FINDING AND PROVING AN IDENTITY FOR K^(C)[p,m] where p=17 and m=1 (quadratic non-residue case)

This worksheet has **Startup Code**

```
> myseeds:=[[15, -3, -1, -2, -1, -2, -1, -2, -1],[27, -2, -2, -3,
  -2, -4, -4, -4, -4]];
```

$$myseeds := [[15, -3, -1, -2, -1, -2, -1, -2, -1], [27, -2, -2, -3, -2, -4, -4, -4, -4]] \quad (1)$$

NOTE 1 : myseeds generates 2*8 = 16 functions. For m=1 we need these 16 functions. Also we need to multiply by **(eta(17*tau)/eta(tau))^(3*k)**, k=-1,1. Thus the list [-3,3] in the plantseeds function is needed.

```
> BIGBAS:=plantseeds(myseeds,[-3,3],17):
```

NOTE 2 : Now to finish getting the basis for m=1 we need to multiply all the functions by f[17,7]/f[17, 8]. This is achieved using the **mult_nv_by_fp_quot** function.

```
> nvLA:=BIGBAS:
> BIGBAS;
```

$$[[15, -3, -1, -2, -1, -2, -1, -2, -1], [15, -1, -3, -2, -1, -1, -2, -2, -1], [15, -1, -2, -3, -2, -1, -1, -1, -2], [15, -1, -1, -2, -3, -2, -2, -1, -1], [15, -2, -2, -1, -1, -3, -1, -1, -2], [15, -2, -1, -1, -2, -1, -3, -1, -2], [15, -2, -2, -1, -2, -1, -1, -3, -1], [15, -1, -1, -1, -1, -2, -2, -2, -3], [27, -2, -2, -3, -2, -4, -4, -4, -4], [27, -4, -2, -4, -2, -4, -3, -4, -2], [27, -4, -4, -2, -4, -2, -2, -4, -3], [27, -2, -4, -4, -2, -3, -4, -4, -2], [27, -4, -3, -2, -4, -2, -4, -2, -4], [27, -3, -4, -4, -4, -2, -2, -2, -4], [27, -4, -4, -2, -3, -4, -2, -2, -4], [27, -2, -2, -4, -4, -4, -4, -3, -2], [-9, 0, 2, 1, 2, 1, 2, 1, 2], [-9, 2, 0, 1, 2, 2, 1, 1, 2], [-9, 2, 1, 0, 1, 2, 2, 2, 1], [-9, 2, 2, 1, 0, 1, 1, 2, 2], [-9, 1, 1, 2, 2, 0, 2, 2, 1], [-9, 1, 2, 2, 1, 2, 0, 2, 1], [-9, 1, 1, 2, 1, 2, 2, 0, 2], [-9, 2, 2, 2, 2, 1, 1, 1, 0], [3, 1, 1, 0, 1, -1, -1, -1, -1], [3, -1, 1, -1, 1, -1, 0, -1, 1], [3, -1, -1, 1, -1, 1, 1, -1, 0], [3, 1, -1, -1, 1, 0, -1, -1, 1], [3, -1, 0, 1, -1, 1, -1, 1, -1], [3, 0, -1, -1, -1, 1, 1, 1, -1], [3, -1, -1, 1, 0, -1, 1, 1, -1], [3, 1, 1, -1, -1, -1, -1, 0, 1], [39, -6, -4, -5, -4, -5, -4, -5, -4], [39, -4, -6, -5, -4, -4, -5, -5, -4], [39, -4, -5, -6, -5, -4, -4, -4, -5], [39, -4, -4, -5, -6, -5, -5, -4, -4], [39, -5, -5, -4, -4, -6, -4, -4, -5], [39, -5, -4, -4, -5, -4, -6, -4, -5], [39, -5, -5, -4, -5, -4, -4, -6, -4], [39, -4, -4, -4, -4, -5, -5, -5, -6], [51, -5, -5, -6, -5, -7, -7, -7, -7], [51, -7, -5, -7, -5, -7, -6, -7, -5], [51, -7, -7, -5, -7, -5, -5, -7, -6], [51, -5, -7, -7, -5, -6, -7, -7, -5], [51, -7, -6, -5, -7, -5, -7, -5, -7], [51, -6, -7, -7, -7, -5, -5, -5, -7], [51, -7, -7, -5, -6, -7, -5, -5, -7], [51, -5, -5, -7, -7, -7, -7, -6, -5]] \quad (2)$$

```
> nvL:=map(nv->mult_nv_by_fp_quot(nv,17,7,8),nvLA):
> nops(nvL);
```

