**FINDING AND PROVING AN IDENTITY FOR K^(C)[p,m] where p=17 and m=12 (quadratic residue case)**

This worksheet has **Startup Code**

```
> myseeds:=[[15, -3, -1, -2, -1, -2, -1, -2, -1],[27, -2, -2, -3,
  -2, -4, -4, -4, -4]];
```

$myseeds := [[15, -3, -1, -2, -1, -2, -1, -2, -1], [27, -2, -2, -3, -2, -4, -4, -4, -4]]$ **(1)**

**NOTE : myseeds** generates 2*8 = 16 functions. For m=1 we need these 16 functions. Also we need to multiply by **(eta(17*tau)/eta(tau))^(3*k)**, k=-1,1. Thus the list [-3,3] in the plantseeds function is needed.

```
> BIGBAS:=plantseeds(myseeds,[-3,3],17):
```

**NOTE 2 :** Now to finish getting the basis for m=1 we need to multiply all the functions by f[17,7]/f[17,5]. This is achieved using the **mult_nv_by_fp_quot** function.

```
> nvLA:=BIGBAS:
> BIGBAS;
```

$[[15, -3, -1, -2, -1, -2, -1, -2, -1], [15, -1, -3, -2, -1, -1, -2, -2, -1], [15, -1, -2, -3, -2, -1, -1, -1, -2], [15, -1, -1, -2, -3, -2, -2, -1, -1], [15, -2, -2, -1, -1, -3, -1, -1, -2], [15, -2, -1, -1, -2, -1, -3, -1, -2], [15, -2, -2, -1, -2, -1, -1, -3, -1], [15, -1, -1, -1, -1, -2, -2, -2, -3], [27, -2, -2, -3, -2, -4, -4, -4, -4], [27, -4, -2, -4, -2, -4, -3, -4, -2], [27, -4, -4, -2, -4, -2, -2, -4, -3], [27, -2, -4, -4, -2, -3, -4, -4, -2], [27, -4, -3, -2, -4, -2, -4, -2, -4], [27, -3, -4, -4, -4, -2, -2, -2, -4], [27, -4, -4, -2, -3, -4, -2, -2, -4], [27, -2, -2, -4, -4, -4, -4, -3, -2], [-9, 0, 2, 1, 2, 1, 2, 1, 2], [-9, 2, 0, 1, 2, 2, 1, 1, 2], [-9, 2, 1, 0, 1, 2, 2, 2, 1], [-9, 2, 2, 1, 0, 1, 1, 2, 2], [-9, 1, 1, 2, 2, 0, 2, 2, 1], [-9, 1, 2, 2, 1, 2, 0, 2, 1], [-9, 1, 1, 2, 1, 2, 2, 0, 2], [-9, 2, 2, 2, 2, 1, 1, 1, 0], [3, 1, 1, 0, 1, -1, -1, -1, -1], [3, -1, 1, -1, 1, -1, 0, -1, 1], [3, -1, -1, 1, -1, 1, 1, -1, 0], [3, 1, -1, -1, 1, 0, -1, -1, 1], [3, -1, 0, 1, -1, 1, -1, 1, -1], [3, 0, -1, -1, -1, 1, 1, 1, -1], [3, -1, -1, 1, 0, -1, 1, 1, -1], [3, 1, 1, -1, -1, -1, -1, 0, 1], [39, -6, -4, -5, -4, -5, -4, -5, -4], [39, -4, -6, -5, -4, -4, -5, -5, -4], [39, -4, -5, -6, -5, -4, -4, -4, -5], [39, -4, -4, -5, -6, -5, -5, -4, -4], [39, -5, -5, -4, -4, -6, -4, -4, -5], [39, -5, -4, -4, -5, -4, -6, -4, -5], [39, -5, -5, -4, -5, -4, -4, -6, -4], [39, -4, -4, -4, -4, -5, -5, -5, -6], [51, -5, -5, -6, -5, -7, -7, -7, -7], [51, -7, -5, -7, -5, -7, -6, -7, -5], [51, -7, -7, -5, -7, -5, -5, -7, -6], [51, -5, -7, -7, -5, -6, -7, -7, -5], [51, -7, -6, -5, -7, -5, -7, -5, -7], [51, -6, -7, -7, -7, -5, -5, -5, -7], [51, -7, -7, -5, -6, -7, -5, -5, -7], [51, -5, -5, -7, -7, -7, -7, -6, -5]]$ **(2)**

```
> nvL:=map(nv->mult_nv_by_fp_quot(nv,17,7,5),nvLA):
> nops(nvL);
```

