FINDING AND PROVING AN IDENTITY FOR K^(C)[p,m] where p=17 and m=0

This worksheet has **Startup Code**

```
> myseeds:=[[15, -3, -1, -2, -1, -2, -1, -2, -1],[27, -2, -2, -3,
  -2, -4, -4, -4, -4]];
```

$$myseeds := [[15, -3, -1, -2, -1, -2, -1, -2, -1], [27, -2, -2, -3, -2, -4, -4, -4, -4]] \quad \textbf{(1)}$$

NOTE : myseeds generates 2*8 = 16 functions. For m=0 we need these 16 functions. Also we need to multiply by **(eta(17*tau)/eta(tau))^(3*k)**, k=1. Thus the list [3] in the plantseeds function is needed.

```
> BIGBAS:=plantseeds(myseeds,[3],17):
  nvL:=BIGBAS:
  do_alg_steps(17,0,nvL);
```

```
-------------------------------------------------
p = 17  and m = 0
STEP 1: check modularity
        modularity checks
-------------------------------------------------
STEP 2: find k0 and divide by j0
        We skip this step since m = 0
-------------------------------------------------
STEP 3: Compute table of ORDS at all cusps for each func
```

"CUSPS: ", [[1, 0], [0, 1], [1, 2], [1, 3], [1, 4], [1, 5], [1, 6], [1, 7], [1, 8], [2, 17], [3, 17], [4, 17], [5, 17], [6, 17], [7, 17], [8, 17]]

"TABLE of ords"

