Parallel oblique projection

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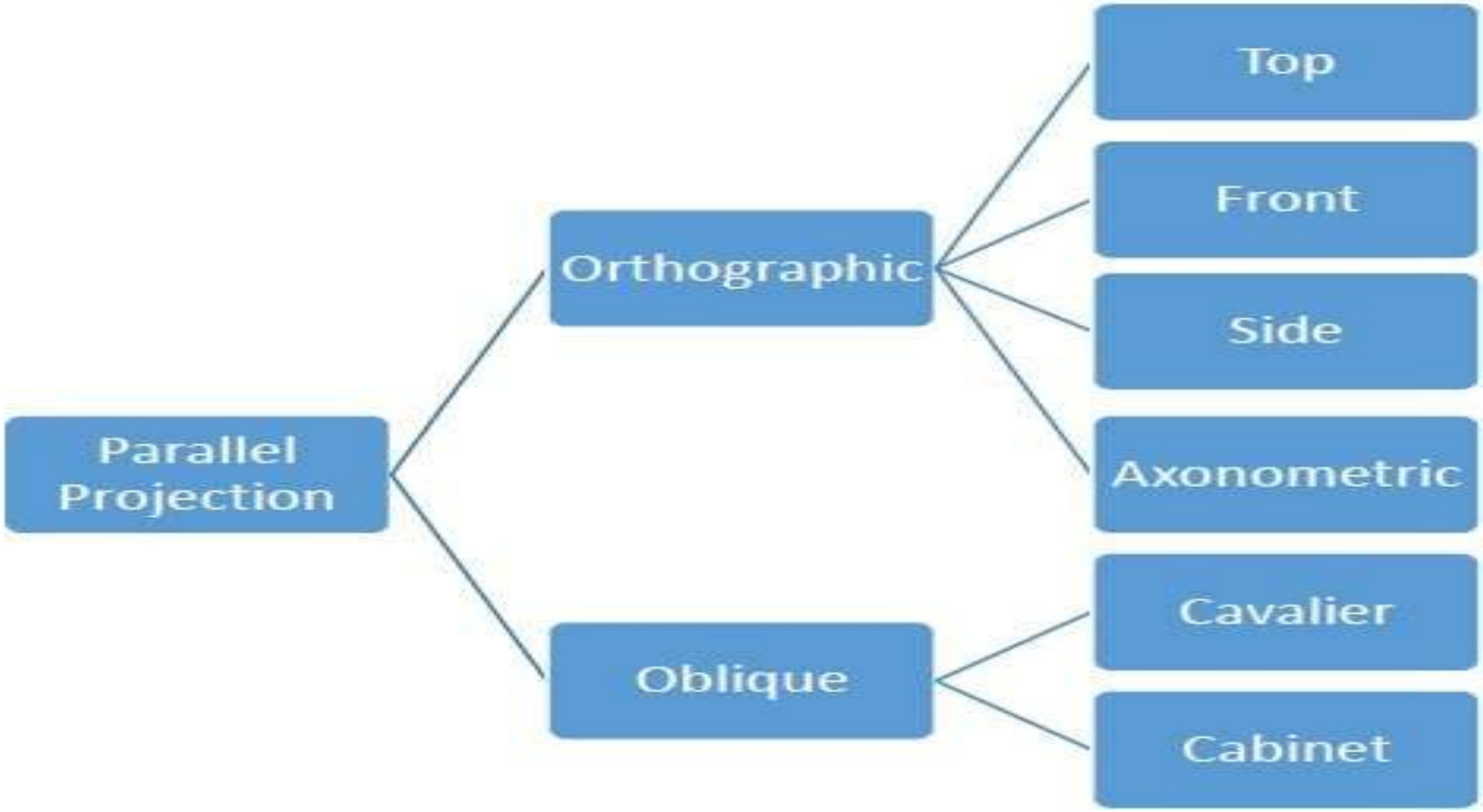
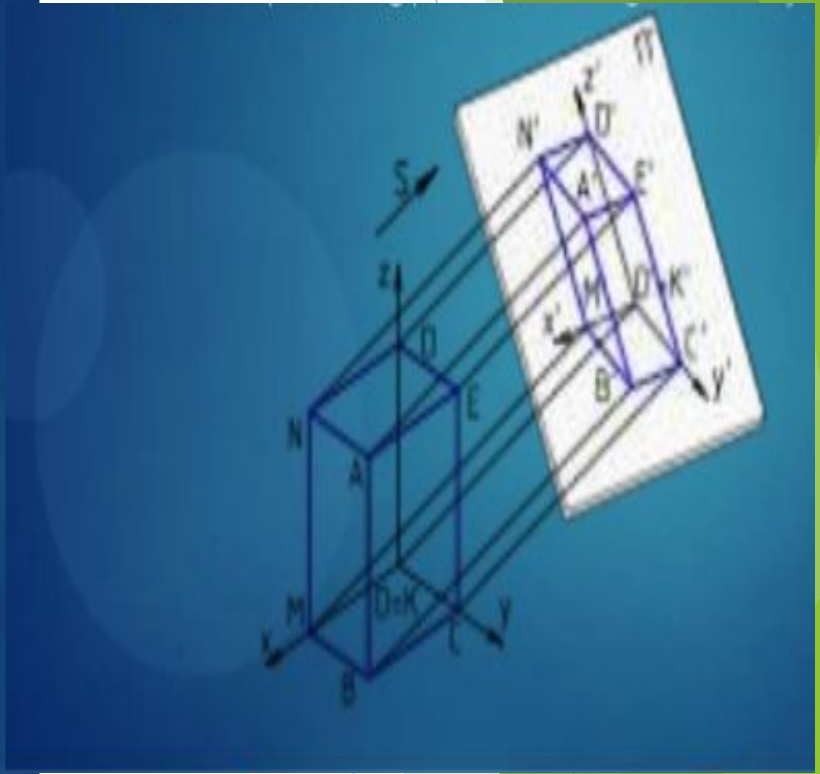

