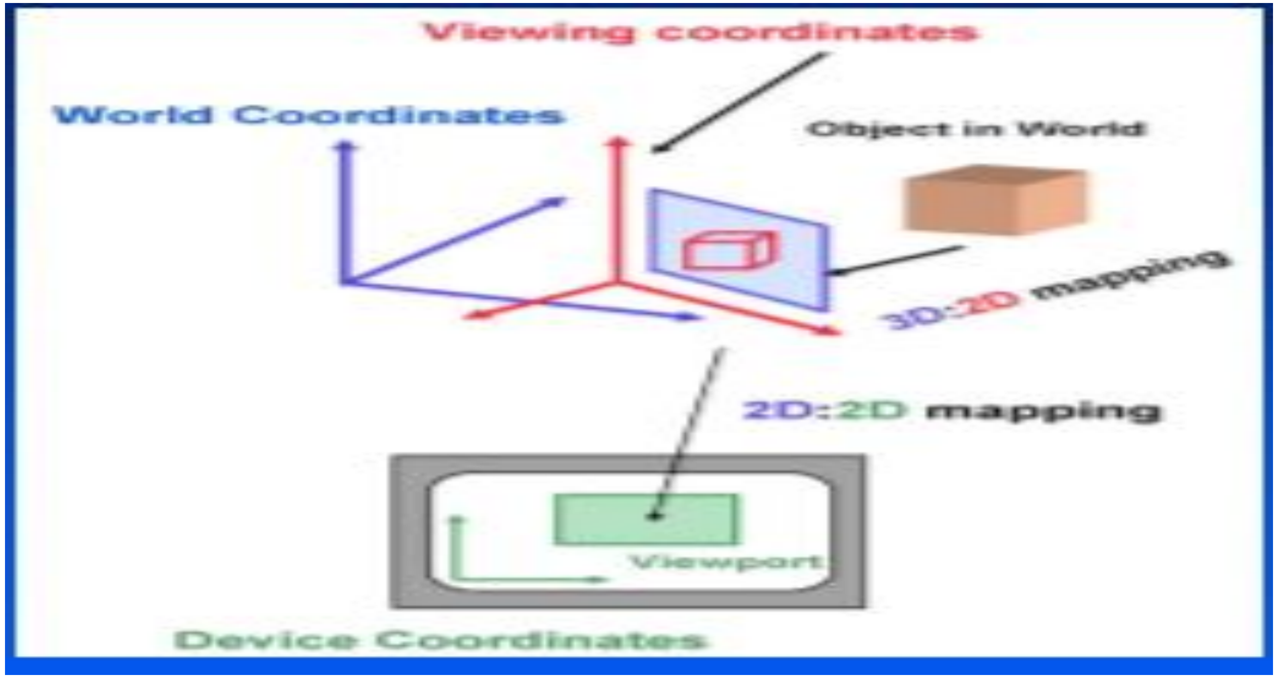
Projections - Orthographic

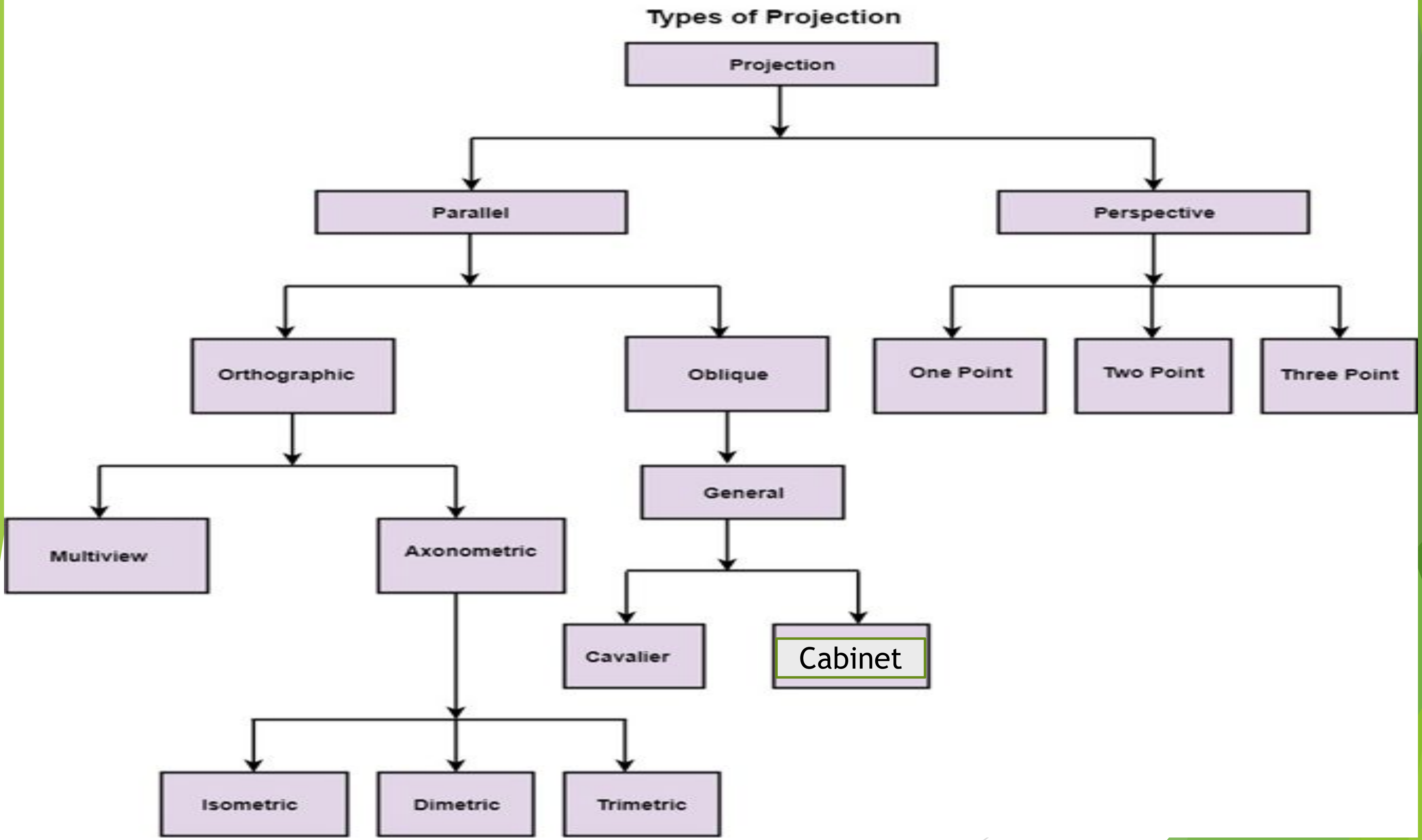
- Date : 8.4.20

Projection

- It is the process of converting a 3D object into a 2D object. It is also defined as mapping or transformation of the object in projection plane or view plane. The view plane is displayed surface

- Diagrammatic representation of projection of an object is as follows




Types of Projection
Projection
Parallel
Perspective
Orthographic
Oblique
One Point
Two Point
Three Point
General
Multiview
Axonometric
Cavalier
Cabinet
Isometric
Dimetric
Trimetric

Definitions

- ***Projection***: a transformation that maps from a higher dimensional space to a lower dimensional space (e.g. 3D->2D)
- ***Center of projection*** **(COP):** the position of the eye or camera with respect to which the projection is performed (also eye point, camera point, proj. reference point)
- **Direction of projection (DOP):** the direction of an eye or camera assumed to be infinite far away.
- ***Projection plane***: in a 3D->2D projection, the plane to which the projection is performed (also view plane)
- **Projectors:** lines from coordinate in original space to coordinate in projected space

- There are two basic ways of projecting objects onto the view plane :
- Parallel projection and
- Perspective projection.

