FINDING AND PROVING AN IDENTITY FOR K^(C)[p,m] where p=19 and m=0

This worksheet has **Startup Code**

```
> myseeds:=[[27, -3, -2, -4, -4, -3, -3, -2, -3, -1], [39, -5, -2,
  -5, -5, -3, -5, -2, -5, -5], [39, -5, -4, -3, -4, -5, -4, -4, -5,
  -3]];
```

$$myseeds := \left[[27, -3, -2, -4, -4, -3, -3, -2, -3, -1], [39, -5, -2, -5, -5, -3, -5, \quad \textbf{(1)} \right.$$
$$\left. -2, -5, -5], [39, -5, -4, -3, -4, -5, -4, -4, -5, -3]] \right]$$

NOTE : myseeds generates $3*9 = 27$ functions. For m=0 we need these 27 functions. Also, we need to multiply by **(eta(19*tau)/eta(tau))^(4*k)**, k=1. Thus the list [4] in the plantseeds function is needed.

```
> BIGBAS:=plantseeds(myseeds,[4],19):
  nvL:=BIGBAS:
  do_alg_steps(19,0,nvL);
----------------------------------------------------
p = 19   and m = 0
STEP 1: check modularity
        modularity checks
----------------------------------------------------
STEP 2: find k0 and divide by j0
        We skip this step since m = 0
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STEP 3: Compute table of ORDS at all cusps for each func
```

"CUSPS: ", $[[1,0],[0,1],[1,2],[1,3],[1,4],[1,5],[1,6],[1,7],[1,8],[1,9],[2,19],[3,19],$
$\quad [4,19],[5,19],[6,19],[7,19],[8,19],[9,19]]$

$$\text{"TABLE of ords"}$$

$$2, -2, -2, -2, -2, -2, -2, -2, -2, -2, 7, 4, 4, 1, 3, 2, 6, 4$$
$$7, -2, -2, -2, -2, -2, -2, -2, -2, -2, 4, 3, 6, 4, 2, 1, 4, 2$$
$$4, -2, -2, -2, -2, -2, -2, -2, -2, -2, 3, 4, 2, 4, 2, 7, 1, 6$$
$$4, -2, -2, -2, -2, -2, -2, -2, -2, -2, 6, 2, 4, 2, 1, 4, 3, 7$$
$$1, -2, -2, -2, -2, -2, -2, -2, -2, -2, 4, 4, 2, 3, 6, 4, 7, 2$$
$$3, -2, -2, -2, -2, -2, -2, -2, -2, -2, 2, 2, 1, 6, 7, 4, 4, 4$$
$$2, -2, -2, -2, -2, -2, -2, -2, -2, -2, 1, 7, 4, 4, 4, 6, 2, 3$$
$$6, -2, -2, -2, -2, -2, -2, -2, -2, -2, 4, 1, 3, 7, 4, 2, 2, 4$$
$$4, -2, -2, -2, -2, -2, -2, -2, -2, -2, 2, 6, 7, 2, 4, 3, 4, 1$$
$$5, -3, -3, -3, -3, -3, -3, -3, -3, -3, 3, 6, 3, 3, 2, 3, 11, 6$$
$$3, -3, -3, -3, -3, -3, -3, -3, -3, -3, 3, 2, 11, 6, 3, 3, 6, 5$$
$$6, -3, -3, -3, -3, -3, -3, -3, -3, -3, 2, 6, 3, 3, 5, 3, 3, 11$$
$$3, -3, -3, -3, -3, -3, -3, -3, -3, -3, 11, 3, 6, 5, 3, 6, 2, 3$$
$$3, -3, -3, -3, -3, -3, -3, -3, -3, -3, 6, 3, 5, 2, 11, 6, 3, 3$$
$$2, -3, -3, -3, -3, -3, -3, -3, -3, -3, 3, 5, 3, 11, 3, 3, 6, 6$$
$$3, -3, -3, -3, -3, -3, -3, -3, -3, -3, 3, 3, 6, 6, 3, 11, 5, 2$$
$$11, -3, -3, -3, -3, -3, -3, -3, -3, -3, 6, 3, 2, 3, 6, 5, 3, 3$$
$$6, -3, -3, -3, -3, -3, -3, -3, -3, -3, 5, 11, 3, 3, 6, 2, 3, 3$$