1, −1, −1, −1, −1, −1, −1, −1, −1, 2, 2, 3, 2, 3, 2, 5

2, −1, −1, −1, −1, −1, −1, −1, −1, 3, 3, 5, 2, 2, 2, 1

2, −1, −1, −1, −1, −1, −1, −1, −1, 3, 5, 2, 2, 1, 3, 2

3, −1, −1, −1, −1, −1, −1, −1, −1, 5, 2, 1, 2, 2, 3, 2

2, −1, −1, −1, −1, −1, −1, −1, −1, 2, 2, 2, 5, 3, 1, 3

3, −1, −1, −1, −1, −1, −1, −1, −1, 2, 1, 2, 3, 2, 5, 2

2, −1, −1, −1, −1, −1, −1, −1, −1, 2, 3, 3, 1, 5, 2, 2

5, −1, −1, −1, −1, −1, −1, −1, −1, 1, 2, 2, 3, 2, 2, 3

7, −2, −2, −2, −2, −2, −2, −2, −2, 3, 2, 4, 3, 2, 2, 5

3, −2, −2, −2, −2, −2, −2, −2, −2, 4, 2, 5, 2, 3, 2, 7

2, −2, −2, −2, −2, −2, −2, −2, −2, 2, 5, 3, 3, 7, 4, 2

4, −2, −2, −2, −2, −2, −2, −2, −2, 5, 3, 7, 2, 2, 2, 3

3, −2, −2, −2, −2, −2, −2, −2, −2, 2, 3, 2, 5, 4, 7, 2

2, −2, −2, −2, −2, −2, −2, −2, −2, 3, 7, 2, 4, 3, 5, 2

2, −2, −2, −2, −2, −2, −2, −2, −2, 2, 4, 2, 7, 5, 3, 3

5, −2, −2, −2, −2, −2, −2, −2, −2, 7, 2, 3, 2, 2, 3, 4

3, −3, −3, −3, −3, −3, −3, −3, −3, 4, 4, 5, 4, 5, 4, 7

4, −3, −3, −3, −3, −3, −3, −3, −3, 5, 5, 7, 4, 4, 4, 3

4, −3, −3, −3, −3, −3, −3, −3, −3, 5, 7, 4, 4, 3, 5, 4

$5, -3, -3, -3, -3, -3, -3, -3, -3, 7, 4, 3, 4, 4, 5, 4$
$4, -3, -3, -3, -3, -3, -3, -3, -3, 4, 4, 4, 7, 5, 3, 5$
$5, -3, -3, -3, -3, -3, -3, -3, -3, 4, 3, 4, 5, 4, 7, 4$
$4, -3, -3, -3, -3, -3, -3, -3, -3, 4, 5, 5, 3, 7, 4, 4$
$7, -3, -3, -3, -3, -3, -3, -3, -3, 3, 4, 4, 5, 4, 4, 5$
$9, -4, -4, -4, -4, -4, -4, -4, -4, 5, 4, 6, 5, 4, 4, 7$
$5, -4, -4, -4, -4, -4, -4, -4, -4, 6, 4, 7, 4, 5, 4, 9$
$4, -4, -4, -4, -4, -4, -4, -4, -4, 4, 7, 5, 5, 9, 6, 4$
$6, -4, -4, -4, -4, -4, -4, -4, -4, 7, 5, 9, 4, 4, 4, 5$
$5, -4, -4, -4, -4, -4, -4, -4, -4, 4, 5, 4, 7, 6, 9, 4$
$4, -4, -4, -4, -4, -4, -4, -4, -4, 5, 9, 4, 6, 5, 7, 4$
$4, -4, -4, -4, -4, -4, -4, -4, -4, 4, 6, 4, 9, 7, 5, 5$
$7, -4, -4, -4, -4, -4, -4, -4, -4, 9, 4, 5, 4, 4, 5, 6$

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STEP 4: Compute LOWER BOUND for ORD of _Kpm at each cusp
```

"TABLE :"

$$\_cusp, \_LOWER\_BOUND\_of\_ORD, \_Kpm, \_at\_cusp$$
$$\_cusp = 0, \_LOWER\_BOUND = -4$$
$$\_cusp = \frac{1}{2}, \_LOWER\_BOUND = 0$$
$$\_cusp = \frac{1}{3}, \_LOWER\_BOUND = 0$$
$$\_cusp = \frac{1}{4}, \_LOWER\_BOUND = 0$$
$$\_cusp = \frac{1}{5}, \_LOWER\_BOUND = 0$$
$$\_cusp = \frac{1}{6}, \_LOWER\_BOUND = 0$$
$$\_cusp = \frac{1}{7}, \_LOWER\_BOUND = 0$$
$$\_cusp = \frac{1}{8}, \_LOWER\_BOUND = 0$$
$$\_cusp = \frac{2}{17}, \_LOWER\_BOUND = \frac{12}{17}$$
$$\_cusp = \frac{3}{17}, \_LOWER\_BOUND = \frac{12}{17}$$
$$\_cusp = \frac{4}{17}, \_LOWER\_BOUND = \frac{12}{17}$$

$$\_cusp = \frac{5}{17}, \_LOWER\_BOUND = \frac{12}{17}$$

$$\_cusp = \frac{6}{17}, \_LOWER\_BOUND = \frac{12}{17}$$

$$\_cusp = \frac{7}{17}, \_LOWER\_BOUND = \frac{12}{17}$$

$$\_cusp = \frac{8}{17}, \_LOWER\_BOUND = \frac{12}{17}$$