Parallel Projection
Orthographic
Oblique
Top
Front
Side
Axonometric
Cavalier
Cabinet

OBLIQUE PROJECTION

- A parallel projection
- Projects a 3d image by the intersection of the parallel rays which is known as projectors.
- Parallel lines of the source object produces a parallel lines in the projected images.
- the projectors intersects the projection in oblique angle to produce the projected image.

Parallel projection of the point(x,y,z) on the xy plane is (x+az,y+bz,0)

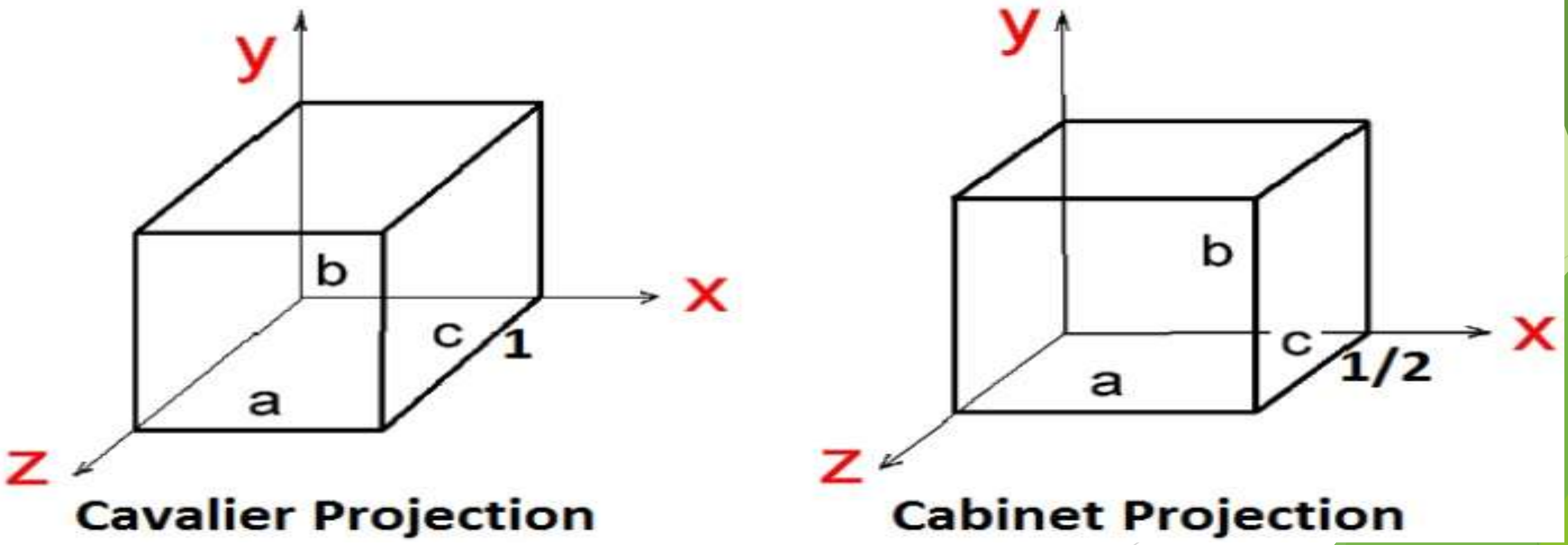
- When a=b=0, the projection is said to be orthographic or orthogonal
- If not the projection is oblique