48 **(3)**

We now have a list of 48 functions in our basis list and we are ready to find an prove the identity for m=12.

```
> nvLq:=nvL2q(nvL,17,100):
> nvLq2:=map(f->series(f/q^(12/17),q,100),nvLq):
> findhom(nvLq2,q,1,0);
```

$$\{\varnothing\} \tag{4}$$

```
> do_alg_steps(17,12,nvL);
-------------------------------------------------
STEP 1: check modularity
        modularity checks
-------------------------------------------------
STEP 2: find k0 and divide by j0
        k0 = 17
-------------------------------------------------
STEP 3: Compute table of ORDS at all cusps for each func
```

"CUSPS: ", [[1, 0], [0, 1], [1, 2], [1, 3], [1, 4], [1, 5], [1, 6], [1, 7], [1, 8], [2, 17], [3, 17], [4, 17], [5, 17], [6, 17], [7, 17], [8, 17]]

"TABLE of ords"

$$2, -2, -2, -2, -2, -2, -2, -2, -2, 2, 2, 2, 2, 2, 2, 2$$
$$3, -2, -2, -2, -2, -2, -2, -2, -2, 3, 3, 4, 2, 1, 2, -2$$
$$3, -2, -2, -2, -2, -2, -2, -2, -2, 3, 5, 1, 2, 0, 3, -1$$
$$4, -2, -2, -2, -2, -2, -2, -2, -2, 5, 2, 0, 2, 1, 3, -1$$
$$3, -2, -2, -2, -2, -2, -2, -2, -2, 2, 2, 1, 5, 2, 1, 0$$
$$4, -2, -2, -2, -2, -2, -2, -2, -2, 2, 1, 1, 3, 1, 5, -1$$
$$3, -2, -2, -2, -2, -2, -2, -2, -2, 2, 3, 2, 1, 4, 2, -1$$
$$6, -2, -2, -2, -2, -2, -2, -2, -2, 1, 2, 1, 3, 1, 2, 0$$
$$8, -3, -3, -3, -3, -3, -3, -3, -3, 3, 2, 3, 3, 1, 2, 2$$
$$4, -3, -3, -3, -3, -3, -3, -3, -3, 4, 2, 4, 2, 2, 2, 4$$
$$3, -3, -3, -3, -3, -3, -3, -3, -3, 2, 5, 2, 3, 6, 4, -1$$
$$5, -3, -3, -3, -3, -3, -3, -3, -3, 5, 3, 6, 2, 1, 2, 0$$
$$4, -3, -3, -3, -3, -3, -3, -3, -3, 2, 3, 1, 5, 3, 7, -1$$
$$3, -3, -3, -3, -3, -3, -3, -3, -3, 3, 7, 1, 4, 2, 5, -1$$
$$3, -3, -3, -3, -3, -3, -3, -3, -3, 2, 4, 1, 7, 4, 3, 0$$
$$6, -3, -3, -3, -3, -3, -3, -3, -3, 7, 2, 2, 2, 1, 3, 1$$
$$0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0$$
$$1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 2, 0, -1, 0, -4$$
$$1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 3, -1, 0, -2, 1, -3$$
$$2, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, -2, 0, -1, 1, -3$$
$$1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 3, 0, -1, -2$$
$$2, 0, 0, 0, 0, 0, 0, 0, 0, -1, -1, 1, -1, 3, -3$$
$$1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, -1, 2, 0, -3$$

