COMPUTER GRAPHICS (RCS-603)

3D TRANSFORMATIONS

3D Transformations

- Homogeneous coordinates: *(x,y,z)=(wx,wy,wz,w)*

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \begin{bmatrix} wx \\ wy \\ wz \\ w \end{bmatrix} \text{for any } \mathrm{w} \neq 0, \text{ usually } \mathrm{w} = 1 \qquad \begin{bmatrix} x/w \\ y/w \\ z/w \end{bmatrix} \Leftarrow \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

- Transformations are now represented as 4x4 matrices

For ex:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D Translation

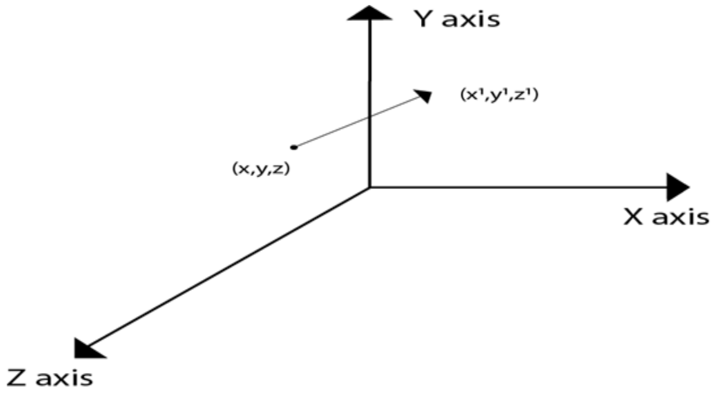
- It is the movement of an object from one position to another position.
- Translation is done using translation vectors.
- There are three vectors in 3D instead of two. These vectors are in x, y, and z directions.
- Translation in the x-direction is represented using T_x. The translation in y-direction is represented using T_y. The translation in the z- direction is represented using T_z.

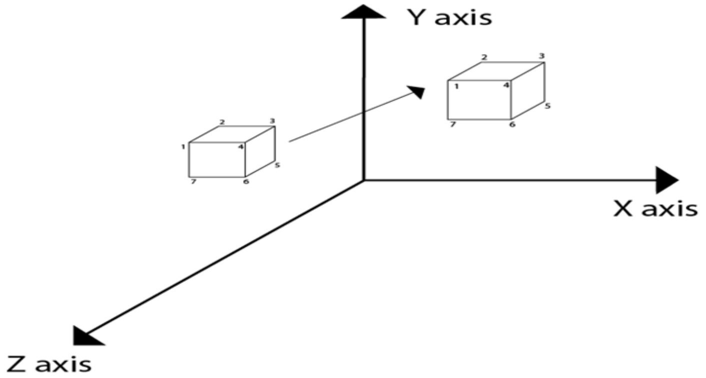
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D Translation

- Three-dimensional transformations are performed by transforming each vertex of the object.
- If an object has five corners, then the translation will be accomplished by translating all five points to new locations.

Following figure 1 shows the translation of point figure 2 shows the translation of the cube.





Example

Question: A point has coordinates in the x, y, z directions (5, 6, 7). The translation is done in the x-direction by 3 coordinate and y direction. Three coordinates and in the z- direction by two coordinates. Shift the object. Find coordinates of the new position.

Solution: Co-ordinate of the point are (5, 6, 7)