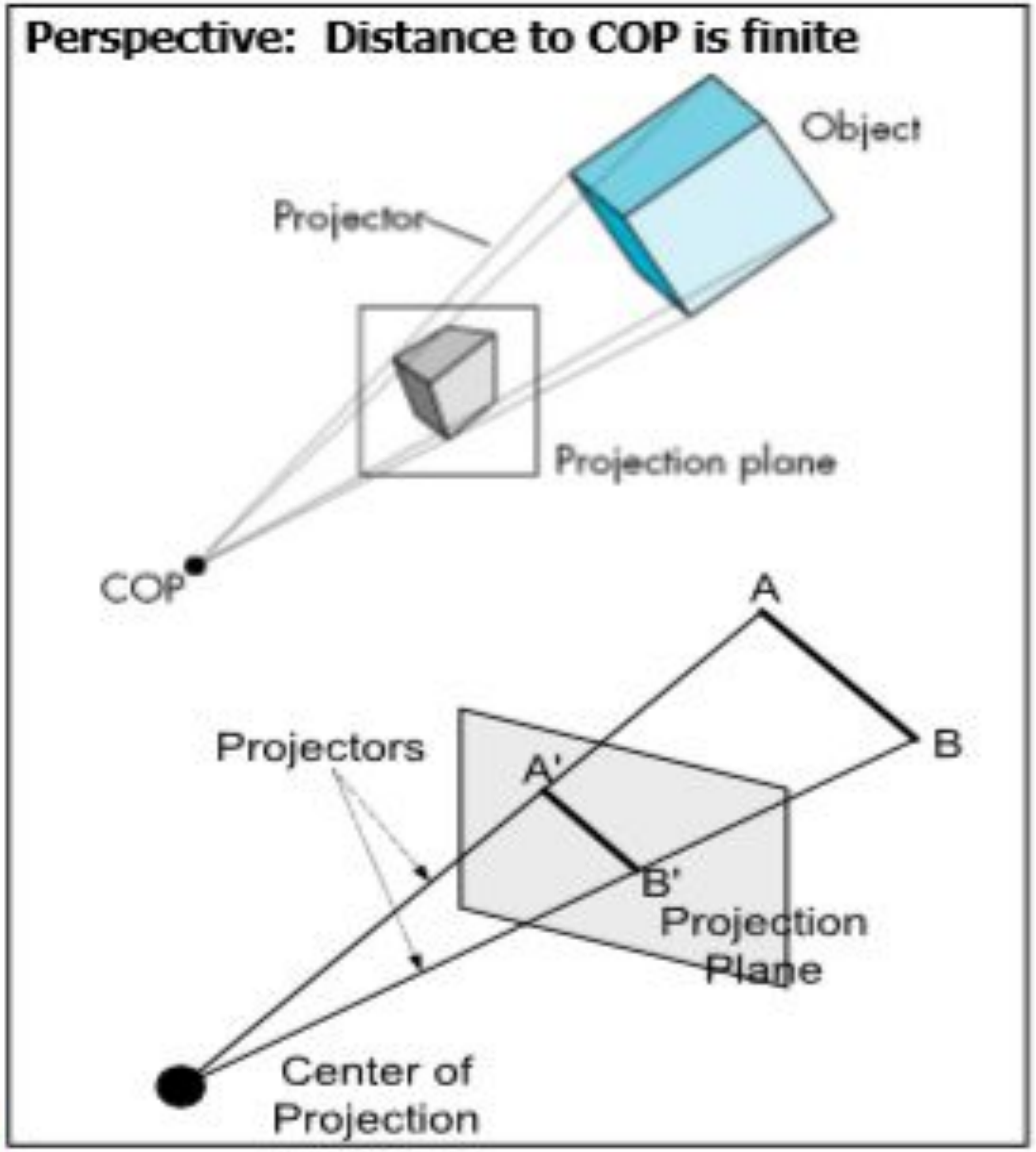
Parallel Projection

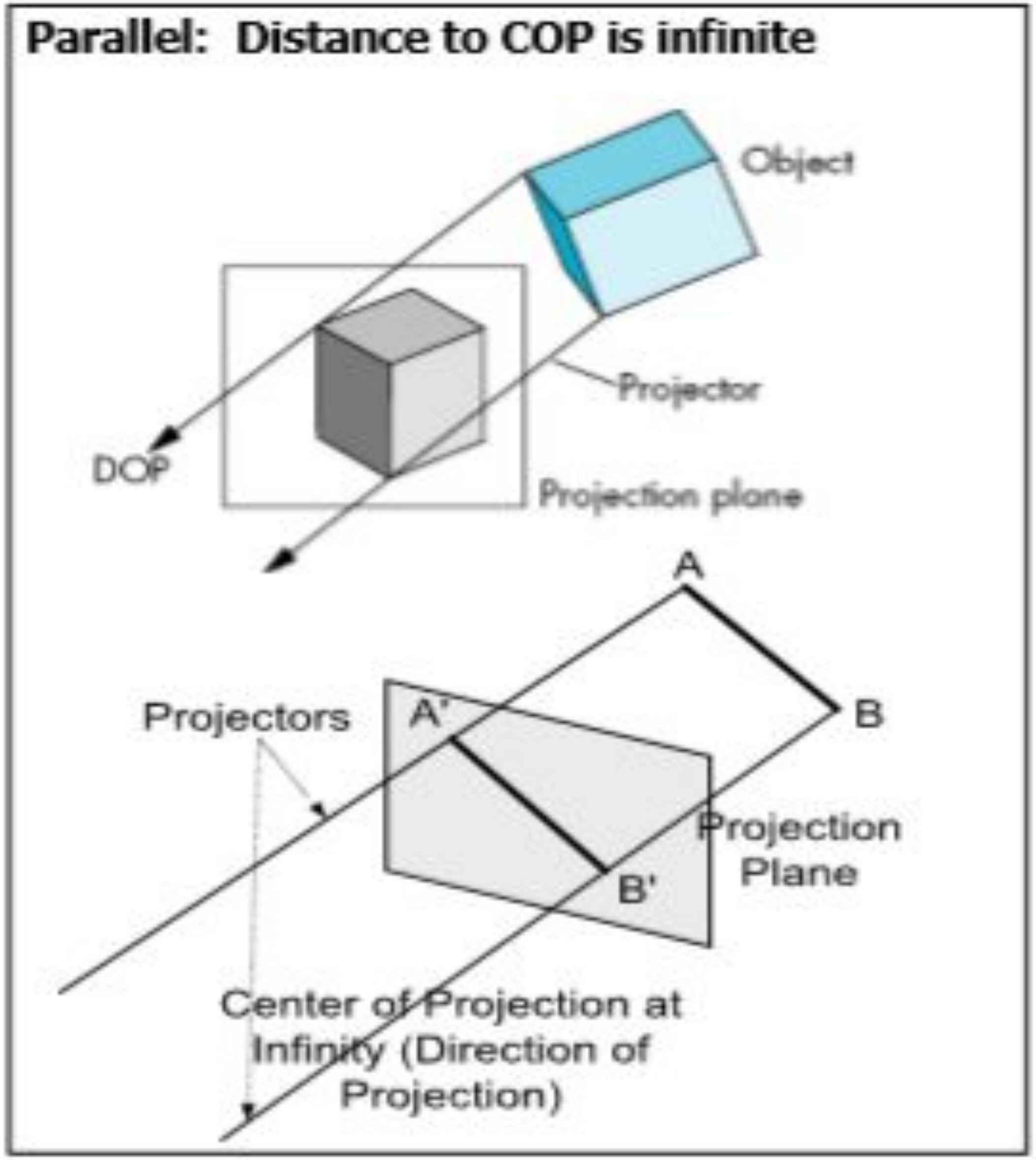
- In parallel projection, z coordinate is discarded and parallel lined from each vertex on the object are extended until they intersect the view plane.
- The point of intersection is the projection of the vertex.
- We connect the projected vertices by line segments which correspond to connections on the original object.

