FINDING AND PROVING AN IDENTITY FOR K^(C)[p,m] where p=13 and m=0

This worksheet has **Startup Code**

```
> myseeds:=[[15,-1,-3,-2,-2,-2,-3]];
```

$$myseeds := [[15, -1, -3, -2, -2, -2, -3]] \tag{1}$$

```
> BIGBAS:=plantseeds(myseeds,[],13):
  nvL:=BIGBAS:
  do_alg_steps(13,0,nvL);
```

```
------------------------------------------------
p = 13  and m = 0
STEP 1: check modularity
        modularity checks
------------------------------------------------
STEP 2: find k0 and divide by j0
        We skip this step since m = 0
------------------------------------------------
STEP 3: Compute table of ORDS at all cusps for each func
```

"CUSPS: ", [[1, 0], [0, 1], [1, 2], [1, 3], [1, 4], [1, 5], [1, 6], [2, 13], [3, 13], [4, 13], [5, 13], [6, 13]]

"TABLE of ords"

$$3, -1, -1, -1, -1, -1, -1, 2, 3, 2, 2, 1$$
$$2, -1, -1, -1, -1, -1, -1, 2, 1, 2, 3, 3$$
$$3, -1, -1, -1, -1, -1, -1, 1, 2, 3, 2, 2$$
$$2, -1, -1, -1, -1, -1, -1, 2, 3, 3, 1, 2$$
$$2, -1, -1, -1, -1, -1, -1, 3, 2, 1, 3, 2$$
$$1, -1, -1, -1, -1, -1, -1, 3, 2, 2, 2, 3$$

```
------------------------------------------------
STEP 4: Compute LOWER BOUND for ORD of _Kpm at each cusp
```

"TABLE :"

$$\_cusp, \_LOWER\_BOUND\_of\_ORD, \_Kpm, \_at\_cusp$$
$$\_cusp = 0, \_LOWER\_BOUND = -1$$
$$\_cusp = \frac{1}{2}, \_LOWER\_BOUND = 0$$
$$\_cusp = \frac{1}{3}, \_LOWER\_BOUND = 0$$
$$\_cusp = \frac{1}{4}, \_LOWER\_BOUND = 0$$
$$\_cusp = \frac{1}{5}, \_LOWER\_BOUND = 0$$
$$\_cusp = \frac{1}{6}, \_LOWER\_BOUND = 0$$
$$\_cusp = \frac{2}{13}, \_LOWER\_BOUND = \frac{7}{13}$$

$$\_cusp = \frac{3}{13}, \_LOWER\_BOUND = \frac{7}{13}$$

$$\_cusp = \frac{4}{13}, \_LOWER\_BOUND = \frac{7}{13}$$

$$\_cusp = \frac{5}{13}, \_LOWER\_BOUND = \frac{7}{13}$$

$$\_cusp = \frac{6}{13}, \_LOWER\_BOUND = \frac{7}{13}$$

```
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STEP 5: Compile LHS vs RHS ORD table at cusps and find constant B
```

"TABLE ORD lower bounds"

$\_cusp, \_width, \_ORD\_LHS, \_ORD\_RHS, \_ORD\_LHS\_minus\_RHS$

$$0, 13, -1, -1, -1$$

$$\frac{1}{2}, 13, 0, -1, -1$$

$$\frac{1}{3}, 13, 0, -1, -1$$

$$\frac{1}{4}, 13, 0, -1, -1$$

$$\frac{1}{5}, 13, 0, -1, -1$$

$$\frac{1}{6}, 13, 0, -1, -1$$

$$\frac{2}{13}, 1, 1, 1, 1$$

$$\frac{3}{13}, 1, 1, 1, 1$$

$$\frac{4}{13}, 1, 1, 1, 1$$

$$\frac{5}{13}, 1, 1, 1, 1$$

$$\frac{6}{13}, 1, 1, 1, 1$$

```
        This implies that B = -1
--------------------------------------------------
STEP 6: Prove and check identity
```

"Coefficients in CKpm identity"

$$\_k = 1, 2\,\zeta^{11} + \zeta^{9} + \zeta^{8} + \zeta^{7} + \zeta^{6} + \zeta^{5} + \zeta^{4} + 2\,\zeta^{2} + 2$$

$$\_k = 2, -\zeta^{11} - \zeta^{10} - \zeta^{7} - \zeta^{6} - \zeta^{3} - \zeta^{2}$$

$$\_k = 3, \zeta^{11} + \zeta^{10} + \zeta^{9} + \zeta^{8} + \zeta^{5} + \zeta^{4} + \zeta^{3} + \zeta^{2} + 1$$

$$\_k = 4, \zeta^{10} - \zeta^{9} + \zeta^{8} + \zeta^{5} - \zeta^{4} + \zeta^{3} + 1$$

$$\_k = 5, -\zeta^{10} - \zeta^{9} - \zeta^{7} - \zeta^{6} - \zeta^{4} - \zeta^{3} - 2$$

$$\_k = 6, -\zeta^{8} - \zeta^{5}$$

"Proving and checking identity"

"IDENTITY CHECKED AND PROVEN"

"IDENTITY checked for ", $\_O\left(q^{\_topq + 1}\right) = \_O\left(q^{154}\right)$

```
and _topq + 1 > -_B + GAMMA1INDEX/12 = 1 + 7 = 8
-------------------------------------------------
```

```
> for k from 1 to 6 do
>   ck:=content(cofs[k],zeta): if ck=1 then print(_c[k]=cofs[k]);
  else  ifacs:=ifactors(ck): xx:=ifacs[2]: if nops(xx)=1 then p:=xx
  [1][1]: j:=xx[1][2]: print(_c[k]=cat(_,p)^j*expand(cofs[k]/ck));
  else print(TROUBLE); fi:fi: od:
```

$$\_c_1 = 2\zeta^{11} + \zeta^{9} + \zeta^{8} + \zeta^{7} + \zeta^{6} + \zeta^{5} + \zeta^{4} + 2\zeta^{2} + 2$$

$$\_c_2 = -\zeta^{11} - \zeta^{10} - \zeta^{7} - \zeta^{6} - \zeta^{3} - \zeta^{2}$$

$$\_c_3 = \zeta^{11} + \zeta^{10} + \zeta^{9} + \zeta^{8} + \zeta^{5} + \zeta^{4} + \zeta^{3} + \zeta^{2} + 1$$

$$\_c_4 = \zeta^{10} - \zeta^{9} + \zeta^{8} + \zeta^{5} - \zeta^{4} + \zeta^{3} + 1$$

$$\_c_5 = -\zeta^{10} - \zeta^{9} - \zeta^{7} - \zeta^{6} - \zeta^{4} - \zeta^{3} - 2$$

$$\_c_6 = -\zeta^{8} - \zeta^{5} \quad \textbf{(2)}$$