Translation vector in x direction = 3

Translation vector in y direction = 3

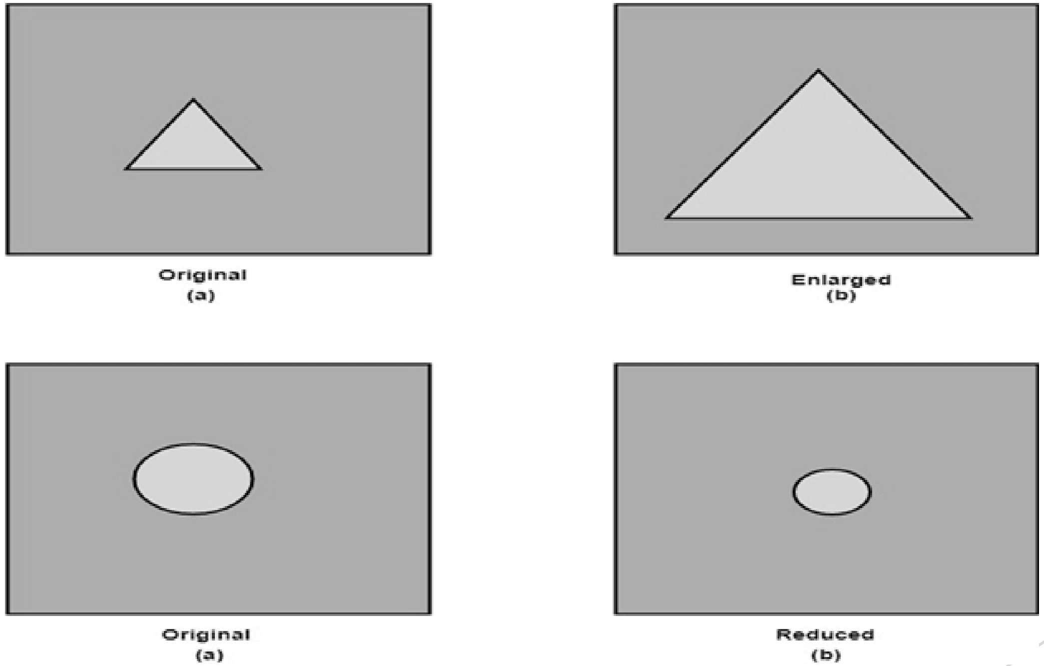
Translation vector in z direction = 2

Transformed Coordinates will be:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 7 \\ 1 \end{bmatrix}$$

Scaling

- Scaling is used to change the size of an object. The size can be increased or decreased. The scaling three factors are required S_x S_y and S_z.
- S_x=Scaling factor in x- direction
  S_y=Scaling factor in y-direction
  S_z=Scaling factor in z-direction



Scaling Matrix

- Matrix for Scaling

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Scaling of an object relative to a fixed point

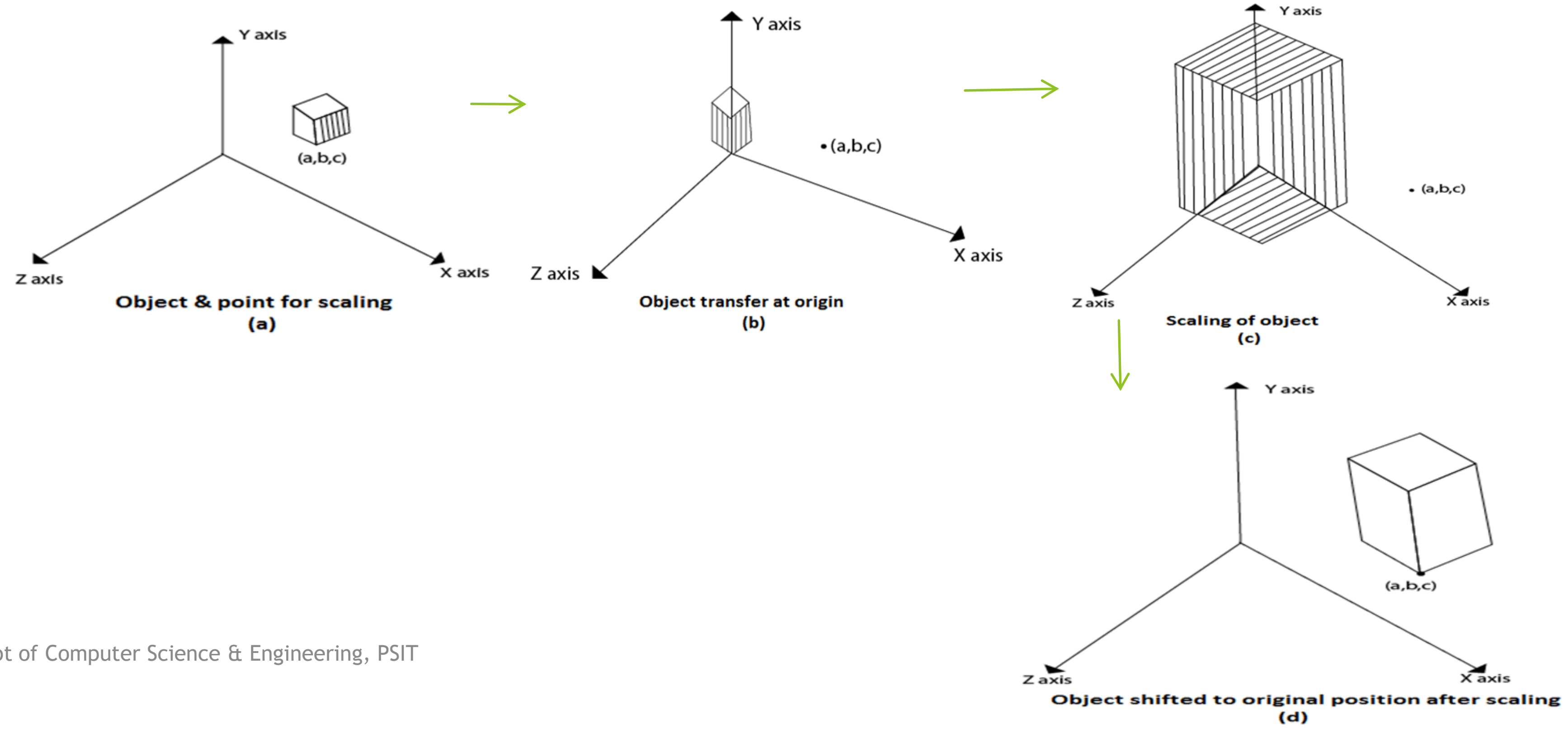
- Following are steps performed when scaling of objects with fixed point (a, b, c):