Prespective Projection

- The perspective projection, on the other hand, produces realistic views but does not preserve relative proportions.
- In perspective projection, the lines of projection are not parallel. Instead, they all coverage at a single point called the **center of projection or projection reference point.**
- The object positions are transformed to the view plane along these converged projection lines and the projected view of an object is determines by calculating the intersection of the converged projection lines with the view plane

Projections

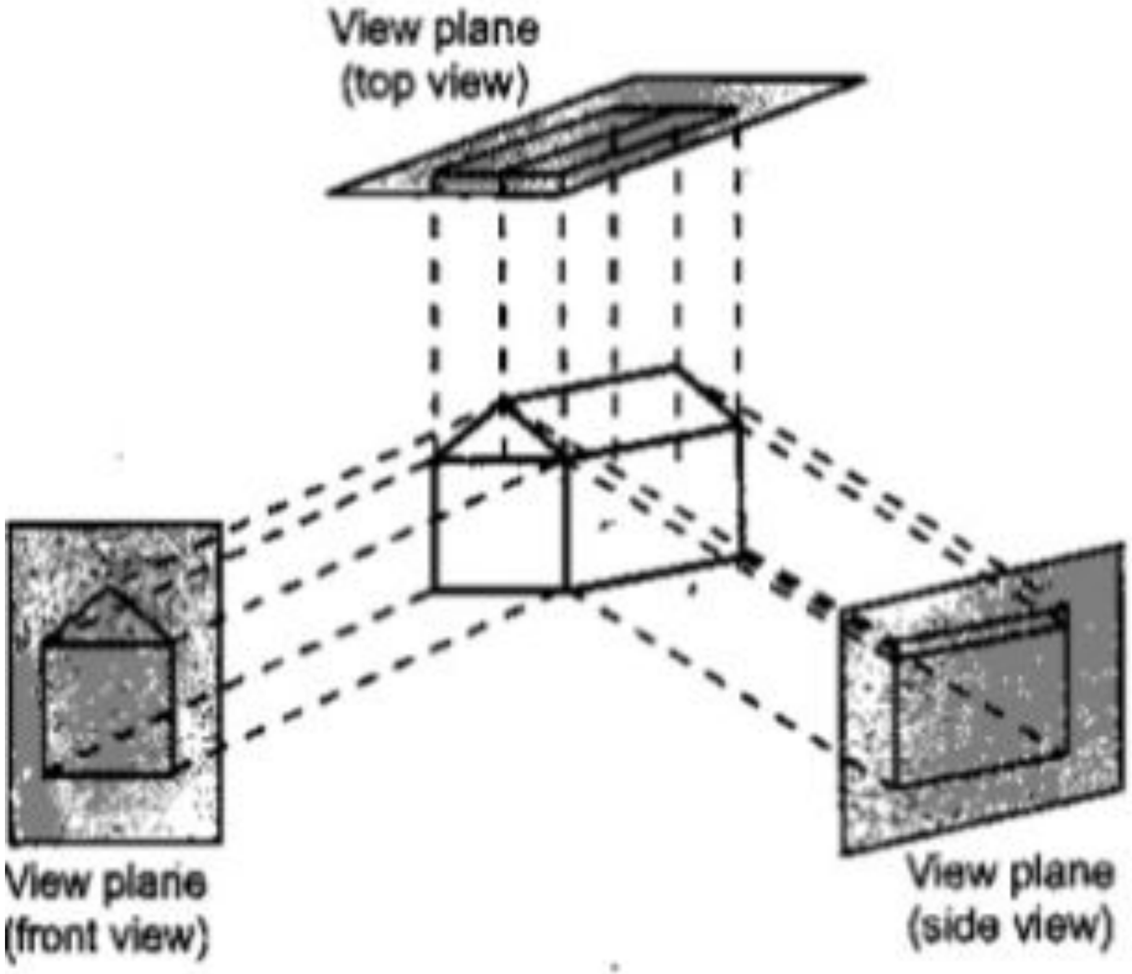




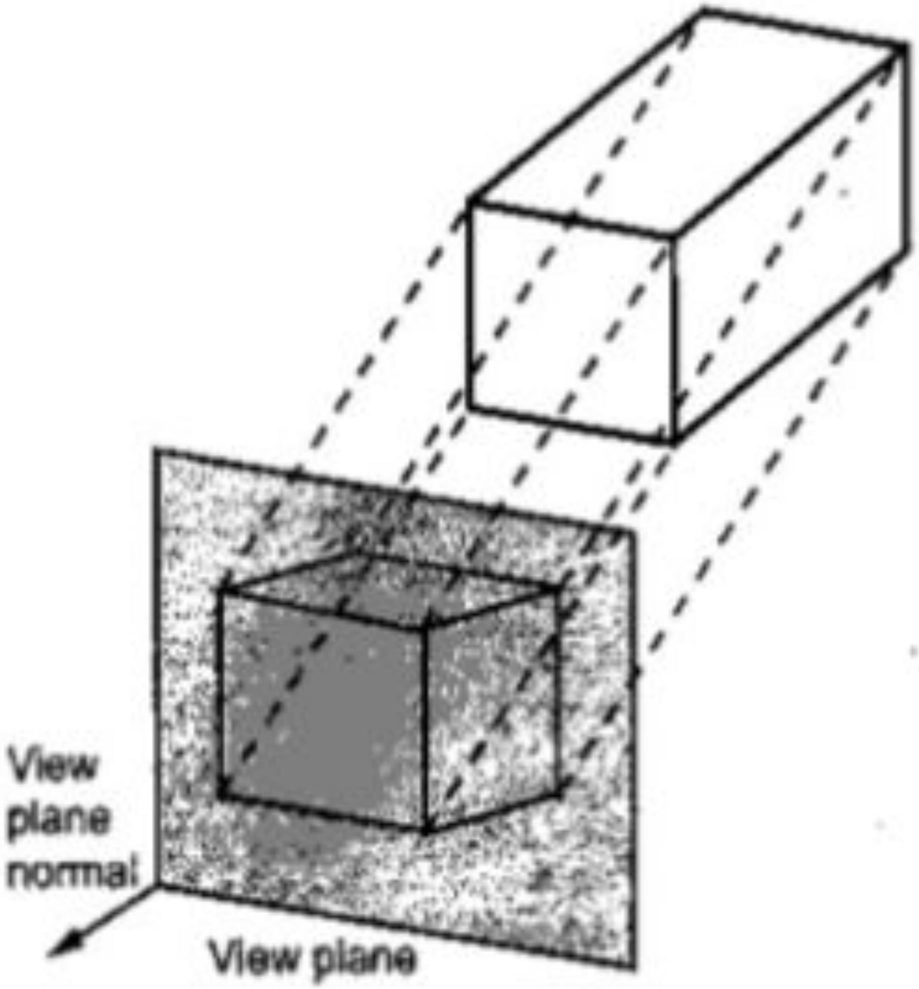
PARRALEL PROJECTIONS

- Orthographic: Direction of projection is orthogonal to the projection plane
  - Elevations: Projection plane is perpendicular to a principal axis
    - Front
    - Top (Plan)
    - Side
  - Axonometric: Projection plane is not orthogonal to a principal axis
  - Isometric: Direction of projection makes equal angles with each principal axis.
- Oblique: Direction of projection is not orthogonal to the projection plane; projection plane is normal to a principal axis
