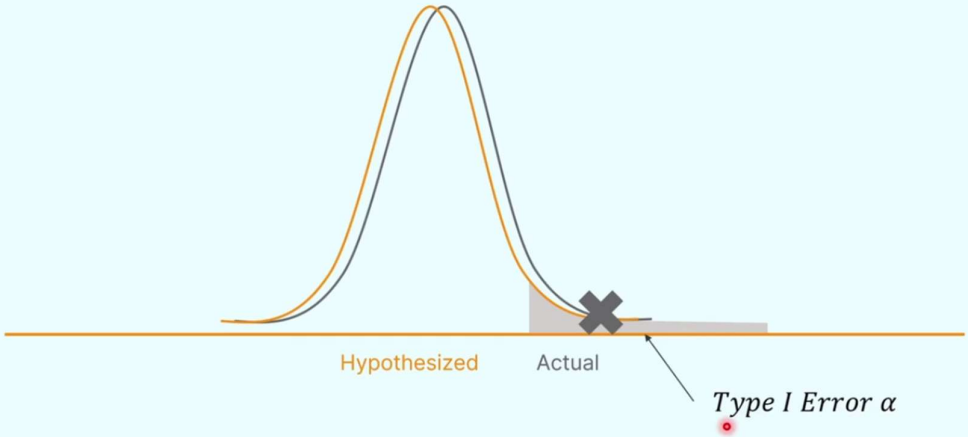
	Accept Null	Reject Null
Null is True	Conclusion is correct	
Null is False		Conclusion is correct

No Error if the decision is to reject null hypothesis



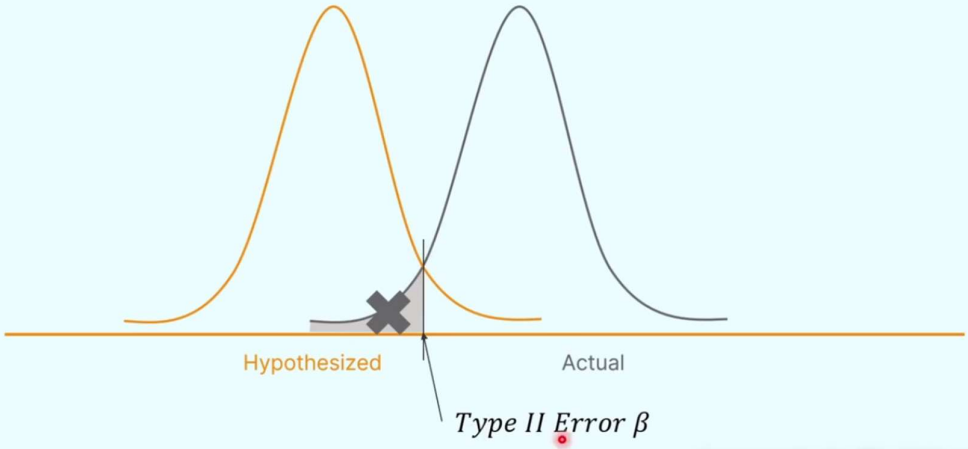
	Accept Null	Reject Null
Null is True	Conclusion is correct	Type I Error Error = α
Null is False		Conclusion is correct

Type I Error if the decision is to reject null hypothesis

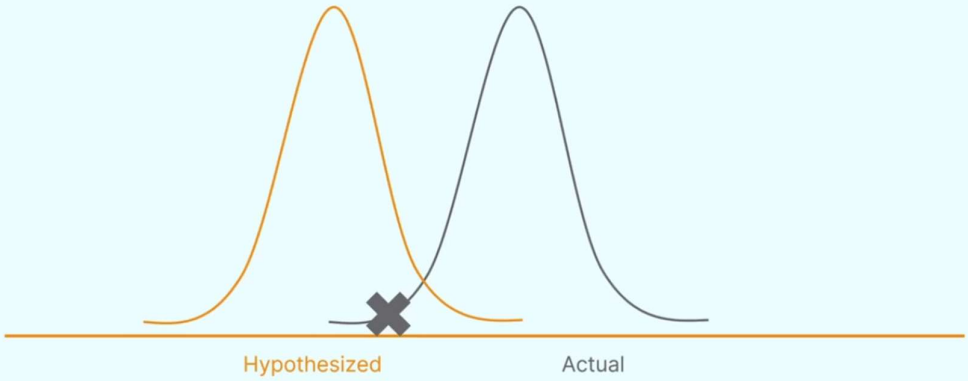


	Accept Null	Reject Null
Null is True	Conclusion is correct	Type I Error Error = α
Null is False	Type II Error Error = β	Conclusion is correct