1. Translate fixed point to the origin
2. Scale the object relative to the origin
3. Translate object back to its original position.

- Note: If all scaling factors $S_x=S_y=S_z$, then scaling is called as uniform scaling. If scaling is done with different scaling vectors, it is called differential scaling.

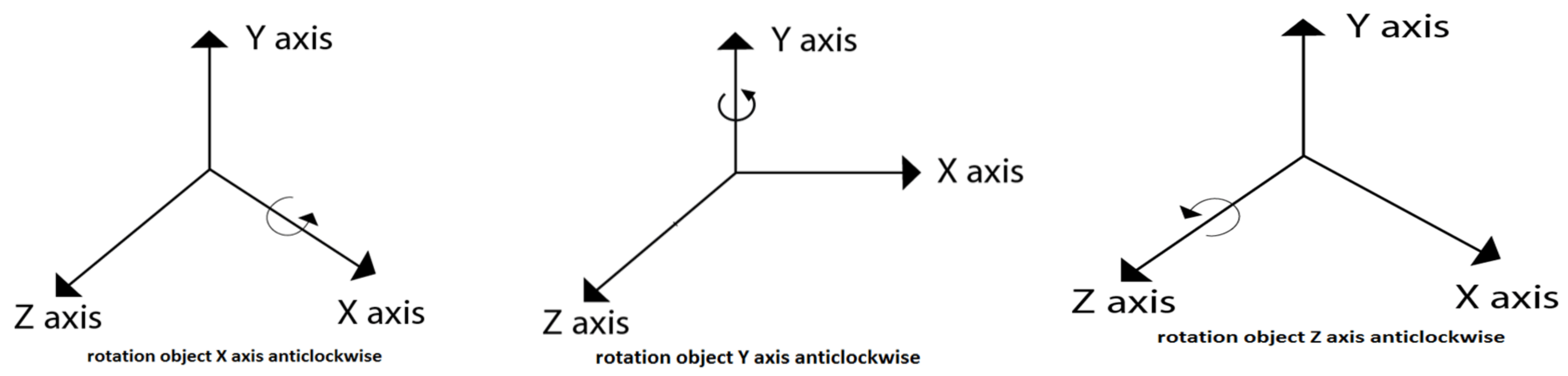
Scaling of an object relative to a fixed point

- In figure (a) point (a, b, c) is shown, and object whose scaling is to done also shown in steps in fig (b), fig (c) and fig (d).