```
------------------------------------------------
STEP 5: Compile LHS vs RHS ORD table at cusps and find constant B
```

"TABLE ORD lower bounds"

$$\_cusp, \_width, \_ORD\_LHS, \_ORD\_RHS, \_ORD\_LHS\_minus\_RHS$$

$$0, 17, -4, -4, -4$$

$$\frac{1}{2}, 17, 0, -4, -4$$

$$\frac{1}{3}, 17, 0, -4, -4$$

$$\frac{1}{4}, 17, 0, -4, -4$$

$$\frac{1}{5}, 17, 0, -4, -4$$

$$\frac{1}{6}, 17, 0, -4, -4$$

$$\frac{1}{7}, 17, 0, -4, -4$$

$$\frac{1}{8}, 17, 0, -4, -4$$

$$\frac{2}{17}, 1, 1, 1, 1$$

$$\frac{3}{17}, 1, 1, 1, 1$$

$$\frac{4}{17}, 1, 1, 1, 1$$

$$\frac{5}{17}, 1, 1, 1, 1$$

$$\frac{6}{17}, 1, 1, 1, 1$$

$$\frac{7}{17}, 1, 1, 1, 1$$

$$\frac{8}{17}, 1, 1, 1, 1$$

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        This implies that B = -25
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STEP 6: Prove and check identity
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"Coefficients in CKpm identity"

$$\_k=1, -\zeta^{15}-\zeta^{13}-\zeta^{11}-\zeta^{10}-\zeta^{9}-\zeta^{8}-\zeta^{7}-\zeta^{6}-\zeta^{4}-\zeta^{2}$$

$$\_k=2, -4\,\zeta^{15}-3\,\zeta^{13}-3\,\zeta^{11}-\zeta^{10}-2\,\zeta^{9}-2\,\zeta^{8}-\zeta^{7}-3\,\zeta^{6}-3\,\zeta^{4}-4\,\zeta^{2}-3$$

$$\_k=3, -\zeta^{15}-3\,\zeta^{14}-2\,\zeta^{12}-3\,\zeta^{11}-2\,\zeta^{9}-2\,\zeta^{8}-3\,\zeta^{6}-2\,\zeta^{5}-3\,\zeta^{3}-\zeta^{2}-4$$

$$\_k=4, \zeta^{15}-2\,\zeta^{13}-\zeta^{12}+\zeta^{11}-\zeta^{9}-\zeta^{8}+\zeta^{6}-\zeta^{5}-2\,\zeta^{4}+\zeta^{2}-2$$

$$\_k=5, 3\,\zeta^{15}+2\,\zeta^{14}+\zeta^{13}+3\,\zeta^{12}+4\,\zeta^{10}+\zeta^{9}+\zeta^{8}+4\,\zeta^{7}+3\,\zeta^{5}+\zeta^{4}+2\,\zeta^{3}+3\,\zeta^{2}+4$$

$$\_k=6, -\zeta^{14}+\zeta^{12}+\zeta^{11}+\zeta^{10}-\zeta^{9}-\zeta^{8}+\zeta^{7}+\zeta^{6}+\zeta^{5}-\zeta^{3}+2$$

$$\_k=7, -\zeta^{15}+\zeta^{14}+\zeta^{13}-2\,\zeta^{12}+2\,\zeta^{10}-\zeta^{9}-\zeta^{8}+2\,\zeta^{7}-2\,\zeta^{5}+\zeta^{4}+\zeta^{3}-\zeta^{2}+1$$

$$\_k=8, \zeta^{15}+\zeta^{14}+2\,\zeta^{13}+\zeta^{12}+\zeta^{11}+\zeta^{10}+\zeta^{7}+\zeta^{6}+\zeta^{5}+2\,\zeta^{4}+\zeta^{3}+\zeta^{2}$$

$$\_k=9, -26\,\zeta^{15}-46\,\zeta^{14}+7\,\zeta^{13}-78\,\zeta^{12}-40\,\zeta^{11}-19\,\zeta^{10}-30\,\zeta^{9}-30\,\zeta^{8}-19\,\zeta^{7}-40\,\zeta^{6}-78\,\zeta^{5}+7\,\zeta^{4}-46\,\zeta^{3}-26\,\zeta^{2}-97$$