$$4, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, -1, 1, -1, 0, -2$$
$$6, -1, -1, -1, -1, -1, -1, -1, -1, 1, 0, 1, 1, -1, 0, 0$$
$$2, -1, -1, -1, -1, -1, -1, -1, -1, 2, 0, 2, 0, 0, 0, 2$$
$$1, -1, -1, -1, -1, -1, -1, -1, -1, 0, 3, 0, 1, 4, 2, -3$$
$$3, -1, -1, -1, -1, -1, -1, -1, -1, 3, 1, 4, 0, -1, 0, -2$$
$$2, -1, -1, -1, -1, -1, -1, -1, -1, 0, 1, -1, 3, 1, 5, -3$$
$$1, -1, -1, -1, -1, -1, -1, -1, -1, 1, 5, -1, 2, 0, 3, -3$$
$$1, -1, -1, -1, -1, -1, -1, -1, -1, 0, 2, -1, 5, 2, 1, -2$$
$$4, -1, -1, -1, -1, -1, -1, -1, -1, 5, 0, 0, 0, -1, 1, -1$$
$$4, -4, -4, -4, -4, -4, -4, -4, -4, 4, 4, 4, 4, 4, 4, 4$$
$$5, -4, -4, -4, -4, -4, -4, -4, -4, 5, 5, 6, 4, 3, 4, 0$$
$$5, -4, -4, -4, -4, -4, -4, -4, -4, 5, 7, 3, 4, 2, 5, 1$$
$$6, -4, -4, -4, -4, -4, -4, -4, -4, 7, 4, 2, 4, 3, 5, 1$$
$$5, -4, -4, -4, -4, -4, -4, -4, -4, 4, 4, 3, 7, 4, 3, 2$$
$$6, -4, -4, -4, -4, -4, -4, -4, -4, 4, 3, 3, 5, 3, 7, 1$$
$$5, -4, -4, -4, -4, -4, -4, -4, -4, 4, 5, 4, 3, 6, 4, 1$$
$$8, -4, -4, -4, -4, -4, -4, -4, -4, 3, 4, 3, 5, 3, 4, 2$$
$$10, -5, -5, -5, -5, -5, -5, -5, -5, 5, 4, 5, 5, 3, 4, 4$$
$$6, -5, -5, -5, -5, -5, -5, -5, -5, 6, 4, 6, 4, 4, 4, 6$$
$$5, -5, -5, -5, -5, -5, -5, -5, -5, 4, 7, 4, 5, 8, 6, 1$$
$$7, -5, -5, -5, -5, -5, -5, -5, -5, 7, 5, 8, 4, 3, 4, 2$$
$$6, -5, -5, -5, -5, -5, -5, -5, -5, 4, 5, 3, 7, 5, 9, 1$$
$$5, -5, -5, -5, -5, -5, -5, -5, -5, 5, 9, 3, 6, 4, 7, 1$$
$$5, -5, -5, -5, -5, -5, -5, -5, -5, 4, 6, 3, 9, 6, 5, 2$$
$$8, -5, -5, -5, -5, -5, -5, -5, -5, 9, 4, 4, 4, 3, 5, 3$$

```
-------------------------------------------------
STEP 4: Compute LOWER BOUND for ORD of _Kpm/_j0 at each cusp
```

"TABLE :"

$$\_cusp, \_LOWER\_BOUND\_of\_ORD, \frac{\_Kpm}{\_j0}, \_at\_cusp$$

$$\_cusp = 0, \_LOWER\_BOUND = -5$$

$$\_cusp = \frac{1}{2}, \_LOWER\_BOUND = -1$$

$$\_cusp = \frac{1}{3}, \_LOWER\_BOUND = -1$$

$$\_cusp = \frac{1}{4}, \_LOWER\_BOUND = -1$$

$$\_cusp = \frac{1}{5}, \_LOWER\_BOUND = -1$$

$$\_cusp = \frac{1}{6}, \_LOWER\_BOUND = -1$$

$$\_cusp = \frac{1}{7}, \_LOWER\_BOUND = -1$$

$$\_cusp = \frac{1}{8}, \_LOWER\_BOUND = -1$$

$$\_cusp = \frac{2}{17}, \_LOWER\_BOUND = -\frac{2}{17}$$

$$\_cusp = \frac{3}{17}, \_LOWER\_BOUND = \frac{23}{17}$$

$$\_cusp = \frac{4}{17}, \_LOWER\_BOUND = \frac{7}{17}$$

$$\_cusp = \frac{5}{17}, \_LOWER\_BOUND = -\frac{16}{17}$$

$$\_cusp = \frac{6}{17}, \_LOWER\_BOUND = \frac{5}{17}$$

$$\_cusp = \frac{7}{17}, \_LOWER\_BOUND = \frac{19}{17}$$

$$\_cusp = \frac{8}{17}, \_LOWER\_BOUND = -\frac{42}{17}$$

```
-------------------------------------------------
STEP 5: Compile LHS vs RHS ORD table at cusps and find constant B
```

