Logistic regression

We

Logit model

- The binary output for new samples can now be easily predicted using the following

$$p(x) = \frac{e^{(\beta_0+\beta_1 X)}}{1 \; + \; e^{(\beta_0+\beta_1 X)}}$$

$\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n$

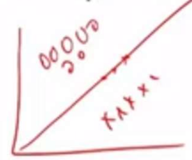
class 1 — 0.5
class 0
0

- If $\beta_0 + \beta_1 X$ is non-negative then we get p>0.5 and Y=1 otherwise we get p<0.5 and Y=0

$\beta_0 + \beta_1 x = 0$

- Decision boundary is the equation $\beta_0 + \beta_1 X$

$p(x) = \frac{e^0}{1+e^0} = \frac{1}{2}$



Example 1

2 dimensional

X_1	X_2
1	1
2	1
3	1
4	1
5	1
1	2
2	2
3	2
4	2
5	2

Class 0

X_1	X_2
6	3
7	3
8	3
9	3
10	3
6	4
7	4
8	4
9	4
10	4

Class 1

X_1	X_2	Class?
1	3	?
2	3	?
4	4	?
5	4	?
3	3	?
6	2	?
9	2	?
8	1	?
7	2	?
10	1	?

Test Data

Results

- Input Features : X_1, X_2
- Classes : 0, 1
- Parameters:

$$\beta_0 = -42.5487$$
$$\beta_{11} = 2.9509$$
$$\beta_{12} = 10.4012$$

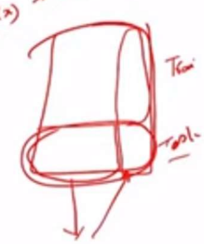
max Loglikelihood

$\beta_0, \beta_1, \beta_2$

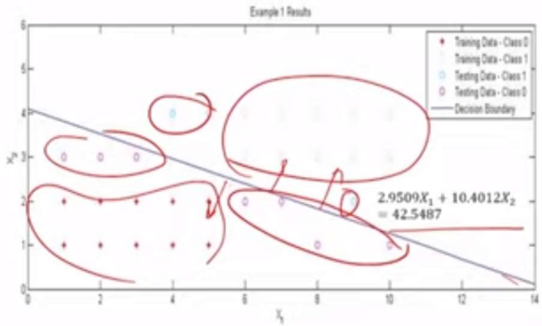
Test Results

X_1	X_2	Prob	Class
1	3	0.0002	0
2	3	0.004	0
4	4	0.999	1
5	4	0.999	1
3	3	0.076	0
6	2	0.0172	0
9	2	0.991	1
8	1	0.0002	0
7	2	0.251	0
10	1	0.0667	0

$f(x) \rightarrow$ Sigmoid



Example 1 solution



Regularization

- General objective
  - $\min_{\theta} -L(\theta)$
  - where $L(\theta) = \left(\sum_{i=1}^{n} y_i \log\left(p(x_i)\right) + (1 - y_i)\log\left(1 - p(x_i)\right)\right)$
- When large number of independent variables are present, logistic regression tends to over-fit
- To prevent over-fitting, we need to penalize the coefficients
- This is known as regularization

$n+1$

$\beta_0 + \beta_{11}x_1 + \beta_{12}x_2$

Regularization

- Regularization helps in building non-complex models that avoids capturing noise in model due to over-fitting
- The objective now becomes
  $\min_{\theta} -L(\theta) + \lambda * h(\theta)$ where λ is regularization parameter and $h(\theta)$ is regularization function
- Depending on $h(\theta)$, the regularization can be classified as L_1 or L_2 type
- $h(\theta) = \theta^T \theta$ for L_2 type regularization
- Larger the value of λ, more is the regularization strength
- Regularization helps the model work better on test data due to the fact that over-fitting is minimized on training data

$(\beta_0 \ \beta_{11} \ \beta_{12})^T \begin{pmatrix} \beta_0 \\ \beta_{11} \\ \beta_{12} \end{pmatrix}$

$\beta_0^2 + \beta_{11}^2 + \beta_{12}^2$