4, −3, −3, −3, −3, −3, −3, −3, −3, −3, 5, 3, 4, 4, 8, 4, 5, 5

5, −3, −3, −3, −3, −3, −3, −3, −3, −3, 4, 8, 5, 5, 4, 4, 3, 4

3, −3, −3, −3, −3, −3, −3, −3, −3, −3, 8, 5, 4, 4, 4, 5, 4, 5

4, −3, −3, −3, −3, −3, −3, −3, −3, −3, 5, 4, 3, 4, 4, 5, 8, 5

4, −3, −3, −3, −3, −3, −3, −3, −3, −3, 5, 4, 4, 8, 5, 3, 5, 4

8, −3, −3, −3, −3, −3, −3, −3, −3, −3, 4, 4, 4, 5, 5, 4, 5, 3

4, −3, −3, −3, −3, −3, −3, −3, −3, −3, 4, 5, 5, 3, 4, 5, 4, 8

5, −3, −3, −3, −3, −3, −3, −3, −3, −3, 3, 4, 8, 5, 5, 4, 4, 4

5, −3, −3, −3, −3, −3, −3, −3, −3, −3, 4, 5, 5, 4, 3, 8, 4, 4

5, −5, −5, −5, −5, −5, −5, −5, −5, −5, 10, 7, 7, 4, 6, 5, 9, 7

10, −5, −5, −5, −5, −5, −5, −5, −5, −5, 7, 6, 9, 7, 5, 4, 7, 5

7, −5, −5, −5, −5, −5, −5, −5, −5, −5, 6, 7, 5, 7, 5, 10, 4, 9

7, −5, −5, −5, −5, −5, −5, −5, −5, −5, 9, 5, 7, 5, 4, 7, 6, 10

4, −5, −5, −5, −5, −5, −5, −5, −5, −5, 7, 7, 5, 6, 9, 7, 10, 5

6, −5, −5, −5, −5, −5, −5, −5, −5, −5, 5, 5, 4, 9, 10, 7, 7, 7

5, −5, −5, −5, −5, −5, −5, −5, −5, −5, 4, 10, 7, 7, 7, 9, 5, 6

9, −5, −5, −5, −5, −5, −5, −5, −5, −5, 7, 4, 6, 10, 7, 5, 5, 7

7, −5, −5, −5, −5, −5, −5, −5, −5, −5, 5, 9, 10, 5, 7, 6, 7, 4

8, −6, −6, −6, −6, −6, −6, −6, −6, −6, 6, 9, 6, 6, 5, 6, 14, 9

6, −6, −6, −6, −6, −6, −6, −6, −6, −6, 6, 5, 14, 9, 6, 6, 9, 8

9, −6, −6, −6, −6, −6, −6, −6, −6, −6, 5, 9, 6, 6, 8, 6, 6, 14

6, −6, −6, −6, −6, −6, −6, −6, −6, −6, 14, 6, 9, 8, 6, 9, 5, 6

6, −6, −6, −6, −6, −6, −6, −6, −6, −6, 9, 6, 8, 5, 14, 9, 6, 6

5, −6, −6, −6, −6, −6, −6, −6, −6, −6, 6, 8, 6, 14, 6, 6, 9, 9

6, −6, −6, −6, −6, −6, −6, −6, −6, −6, 6, 6, 9, 9, 6, 14, 8, 5

14, −6, −6, −6, −6, −6, −6, −6, −6, −6, 9, 6, 5, 6, 9, 8, 6, 6

9, −6, −6, −6, −6, −6, −6, −6, −6, −6, 8, 14, 6, 6, 9, 5, 6, 6

7, −6, −6, −6, −6, −6, −6, −6, −6, −6, 8, 6, 7, 7, 11, 7, 8, 8

8, −6, −6, −6, −6, −6, −6, −6, −6, −6, 7, 11, 8, 8, 7, 7, 6, 7

6, −6, −6, −6, −6, −6, −6, −6, −6, −6, 11, 8, 7, 7, 7, 8, 7, 8

7, −6, −6, −6, −6, −6, −6, −6, −6, −6, 8, 7, 6, 7, 7, 8, 11, 8

7, −6, −6, −6, −6, −6, −6, −6, −6, −6, 8, 7, 7, 11, 8, 6, 8, 7

11, −6, −6, −6, −6, −6, −6, −6, −6, −6, 7, 7, 7, 8, 8, 7, 8, 6

7, −6, −6, −6, −6, −6, −6, −6, −6, −6, 7, 8, 8, 6, 7, 8, 7, 11

8, −6, −6, −6, −6, −6, −6, −6, −6, −6, 6, 7, 11, 8, 8, 7, 7, 7

8, −6, −6, −6, −6, −6, −6, −6, −6, −6, 7, 8, 8, 7, 6, 11, 7, 7

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STEP 4: Compute LOWER BOUND for ORD of _Kpm at each cusp
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$\_cusp, \_LOWER\_BOUND\_of\_ORD, \_Kpm, \_at\_cusp$

$\_cusp = 0, \_LOWER\_BOUND = -6$

$\_cusp = \dfrac{1}{2}, \_LOWER\_BOUND = 0$

$\_cusp = \dfrac{1}{3}, \_LOWER\_BOUND = 0$

$\_cusp = \dfrac{1}{4}, \_LOWER\_BOUND = 0$

$\_cusp = \dfrac{1}{5}, \_LOWER\_BOUND = 0$

$\_cusp = \dfrac{1}{6}, \_LOWER\_BOUND = 0$

$\_cusp = \dfrac{1}{7}, \_LOWER\_BOUND = 0$

$\_cusp = \dfrac{1}{8}, \_LOWER\_BOUND = 0$

$\_cusp = \dfrac{1}{9}, \_LOWER\_BOUND = 0$

$\_cusp = \dfrac{2}{19}, \_LOWER\_BOUND = \dfrac{15}{19}$

$\_cusp = \dfrac{3}{19}, \_LOWER\_BOUND = \dfrac{15}{19}$

$\_cusp = \dfrac{4}{19}, \_LOWER\_BOUND = \dfrac{15}{19}$

$\_cusp = \dfrac{5}{19}, \_LOWER\_BOUND = \dfrac{15}{19}$

$\_cusp = \dfrac{6}{19}, \_LOWER\_BOUND = \dfrac{15}{19}$

$\_cusp = \dfrac{7}{19}, \_LOWER\_BOUND = \dfrac{15}{19}$

$\_cusp = \dfrac{8}{19}, \_LOWER\_BOUND = \dfrac{15}{19}$

$\_cusp = \dfrac{9}{19}, \_LOWER\_BOUND = \dfrac{15}{19}$

STEP 5: Compile LHS vs RHS ORD table at cusps and find constant B

"TABLE ORD lower bounds"