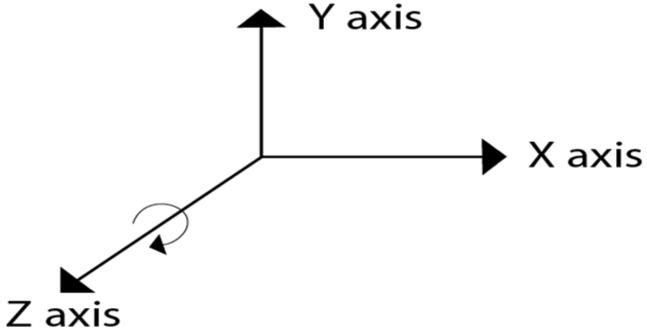


3D Rotation

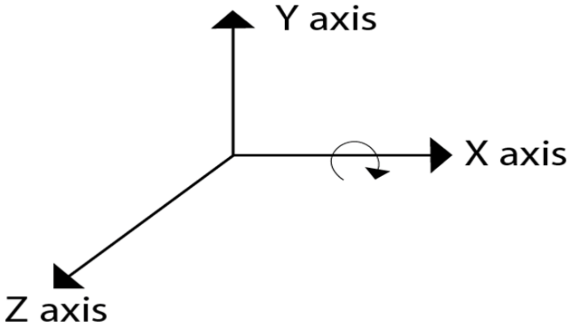
- Rotation in 3D is about an *axis* in 3D space passing through the origin
- It is moving of an object about an angle.
- Movement can be anticlockwise or clockwise. 3D rotation is complex as compared to the 2D rotation.
- For 2D we describe the angle of rotation, but for a 3D angle of rotation an axis of rotation are required. The axis can be either x or y or z.



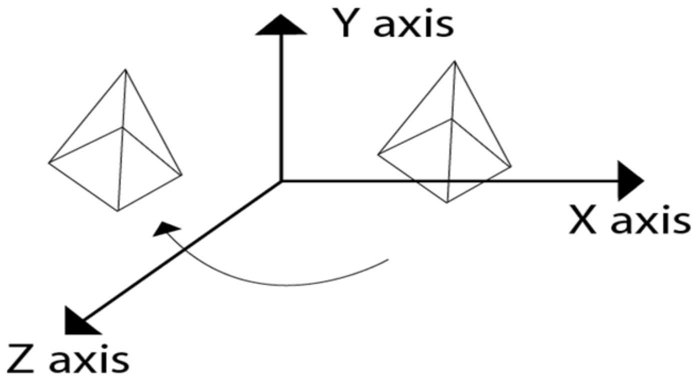
3D Rotation

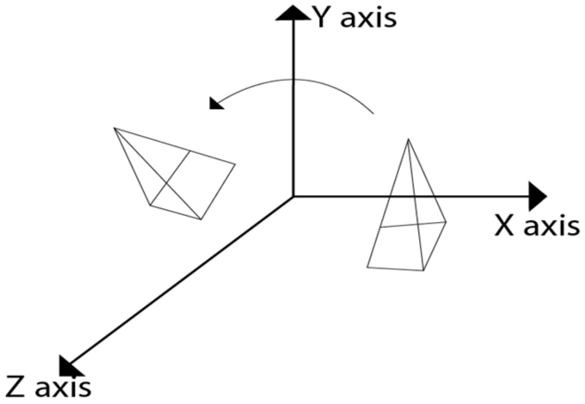


rotation about Z axis clockwise



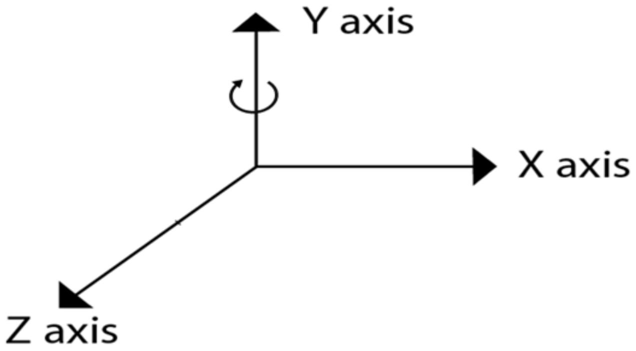
rotation about X axis clockwise





Rotation about Arbitrary Axis

- When the object is rotated about an axis that is not parallel to any one of co-ordinate axis, i.e., x, y, z. Then additional transformations are required. First of all, alignment is needed, and then the object is being back to the original position. Following steps are required
- Translate the object to the origin
- Rotate object so that axis of object coincide with any of coordinate axis.
- Perform rotation about co-ordinate axis with whom coinciding is done.
- Apply inverse rotation to bring rotation back to the original position.