  - Cavalier: Direction of projection makes a 45° angle with the projection plane
  - Cabinet: Direction of projection makes a 63.4° angle with the projection plane

MULTIVIEW means FRONT , TOP and Side view



(a) Orthographic parallel projection



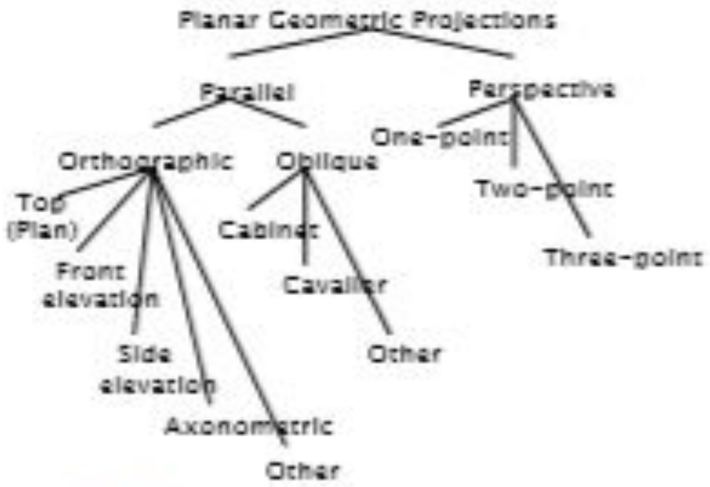
(b) Oblique parallel projection

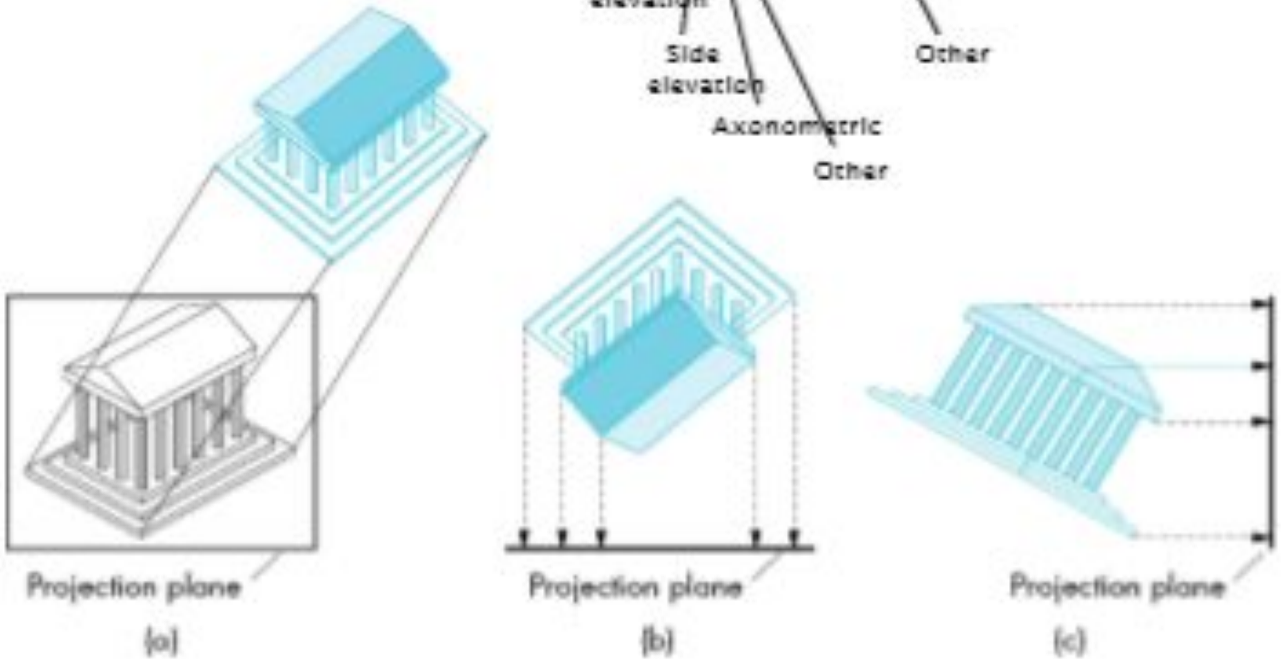
ORTHOGRAPHIC PROJECTION

- The orthographic projection can display more than one face of an object. Such as orthographic projection is called **axonometric orthographic projection.**
- It uses projection planes (view planes) that are not normal to a principle axis.
- They resemble the perspective projection in this way, but differ in that the foreshortening is uniform rather than being related to the distance from the center of projection.
- Parallelism of lines is preserved but angles are not.
- The most commonly used axonometric orthographic projection is the **isometric** projection.
- The isometric projection can be generated by aligning the view plane so that it intersects each coordinate axis in which the object is defined at the same distance from the origin.