OBLIQUE PROJECTION

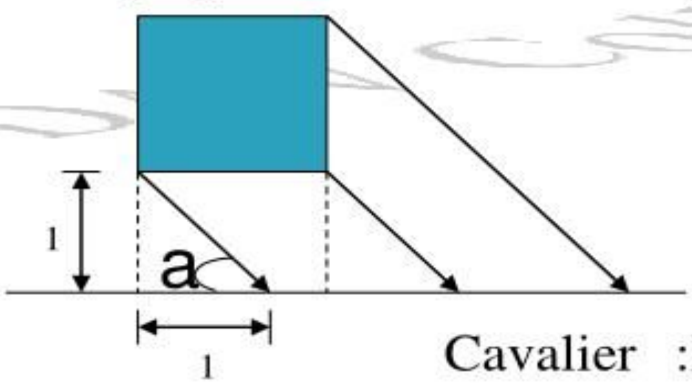
- In oblique projection, the direction of projection is not normal to the projection of plane. In oblique projection, we can view the object better than orthographic projection.
- Oblique projections are rarely used as they are unconvincing to eyes
- There are two types of oblique projections − **Cavalier** and **Cabinet**.
- The Cavalier projection makes 45° angle with the projection plane. The projection of a line perpendicular to the view plane has the same length as the line itself in Cavalier projection. In a cavalier projection, the foreshortening factors for all three principal directions are equal.
- The Cabinet projection makes 63.4° angle with the projection plane
- In Cabinet projection, lines perpendicular to the viewing surface are projected at ½ their actual length.

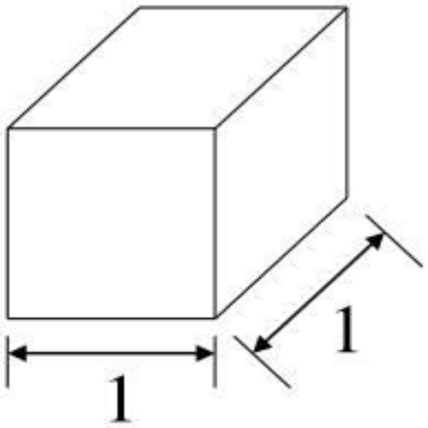
Both the projections are shown in the following figure −



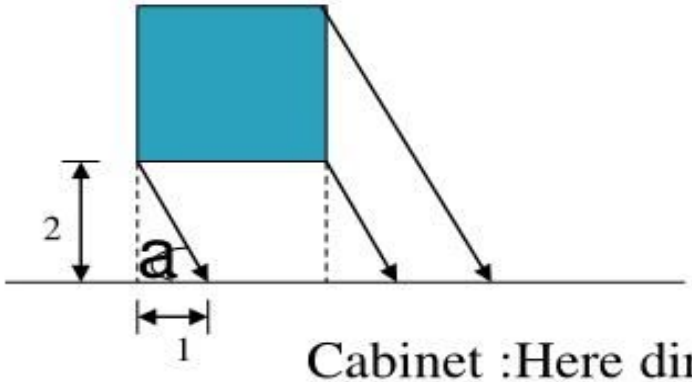
Oblique projections

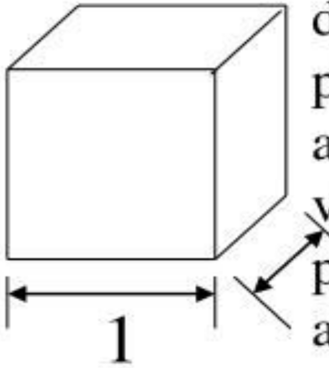
- Projection lines are at an angle to the view plane i.e. Projection lines are not perpendicular to view plane.





Cavalier :Here direction of projection makes 45 degrees angle with view plane and the projection of a line perpendicular to view plane has same length as the line.





Cabinet :Here direction of projection makes 63.4 degrees angle with view plane and the projection and lines perpendicular to viewing surface are projected at one half their actual length.

