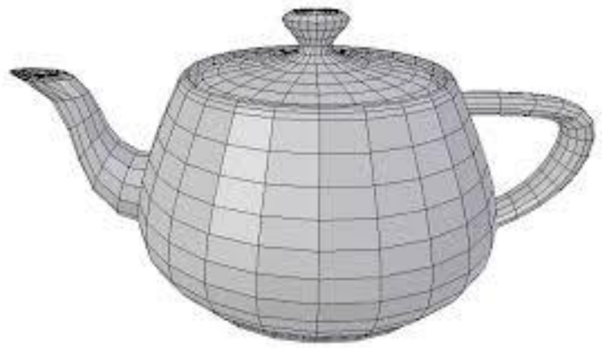
COMPUTER GRAPHICS (RCS-603)

POLYGON SURFACE, POLYGON TABLES, PLANE EQUATIONS & POLYGON MESHES

Polygon Surfaces

- Objects are represented as a collection of surfaces. 3D object representation is divided into two categories.
- **Boundary Representations:** B−reps − It describes a 3D object as a set of surfaces that separates the object interior from the environment.
- **Space-partitioning representations** − It is used to describe interior properties, by partitioning the spatial region containing an object into a set of small, non-overlapping, contiguous solids usually cubes.



Quadratic Surfaces

Introduction: A frequently used class of objects is the quadric surfaces, which are described with second-degree equations (quadratics). They include spheres, ellipsoids, tori,paraboloids, and hyperboloids. Quadric surfaces, particularly spheres and ellipsoids, are common elements of graphics scenes, and they are often available in graphics packages as primitives horn which more complex objects can be constructed.

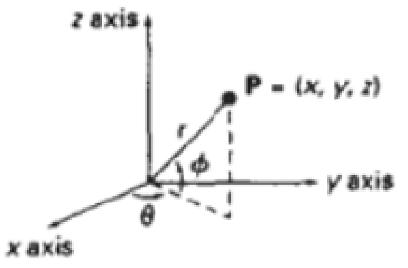


Figure 10-8
Parametric coordinate position (r, θ, ϕ) on the surface of a sphere with radius r.

Sphere:

In Cartesian coordinates, a spherical surface with radius r centered on the coordinate origin is defined as the set of points (x, y, z) that satisfy the equation

$X^2\ y^2\ z^2 = r^2$

We can also describe the spherical surface in parametric form, using latitude and longitude angles (Fig. 10-8):

Quadratic Surfaces

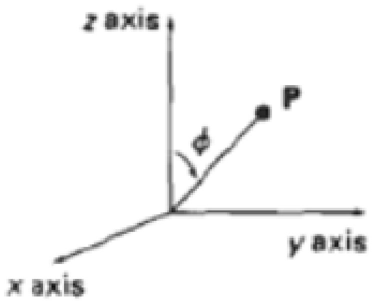


Figure 10-9
Spherical coordinate parameters (r, θ, ϕ), using colatitude for angle ϕ.

$$x = r\cos\phi\cos\theta, \qquad -\pi/2 \le \phi \le \pi/2$$

$$y = r\cos\phi\sin\theta, \qquad -\pi \le \theta \le \pi$$

$$z = r\sin\phi$$

The parametric representation inEqs. 10-8 provides a symmetric range for the angular parameters **θ**and φ. alternatively, we could write the parametric colatitude (Fig. 10-9). Then, φis defined over the range 0≤φ≤ π and **θ**is often taken in the range 0 ≤**θ≤ 2**π. We could also set up the representation using parameters u and v, defined over the range from 0 to 1 by substituting φ= πu and **θ**= 2πv.

Quadratic Surfaces

Ellipsoid: An ellipsoidal surface can be described as an extension of a spherical surface, where the radii in three mutually perpendicular directions can have different values (Fig. 10-10). The Cartesian representation for points over the surface of an ellipsoid centered on the origin is

$$\left(\frac{x}{r_x}\right)^2 + \left(\frac{y}{r_y}\right)^2 + \left(\frac{z}{r_z}\right)^2 = 1$$

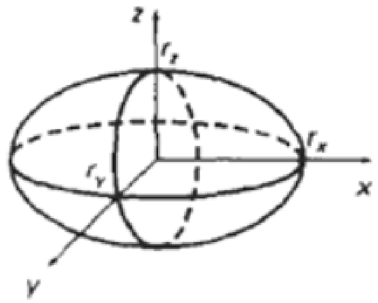


Figure 10-10
An ellipsoid with radii r_x, r_y, and r_z centered on the coordinate origin.