"TABLE  ORD lower bounds"

$$\_cusp, \_width, \_ORD\_LHS, \_ORD\_RHS, \_ORD\_LHS\_minus\_RHS$$

$$0, 17, -5, -5, -5$$

$$\frac{1}{2}, 17, -1, -5, -5$$

$$\frac{1}{3}, 17, -1, -5, -5$$

$$\frac{1}{4}, 17, -1, -5, -5$$

$$\frac{1}{5}, 17, -1, -5, -5$$

$$\frac{1}{6}, 17, -1, -5, -5$$

$$\frac{1}{7}, 17, -1, -5, -5$$

$$\frac{1}{8}, 17, -1, -5, -5$$

$$\frac{2}{17}, 1, 0, -1, -1$$

$$\frac{3}{17}, 1, 2, -1, -1$$

$$\frac{4}{17}, 1, 1, -2, -2$$

$$\frac{5}{17}, 1, 0, -1, -1$$

$$\frac{6}{17}, 1, 1, -2, -2$$

$$\frac{7}{17}, 1, 2, -1, -1$$

$$\frac{8}{17}, 1, -2, -4, -4$$

```
        This implies that B = -52
-------------------------------------------------
STEP 6: Prove and check identity
```

"Coefficients in CKpm identity"

$$\_k = 1, -7\zeta^{15} + 13\zeta^{14} + 9\zeta^{13} - 9\zeta^{12} + 5\zeta^{11} + 15\zeta^{10} - 3\zeta^{9} - 3\zeta^{8} + 15\zeta^{7} + 5\zeta^{6} - 9\zeta^{5} + 9\zeta^{4} + 13\zeta^{3} - 7\zeta^{2} + 18$$

$$\_k = 2, 14\zeta^{14} + 16\zeta^{13} - 7\zeta^{12} + 10\zeta^{11} + 18\zeta^{10} + \zeta^{9} + \zeta^{8} + 18\zeta^{7} + 10\zeta^{6} - 7\zeta^{5} + 16\zeta^{4} + 14\zeta^{3} + 25$$

$$\_k = 3, 7\zeta^{15} - 14\zeta^{14} - 9\zeta^{13} + 8\zeta^{12} - 5\zeta^{11} - 15\zeta^{10} + 3\zeta^{9} + 3\zeta^{8} - 15\zeta^{7} - 5\zeta^{6} + 8\zeta^{5} - 9\zeta^{4} - 14\zeta^{3} + 7\zeta^{2} - 17$$

$$\_k = 4, 7\zeta^{15} - 14\zeta^{14} - 9\zeta^{13} + 8\zeta^{12} - 5\zeta^{11} - 15\zeta^{10} + 3\zeta^{9} + 3\zeta^{8} - 15\zeta^{7} - 5\zeta^{6} + 8\zeta^{5} - 9\zeta^{4} - 14\zeta^{3} + 7\zeta^{2} - 17$$

$$\_k = 5, \zeta^{14} + \zeta^{12} + \zeta^{5} + \zeta^{3}$$

$$\_k = 6, \zeta^{14} + \zeta^{12} + \zeta^{5} + \zeta^{3}$$

$$\_k = 7, 75\zeta^{15} - 130\zeta^{14} - 93\zeta^{13} + 93\zeta^{12} - 46\zeta^{11} - 157\zeta^{10} + 43\zeta^{9} + 43\zeta^{8} - 157\zeta^{7} - 46\zeta^{6} + 93\zeta^{5} - 93\zeta^{4} - 130\zeta^{3} + 75\zeta^{2} - 164$$

$$\_k = 8, 6\zeta^{15} + 12\zeta^{14} + 15\zeta^{13} + 15\zeta^{12} + 9\zeta^{11} + 2\zeta^{10} - \zeta^{9} - \zeta^{8} + 2\zeta^{7} + 9\zeta^{6} + 15\zeta^{5} + 15\zeta^{4} + 12\zeta^{3} + 6\zeta^{2} - 3$$

$$\_k = 9, 26\zeta^{15} - 137\zeta^{14} - 127\zeta^{13} + 23\zeta^{12} - 50\zeta^{11} - 157\zeta^{10} + 36\zeta^{9} + 36\zeta^{8} - 157\zeta^{7} - 50\zeta^{6}$$

