In the previous video, we learned about the so-called overall model test (OMT) that can be used to detect whether an error is present in the mathematical model or not. The OMT test statistic is given by:

$$\underline{T} = \underline{\hat{e}}^T Q_{yy}^{-1} \underline{\hat{e}}.$$

If the observations are normally distributed, it can be shown that the test statistic $\underline{T}$ follows the central χ^2-distribution (chi-square) with $m-n$ degrees of freedom (where m is the number of observations, and n is the number of unknowns) :

$$\underline{T} \sim \chi^2\left(m-n, 0\right).$$

Note that the χ^2-distribution is defined for degrees of freedom equal to one or larger. So, in case of no redundancy (i.e., $m=n$), the degrees of freedom is zero, and the test is not applicable. Knowing the distribution, it is possible to evaluate the probability that $\underline{T}$ is larger than a certain value. The idea is then to choose a small probability (see the plot below), called α and to find the corresponding value K such that:

$$P\left(\underline{T} > K\right) = \alpha$$

If we find a T that is larger than K, the OMT is rejected, meaning it is likely that there is an error in the assumed mathematical model, since it is unlikely that T is larger than K.

The parameter α is called the *level of significance,* and it is usually chosen to be a small number (for example 0.05 or 0.001, depending on the application). In fact, the level of significance is the probability that the overall model test is **mistakenly** or **falsely** rejected. Note that, **by choosing a smaller α, although the chance of *false-rejection* of the OMT is reduced, the chance of its *false-acceptance* increases.** So there is a trade-off between these two probabilities.

After choosing the level of significance, the threshold K (or the critical value) corresponding to the chosen α can be obtained from the χ^2 distribution function. In the following, you find more information how to compute the critical value for a chosen α.

Computing the critical value in Matlab

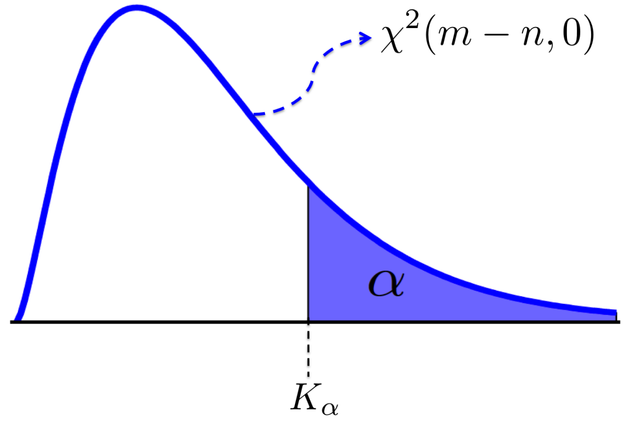
In Matlab, you can use the function `'chi2inv'` to define the critical value. For example for $\alpha = 0.05$ and $m-n=10$, you can use

`chi2inv(1-0.05,10)` which gives 18.307.

Computing the critical value from the χ^2 table

The table below shows the critical value K_α of a random variable with a $\chi^2\left(m-n, 0\right)$-distribution for some α values and different degrees of freedom.

As an example, if $\alpha = 0.05$ and $m-n=10$, the critical value will be $K_{0.05} = 18.307$ (For this example, the $\alpha = 0.05$, $m-n=10$, and $K_{0.05}$ have been highlighted in the following table with the red boxes).



α / $m-n$	$\boxed{0.0500}$	0.0250	0.0100	0.0050	0.0010
1	3.8415	5.0239	6.6349	7.8794	10.8276
2	5.9915	7.3778	9.2103	10.5966	13.8155
3	7.8147	9.3484	11.3449	12.8382	16.2662
4	9.4877	11.1433	13.2767	14.8603	18.4668
5	11.0705	12.8325	15.0863	16.7496	20.5150
6	12.5916	14.4494	16.8119	18.5476	22.4577
7	14.0671	16.0128	18.4753	20.2777	24.3219
8	15.5073	17.5345	20.0902	21.9550	26.1245
9	16.9190	19.0228	21.6660	23.5894	27.8772
$\boxed{10}$	$\boxed{18.3070}$	20.4832	23.2093	25.1882	29.5883
11	19.6751	21.9200	24.7250	26.7568	31.2641
12	21.0261	23.3367	26.2170	28.2995	32.9095
13	22.3620	24.7356	27.6882	29.8195	34.5282
14	23.6848	26.1189	29.1412	31.3193	36.1233
15	24.9958	27.4884	30.5779	32.8013	37.6973
16	26.2962	28.8454	31.9999	34.2672	39.2524
17	27.5871	30.1910	33.4087	35.7185	40.7902
18	28.8693	31.5264	34.8053	37.1565	42.3124
19	30.1435	32.8523	36.1909	38.5823	43.8202
20	31.4104	34.1696	37.5662	39.9968	45.3147
21	32.6706	35.4789	38.9322	41.4011	46.7970
22	33.9244	36.7807	40.2894	42.7957	48.2679
23	35.1725	38.0756	41.6384	44.1813	49.7282
24	36.4150	39.3641	42.9798	45.5585	51.1786
25	37.6525	40.6465	44.3141	46.9279	52.6197
26	38.8851	41.9232	45.6417	48.2899	54.0520
27	40.1133	43.1945	46.9629	49.6449	55.4760
28	41.3371	44.4608	48.2782	50.9934	56.8923
29	42.5570	45.7223	49.5879	52.3356	58.3012
30	43.7730	46.9792	50.8922	53.6720	59.7031
40	55.7585	59.3417	63.6907	66.7660	73.4020
50	67.5048	71.4202	76.1539	79.4900	86.6608
60	79.0819	83.2977	88.3794	91.9517	99.6072
70	90.5312	95.0232	100.4252	104.2149	112.3169
80	101.8795	106.6286	112.3288	116.3211	124.8392
90	113.1453	118.1359	124.1163	128.2989	137.2084
100	124.3421	129.5612	135.8067	140.1695	149.4493