rotation object Y axis clockwise

Rotation about Arbitrary Axis

- Following figure show the original position of object and position of object after rotation about the x-axis
- 5. Apply inverse translation to bring rotation axis to the original position.
- For such transformations, composite transformations are required. All the above steps are applied on points P' and P".Each step is explained using a separate figure.
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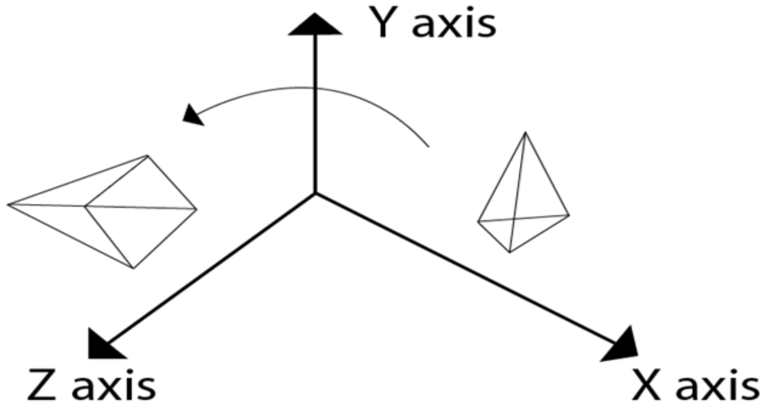
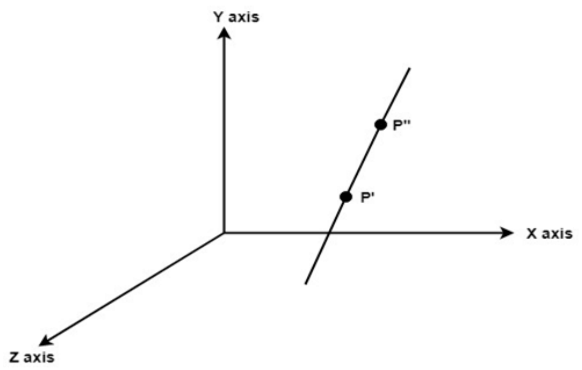
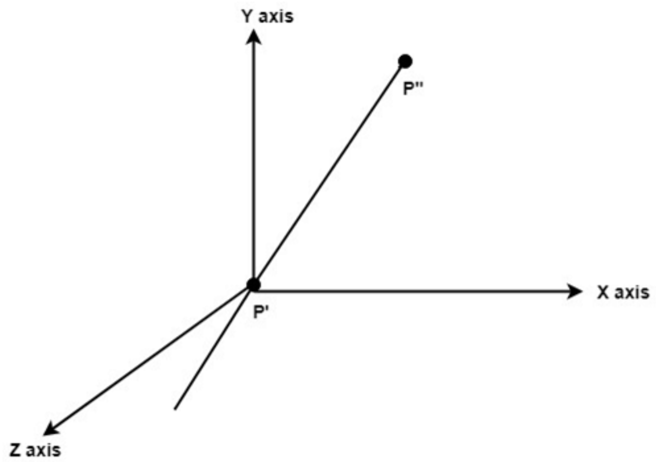


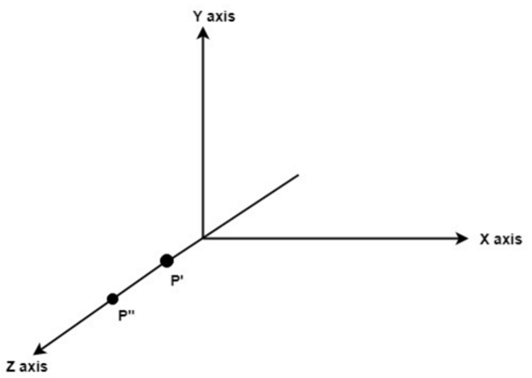
Step1: Initial position of P' and P" is shown