$\_cusp, \_width, \_ORD\_LHS, \_ORD\_RHS, \_ORD\_LHS\_minus\_RHS$

$0, 19, -6, -6, -6$

$$\frac{1}{2}, 19, 0, -6, -6$$

$$\frac{1}{3}, 19, 0, -6, -6$$

$$\frac{1}{4}, 19, 0, -6, -6$$

$$\frac{1}{5}, 19, 0, -6, -6$$

$$\frac{1}{6}, 19, 0, -6, -6$$

$$\frac{1}{7}, 19, 0, -6, -6$$

$$\frac{1}{8}, 19, 0, -6, -6$$

$$\frac{1}{9}, 19, 0, -6, -6$$

$$\frac{2}{19}, 1, 1, 1, 1$$

$$\frac{3}{19}, 1, 1, 1, 1$$

$$\frac{4}{19}, 1, 1, 1, 1$$

$$\frac{5}{19}, 1, 1, 1, 1$$

$$\frac{6}{19}, 1, 1, 1, 1$$

$$\frac{7}{19}, 1, 1, 1, 1$$

$$\frac{8}{19}, 1, 1, 1, 1$$

$$\frac{9}{19}, 1, 1, 1, 1$$

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        This implies that B = -46
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STEP 6: Prove and check identity
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"Coefficients in CKpm identity"

$$\_k = 1, \; -\zeta^{17} + \zeta^{16} + \zeta^{15} - \zeta^{13} + \zeta^{12} + 2\zeta^{11} - \zeta^{10} - \zeta^{9} + 2\zeta^{8} + \zeta^{7} - \zeta^{6} + \zeta^{4} + \zeta^{3} - \zeta^{2} + 2$$

$$\_k = 2, \; \zeta^{17} + 2\zeta^{16} - \zeta^{15} + 2\zeta^{14} + \zeta^{13} - \zeta^{12} + 3\zeta^{11} + 3\zeta^{8} - \zeta^{7} + \zeta^{6} + 2\zeta^{5} - \zeta^{4} + 2\zeta^{3} + \zeta^{2} + 2$$

$$\_k = 3, \; -3\zeta^{17} - 2\zeta^{16} - \zeta^{15} - 3\zeta^{14} - 4\zeta^{12} - \zeta^{11} - 2\zeta^{10} - 2\zeta^{9} - \zeta^{8} - 4\zeta^{7} - 3\zeta^{5} - \zeta^{4} - 2\zeta^{3}$$

$$-3\,\zeta^2 - 4$$

$$\_k = 4,\ -\zeta^{17} + 2\,\zeta^{16} - \zeta^{14} + 2\,\zeta^{13} + \zeta^{12} - \zeta^{11} + \zeta^{10} + \zeta^{9} - \zeta^{8} + \zeta^{7} + 2\,\zeta^{6} - \zeta^{5} + 2\,\zeta^{3} - \zeta^{2} + 2$$

$$\_k = 5,\ \zeta^{17} + \zeta^{15} + \zeta^{4} + \zeta^{2} + 1$$

$$\_k = 6,\ -\zeta^{17} - \zeta^{16} + \zeta^{15} + \zeta^{14} - \zeta^{12} - \zeta^{11} + \zeta^{10} + \zeta^{9} - \zeta^{8} - \zeta^{7} + \zeta^{5} + \zeta^{4} - \zeta^{3} - \zeta^{2} + 1$$

$$\_k = 7,\ -\zeta^{16} - \zeta^{15} - \zeta^{14} - \zeta^{13} - \zeta^{12} - \zeta^{7} - \zeta^{6} - \zeta^{5} - \zeta^{4} - \zeta^{3}$$

$$\_k = 8,\ 2\,\zeta^{17} + 2\,\zeta^{16} + 2\,\zeta^{15} + \zeta^{14} + \zeta^{12} + \zeta^{11} + 2\,\zeta^{10} + 2\,\zeta^{9} + \zeta^{8} + \zeta^{7} + \zeta^{5} + 2\,\zeta^{4} + 2\,\zeta^{3} + 2\,\zeta^{2}$$
$$+ 1$$

$$\_k = 9,\ -4\,\zeta^{17} - \zeta^{16} - 4\,\zeta^{15} - \zeta^{14} - 3\,\zeta^{13} - 2\,\zeta^{12} - 3\,\zeta^{11} - 2\,\zeta^{10} - 2\,\zeta^{9} - 3\,\zeta^{8} - 2\,\zeta^{7} - 3\,\zeta^{6} - \zeta^{5}$$
$$- 4\,\zeta^{4} - \zeta^{3} - 4\,\zeta^{2} - 5$$