Axonometric Projections

- Projection plane can have any orientation to object
- Parallel lines preserved
- Angles are not preserved
- Foreshortening:
  - Length is shorter in image space than in object space
  - Uniform Foreshortening (Perspective projections: foreshortening is dependent on distance from object to COP)

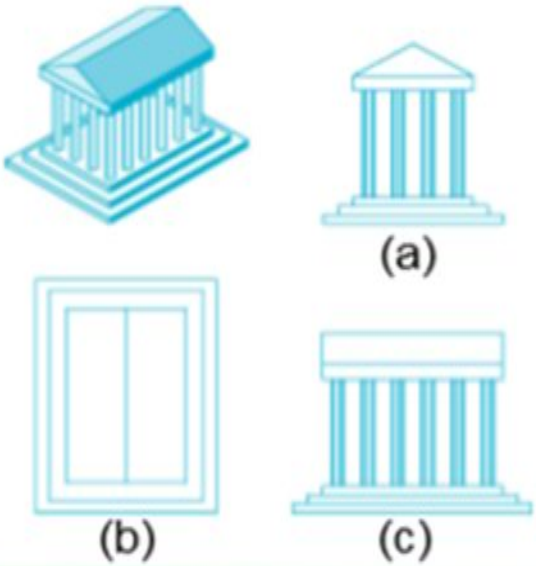




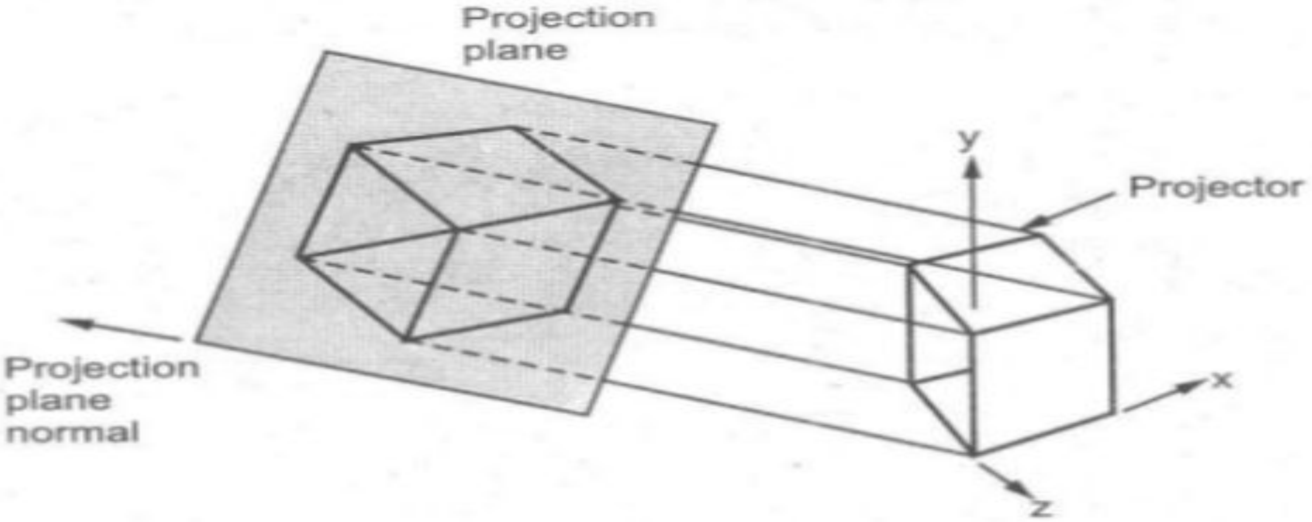
Dimetric Trimetric Isometric

- Isometric: Symmetric to three faces
- Dimetric: Symmetric to two faces
- Trimetric: General Axonometric case

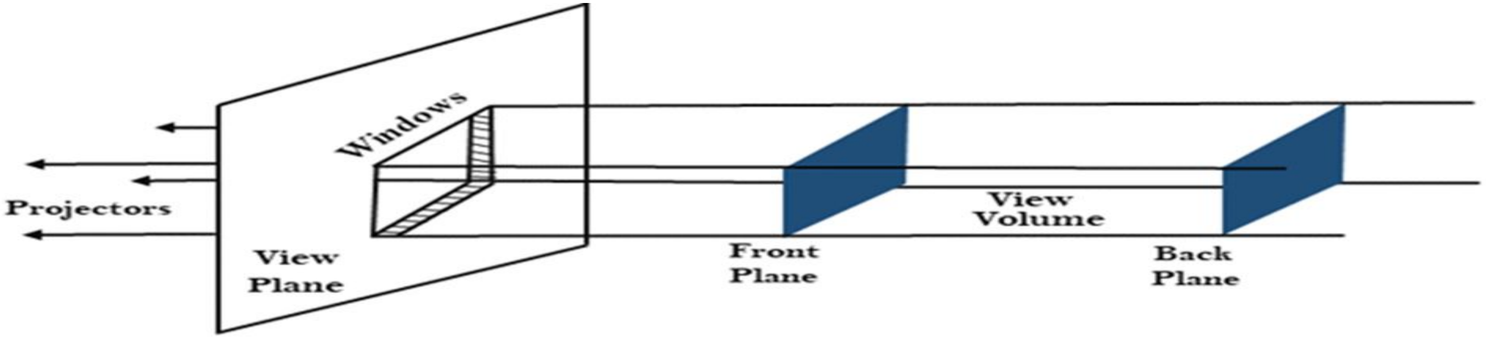
- Multiview
  - Projection Plane Parallel to Principle Faces
  - Classical Drafting Views
  - Preserves both distance and angles
  - Suitable to Object Views, not scenes
  - (a): Front-Elevation
  - (b): Top or Plan-Elevation
  - (c): Side-Elevation