Difference between Cabinet and Cavalier Projection

CABINET PROJECTION

- Used by furniture industry.
- One face of the object is parallel to the viewing plane
- Third axis is projected at angle 30 or 45 degree
- The projection length of the receding lines are half
- Direction of projection makes 63.4 degree angle with view plane

CAVALIER PROJECTION

- Also known as high view point
- Length along z axis remains unscaled
- Y axis is drawn diagonally making an angle of 30 or 45 degree with x axis
- X and Z are perpendicular on those area are 1:1 scaled i.e same as original size
- Direction of projection makes 45 degree angle with view plane

If the plane facing the viewer is *xy*, and the receding axis is *z*, then a point *P* (x,y,z) is projected like this and result is $P' = (X', Y', Z')$

A projected point has the following transformation formula

$$X' = X + 0.5 * Z * \cos(\alpha)$$
$$Y' = Y + 0.5 * Z * \sin(\alpha)$$
$$Z' = 0$$

α is the mentioned angle

Find the general form of an oblique projection onto the xy plane.

SOLUTION

Refer to Fig. 7-24. Oblique projections (to the xy plane) can be specified by a number f and an angle θ. The number f prescribes the ratio that any line L perpendicular to the xy plane will be foreshortened after projection. The angle θ is the angle that the projection of any line perpendicular to the xy plane makes with the (positive) x axis.

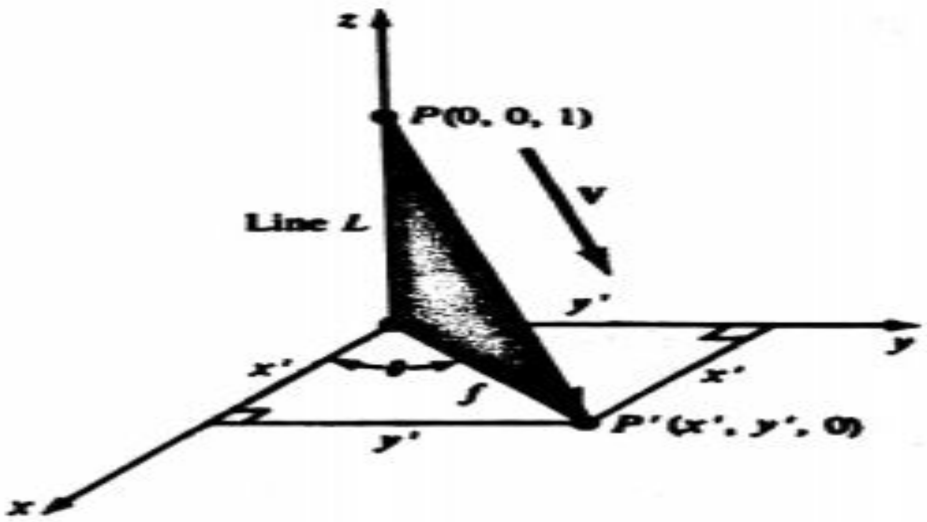


Fig. 7-24

To determine the projection transformation, we need to find the direction vector **V**. From Fig. 7-24, with line L of length 1, we see that the vector $\overline{P'P}$ has the same direction as **V**. We choose **V** to be this vector:

$$\mathbf{V} = \overline{P'P} = x'\mathbf{I} + y'\mathbf{J} - \mathbf{K} \qquad (= a\mathbf{I} + b\mathbf{J} + c\mathbf{K})$$

From Fig. 7-24 we find $a = x' = f\cos\theta$, $b = y' = f\sin\theta$, and $c = -1$.

From Prob. 7.10, the required transformation is

$$Par_{\mathbf{V}} = \begin{pmatrix} 1 & 0 & f\cos\theta & 0 \\ 0 & 1 & f\sin\theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Find the transformation for (a) cavalier with $\theta = 45°$ and (b) cabinet projections with $\theta = 30°$. (c) Draw the projection of the unit cube for each transformation.

SOLUTION

(a) A cavalier projection is an oblique projection where there is no foreshortening of lines perpendicular to the xy plane. From Prob. 7.12 we then see that $f = 1$. With $\theta = 45°$, we have

$$Par_{\mathbf{V}_1} = \begin{pmatrix} 1 & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 1 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(b) A cabinet projection is an oblique projection with $f = \frac{1}{2}$. With $\theta = 30°$, we have

$$Par_{\mathbf{V}_2} = \begin{pmatrix} 1 & 0 & \frac{\sqrt{3}}{4} & 0 \\ 0 & 1 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

To construct the projections, we represent the vertices of the unit cube by a matrix whose columns are homogeneous coordinates of the vertices (see Prob. 7.1):

$$V = (ABCDEFGH) = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

(c) To draw the cavalier projection, we find the image coordinates by applying the transformation matrix $Par_{\mathbf{V}_1}$ to the coordinate matrix $\mathbf{V}$:

$$Par_{\mathbf{V}_1} \cdot V = \begin{pmatrix} 0 & 1 & 1 & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1+\frac{\sqrt{2}}{2} & 1+\frac{\sqrt{2}}{2} \\ 0 & 0 & 1 & 1 & 1+\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1+\frac{\sqrt{2}}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

The image coordinates are then

$$A' = (0,0,0) \qquad E' = \left(\frac{\sqrt{2}}{2}, 1+\frac{\sqrt{2}}{2}, 0\right)$$
$$B' = (1,0,0) \qquad F' = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)$$
$$C' = (1,1,0) \qquad G' = \left(1+\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)$$
$$D' = (0,1,0) \qquad H' = \left(1+\frac{\sqrt{2}}{2}, 1+\frac{\sqrt{2}}{2}, 0\right)$$

Refer to Fig. 7-25.