$$\_k = 10,\ -9\,\zeta^{17} + 82\,\zeta^{16} + 21\,\zeta^{15} - 21\,\zeta^{14} + 68\,\zeta^{13} + 40\,\zeta^{12} - 28\,\zeta^{11} + 58\,\zeta^{10} + 58\,\zeta^{9} - 28\,\zeta^{8}$$
$$+ 40\,\zeta^{7} + 68\,\zeta^{6} - 21\,\zeta^{5} + 21\,\zeta^{4} + 82\,\zeta^{3} - 9\,\zeta^{2} + 86$$

$$\_k = 11,\ 23\,\zeta^{17} + 47\,\zeta^{16} + 37\,\zeta^{15} + 5\,\zeta^{14} - 12\,\zeta^{13} - 14\,\zeta^{12} + 16\,\zeta^{11} + 44\,\zeta^{10} + 44\,\zeta^{9} + 16\,\zeta^{8}$$
$$- 14\,\zeta^{7} - 12\,\zeta^{6} + 5\,\zeta^{5} + 37\,\zeta^{4} + 47\,\zeta^{3} + 23\,\zeta^{2} - 19$$

$$\_k = 12,\ -18\,\zeta^{17} - 27\,\zeta^{16} - 40\,\zeta^{15} - 39\,\zeta^{14} - 34\,\zeta^{13} - 21\,\zeta^{12} - 9\,\zeta^{11} + 3\,\zeta^{10} + 3\,\zeta^{9} - 9\,\zeta^{8} - 21\,\zeta^{7}$$
$$- 34\,\zeta^{6} - 39\,\zeta^{5} - 40\,\zeta^{4} - 27\,\zeta^{3} - 18\,\zeta^{2} + 1$$

$$\_k = 13,\ -90\,\zeta^{17} - 49\,\zeta^{16} - 31\,\zeta^{15} - 110\,\zeta^{14} + 6\,\zeta^{13} - 117\,\zeta^{12} - 10\,\zeta^{11} - 73\,\zeta^{10} - 73\,\zeta^{9} - 10\,\zeta^{8}$$
$$- 117\,\zeta^{7} + 6\,\zeta^{6} - 110\,\zeta^{5} - 31\,\zeta^{4} - 49\,\zeta^{3} - 90\,\zeta^{2} - 122$$

$$\_k = 14,\ -51\,\zeta^{17} + 14\,\zeta^{16} + 49\,\zeta^{15} - 20\,\zeta^{14} - 42\,\zeta^{13} + 31\,\zeta^{12} + 35\,\zeta^{11} - 30\,\zeta^{10} - 30\,\zeta^{9} + 35\,\zeta^{8}$$
$$+ 31\,\zeta^{7} - 42\,\zeta^{6} - 20\,\zeta^{5} + 49\,\zeta^{4} + 14\,\zeta^{3} - 51\,\zeta^{2} + 48$$

$$\_k = 15,\ -7\,\zeta^{17} - 6\,\zeta^{16} - 14\,\zeta^{15} - 12\,\zeta^{14} - 17\,\zeta^{13} - 21\,\zeta^{12} - 26\,\zeta^{11} - 24\,\zeta^{10} - 24\,\zeta^{9} - 26\,\zeta^{8}$$
$$- 21\,\zeta^{7} - 17\,\zeta^{6} - 12\,\zeta^{5} - 14\,\zeta^{4} - 6\,\zeta^{3} - 7\,\zeta^{2} - 8$$

$$\_k = 16,\ -128\,\zeta^{17} - 3\,\zeta^{16} - 117\,\zeta^{15} - 18\,\zeta^{14} - 103\,\zeta^{13} - 37\,\zeta^{12} - 77\,\zeta^{11} - 61\,\zeta^{10} - 61\,\zeta^{9} - 77\,\zeta^{8}$$
$$- 37\,\zeta^{7} - 103\,\zeta^{6} - 18\,\zeta^{5} - 117\,\zeta^{4} - 3\,\zeta^{3} - 128\,\zeta^{2} - 126$$

$$\_k = 17,\ 43\,\zeta^{17} + 37\,\zeta^{16} - 12\,\zeta^{15} - 31\,\zeta^{14} + 15\,\zeta^{13} + 49\,\zeta^{12} + 25\,\zeta^{11} - 24\,\zeta^{10} - 24\,\zeta^{9} + 25\,\zeta^{8}$$
$$+ 49\,\zeta^{7} + 15\,\zeta^{6} - 31\,\zeta^{5} - 12\,\zeta^{4} + 37\,\zeta^{3} + 43\,\zeta^{2} - 36$$

$$\_k = 18,\ 35\,\zeta^{17} + 91\,\zeta^{16} - 21\,\zeta^{15} + 74\,\zeta^{14} + 61\,\zeta^{13} - 12\,\zeta^{12} + 96\,\zeta^{11} + 18\,\zeta^{10} + 18\,\zeta^{9} + 96\,\zeta^{8}$$
$$- 12\,\zeta^{7} + 61\,\zeta^{6} + 74\,\zeta^{5} - 21\,\zeta^{4} + 91\,\zeta^{3} + 35\,\zeta^{2} + 104$$

$$\_k = 19,\ -115\,\zeta^{17} - 398\,\zeta^{16} - 269\,\zeta^{15} - 196\,\zeta^{14} + 262\,\zeta^{13} - 25\,\zeta^{12} - 106\,\zeta^{11} - 354\,\zeta^{10} - 354\,\zeta^{9}$$
$$- 106\,\zeta^{8} - 25\,\zeta^{7} + 262\,\zeta^{6} - 196\,\zeta^{5} - 269\,\zeta^{4} - 398\,\zeta^{3} - 115\,\zeta^{2} + 6$$

$$\_k = 20,\ 843\,\zeta^{17} + 531\,\zeta^{16} + 183\,\zeta^{15} + 904\,\zeta^{14} + 93\,\zeta^{13} + 913\,\zeta^{12} + 199\,\zeta^{11} + 559\,\zeta^{10} + 559\,\zeta^{9}$$