$+ 23\,\zeta^5 - 127\,\zeta^4 - 137\,\zeta^3 + 26\,\zeta^2 - 129$

$\_k = 10, -207\,\zeta^{15} + 312\,\zeta^{14} + 218\,\zeta^{13} - 273\,\zeta^{12} + 127\,\zeta^{11} + 379\,\zeta^{10} - 104\,\zeta^9 - 104\,\zeta^8 + 379\,\zeta^7$
$+ 127\,\zeta^6 - 273\,\zeta^5 + 218\,\zeta^4 + 312\,\zeta^3 - 207\,\zeta^2 + 439$

$\_k = 11, 122\,\zeta^{15} - 222\,\zeta^{14} - 187\,\zeta^{13} + 273\,\zeta^{12} - 104\,\zeta^{11} - 357\,\zeta^{10} + 57\,\zeta^9 + 57\,\zeta^8 - 357\,\zeta^7$
$- 104\,\zeta^6 + 273\,\zeta^5 - 187\,\zeta^4 - 222\,\zeta^3 + 122\,\zeta^2 - 439$

$\_k = 12, 188\,\zeta^{15} - 324\,\zeta^{14} - 258\,\zeta^{13} + 254\,\zeta^{12} - 138\,\zeta^{11} - 389\,\zeta^{10} + 106\,\zeta^9 + 106\,\zeta^8 - 389\,\zeta^7$
$- 138\,\zeta^6 + 254\,\zeta^5 - 258\,\zeta^4 - 324\,\zeta^3 + 188\,\zeta^2 - 425$

$\_k = 13, 119\,\zeta^{15} + 32\,\zeta^{14} + 127\,\zeta^{13} + 95\,\zeta^{12} + 90\,\zeta^{11} + 12\,\zeta^{10} + 44\,\zeta^9 + 44\,\zeta^8 + 12\,\zeta^7 + 90\,\zeta^6$
$+ 95\,\zeta^5 + 127\,\zeta^4 + 32\,\zeta^3 + 119\,\zeta^2 + 50$

$\_k = 14, -35\,\zeta^{15} + 158\,\zeta^{14} + 174\,\zeta^{13} - 128\,\zeta^{12} + 98\,\zeta^{11} + 226\,\zeta^{10} - 26\,\zeta^9 - 26\,\zeta^8 + 226\,\zeta^7 + 98\,\zeta^6$
$- 128\,\zeta^5 + 174\,\zeta^4 + 158\,\zeta^3 - 35\,\zeta^2 + 290$

$\_k = 15, -150\,\zeta^{15} + 152\,\zeta^{14} + 68\,\zeta^{13} - 218\,\zeta^{12} + 26\,\zeta^{11} + 234\,\zeta^{10} - 76\,\zeta^9 - 76\,\zeta^8 + 234\,\zeta^7 + 26\,\zeta^6$
$- 218\,\zeta^5 + 68\,\zeta^4 + 152\,\zeta^3 - 150\,\zeta^2 + 251$

$\_k = 16, 224\,\zeta^{15} - 463\,\zeta^{14} - 362\,\zeta^{13} + 286\,\zeta^{12} - 184\,\zeta^{11} - 539\,\zeta^{10} + 137\,\zeta^9 + 137\,\zeta^8 - 539\,\zeta^7$
$- 184\,\zeta^6 + 286\,\zeta^5 - 362\,\zeta^4 - 463\,\zeta^3 + 224\,\zeta^2 - 561$

$\_k = 17, 0$

$\_k = 18, 0$

$\_k = 19, 0$

$\_k = 20, 0$

$\_k = 21, 0$

$\_k = 22, 0$

$\_k = 23, 0$

$\_k = 24, 0$

$\_k = 25, 0$

$\_k = 26, 7\,\zeta^{15} - 14\,\zeta^{14} - 9\,\zeta^{13} + 8\,\zeta^{12} - 5\,\zeta^{11} - 15\,\zeta^{10} + 3\,\zeta^9 + 3\,\zeta^8 - 15\,\zeta^7 - 5\,\zeta^6 + 8\,\zeta^5 - 9\,\zeta^4$
$- 14\,\zeta^3 + 7\,\zeta^2 - 17$