$$48 \quad (3)$$

We now have a list of 48 functions in our basis list and we are ready to find an prove the identity for m=1.

```
> nvLq:=nvL2q(nvL,17,100):
> nvLq2:=map(f->series(f/q^(1/17),q,100),nvLq):
> findhom(nvLq2,q,1,0);
```

$$\{\varnothing\} \tag{4}$$

```
> do_alg_steps(17,1,nvL);
--------------------------------------------------
STEP 1: check modularity
        modularity checks
--------------------------------------------------
STEP 2: find k0 and divide by j0
        k0 = 17
--------------------------------------------------
STEP 3: Compute table of ORDS at all cusps for each func
```

"CUSPS: ", [[1, 0], [0, 1], [1, 2], [1, 3], [1, 4], [1, 5], [1, 6], [1, 7], [1, 8], [2, 17], [3, 17], [4, 17], [5, 17], [6, 17], [7, 17], [8, 17]]

"TABLE of ords"

$2, -2, -2, -2, -2, -2, -2, -2, -2, 2, 2, 2, 2, 2, 2, 2$

$3, -2, -2, -2, -2, -2, -2, -2, -2, 3, 3, 4, 2, 1, 2, -2$

$3, -2, -2, -2, -2, -2, -2, -2, -2, 3, 5, 1, 2, 0, 3, -1$

$4, -2, -2, -2, -2, -2, -2, -2, -2, 5, 2, 0, 2, 1, 3, -1$

$3, -2, -2, -2, -2, -2, -2, -2, -2, 2, 2, 1, 5, 2, 1, 0$

$4, -2, -2, -2, -2, -2, -2, -2, -2, 2, 1, 1, 3, 1, 5, -1$

$3, -2, -2, -2, -2, -2, -2, -2, -2, 2, 3, 2, 1, 4, 2, -1$

$6, -2, -2, -2, -2, -2, -2, -2, -2, 1, 2, 1, 3, 1, 2, 0$

$8, -3, -3, -3, -3, -3, -3, -3, -3, 3, 2, 3, 3, 1, 2, 2$

$4, -3, -3, -3, -3, -3, -3, -3, -3, 4, 2, 4, 2, 2, 2, 4$

$3, -3, -3, -3, -3, -3, -3, -3, -3, 2, 5, 2, 3, 6, 4, -1$

$5, -3, -3, -3, -3, -3, -3, -3, -3, 5, 3, 6, 2, 1, 2, 0$

$4, -3, -3, -3, -3, -3, -3, -3, -3, 2, 3, 1, 5, 3, 7, -1$

$3, -3, -3, -3, -3, -3, -3, -3, -3, 3, 7, 1, 4, 2, 5, -1$

$3, -3, -3, -3, -3, -3, -3, -3, -3, 2, 4, 1, 7, 4, 3, 0$

$6, -3, -3, -3, -3, -3, -3, -3, -3, 7, 2, 2, 2, 1, 3, 1$

$0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0$

$1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 2, 0, -1, 0, -4$

$1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, -1, 0, -2, 1, -3$