$+\ 199\ \zeta^8 + 913\ \zeta^7 + 93\ \zeta^6 + 904\ \zeta^5 + 183\ \zeta^4 + 531\ \zeta^3 + 843\ \zeta^2 + 916$

$\_k = 21,\ 256\ \zeta^{17} - 33\ \zeta^{16} + 25\ \zeta^{15} + 60\ \zeta^{14} + 57\ \zeta^{13} + 16\ \zeta^{12} - 94\ \zeta^{11} - 28\ \zeta^{10} - 28\ \zeta^9 - 94\ \zeta^8$
$+\ 16\ \zeta^7 + 57\ \zeta^6 + 60\ \zeta^5 + 25\ \zeta^4 - 33\ \zeta^3 + 256\ \zeta^2 + 254$

$\_k = 22,\ 319\ \zeta^{17} + 90\ \zeta^{16} - 119\ \zeta^{15} - 292\ \zeta^{14} - 49\ \zeta^{13} + 352\ \zeta^{12} + 150\ \zeta^{11} - 207\ \zeta^{10} - 207\ \zeta^9$
$+\ 150\ \zeta^8 + 352\ \zeta^7 - 49\ \zeta^6 - 292\ \zeta^5 - 119\ \zeta^4 + 90\ \zeta^3 + 319\ \zeta^2 - 402$

$\_k = 23,\ 279\ \zeta^{17} + 554\ \zeta^{16} - 163\ \zeta^{15} + 611\ \zeta^{14} + 373\ \zeta^{13} - 119\ \zeta^{12} + 750\ \zeta^{11} - 5\ \zeta^{10} - 5\ \zeta^9$
$+\ 750\ \zeta^8 - 119\ \zeta^7 + 373\ \zeta^6 + 611\ \zeta^5 - 163\ \zeta^4 + 554\ \zeta^3 + 279\ \zeta^2 + 692$

$\_k = 24,\ 823\ \zeta^{17} - 32\ \zeta^{16} + 913\ \zeta^{15} - 81\ \zeta^{14} + 673\ \zeta^{13} + 269\ \zeta^{12} + 579\ \zeta^{11} + 297\ \zeta^{10} + 297\ \zeta^9$
$+\ 579\ \zeta^8 + 269\ \zeta^7 + 673\ \zeta^6 - 81\ \zeta^5 + 913\ \zeta^4 - 32\ \zeta^3 + 823\ \zeta^2 + 788$

$\_k = 25,\ -353\ \zeta^{17} + 240\ \zeta^{16} + 352\ \zeta^{15} - 61\ \zeta^{14} - 202\ \zeta^{13} + 183\ \zeta^{12} + 377\ \zeta^{11} - 114\ \zeta^{10} - 114\ \zeta^9$
$+\ 377\ \zeta^8 + 183\ \zeta^7 - 202\ \zeta^6 - 61\ \zeta^5 + 352\ \zeta^4 + 240\ \zeta^3 - 353\ \zeta^2 + 641$

$\_k = 26,\ 195\ \zeta^{17} + 70\ \zeta^{16} + 334\ \zeta^{15} + 332\ \zeta^{14} + 301\ \zeta^{13} + 163\ \zeta^{12} + 151\ \zeta^{11} - 44\ \zeta^{10} - 44\ \zeta^9$
$+\ 151\ \zeta^8 + 163\ \zeta^7 + 301\ \zeta^6 + 332\ \zeta^5 + 334\ \zeta^4 + 70\ \zeta^3 + 195\ \zeta^2 + 22$

$\_k = 27,\ -197\ \zeta^{17} + 536\ \zeta^{16} - 16\ \zeta^{15} - 137\ \zeta^{14} + 382\ \zeta^{13} + 334\ \zeta^{12} - 368\ \zeta^{11} + 329\ \zeta^{10} + 329\ \zeta^9$
$-\ 368\ \zeta^8 + 334\ \zeta^7 + 382\ \zeta^6 - 137\ \zeta^5 - 16\ \zeta^4 + 536\ \zeta^3 - 197\ \zeta^2 + 577$

$\_k = 28,\ -2090\ \zeta^{17} + 1083\ \zeta^{16} + 1748\ \zeta^{15} - 1216\ \zeta^{14} - 969\ \zeta^{13} + 1957\ \zeta^{12} + 1615\ \zeta^{11} - 1330\ \zeta^{10}$
$-\ 1330\ \zeta^9 + 1615\ \zeta^8 + 1957\ \zeta^7 - 969\ \zeta^6 - 1216\ \zeta^5 + 1748\ \zeta^4 + 1083\ \zeta^3 - 2090\ \zeta^2 + 2375$