Type II Error if the decision is to accept null hypothesis

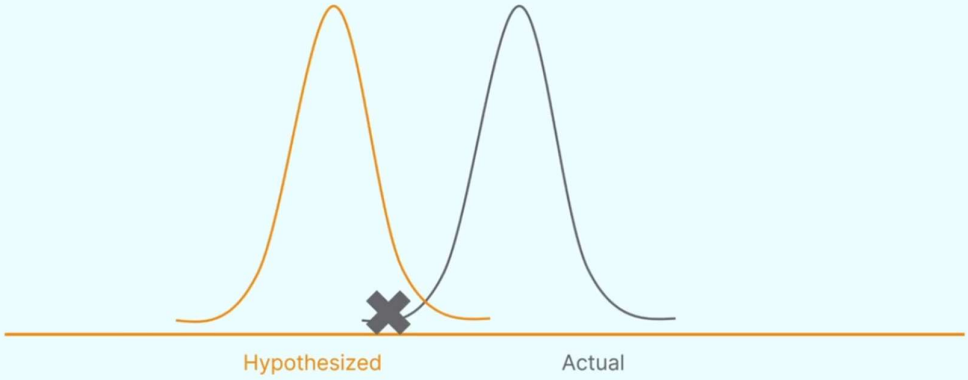


Type II Error if the decision is to accept null hypothesis



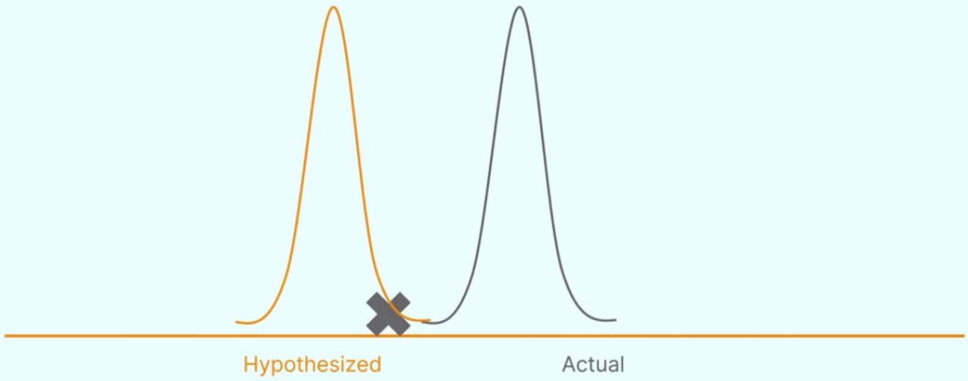
If number of samples is increased

Type II Error if the decision is to accept null hypothesis



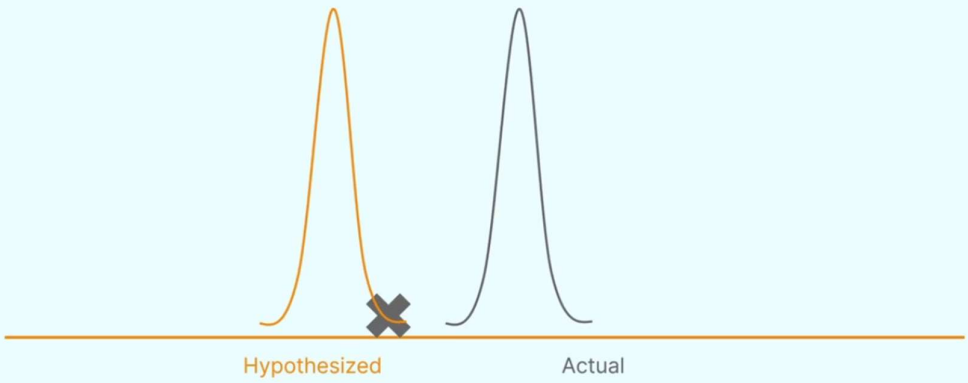
If number of samples is increased

Type II Error if the decision is to accept null hypothesis



If number of samples is increased

Type II Error if the decision is to accept null hypothesis



If number of samples is increased