And a parametric representation for the ellipsoid in terms of the latitude angle 4 and the longitude angle 0 in Fig. 10-8 is

$$x = r_x \cos\phi \cos\theta, \qquad -\pi/2 \le \phi \le \pi/2$$

$$y = r_y \cos\phi \sin\theta, \qquad -\pi \le \theta \le \pi$$

$$z = r_z \sin\phi$$

Blobby Objects

- Some objects don not maintain a fixed shape, but change their surface characteristics in certain motions or when in proximity to other objects. Ex: molecular structures, water droplets, and other liquid effects, melting objects and muscle shape in human body (Shapes that show a certain degree of *fluidity*).
- These characteristics cannot be adequately described simply with basic spherical or ellipsoid shapes.
- One way to represent such objects is to model them as objects of Gaussian density functions or ‘bumps’.

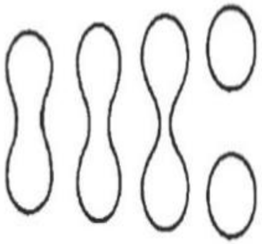


Figure 10-14
Molecular bonding. As two molecules move away from each other, the surface shapes stretch, snap, and finally contract into spheres.

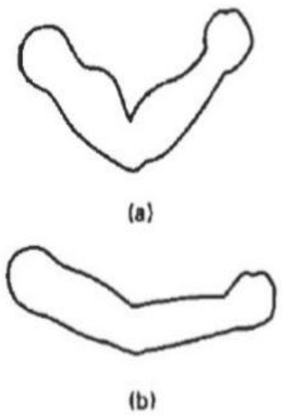
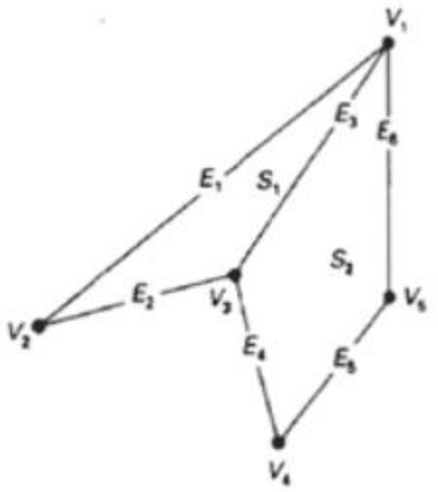


Figure 10-15
Blobby muscle shapes in a human arm.

Polygon Tables

- We specify polygon surfaces with a set of vertex coordinates and associated attribute parameters.
- As info for each polygon is input, the data are placed into tables that are to be used in the subsequent processing, display and manipulation of the objects in a scene.
- Polygon data tables can be organized into two groups: Geometric Tables and Attribute Tables.
- Geometric data tables contain vertex coordinates and parameters to identify the spatial orientation of the polygon surfaces.
- Attribute info for an object includes parameters specifying the degree of transparency of the object and its surface reflectivity and texture characteristics.

Polygon Tables



VERTEX TABLE	
V_1:	x_1, y_1, z_1
V_2:	x_2, y_2, z_2
V_3:	x_3, y_3, z_3
V_4:	x_4, y_4, z_4
V_5:	x_5, y_5, z_5

EDGE TABLE	
E_1:	V_1, V_2
E_2:	V_2, V_3
E_3:	V_3, V_1
E_4:	V_3, V_4
E_5:	V_4, V_5
E_6:	V_5, V_1

POLYGON-SURFACE TABLE	
S_1:	E_1, E_2, E_3
S_2:	E_3, E_4, E_5, E_6

Figure 10-2
Geometric data table representation for two adjacent polygon surfaces, formed with six edges and five vertices.

- A convenient way to store geometric data is to create three lists: a vertex table, an edge table and a polygon table.
- Coordinate values for each vertex in an object is stored in the vertex table.
- The edge table contains pointers back into the vertex table to identify the vertices for each polygon edge.
- The polygon table contains pointers back into the edge table to identify the edges for each polygon.

Polygon Tables

- Additional geometric info is stored the data tables that includes slopes of each edge and coordinate extents for each polygon. As vertices are input, we can calculate edge slopes, and we can scan the coordinate values to identify the minimum and maximum x, y and z values for individual polygons.
- Edge slopes and bounding box info for polygons are required in subsequent processing for example in surface rendering. Coordinate extents are used in some visible surface determination algorithms.

E_1:	V_1, V_2, S_1
E_2:	V_2, V_3, S_1
E_3:	V_3, V_1, S_1, S_2
E_4:	V_3, V_4, S_2
E_5:	V_4, V_5, S_2
E_6:	V_5, V_1, S_2

Figure 10-3
Edge table for the surfaces of Fig. 10-2 expanded to include pointers to the polygon table.