$2, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, -2, 0, -1, 1, -3$

$1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 3, 0, -1, -2$

$2, 0, 0, 0, 0, 0, 0, 0, 0, -1, -1, 1, -1, 3, -3$

$1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, -1, 2, 0, -3$

$$4, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, -1, 1, -1, 0, -2$$
$$6, -1, -1, -1, -1, -1, -1, -1, -1, 1, 0, 1, 1, -1, 0, 0$$
$$2, -1, -1, -1, -1, -1, -1, -1, -1, 2, 0, 2, 0, 0, 0, 2$$
$$1, -1, -1, -1, -1, -1, -1, -1, -1, 0, 3, 0, 1, 4, 2, -3$$
$$3, -1, -1, -1, -1, -1, -1, -1, -1, 3, 1, 4, 0, -1, 0, -2$$
$$2, -1, -1, -1, -1, -1, -1, -1, -1, 0, 1, -1, 3, 1, 5, -3$$
$$1, -1, -1, -1, -1, -1, -1, -1, -1, 1, 5, -1, 2, 0, 3, -3$$
$$1, -1, -1, -1, -1, -1, -1, -1, -1, 0, 2, -1, 5, 2, 1, -2$$
$$4, -1, -1, -1, -1, -1, -1, -1, -1, 5, 0, 0, 0, -1, 1, -1$$
$$4, -4, -4, -4, -4, -4, -4, -4, -4, 4, 4, 4, 4, 4, 4, 4$$
$$5, -4, -4, -4, -4, -4, -4, -4, -4, 5, 5, 6, 4, 3, 4, 0$$
$$5, -4, -4, -4, -4, -4, -4, -4, -4, 5, 7, 3, 4, 2, 5, 1$$
$$6, -4, -4, -4, -4, -4, -4, -4, -4, 7, 4, 2, 4, 3, 5, 1$$
$$5, -4, -4, -4, -4, -4, -4, -4, -4, 4, 4, 3, 7, 4, 3, 2$$
$$6, -4, -4, -4, -4, -4, -4, -4, -4, 4, 3, 3, 5, 3, 7, 1$$
$$5, -4, -4, -4, -4, -4, -4, -4, -4, 4, 5, 4, 3, 6, 4, 1$$
$$8, -4, -4, -4, -4, -4, -4, -4, -4, 3, 4, 3, 5, 3, 4, 2$$
$$10, -5, -5, -5, -5, -5, -5, -5, -5, 5, 4, 5, 5, 3, 4, 4$$
$$6, -5, -5, -5, -5, -5, -5, -5, -5, 6, 4, 6, 4, 4, 4, 6$$
$$5, -5, -5, -5, -5, -5, -5, -5, -5, 4, 7, 4, 5, 8, 6, 1$$
$$7, -5, -5, -5, -5, -5, -5, -5, -5, 7, 5, 8, 4, 3, 4, 2$$
$$6, -5, -5, -5, -5, -5, -5, -5, -5, 4, 5, 3, 7, 5, 9, 1$$
$$5, -5, -5, -5, -5, -5, -5, -5, -5, 5, 9, 3, 6, 4, 7, 1$$
$$5, -5, -5, -5, -5, -5, -5, -5, -5, 4, 6, 3, 9, 6, 5, 2$$
$$8, -5, -5, -5, -5, -5, -5, -5, -5, 9, 4, 4, 4, 3, 5, 3$$

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STEP 4: Compute LOWER BOUND for ORD of _Kpm/_j0 at each cusp
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"TABLE :"