$\_k = 29,\ -437\ \zeta^{17} + 2717\ \zeta^{16} - 2489\ \zeta^{15} + 1843\ \zeta^{14} + 1558\ \zeta^{13} - 1862\ \zeta^{12} + 3173\ \zeta^{11} - 532\ \zeta^{10}$
$-\ 532\ \zeta^9 + 3173\ \zeta^8 - 1862\ \zeta^7 + 1558\ \zeta^6 + 1843\ \zeta^5 - 2489\ \zeta^4 + 2717\ \zeta^3 - 437\ \zeta^2 + 2527$

$\_k = 30,\ -3344\ \zeta^{17} - 3002\ \zeta^{16} - 1957\ \zeta^{15} - 4560\ \zeta^{14} - 171\ \zeta^{13} - 5662\ \zeta^{12} - 722\ \zeta^{11} - 3249\ \zeta^{10}$
$-\ 3249\ \zeta^9 - 722\ \zeta^8 - 5662\ \zeta^7 - 171\ \zeta^6 - 4560\ \zeta^5 - 1957\ \zeta^4 - 3002\ \zeta^3 - 3344\ \zeta^2 - 5928$

$\_k = 31,\ 76\ \zeta^{17} + 4047\ \zeta^{16} + 1235\ \zeta^{15} - 532\ \zeta^{14} + 2660\ \zeta^{13} + 3078\ \zeta^{12} - 513\ \zeta^{11} + 2546\ \zeta^{10}$
$+\ 2546\ \zeta^9 - 513\ \zeta^8 + 3078\ \zeta^7 + 2660\ \zeta^6 - 532\ \zeta^5 + 1235\ \zeta^4 + 4047\ \zeta^3 + 76\ \zeta^2 + 4408$

$\_k = 32,\ -152\ \zeta^{17} - 779\ \zeta^{16} - 133\ \zeta^{15} - 608\ \zeta^{14} - 874\ \zeta^{13} - 1235\ \zeta^{12} - 1995\ \zeta^{11} - 247\ \zeta^{10}$
$-\ 247\ \zeta^9 - 1995\ \zeta^8 - 1235\ \zeta^7 - 874\ \zeta^6 - 608\ \zeta^5 - 133\ \zeta^4 - 779\ \zeta^3 - 152\ \zeta^2 - 950$

$\_k = 33,\ -969\ \zeta^{17} - 1786\ \zeta^{16} + 1862\ \zeta^{15} + 874\ \zeta^{14} - 456\ \zeta^{13} - 1748\ \zeta^{12} - 1178\ \zeta^{11} + 1501\ \zeta^{10}$
$+\ 1501\ \zeta^9 - 1178\ \zeta^8 - 1748\ \zeta^7 - 456\ \zeta^6 + 874\ \zeta^5 + 1862\ \zeta^4 - 1786\ \zeta^3 - 969\ \zeta^2 + 1634$

$\_k = 34,\ -1748\ \zeta^{17} - 2318\ \zeta^{16} - 3078\ \zeta^{15} - 2280\ \zeta^{14} - 2299\ \zeta^{13} - 2489\ \zeta^{12} - 1121\ \zeta^{11} - 988\ \zeta^{10}$
$-\ 988\ \zeta^9 - 1121\ \zeta^8 - 2489\ \zeta^7 - 2299\ \zeta^6 - 2280\ \zeta^5 - 3078\ \zeta^4 - 2318\ \zeta^3 - 1748\ \zeta^2 - 2736$

$\_k = 35,\ 1653\,\zeta^{17} + 1425\,\zeta^{16} + 3040\,\zeta^{15} + 456\,\zeta^{14} + 38\,\zeta^{13} - 133\,\zeta^{12} + 551\,\zeta^{11} + 2413\,\zeta^{10}$
$\qquad + 2413\,\zeta^{9} + 551\,\zeta^{8} - 133\,\zeta^{7} + 38\,\zeta^{6} + 456\,\zeta^{5} + 3040\,\zeta^{4} + 1425\,\zeta^{3} + 1653\,\zeta^{2} - 114$

$\_k = 36,\ -5757\,\zeta^{17} - 741\,\zeta^{16} - 5662\,\zeta^{15} - 1197\,\zeta^{14} - 4579\,\zeta^{13} - 3040\,\zeta^{12} - 2584\,\zeta^{11} - 3287\,\zeta^{10}$
$\qquad - 3287\,\zeta^{9} - 2584\,\zeta^{8} - 3040\,\zeta^{7} - 4579\,\zeta^{6} - 1197\,\zeta^{5} - 5662\,\zeta^{4} - 741\,\zeta^{3} - 5757\,\zeta^{2} - 6232$

$\_k = 37,\ 893\,\zeta^{17} + 2812\,\zeta^{16} - 608\,\zeta^{15} + 4522\,\zeta^{14} + 2945\,\zeta^{13} - 418\,\zeta^{12} + 798\,\zeta^{11} + 2337\,\zeta^{10}$
$\qquad + 2337\,\zeta^{9} + 798\,\zeta^{8} - 418\,\zeta^{7} + 2945\,\zeta^{6} + 4522\,\zeta^{5} - 608\,\zeta^{4} + 2812\,\zeta^{3} + 893\,\zeta^{2} + 5928$