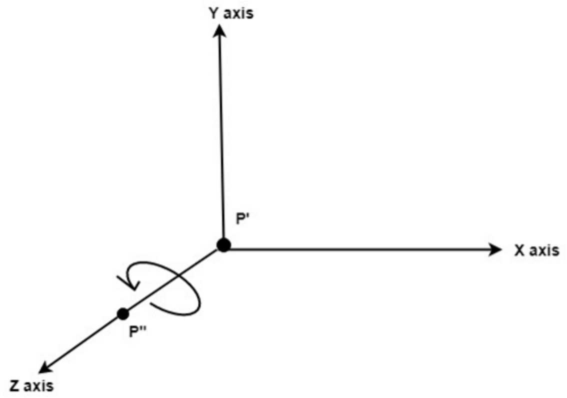
Step2: Translate object P' to origin



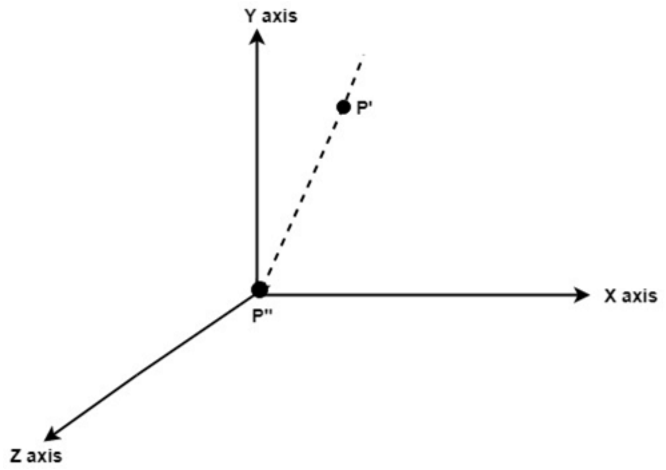
Step3: Rotate P" to z axis so that it aligns along the z-axis



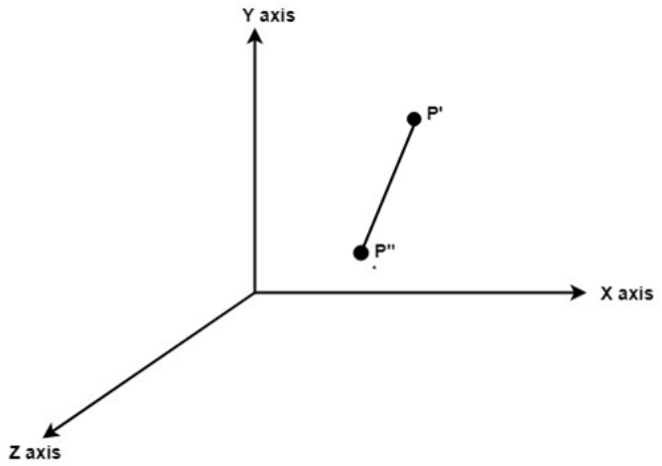
Step4: Rotate about around z- axis



Step5: Rotate axis to the original position



Step6: Translate axis to the original position.



Matrix Representations

- Matrix for representing three-dimensional rotations about the Z axis

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- Matrix for representing three-dimensional rotations about the X axis

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Matrix Representations

- Matrix for representing three-dimensional rotations about the Y axis

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Reflection

- It is also called a mirror image of an object.
- For this reflection axis and reflection of plane is selected. Three-dimensional reflections are similar to two dimensions.
- Reflection is 180° about the given axis.
- For reflection, plane is selected (xy, xz or yz).

Following matrices show reflection respect to all these three planes.

Reflection matrix for reflection relative to XY plane

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Reflection matrix for reflection relative to YZ plane

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Reflection matrix for reflection relative to ZX plane

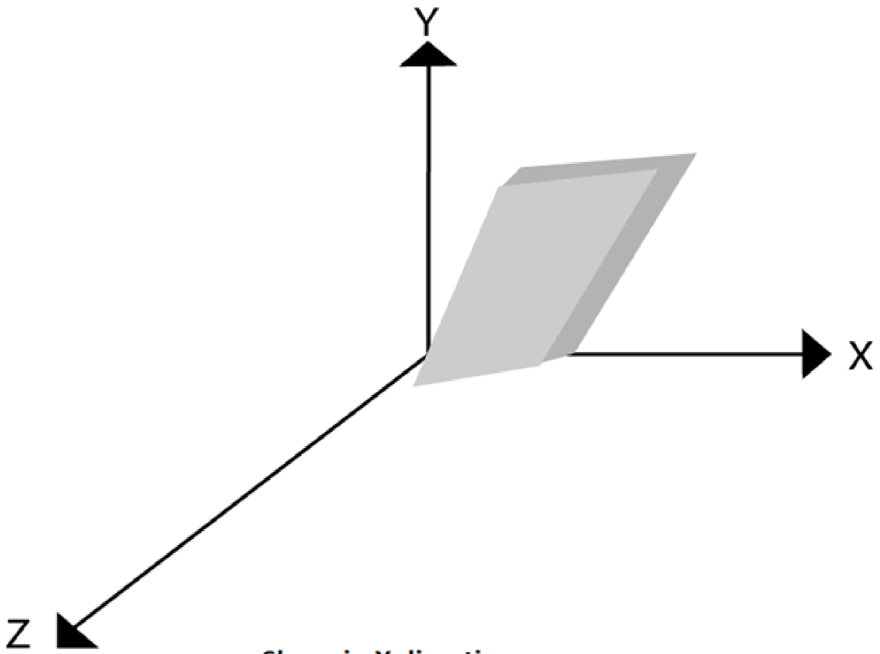
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Shearing