$$\_cusp, \_LOWER\_BOUND\_of\_ORD, \frac{\_Kpm}{\_j0}, \_at\_cusp$$

$$\_cusp = 0, \_LOWER\_BOUND = -5$$

$$\_cusp = \frac{1}{2}, \_LOWER\_BOUND = -1$$

$$\_cusp = \frac{1}{3}, \_LOWER\_BOUND = -1$$

$$\_cusp = \frac{1}{4}, \_LOWER\_BOUND = -1$$

$$\_cusp = \frac{1}{5}, \_LOWER\_BOUND = -1$$

$$\_cusp = \frac{1}{6}, \_LOWER\_BOUND = -1$$

$$\_cusp = \frac{1}{7}, \_LOWER\_BOUND = -1$$

$$\_cusp = \frac{1}{8}, \_LOWER\_BOUND = -1$$

$$\_cusp = \frac{2}{17}, \_LOWER\_BOUND = \frac{25}{17}$$

$$\_cusp = \frac{3}{17}, \_LOWER\_BOUND = \frac{3}{17}$$

$$\_cusp = \frac{4}{17}, \_LOWER\_BOUND = \frac{13}{17}$$

$$\_cusp = \frac{5}{17}, \_LOWER\_BOUND = -\frac{13}{17}$$

$$\_cusp = \frac{6}{17}, \_LOWER\_BOUND = \frac{10}{17}$$

$$\_cusp = \frac{7}{17}, \_LOWER\_BOUND = -\frac{3}{17}$$

$$\_cusp = \frac{8}{17}, \_LOWER\_BOUND = -\frac{35}{17}$$

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STEP 5: Compile LHS vs RHS ORD table at cusps and find constant B
```

"TABLE ORD lower bounds"

$$\_cusp, \_width, \_ORD\_LHS, \_ORD\_RHS, \_ORD\_LHS\_minus\_RHS$$

$$0, 17, -5, -5, -5$$

$$\frac{1}{2}, 17, -1, -5, -5$$

$$\frac{1}{3}, 17, -1, -5, -5$$

$$\frac{1}{4}, 17, -1, -5, -5$$

$$\frac{1}{5}, 17, -1, -5, -5$$

$$\frac{1}{6}, 17, -1, -5, -5$$

$$\frac{1}{7}, 17, -1, -5, -5$$

$$\frac{1}{8}, 17, -1, -5, -5$$

$$\frac{2}{17}, 1, 2, -1, -1$$

$$\frac{3}{17}, 1, 1, -1, -1$$

$$\frac{4}{17}, 1, 1, -2, -2$$

$$\frac{5}{17}, 1, 0, -1, -1$$

$$\frac{6}{17}, 1, 1, -2, -2$$

$$\frac{7}{17}, 1, 0, -1, -1$$

$$\frac{8}{17}, 1, -2, -4, -4$$