$\_k = 38,\ -1406\,\zeta^{17} + 3553\,\zeta^{16} - 1539\,\zeta^{15} + 323\,\zeta^{14} - 228\,\zeta^{13} + 228\,\zeta^{12} - 1026\,\zeta^{11} + 2565\,\zeta^{10}$
$\qquad + 2565\,\zeta^{9} - 1026\,\zeta^{8} + 228\,\zeta^{7} - 228\,\zeta^{6} + 323\,\zeta^{5} - 1539\,\zeta^{4} + 3553\,\zeta^{3} - 1406\,\zeta^{2} + 3724$

$\_k = 39,\ -2983\,\zeta^{17} + 1767\,\zeta^{16} + 418\,\zeta^{15} + 1539\,\zeta^{14} - 1957\,\zeta^{13} + 513\,\zeta^{12} + 38\,\zeta^{11} + 988\,\zeta^{10}$
$\qquad + 988\,\zeta^{9} + 38\,\zeta^{8} + 513\,\zeta^{7} - 1957\,\zeta^{6} + 1539\,\zeta^{5} + 418\,\zeta^{4} + 1767\,\zeta^{3} - 2983\,\zeta^{2} + 5130$

$\_k = 40,\ -3401\,\zeta^{17} + 1311\,\zeta^{16} + 380\,\zeta^{15} - 2014\,\zeta^{14} - 1254\,\zeta^{13} - 1634\,\zeta^{12} - 228\,\zeta^{11} + 931\,\zeta^{10}$
$\qquad + 931\,\zeta^{9} - 228\,\zeta^{8} - 1634\,\zeta^{7} - 1254\,\zeta^{6} - 2014\,\zeta^{5} + 380\,\zeta^{4} + 1311\,\zeta^{3} - 3401\,\zeta^{2} - 1539$

$\_k = 41,\ -3192\,\zeta^{17} + 3344\,\zeta^{16} - 1406\,\zeta^{15} + 1387\,\zeta^{14} - 3230\,\zeta^{13} - 380\,\zeta^{12} - 1178\,\zeta^{11} + 779\,\zeta^{10}$
$\qquad + 779\,\zeta^{9} - 1178\,\zeta^{8} - 380\,\zeta^{7} - 3230\,\zeta^{6} + 1387\,\zeta^{5} - 1406\,\zeta^{4} + 3344\,\zeta^{3} - 3192\,\zeta^{2} + 171$

$\_k = 42,\ 3952\,\zeta^{17} + 2375\,\zeta^{16} + 228\,\zeta^{15} + 3629\,\zeta^{14} + 3724\,\zeta^{13} + 608\,\zeta^{12} + 3762\,\zeta^{11} - 380\,\zeta^{10}$
$\qquad - 380\,\zeta^{9} + 3762\,\zeta^{8} + 608\,\zeta^{7} + 3724\,\zeta^{6} + 3629\,\zeta^{5} + 228\,\zeta^{4} + 2375\,\zeta^{3} + 3952\,\zeta^{2} + 2831$

$\_k = 43,\ -285\,\zeta^{17} + 3496\,\zeta^{16} - 1634\,\zeta^{15} + 1729\,\zeta^{14} + 1710\,\zeta^{13} + 1539\,\zeta^{12} - 2052\,\zeta^{11} + 1159\,\zeta^{10}$
$\qquad + 1159\,\zeta^{9} - 2052\,\zeta^{8} + 1539\,\zeta^{7} + 1710\,\zeta^{6} + 1729\,\zeta^{5} - 1634\,\zeta^{4} + 3496\,\zeta^{3} - 285\,\zeta^{2} + 3116$

$\_k = 44,\ 969\,\zeta^{17} - 1634\,\zeta^{16} + 3534\,\zeta^{15} - 4579\,\zeta^{14} + 1767\,\zeta^{13} - 1406\,\zeta^{12} + 1995\,\zeta^{11} - 475\,\zeta^{10}$
$\qquad - 475\,\zeta^{9} + 1995\,\zeta^{8} - 1406\,\zeta^{7} + 1767\,\zeta^{6} - 4579\,\zeta^{5} + 3534\,\zeta^{4} - 1634\,\zeta^{3} + 969\,\zeta^{2} - 1425$

$\_k = 45,\ 7087\,\zeta^{17} + 1824\,\zeta^{16} + 513\,\zeta^{15} + 5548\,\zeta^{14} + 3325\,\zeta^{13} + 3534\,\zeta^{12} + 2147\,\zeta^{11} + 2755\,\zeta^{10}$
$\qquad + 2755\,\zeta^{9} + 2147\,\zeta^{8} + 3534\,\zeta^{7} + 3325\,\zeta^{6} + 5548\,\zeta^{5} + 513\,\zeta^{4} + 1824\,\zeta^{3} + 7087\,\zeta^{2} + 4294$

$\_k = 46,\ 551\,\zeta^{17} - 9310\,\zeta^{16} + 7828\,\zeta^{15} - 10659\,\zeta^{14} - 57\,\zeta^{13} - 2831\,\zeta^{12} - 3686\,\zeta^{11} + 4522\,\zeta^{10}$
$\qquad + 4522\,\zeta^{9} - 3686\,\zeta^{8} - 2831\,\zeta^{7} - 57\,\zeta^{6} - 10659\,\zeta^{5} + 7828\,\zeta^{4} - 9310\,\zeta^{3} + 551\,\zeta^{2} - 2318$