$\_k = 27, \zeta^{14} + \zeta^{12} + \zeta^5 + \zeta^3$

$\_k = 28, -7\,\zeta^{15} + 14\,\zeta^{14} + 9\,\zeta^{13} - 8\,\zeta^{12} + 5\,\zeta^{11} + 15\,\zeta^{10} - 3\,\zeta^9 - 3\,\zeta^8 + 15\,\zeta^7 + 5\,\zeta^6 - 8\,\zeta^5 + 9\,\zeta^4$
$+ 14\,\zeta^3 - 7\,\zeta^2 + 17$

$\_k = 29, \zeta^{14} + \zeta^{12} + \zeta^5 + \zeta^3$

$\_k = 30, 0$

$\_k = 31, -\zeta^{14} - \zeta^{12} - \zeta^5 - \zeta^3$

$\_k = 32, -7\,\zeta^{15} + 14\,\zeta^{14} + 9\,\zeta^{13} - 8\,\zeta^{12} + 5\,\zeta^{11} + 15\,\zeta^{10} - 3\,\zeta^{9} - 3\,\zeta^{8} + 15\,\zeta^{7} + 5\,\zeta^{6} - 8\,\zeta^{5} + 9\,\zeta^{4} + 14\,\zeta^{3} - 7\,\zeta^{2} + 17$

$\_k = 33, 187\,\zeta^{15} - 374\,\zeta^{14} - 289\,\zeta^{13} + 170\,\zeta^{12} - 187\,\zeta^{11} - 408\,\zeta^{10} + 85\,\zeta^{9} + 85\,\zeta^{8} - 408\,\zeta^{7} - 187\,\zeta^{6} + 170\,\zeta^{5} - 289\,\zeta^{4} - 374\,\zeta^{3} + 187\,\zeta^{2} - 391$

$\_k = 34, 612\,\zeta^{15} - 272\,\zeta^{14} + 153\,\zeta^{13} + 340\,\zeta^{12} + 187\,\zeta^{11} - 221\,\zeta^{10} + 340\,\zeta^{9} + 340\,\zeta^{8} - 221\,\zeta^{7} + 187\,\zeta^{6} + 340\,\zeta^{5} + 153\,\zeta^{4} - 272\,\zeta^{3} + 612\,\zeta^{2} + 34$

$\_k = 35, -238\,\zeta^{15} + 170\,\zeta^{14} + 119\,\zeta^{13} - 238\,\zeta^{12} - 34\,\zeta^{11} + 306\,\zeta^{10} - 170\,\zeta^{9} - 170\,\zeta^{8} + 306\,\zeta^{7} - 34\,\zeta^{6} - 238\,\zeta^{5} + 119\,\zeta^{4} + 170\,\zeta^{3} - 238\,\zeta^{2} + 170$

$\_k = 36, -170\,\zeta^{15} + 255\,\zeta^{14} + 85\,\zeta^{13} - 204\,\zeta^{12} + 68\,\zeta^{11} + 289\,\zeta^{10} - 136\,\zeta^{9} - 136\,\zeta^{8} + 289\,\zeta^{7} + 68\,\zeta^{6} - 204\,\zeta^{5} + 85\,\zeta^{4} + 255\,\zeta^{3} - 170\,\zeta^{2} + 204$

$\_k = 37, -85\,\zeta^{15} - 17\,\zeta^{14} + 51\,\zeta^{13} - 68\,\zeta^{12} + 51\,\zeta^{11} - 102\,\zeta^{10} - 51\,\zeta^{9} - 51\,\zeta^{8} - 102\,\zeta^{7} + 51\,\zeta^{6} - 68\,\zeta^{5} + 51\,\zeta^{4} - 17\,\zeta^{3} - 85\,\zeta^{2} - 136$

$\_k = 38, 17\,\zeta^{15} + 85\,\zeta^{14} + 153\,\zeta^{13} + 34\,\zeta^{12} + 136\,\zeta^{11} + 102\,\zeta^{10} - 34\,\zeta^{9} - 34\,\zeta^{8} + 102\,\zeta^{7} + 136\,\zeta^{6} + 34\,\zeta^{5} + 153\,\zeta^{4} + 85\,\zeta^{3} + 17\,\zeta^{2} + 170$