```
        This implies that B = -52
-------------------------------------------------
STEP 6: Prove and check identity
```

"Coefficients in CKpm identity"

$$\_k = 1, 7\,\zeta^{15} - 6\,\zeta^{14} - 5\,\zeta^{13} + 7\,\zeta^{12} - 2\,\zeta^{11} - 8\,\zeta^{10} + 3\,\zeta^{9} + 3\,\zeta^{8} - 8\,\zeta^{7} - 2\,\zeta^{6} + 7\,\zeta^{5} - 5\,\zeta^{4} - 6\,\zeta^{3} + 7\,\zeta^{2} - 6$$

$$\_k = 2, 8\,\zeta^{15} - 6\,\zeta^{14} - 4\,\zeta^{13} + 8\,\zeta^{12} - \zeta^{11} - 6\,\zeta^{10} + 4\,\zeta^{9} + 4\,\zeta^{8} - 6\,\zeta^{7} - \zeta^{6} + 8\,\zeta^{5} - 4\,\zeta^{4} - 6\,\zeta^{3} + 8\,\zeta^{2} - 5$$

$$\_k = 3, -5\,\zeta^{15} + 7\,\zeta^{14} + 6\,\zeta^{13} - 6\,\zeta^{12} + 2\,\zeta^{11} + 9\,\zeta^{10} - 3\,\zeta^{9} - 3\,\zeta^{8} + 9\,\zeta^{7} + 2\,\zeta^{6} - 6\,\zeta^{5} + 6\,\zeta^{4} + 7\,\zeta^{3} - 5\,\zeta^{2} + 9$$

$$\_k = 4, -5\,\zeta^{15} + 7\,\zeta^{14} + 6\,\zeta^{13} - 6\,\zeta^{12} + 2\,\zeta^{11} + 9\,\zeta^{10} - 3\,\zeta^{9} - 3\,\zeta^{8} + 9\,\zeta^{7} + 2\,\zeta^{6} - 6\,\zeta^{5} + 6\,\zeta^{4} + 7\,\zeta^{3} - 5\,\zeta^{2} + 9$$

$$\_k = 5, -36\,\zeta^{15} - 25\,\zeta^{14} - 7\,\zeta^{13} - 49\,\zeta^{12} + 4\,\zeta^{11} - 43\,\zeta^{10} - 16\,\zeta^{9} - 16\,\zeta^{8} - 43\,\zeta^{7} + 4\,\zeta^{6} - 49\,\zeta^{5} - 7\,\zeta^{4} - 25\,\zeta^{3} - 36\,\zeta^{2} - 50$$

$$\_k = 6, -7\,\zeta^{15} - 9\,\zeta^{14} - 5\,\zeta^{13} + \zeta^{12} + 3\,\zeta^{11} - 3\,\zeta^{10} - 8\,\zeta^{9} - 8\,\zeta^{8} - 3\,\zeta^{7} + 3\,\zeta^{6} + \zeta^{5} - 5\,\zeta^{4} - 9\,\zeta^{3} - 7\,\zeta^{2} + 2$$

$$\_k = 7, -54\,\zeta^{15} + 91\,\zeta^{14} + 64\,\zeta^{13} - 66\,\zeta^{12} + 34\,\zeta^{11} + 106\,\zeta^{10} - 30\,\zeta^{9} - 30\,\zeta^{8} + 106\,\zeta^{7} + 34\,\zeta^{6} - 66\,\zeta^{5} + 64\,\zeta^{4} + 91\,\zeta^{3} - 54\,\zeta^{2} + 113$$

$$\_k = 8, -2\,\zeta^{15} - \zeta^{14} - \zeta^{13} - 2\,\zeta^{12} - 2\,\zeta^{10} - \zeta^{9} - \zeta^{8} - 2\,\zeta^{7} - 2\,\zeta^{5} - \zeta^{4} - \zeta^{3} - 2\,\zeta^{2} - 3$$

$$\_k = 9, 142\,\zeta^{15} + 246\,\zeta^{14} + 107\,\zeta^{13} + 136\,\zeta^{12} + 3\,\zeta^{11} + 283\,\zeta^{10} + 91\,\zeta^{9} + 91\,\zeta^{8} + 283\,\zeta^{7} + 3\,\zeta^{6} + 136\,\zeta^{5} + 107\,\zeta^{4} + 246\,\zeta^{3} + 142\,\zeta^{2} + 296$$