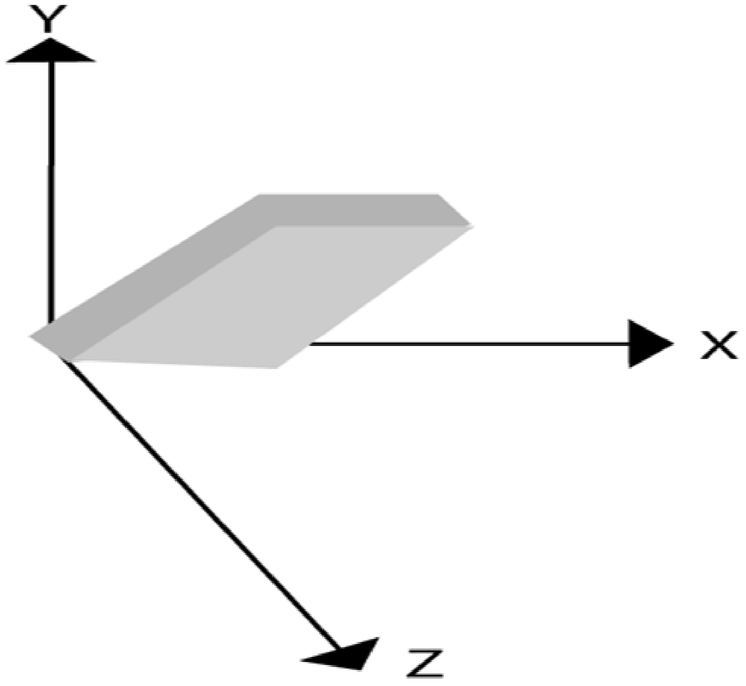
- It is change in the shape of the object.
- It is also called as deformation.
- Change can be in the x -direction or y -direction or both directions in case of 2D.
- If shear occurs in both directions, the object will be distorted.
- But in 3D shear can occur in three directions.

Matrix for shear

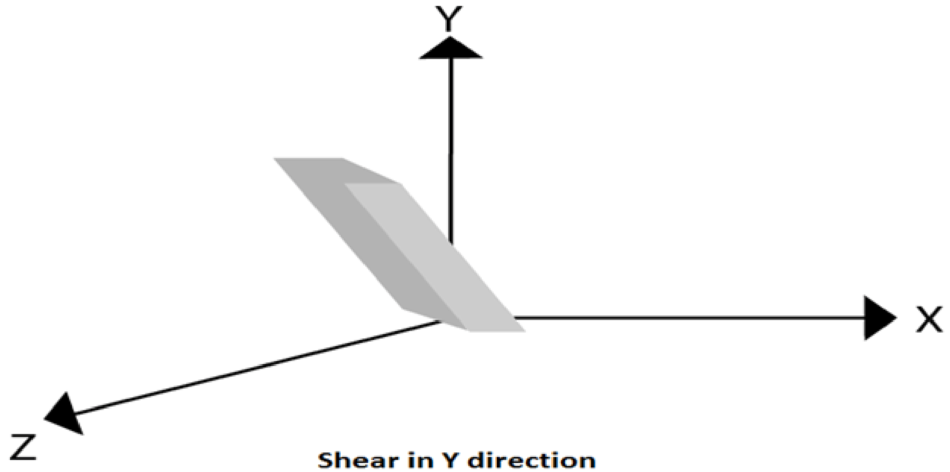
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



Shear in X direction
(a)



Shear in X, Y direction
(b)



Shear in Y direction
(c)

Thank You!