Polygon Tables

- As geometric data tables contain extensive data for complex objects, it's important to check it for consistency and completeness.
- When vertex, polygon and edge definitions are specified, it's possible that input errors are made (distortion is produced in such cases).
- Error checking is easier when these three tables provide most information.
- Some of the tests that could be performed by a graphics package are:

1. Every vertex is listed as an end point for at least two edges;
2. Every edge is a part of at least one polygon;
3. Every polygon is closed;
4. Each polygon shares at least one edge;
5. And if the edge table contains pointers back to polygons, every edge referenced by a polygon pointer has a reciprocal pointer back to the polygon.

Plane Equations

Equation of a Plane

A plane in 3-space has the equation

$$ax + by + cz = d,$$

where at least one of the numbers a, b, c must be nonzero.

As for the line, if the equation is multiplied by any nonzero constant k to get the equation kax + kby + kcz = kd, the plane of solutions is the same.

If c is not zero, it is often useful to think of the plane as the graph of a function z of x and y. The equation can be rearranged like this:

$$z = -(a/c)x + (-b/c)\, y + d/c$$

Another useful choice, when d is not zero, is to divide by d so that the constant term = 1.

$$(a/d)x + (b/d)y + (c/d)z = 1.$$

Another useful form of the equation is to **divide by |(a,b,c)|, the square root of $a^2 + b^2 + c^2$.** This choice will be explained in the **Normal Vector section.**

Exercise: Where does the plane ax + by + cz = d intersect the coordinate axes?

Exercise: What is special about the equation of a plane that passes through 0.

Plane Equations*

Finding the equation of a plane through 3 points in space

Given points P, Q, R in space, find the equation of the plane through the 3 points.

Example: P = (1, 1, 1), Q = (1, 2, 0), R = (-1, 2, 1). We seek the coefficients of an equation $ax + by + cz = d$, where P, Q and R satisfy the equations, thus:

$$a + b + c = d$$
$$a + 2b + 0c = d$$
$$-a + 2b + c = d$$

Subtracting the first equation from the second and then adding the first equation to the third, we eliminate a to get

$$b - c = 0$$
$$4b + c = 2d$$

Adding the equations gives $5b = 2d$, or $b = (2/5)d$, then solving for $c = b = (2/5)d$ and then $a = d - b - c = (1/5)d$.

So the equation (with a nonzero constant left in to choose) is $d(1/5)x + d(2/5)y + d(2/5)z = d$, so one choice of constant gives

$$x + 2y + 2z = 5$$

or another choice would be $(1/5)x + (2/5)y + (2/5)z = 1$

Given the coordinates of P, Q, R, there is a formula for the coefficients of the plane that uses determinants or **cross product**.

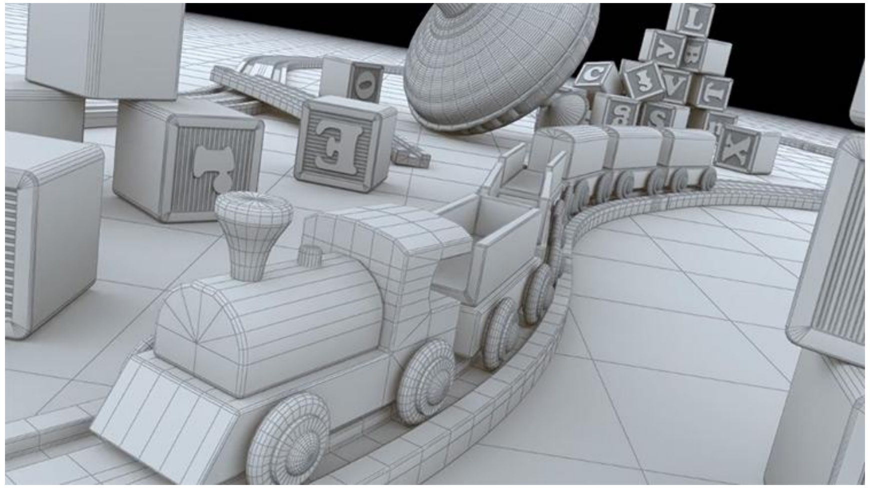
Exercise. What is equation of the plane through the points I, J, K?

Exercise: What is the equation of the plane through (1, 1, 1), (-1, 1, -1), and (1, -1, -1)?

*Find extra reference material at: https://web.ma.utexas.edu/users/m408m/Display12-5-3.shtml

Polygon Meshes

- In computer graphics a polygon mesh is the **collection of vertices, edges, polygons and faces** that make up a 3D object; or form the surface or skin of an object.
- Each vertex in the polygon mesh stores x, y, and z coordinate information.
- Then each face of that polygon contains surface information which is used by the rendering engine to calculate lightning and shadows (among other things).
- Polygon meshes can be used to model almost any object.



Thank You!