Isometric projection of an object onto a viewing plane



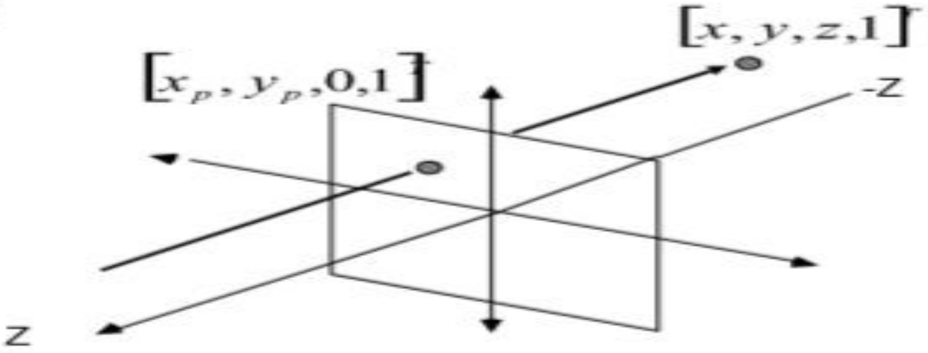
Viewing volume in orthogonal planes



(a) Viewing Volume in orthographic projection

Orthogonal Projections

- DOP parallel to z-axis
- Looking down negative z
- Display Plane at z=0
- Special Case of Perspective Projection



$$x_p = x, \quad y_p = y, \quad z_p = 0$$

$$p_{orth} = M_{orth} p = \begin{bmatrix} x_p \\ y_p \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Difference between Multiview and axonometric projections

MULTIVIEW

Preserves both distances and angles

- Shapes preserved
- Can be used for measurements
  - *Building plans*
  - *Manuals*

Cannot see what object really looks like because many surfaces hidden from view

- Often we add the isometric

AXONOMETRIC

Lines are scaled (foreshortened) but can find scaling factors

Lines preserved but angles are not

- Projection of a circle in a plane not parallel to the projection plane is an ellipse

Can see three principal faces of a box-like object

Some optical illusions possible

- Parallel lines appear to diverge

Does not look real because far objects are scaled the same as near objects

Used in CAD applications

Derive the general equation of parallel projection onto a given view plane in the direction of a given projector **V** (see Fig. 7-23).

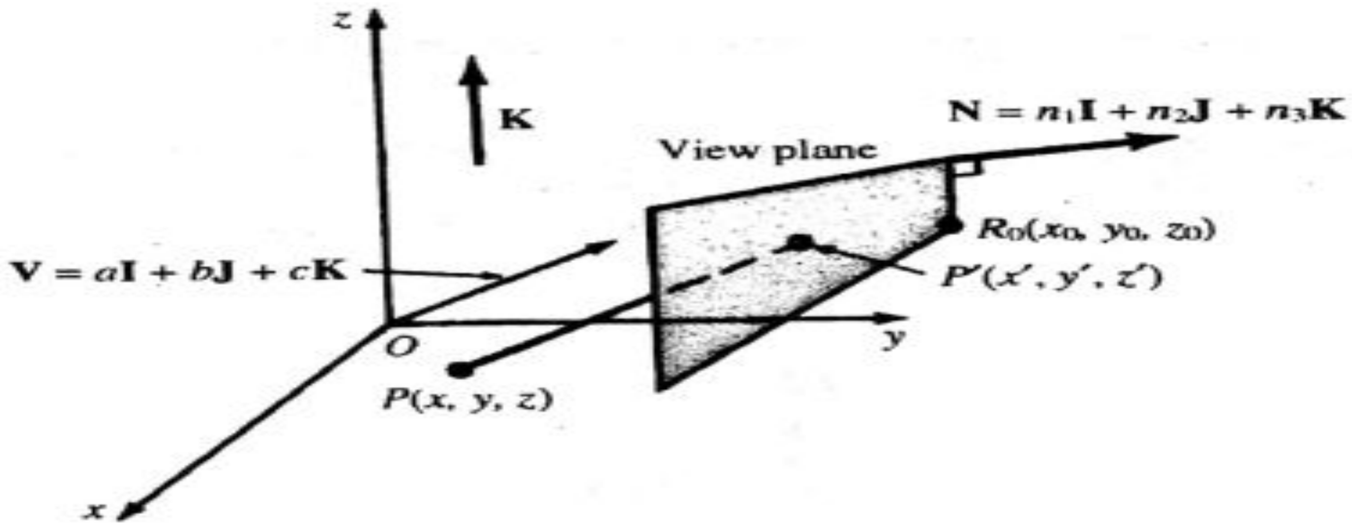


Fig. 7-23

SOLUTION

We reduce the problem to parallel projection onto the xy plane in the direction of the projector $\mathbf{V} = a\mathbf{I} + b\mathbf{J} + c\mathbf{K}$ by means of these steps:

1. Translate the view reference point R_0 of the view plane to the origin using the translation matrix T_{-R_0}.
2. Perform an alignment transformation $A_\mathbf{N}$ so that the view normal vector **N** of the view plane points in the direction **K** of the normal to the xy plane. The direction of projection vector **V** is transformed to a new vector $\mathbf{V}' = A_\mathbf{N}\mathbf{V}$.
3. Project onto the xy plane using $Par_{\mathbf{V}'}$.
4. Perform the inverse of steps 2 and 1. So finally $Par_{\mathbf{V},\mathbf{N},R_0} = T_{-R_0}^{-1} \cdot A_\mathbf{N}^{-1} \cdot Par_{\mathbf{V}'} \cdot A_\mathbf{N} \cdot T_{-R_0}$. From what we learned in Chap. 6, we know that

$$T_{-R_0} = \begin{pmatrix} 1 & 0 & 0 & -x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & 1 & -z_0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and further from Chap. 6, Prob. 6.2, where $\lambda = \sqrt{n_2^2 + n_3^2}$ and $\lambda \neq 0$, that

$$A_{\mathbf{N}} = \begin{pmatrix} \frac{\lambda}{|\mathbf{N}|} & \frac{-n_1 n_2}{\lambda|\mathbf{N}|} & \frac{-n_1 n_3}{\lambda|\mathbf{N}|} & 0 \\ 0 & \frac{n_3}{\lambda} & \frac{-n_2}{\lambda} & 0 \\ \frac{n_1}{|\mathbf{N}|} & \frac{n_2}{|\mathbf{N}|} & \frac{n_3}{|\mathbf{N}|} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Then, after multiplying, we find

$$Par_{\mathbf{V},\mathbf{N},R_0} = \begin{pmatrix} d_1 - an_1 & -an_2 & -an_3 & ad_0 \\ -bn_1 & d_1 - bn_2 & -bn_3 & bd_0 \\ -cn_1 & -cn_2 & d_1 - cn_3 & cd_0 \\ 0 & 0 & 0 & d_1 \end{pmatrix}$$

Here $d_0 = n_1x_0 + n_2y_0 + n_3z_0$ and $d_1 = n_1a + n_2b + n_3c$. An alternative and much easier method to derive this matrix is by finding the intersection of the projector through P with the equation of the view plane (see Prob. A2.14).

Derive the equations of parallel projection onto the xy plane in the direction of projection $\mathbf{V} = a\mathbf{I} + b\mathbf{J} + c\mathbf{K}$.

SOLUTION

From Fig. 7-22 we see that the vectors $\mathbf{V}$ and $\overline{\mathbf{PP'}}$ have the same direction. This means that $\overline{\mathbf{PP'}} = k\mathbf{V}$. Comparing components, we see that

$$x' - x = ka \qquad y' - y = kb \qquad z' - z = kc$$

So

$$k = -\frac{z}{c} \qquad x' = x - \frac{a}{c}z \qquad \text{and} \qquad y' = y - \frac{b}{c}z$$

In 3×3 matrix form, this is

$$Par_{\mathbf{V}} = \begin{pmatrix} 1 & 0 & -\frac{a}{c} \\ 0 & 1 & -\frac{b}{c} \\ 0 & 0 & 0 \end{pmatrix}$$

and so $P' = Par_{\mathbf{V}} \cdot P$.

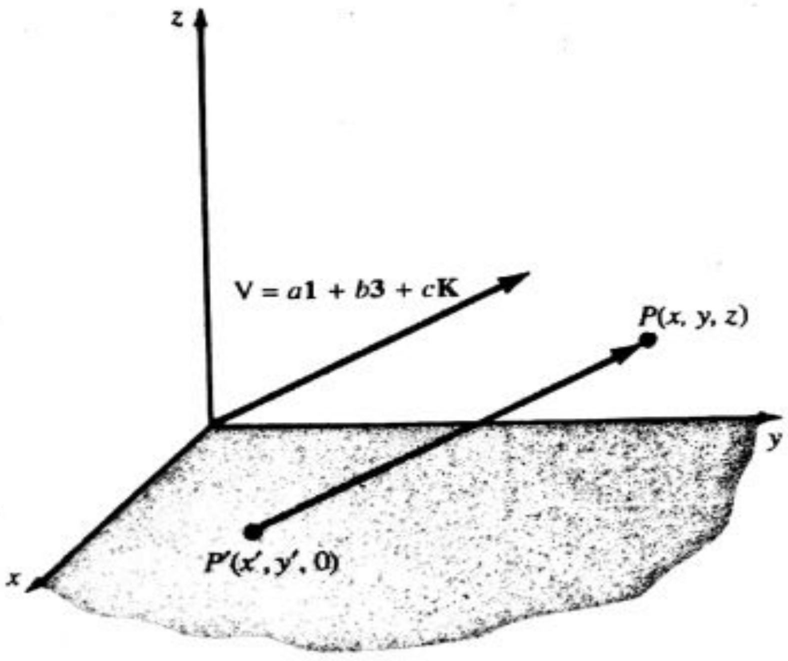


Fig. 7-22