$$\_k = 10, 348\,\zeta^{15} - 87\,\zeta^{14} - 130\,\zeta^{13} + 376\,\zeta^{12} - 120\,\zeta^{11} - 124\,\zeta^{10} + 212\,\zeta^{9} + 212\,\zeta^{8} - 124\,\zeta^{7} - 120\,\zeta^{6} + 376\,\zeta^{5} - 130\,\zeta^{4} - 87\,\zeta^{3} + 348\,\zeta^{2} - 134$$

$$\_k = 11, -208\,\zeta^{15} + 175\,\zeta^{14} + 112\,\zeta^{13} - 233\,\zeta^{12} + 79\,\zeta^{11} + 179\,\zeta^{10} - 99\,\zeta^{9} - 99\,\zeta^{8} + 179\,\zeta^{7} + 79\,\zeta^{6} - 233\,\zeta^{5} + 112\,\zeta^{4} + 175\,\zeta^{3} - 208\,\zeta^{2} + 160$$

$$\_k = 12, -345\,\zeta^{15} + 119\,\zeta^{14} + 147\,\zeta^{13} - 436\,\zeta^{12} + 110\,\zeta^{11} + 90\,\zeta^{10} - 177\,\zeta^{9} - 177\,\zeta^{8} + 90\,\zeta^{7} + 110\,\zeta^{6} - 436\,\zeta^{5} + 147\,\zeta^{4} + 119\,\zeta^{3} - 345\,\zeta^{2} + 66$$

$$\_k = 13, -58\,\zeta^{15} - 58\,\zeta^{14} + 17\,\zeta^{13} - 95\,\zeta^{12} + 47\,\zeta^{11} - 58\,\zeta^{10} - 37\,\zeta^{9} - 37\,\zeta^{8} - 58\,\zeta^{7} + 47\,\zeta^{6} - 95\,\zeta^{5} + 17\,\zeta^{4} - 58\,\zeta^{3} - 58\,\zeta^{2} - 26$$

$$\_k = 14, 85\,\zeta^{15} - 155\,\zeta^{14} - 76\,\zeta^{13} + 85\,\zeta^{12} - 34\,\zeta^{11} - 163\,\zeta^{10} + 28\,\zeta^{9} + 28\,\zeta^{8} - 163\,\zeta^{7} - 34\,\zeta^{6} + 85\,\zeta^{5} - 76\,\zeta^{4} - 155\,\zeta^{3} + 85\,\zeta^{2} - 135$$

$$\_k = 15, 186\,\zeta^{15} - 73\,\zeta^{14} - 88\,\zeta^{13} + 230\,\zeta^{12} - 91\,\zeta^{11} - 73\,\zeta^{10} + 91\,\zeta^{9} + 91\,\zeta^{8} - 73\,\zeta^{7} - 91\,\zeta^{6} + 230\,\zeta^{5} - 88\,\zeta^{4} - 73\,\zeta^{3} + 186\,\zeta^{2} - 75$$

$$\_k = 16, -196\,\zeta^{15} + 370\,\zeta^{14} + 261\,\zeta^{13} - 303\,\zeta^{12} + 107\,\zeta^{11} + 378\,\zeta^{10} - 81\,\zeta^{9} - 81\,\zeta^{8} + 378\,\zeta^{7} + 107\,\zeta^{6} - 303\,\zeta^{5} + 261\,\zeta^{4} + 370\,\zeta^{3} - 196\,\zeta^{2} + 356$$

$$\_k = 17, 0$$

$$\_k = 18, 0$$

$$\_k = 19, 0$$

$$\_k = 20, 0$$

$$\_k = 21, 0$$

$$\_k = 22, 0$$

$$\_k = 23, 0$$

$$\_k = 24, 0$$

$$\_k = 25, -2\,\zeta^{15} - \zeta^{14} - \zeta^{13} - 2\,\zeta^{12} - 2\,\zeta^{10} - \zeta^{9} - \zeta^{8} - 2\,\zeta^{7} - 2\,\zeta^{5} - \zeta^{4} - \zeta^{3} - 2\,\zeta^{2} - 3$$

$$\_k = 26, -7\,\zeta^{15} + 6\,\zeta^{14} + 5\,\zeta^{13} - 8\,\zeta^{12} + 2\,\zeta^{11} + 7\,\zeta^{10} - 4\,\zeta^{9} - 4\,\zeta^{8} + 7\,\zeta^{7} + 2\,\zeta^{6} - 8\,\zeta^{5} + 5\,\zeta^{4} + 6\,\zeta^{3} - 7\,\zeta^{2} + 6$$

$$\_k = 27, 0$$