$\_k = 47,\ 7904\,\zeta^{17} - 7885\,\zeta^{16} - 1349\,\zeta^{15} - 1406\,\zeta^{14} + 2033\,\zeta^{13} - 5643\,\zeta^{12} + 5719\,\zeta^{11} - 1121\,\zeta^{10}$
$\qquad - 1121\,\zeta^{9} + 5719\,\zeta^{8} - 5643\,\zeta^{7} + 2033\,\zeta^{6} - 1406\,\zeta^{5} - 1349\,\zeta^{4} - 7885\,\zeta^{3} + 7904\,\zeta^{2} - 10583$

$\_k = 48,\ 2261\,\zeta^{17} - 10431\,\zeta^{16} + 2831\,\zeta^{15} - 8398\,\zeta^{14} - 3458\,\zeta^{13} + 7068\,\zeta^{12} - 1121\,\zeta^{11} - 6764\,\zeta^{10}$
$\qquad - 6764\,\zeta^{9} - 1121\,\zeta^{8} + 7068\,\zeta^{7} - 3458\,\zeta^{6} - 8398\,\zeta^{5} + 2831\,\zeta^{4} - 10431\,\zeta^{3} + 2261\,\zeta^{2} - 7657$

$\_k = 49,\ -13870\,\zeta^{17} - 3382\,\zeta^{16} - 3952\,\zeta^{15} - 5624\,\zeta^{14} - 17499\,\zeta^{13} - 3439\,\zeta^{12} - 11780\,\zeta^{11}$

$$- 2318\, \zeta^{10} - 2318\, \zeta^{9} - 11780\, \zeta^{8} - 3439\, \zeta^{7} - 17499\, \zeta^{6} - 5624\, \zeta^{5} - 3952\, \zeta^{4} - 3382\, \zeta^{3}$$
$$- 13870\, \zeta^{2} - 13813$$

$\_k = 50,\ 5358\, \zeta^{17} - 4788\, \zeta^{16} + 11514\, \zeta^{15} - 8246\, \zeta^{14} + 6479\, \zeta^{13} - 3952\, \zeta^{12} + 5871\, \zeta^{11} - 3667\, \zeta^{10}$
$$- 3667\, \zeta^{9} + 5871\, \zeta^{8} - 3952\, \zeta^{7} + 6479\, \zeta^{6} - 8246\, \zeta^{5} + 11514\, \zeta^{4} - 4788\, \zeta^{3} + 5358\, \zeta^{2} - 7391$$

$\_k = 51,\ -12616\, \zeta^{17} + 6289\, \zeta^{16} - 5643\, \zeta^{15} - 11210\, \zeta^{14} - 836\, \zeta^{13} - 7828\, \zeta^{12} - 1957\, \zeta^{11}$
$$- 1064\, \zeta^{10} - 1064\, \zeta^{9} - 1957\, \zeta^{8} - 7828\, \zeta^{7} - 836\, \zeta^{6} - 11210\, \zeta^{5} - 5643\, \zeta^{4} + 6289\, \zeta^{3} - 12616\, \zeta^{2}$$
$$- 1387$$

$\_k = 52,\ 3686\, \zeta^{17} - 4807\, \zeta^{16} - 3439\, \zeta^{15} + 9310\, \zeta^{14} + 1349\, \zeta^{13} - 1349\, \zeta^{12} - 608\, \zeta^{11} - 285\, \zeta^{10}$
$$- 285\, \zeta^{9} - 608\, \zeta^{8} - 1349\, \zeta^{7} + 1349\, \zeta^{6} + 9310\, \zeta^{5} - 3439\, \zeta^{4} - 4807\, \zeta^{3} + 3686\, \zeta^{2} + 9253$$

$\_k = 53,\ -17803\, \zeta^{17} - 13547\, \zeta^{16} - 3686\, \zeta^{15} - 13604\, \zeta^{14} - 14117\, \zeta^{13} - 11514\, \zeta^{12} - 2337\, \zeta^{11}$
$$- 13832\, \zeta^{10} - 13832\, \zeta^{9} - 2337\, \zeta^{8} - 11514\, \zeta^{7} - 14117\, \zeta^{6} - 13604\, \zeta^{5} - 3686\, \zeta^{4} - 13547\, \zeta^{3}$$
$$- 17803\, \zeta^{2} - 19475$$

$\_k = 54,\ 4199\, \zeta^{17} - 5035\, \zeta^{16} - 7068\, \zeta^{15} - 4199\, \zeta^{14} + 2242\, \zeta^{13} - 3686\, \zeta^{12} - 3629\, \zeta^{11} - 7353\, \zeta^{10}$
$$- 7353\, \zeta^{9} - 3629\, \zeta^{8} - 3686\, \zeta^{7} + 2242\, \zeta^{6} - 4199\, \zeta^{5} - 7068\, \zeta^{4} - 5035\, \zeta^{3} + 4199\, \zeta^{2} - 15561$$

"Proving and checking identity"

"IDENTITY CHECKED AND PROVEN"

"IDENTITY checked for ", $\_O\!\left(q^{-topq\, +\, 1}\right) = \_O\!\left(q^{106}\right)$

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and _topq + 1 > -_B + GAMMA1INDEX/12 = 46 + 15 = 61
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