To draw the cabinet projection:

$$Par_{\mathbf{V}_2} \cdot \mathbf{V} = \begin{pmatrix} 0 & 1 & 1 & 0 & \frac{\sqrt{3}}{4} & \frac{\sqrt{3}}{4} & 1+\frac{\sqrt{3}}{4} & 1+\frac{\sqrt{3}}{4} \\ 0 & 0 & 1 & 1 & 1\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 1\frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

The image coordinates are then (see Fig. 7-26)

$$A' = (0,0,0) \qquad E' = \left(\frac{\sqrt{3}}{4}, 1\frac{1}{4}, 0\right)$$
$$B' = (1,0,0) \qquad F' = \left(\frac{\sqrt{3}}{4}, \frac{1}{4}, 0\right)$$
$$C' = (1,1,0) \qquad G' = \left(1+\frac{\sqrt{3}}{4}, \frac{1}{4}, 0\right)$$
$$D' = (0,1,0) \qquad H'\left(1+\frac{\sqrt{3}}{4}, 1\frac{1}{4}, 0\right)$$

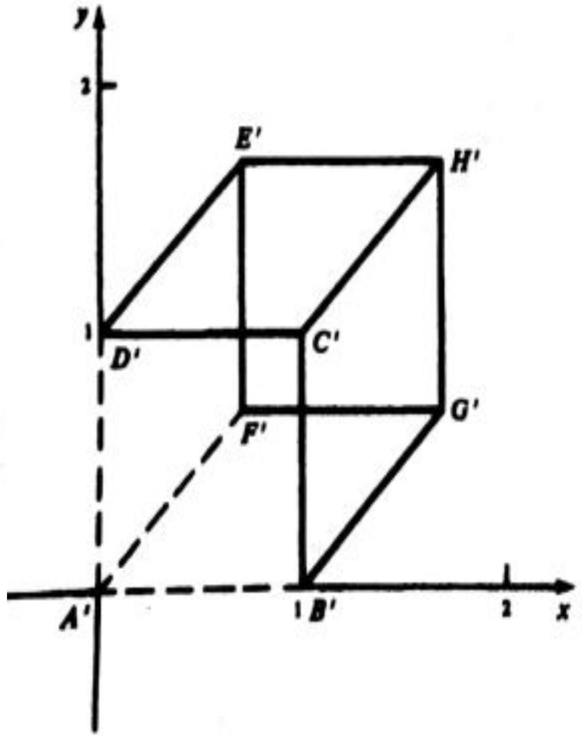


Fig. 7-25

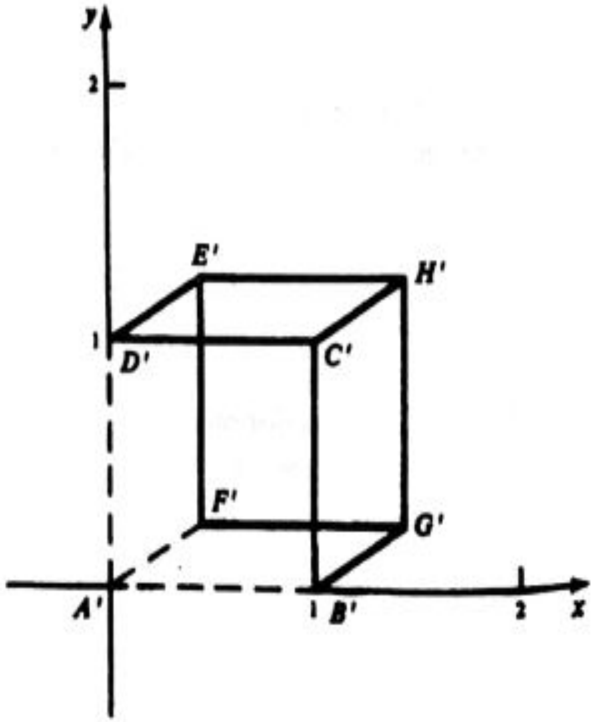


Fig. 7-26