$$\_k=10, -15\,\zeta^{15}-32\,\zeta^{14}-21\,\zeta^{13}-46\,\zeta^{12}-28\,\zeta^{11}-65\,\zeta^{10}+2\,\zeta^{9}+2\,\zeta^{8}-65\,\zeta^{7}-28\,\zeta^{6}-46\,\zeta^{5}-21\,\zeta^{4}-32\,\zeta^{3}-15\,\zeta^{2}-100$$

$$\_k=11, 57\,\zeta^{15}+24\,\zeta^{14}+78\,\zeta^{13}+38\,\zeta^{12}+50\,\zeta^{11}+45\,\zeta^{10}+34\,\zeta^{9}+34\,\zeta^{8}+45\,\zeta^{7}+50\,\zeta^{6}+38\,\zeta^{5}+78\,\zeta^{4}+24\,\zeta^{3}+57\,\zeta^{2}+62$$

$$\_k=12, 34\,\zeta^{15}+14\,\zeta^{14}+38\,\zeta^{13}+27\,\zeta^{12}-12\,\zeta^{11}+92\,\zeta^{10}-10\,\zeta^{9}-10\,\zeta^{8}+92\,\zeta^{7}-12\,\zeta^{6}+27\,\zeta^{5}+38\,\zeta^{4}+14\,\zeta^{3}+34\,\zeta^{2}+59$$

$$\_k=13, -71\,\zeta^{15}+7\,\zeta^{14}-27\,\zeta^{13}+21\,\zeta^{12}-47\,\zeta^{11}-31\,\zeta^{10}-23\,\zeta^{9}-23\,\zeta^{8}-31\,\zeta^{7}-47\,\zeta^{6}+21\,\zeta^{5}-27\,\zeta^{4}+7\,\zeta^{3}-71\,\zeta^{2}-32$$

$$\_k=14, -72\,\zeta^{15}-14\,\zeta^{14}-46\,\zeta^{13}-7\,\zeta^{12}-58\,\zeta^{11}-52\,\zeta^{10}-24\,\zeta^{9}-24\,\zeta^{8}-52\,\zeta^{7}-58\,\zeta^{6}-7\,\zeta^{5}-46\,\zeta^{4}-14\,\zeta^{3}-72\,\zeta^{2}-32$$

$$\_k=15, 21\,\zeta^{15}-14\,\zeta^{14}-33\,\zeta^{13}-38\,\zeta^{12}+7\,\zeta^{11}-7\,\zeta^{10}-16\,\zeta^{9}-16\,\zeta^{8}-7\,\zeta^{7}+7\,\zeta^{6}-38\,\zeta^{5}-33\,\zeta^{4}-14\,\zeta^{3}+21\,\zeta^{2}-27$$

$$\_k=16, 12\,\zeta^{15}+13\,\zeta^{14}+38\,\zeta^{13}-33\,\zeta^{12}-20\,\zeta^{11}+59\,\zeta^{10}-10\,\zeta^{9}-10\,\zeta^{8}+59\,\zeta^{7}-20\,\zeta^{6}-33\,\zeta^{5}+38\,\zeta^{4}+13\,\zeta^{3}+12\,\zeta^{2}+1$$

$$\_k=17, -17\,\zeta^{15}-34\,\zeta^{13}-17\,\zeta^{12}-51\,\zeta^{11}-68\,\zeta^{9}-68\,\zeta^{8}-51\,\zeta^{6}-17\,\zeta^{5}-34\,\zeta^{4}-17\,\zeta^{2}+85$$

$$\_k=18,\ -306\,\zeta^{15}-17\,\zeta^{14}-255\,\zeta^{13}-51\,\zeta^{12}-272\,\zeta^{11}-51\,\zeta^{10}-204\,\zeta^{9}-204\,\zeta^{8}-51\,\zeta^{7}-272\,\zeta^{6}-51\,\zeta^{5}-255\,\zeta^{4}-17\,\zeta^{3}-306\,\zeta^{2}-289$$

$$\_k=19,\ 136\,\zeta^{15}+221\,\zeta^{14}-17\,\zeta^{13}+136\,\zeta^{12}+255\,\zeta^{11}-34\,\zeta^{10}+187\,\zeta^{9}+187\,\zeta^{8}-34\,\zeta^{7}+255\,\zeta^{6}+136\,\zeta^{5}-17\,\zeta^{4}+221\,\zeta^{3}+136\,\zeta^{2}+255$$

$$\_k=20,\ 170\,\zeta^{15}+34\,\zeta^{14}-85\,\zeta^{13}+170\,\zeta^{11}+51\,\zeta^{10}-34\,\zeta^{9}-34\,\zeta^{8}+51\,\zeta^{7}+170\,\zeta^{6}-85\,\zeta^{4}+34\,\zeta^{3}+170\,\zeta^{2}-34$$

$$\_k=21,\ -204\,\zeta^{15}-119\,\zeta^{14}-255\,\zeta^{12}+17\,\zeta^{11}-255\,\zeta^{10}-51\,\zeta^{9}-51\,\zeta^{8}-255\,\zeta^{7}+17\,\zeta^{6}-255\,\zeta^{5}-119\,\zeta^{3}-204\,\zeta^{2}-289$$