$$\_k = 28, 7\,\zeta^{15} - 6\,\zeta^{14} - 5\,\zeta^{13} + 8\,\zeta^{12} - 2\,\zeta^{11} - 7\,\zeta^{10} + 4\,\zeta^{9} + 4\,\zeta^{8} - 7\,\zeta^{7} - 2\,\zeta^{6} + 8\,\zeta^{5} - 5\,\zeta^{4} - 6\,\zeta^{3} + 7\,\zeta^{2} - 6$$

$$\_k = 29, 0$$

$$\_k = 30, 0$$

$$\_k = 31, 0$$

$\_k = 32, 5\,\zeta^{15} - 7\,\zeta^{14} - 6\,\zeta^{13} + 6\,\zeta^{12} - 2\,\zeta^{11} - 9\,\zeta^{10} + 3\,\zeta^{9} + 3\,\zeta^{8} - 9\,\zeta^{7} - 2\,\zeta^{6} + 6\,\zeta^{5} - 6\,\zeta^{4} - 7\,\zeta^{3} + 5\,\zeta^{2} - 9$

$\_k = 33, -323\,\zeta^{15} + 85\,\zeta^{14} + 119\,\zeta^{13} - 408\,\zeta^{12} + 68\,\zeta^{11} + 68\,\zeta^{10} - 255\,\zeta^{9} - 255\,\zeta^{8} + 68\,\zeta^{7} + 68\,\zeta^{6} - 408\,\zeta^{5} + 119\,\zeta^{4} + 85\,\zeta^{3} - 323\,\zeta^{2} + 136$

$\_k = 34, -34\,\zeta^{15} + 187\,\zeta^{14} + 357\,\zeta^{13} - 306\,\zeta^{12} + 255\,\zeta^{11} + 238\,\zeta^{10} - 17\,\zeta^{9} - 17\,\zeta^{8} + 238\,\zeta^{7} + 255\,\zeta^{6} - 306\,\zeta^{5} + 357\,\zeta^{4} + 187\,\zeta^{3} - 34\,\zeta^{2} + 374$

$\_k = 35, 51\,\zeta^{15} - 272\,\zeta^{14} - 170\,\zeta^{13} + 85\,\zeta^{12} - 119\,\zeta^{11} - 272\,\zeta^{10} + 17\,\zeta^{9} + 17\,\zeta^{8} - 272\,\zeta^{7} - 119\,\zeta^{6} + 85\,\zeta^{5} - 170\,\zeta^{4} - 272\,\zeta^{3} + 51\,\zeta^{2} - 374$

$\_k = 36, -272\,\zeta^{14} - 187\,\zeta^{13} + 119\,\zeta^{12} - 85\,\zeta^{11} - 306\,\zeta^{10} - 306\,\zeta^{7} - 85\,\zeta^{6} + 119\,\zeta^{5} - 187\,\zeta^{4} - 272\,\zeta^{3} - 272$

$\_k = 37, 714\,\zeta^{15} + 527\,\zeta^{14} + 153\,\zeta^{13} + 918\,\zeta^{12} - 68\,\zeta^{11} + 833\,\zeta^{10} + 357\,\zeta^{9} + 357\,\zeta^{8} + 833\,\zeta^{7} - 68\,\zeta^{6} + 918\,\zeta^{5} + 153\,\zeta^{4} + 527\,\zeta^{3} + 714\,\zeta^{2} + 935$

$\_k = 38, 255\,\zeta^{15} + 204\,\zeta^{14} + 119\,\zeta^{13} - 85\,\zeta^{11} + 17\,\zeta^{10} + 272\,\zeta^{9} + 272\,\zeta^{8} + 17\,\zeta^{7} - 85\,\zeta^{6} + 119\,\zeta^{4} + 204\,\zeta^{3} + 255\,\zeta^{2} - 119$

$\_k = 39, 714\,\zeta^{15} - 1037\,\zeta^{14} - 799\,\zeta^{13} + 935\,\zeta^{12} - 425\,\zeta^{11} - 1173\,\zeta^{10} + 391\,\zeta^{9} + 391\,\zeta^{8} - 1173\,\zeta^{7} - 425\,\zeta^{6} + 935\,\zeta^{5} - 799\,\zeta^{4} - 1037\,\zeta^{3} + 714\,\zeta^{2} - 1258$