$\_k = 39, -850\,\zeta^{15} + 1377\,\zeta^{14} + 952\,\zeta^{13} - 935\,\zeta^{12} + 459\,\zeta^{11} + 1581\,\zeta^{10} - 493\,\zeta^{9} - 493\,\zeta^{8} + 1581\,\zeta^{7} + 459\,\zeta^{6} - 935\,\zeta^{5} + 952\,\zeta^{4} + 1377\,\zeta^{3} - 850\,\zeta^{2} + 1598$

$\_k = 40, 34\,\zeta^{15} + 306\,\zeta^{14} + 340\,\zeta^{13} + 204\,\zeta^{12} + 204\,\zeta^{11} + 119\,\zeta^{10} - 68\,\zeta^{9} - 68\,\zeta^{8} + 119\,\zeta^{7} + 204\,\zeta^{6} + 204\,\zeta^{5} + 340\,\zeta^{4} + 306\,\zeta^{3} + 34\,\zeta^{2} + 34$

$\_k = 41, -289\,\zeta^{15} + 289\,\zeta^{14} + 289\,\zeta^{13} - 867\,\zeta^{12} + 867\,\zeta^{10} - 289\,\zeta^{9} - 289\,\zeta^{8} + 867\,\zeta^{7} - 867\,\zeta^{5} + 289\,\zeta^{4} + 289\,\zeta^{3} - 289\,\zeta^{2} + 867$

$\_k = 42, 578\,\zeta^{15} - 289\,\zeta^{14} + 289\,\zeta^{13} + 289\,\zeta^{9} + 289\,\zeta^{8} + 289\,\zeta^{4} - 289\,\zeta^{3} + 578\,\zeta^{2} + 289$

$\_k = 43, -867\,\zeta^{15} + 1156\,\zeta^{14} + 1156\,\zeta^{13} - 867\,\zeta^{12} + 289\,\zeta^{11} + 1445\,\zeta^{10} - 578\,\zeta^{9} - 578\,\zeta^{8} + 1445\,\zeta^{7} + 289\,\zeta^{6} - 867\,\zeta^{5} + 1156\,\zeta^{4} + 1156\,\zeta^{3} - 867\,\zeta^{2} + 1156$

$\_k = 44, -867\,\zeta^{15} - 578\,\zeta^{13} - 578\,\zeta^{12} - 289\,\zeta^{11} - 578\,\zeta^{9} - 578\,\zeta^{8} - 289\,\zeta^{6} - 578\,\zeta^{5} - 578\,\zeta^{4} - 867\,\zeta^{2} - 578$

$\_k = 45, 289\,\zeta^{14} + 578\,\zeta^{13} - 289\,\zeta^{12} + 578\,\zeta^{11} + 289\,\zeta^{10} + 289\,\zeta^{7} + 578\,\zeta^{6} - 289\,\zeta^{5} + 578\,\zeta^{4} + 289\,\zeta^{3} + 867$

$\_k = 46, 289\,\zeta^{15} - 578\,\zeta^{14} - 578\,\zeta^{13} + 289\,\zeta^{12} - 867\,\zeta^{10} + 289\,\zeta^{9} + 289\,\zeta^{8} - 867\,\zeta^{7} + 289\,\zeta^{5} - 578\,\zeta^{4} - 578\,\zeta^{3} + 289\,\zeta^{2} - 289$

$$\_k = 47,\ 578\,\zeta^{15} - 867\,\zeta^{14} - 1156\,\zeta^{13} + 867\,\zeta^{12} - 578\,\zeta^{11} - 1156\,\zeta^{10} + 289\,\zeta^{9} + 289\,\zeta^{8} - 1156\,\zeta^{7} - 578\,\zeta^{6} + 867\,\zeta^{5} - 1156\,\zeta^{4} - 867\,\zeta^{3} + 578\,\zeta^{2} - 1445$$

$$\_k = 48,\ -867\,\zeta^{15} + 578\,\zeta^{14} - 1156\,\zeta^{12} + 867\,\zeta^{10} - 578\,\zeta^{9} - 578\,\zeta^{8} + 867\,\zeta^{7} - 1156\,\zeta^{5} + 578\,\zeta^{3} - 867\,\zeta^{2} + 578$$

"Proving and checking identity"

"IDENTITY CHECKED AND PROVEN"

"IDENTITY checked for ", $\_O\left(q^{\_topq+1}\right) = \_O\left(q^{118}\right)$

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and _topq + 1 > -_B = 52
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