$$\_k=22,\ 17\,\zeta^{15}+85\,\zeta^{14}+51\,\zeta^{13}-34\,\zeta^{12}-119\,\zeta^{11}+68\,\zeta^{9}+68\,\zeta^{8}-119\,\zeta^{6}-34\,\zeta^{5}+51\,\zeta^{4}+85\,\zeta^{3}+17\,\zeta^{2}-136$$

$$\_k=23,\ 102\,\zeta^{15}-136\,\zeta^{14}-34\,\zeta^{13}+85\,\zeta^{12}+17\,\zeta^{11}-170\,\zeta^{10}+68\,\zeta^{9}+68\,\zeta^{8}-170\,\zeta^{7}+17\,\zeta^{6}+85\,\zeta^{5}-34\,\zeta^{4}-136\,\zeta^{3}+102\,\zeta^{2}-153$$

$$\_k=24,\ 34\,\zeta^{14}+136\,\zeta^{13}+34\,\zeta^{12}+17\,\zeta^{11}-17\,\zeta^{10}-17\,\zeta^{9}-17\,\zeta^{8}-17\,\zeta^{7}+17\,\zeta^{6}+34\,\zeta^{5}+136\,\zeta^{4}+34\,\zeta^{3}-85$$

$$\_k=25,\ -578\,\zeta^{13}-289\,\zeta^{10}-289\,\zeta^{7}-578\,\zeta^{4}$$

$$\_k=26,\ -578\,\zeta^{15}-578\,\zeta^{13}-289\,\zeta^{11}-289\,\zeta^{10}-289\,\zeta^{9}-289\,\zeta^{8}-289\,\zeta^{7}-289\,\zeta^{6}-578\,\zeta^{4}-578\,\zeta^{2}$$

$$\_k=27,\ 289\,\zeta^{12}+289\,\zeta^{11}-578\,\zeta^{10}+289\,\zeta^{9}+289\,\zeta^{8}-578\,\zeta^{7}+289\,\zeta^{6}+289\,\zeta^{5}$$

$$\_k=28,\ 578\,\zeta^{15}+289\,\zeta^{13}-289\,\zeta^{12}+578\,\zeta^{11}+289\,\zeta^{9}+289\,\zeta^{8}+578\,\zeta^{6}-289\,\zeta^{5}+289\,\zeta^{4}+578\,\zeta^{2}$$

$$\_k=29,\ -578\,\zeta^{15}-289\,\zeta^{14}-289\,\zeta^{13}-578\,\zeta^{12}-578\,\zeta^{11}-578\,\zeta^{10}-289\,\zeta^{9}-289\,\zeta^{8}-578\,\zeta^{7}-578\,\zeta^{6}-578\,\zeta^{5}-289\,\zeta^{4}-289\,\zeta^{3}-578\,\zeta^{2}-1156$$

$$\_k=30,\ -289\,\zeta^{15}-289\,\zeta^{14}-578\,\zeta^{12}-578\,\zeta^{11}-289\,\zeta^{9}-289\,\zeta^{8}-578\,\zeta^{6}-578\,\zeta^{5}-289\,\zeta^{3}-289\,\zeta^{2}-578$$

$$\_k=31,\ 578\,\zeta^{15}+289\,\zeta^{14}+289\,\zeta^{13}+289\,\zeta^{12}+289\,\zeta^{11}+867\,\zeta^{10}+867\,\zeta^{7}+289\,\zeta^{6}+289\,\zeta^{5}+289\,\zeta^{4}+289\,\zeta^{3}+578\,\zeta^{2}+867$$

$$\_k=32,\ 289\,\zeta^{15}-289\,\zeta^{14}-289\,\zeta^{13}-289\,\zeta^{12}+289\,\zeta^{11}-289\,\zeta^{10}-289\,\zeta^{7}+289\,\zeta^{6}-289\,\zeta^{5}-289\,\zeta^{4}-289\,\zeta^{3}+289\,\zeta^{2}-289$$

"Proving and checking identity"

"IDENTITY CHECKED AND PROVEN"

"IDENTITY checked for ", $\_O\left(q^{\_topq + 1}\right) = \_O\left(q^{118}\right)$

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and _topq + 1 > -_B + GAMMA1INDEX/12 = 25 + 12 = 37
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