$\_k = 40, 204\,\zeta^{15} + 85\,\zeta^{14} - 17\,\zeta^{13} + 204\,\zeta^{12} - 68\,\zeta^{11} + 119\,\zeta^{10} + 136\,\zeta^{9} + 136\,\zeta^{8} + 119\,\zeta^{7} - 68\,\zeta^{6} + 204\,\zeta^{5} - 17\,\zeta^{4} + 85\,\zeta^{3} + 204\,\zeta^{2} + 119$

$\_k = 41, 289\,\zeta^{15} - 867\,\zeta^{14} - 289\,\zeta^{13} + 289\,\zeta^{12} - 289\,\zeta^{11} - 867\,\zeta^{10} - 867\,\zeta^{7} - 289\,\zeta^{6} + 289\,\zeta^{5} - 289\,\zeta^{4} - 867\,\zeta^{3} + 289\,\zeta^{2} - 867$

$\_k = 42, -289\,\zeta^{14} + 289\,\zeta^{13} - 289\,\zeta^{12} + 289\,\zeta^{11} - 289\,\zeta^{9} - 289\,\zeta^{8} + 289\,\zeta^{6} - 289\,\zeta^{5} + 289\,\zeta^{4} - 289\,\zeta^{3} + 289$

$\_k = 43, 867\,\zeta^{15} - 1156\,\zeta^{14} - 867\,\zeta^{13} + 1156\,\zeta^{12} - 578\,\zeta^{11} - 867\,\zeta^{10} + 289\,\zeta^{9} + 289\,\zeta^{8} - 867\,\zeta^{7} - 578\,\zeta^{6} + 1156\,\zeta^{5} - 867\,\zeta^{4} - 1156\,\zeta^{3} + 867\,\zeta^{2} - 1156$

$\_k = 44, 289\,\zeta^{15} + 289\,\zeta^{14} - 289\,\zeta^{13} + 867\,\zeta^{12} - 578\,\zeta^{11} + 289\,\zeta^{10} + 289\,\zeta^{9} + 289\,\zeta^{8} + 289\,\zeta^{7} - 578\,\zeta^{6} + 867\,\zeta^{5} - 289\,\zeta^{4} + 289\,\zeta^{3} + 289\,\zeta^{2}$

$\_k = 45, 1445\,\zeta^{15} + 578\,\zeta^{14} + 289\,\zeta^{13} + 1156\,\zeta^{12} + 578\,\zeta^{10} + 867\,\zeta^{9} + 867\,\zeta^{8} + 578\,\zeta^{7} + 1156\,\zeta^{5} + 289\,\zeta^{4} + 578\,\zeta^{3} + 1445\,\zeta^{2} + 867$

$\_k = 46, 289\,\zeta^{15} + 1156\,\zeta^{14} + 578\,\zeta^{13} + 289\,\zeta^{11} + 867\,\zeta^{10} + 289\,\zeta^{9} + 289\,\zeta^{8} + 867\,\zeta^{7} + 289\,\zeta^{6}$

$+ 578\,\zeta^4 + 1156\,\zeta^3 + 289\,\zeta^2 + 1156$

$\_k = 47, -1734\,\zeta^{15} + 289\,\zeta^{14} + 289\,\zeta^{13} - 1734\,\zeta^{12} + 289\,\zeta^{11} - 867\,\zeta^9 - 867\,\zeta^8 + 289\,\zeta^6 - 1734\,\zeta^5$
$+ 289\,\zeta^4 + 289\,\zeta^3 - 1734\,\zeta^2$

$\_k = 48, 578\,\zeta^{15} - 578\,\zeta^{14} - 578\,\zeta^{13} + 1156\,\zeta^{12} - 578\,\zeta^{11} - 578\,\zeta^{10} + 289\,\zeta^9 + 289\,\zeta^8 - 578\,\zeta^7$
$- 578\,\zeta^6 + 1156\,\zeta^5 - 578\,\zeta^4 - 578\,\zeta^3 + 578\,\zeta^2 - 578$

"Proving and checking identity"

"IDENTITY CHECKED AND PROVEN"

"IDENTITY checked for ", $\_O\left(q^{-topq+1}\right) = \_O\left(q^{118}\right)$

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and _topq + 1 > -_B = 52
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