

# Lab 9 Solution

## Problem 1

Minimize

$$f(x, y) = -xy$$

subject to

$$x + y^2 \leq 2 \text{ and } x, y \geq 0$$

```
# Load the nloptr library
library(nloptr)

# Objective function
func <- function(x) {
  c(-x[1] * x[2])
}
```

The above code defines the objective function that needs to be minimized.

```
# Nonlinear inequality constraints:

eval_g_ineq <- function(x) {
  c(x[1] + x[2]^2 - 2, -x[1], -x[2])
}
```

The above code defines the inequality constraints subject to which the objective function is to be minimized.

```
# Run optimization using NLOPT_GN_ISRES
res <- nloptr(
  x0 = c(0.5, 0.5),           # Feasible start point
  eval_f = func,
  eval_g_ineq = eval_g_ineq,
```

```

lb = c(0, 0),                # Lower bounds for x and y
ub = c(10, 10),             # Large but finite upper bounds
opts = list(
  "algorithm" = "NLOPT_GN_ISRES",
  "xtol_rel" = 1e-8, "maxeval" = 1e4
)
)

```

The variable **res** stores the required output obtained after minimization of the given function.

```
res
```

Call:

```

nloptr(x0 = c(0.5, 0.5), eval_f = func, lb = c(0, 0), ub = c(10,
  10), eval_g_ineq = eval_g_ineq, opts = list(algorithm = "NLOPT_GN_ISRES",
  xtol_rel = 1e-08, maxeval = 10000))

```

Minimization using NLOpt version 2.7.1

NLOpt solver status: 5 ( NLOPT\_MAXEVAL\_REACHED: Optimization stopped because maxeval (above) was reached. )

```

Number of Iterations.....: 10000
Termination conditions:  xtol_rel: 1e-08    maxeval: 10000
Number of inequality constraints:  3
Number of equality constraints:    0
Current value of objective function: -1.08862650968763
Current value of controls: 1.330452 0.8182383

```

From the above, we can see that the minimum value of the function for relative tolerance  $10^{-8}$  and maximum number of function evaluations as  $10^4$  is quite close to the actual minimum value of the function i.e.  $-1.088662$ . The function attains its minimum value at  $(\frac{4}{3}, \sqrt{\frac{2}{3}})$ .

## Problem 2

Minimize

$$f(x, y) = 2x + y$$

subject to

$$\sqrt{x^2 + y^2} \leq 2, x \geq 0 \text{ and } y \geq 0.5x - 1$$

```
# Objective function
func <- function(x) {
  2 * x[1] + x[2]
}
```

The above code defines the objective function that needs to be minimized.

```
# Inequality Constraints:

eval_g_ineq <- function(x) {
  c(sqrt(x[1]^2 + x[2]^2) - 2,
    -x[1],
    (0.5 * x[1] - 1) - x[2]      # y = 0.5x - 1
  )
}
```

The above code defines the inequality constraints subject to which the objective function is to be minimized.

```
res <- nloptr(
  x0 = c(1, 1), # start point satisfying constraints approximately
  eval_f = func,
  eval_g_ineq = eval_g_ineq,
  lb = c(0, -3),
  ub = c(10, 10),
  opts = list(
    "algorithm" = "NLOPT_GN_ISRES",
    "xtol_rel" = 1e-8, "maxeval" = 10000
  )
)
```

The variable **res** stores the required output obtained after minimization of the given function.

```
res
```

Call:

```
nloptr(x0 = c(1, 1), eval_f = func, lb = c(0, -3), ub = c(10,
  10), eval_g_ineq = eval_g_ineq, opts = list(algorithm = "NLOPT_GN_ISRES",
  xtol_rel = 1e-08, maxeval = 10000))
```

Minimization using NLOpt version 2.7.1

NLOpt solver status: 5 ( NLOPT\_MAXEVAL\_REACHED: Optimization stopped because maxeval (above) was reached. )

Number of Iterations.....: 10000  
Termination conditions: xtol\_rel: 1e-08      maxeval: 10000  
Number of inequality constraints: 3  
Number of equality constraints: 0  
Current value of objective function: -0.999994258712022  
Current value of controls: 2.185729e-06 -0.9999986

From the above, we can see that the minimum value of the function for relative tolerance  $10^{-8}$  and maximum number of function evaluations as  $10^4$  is quite close to the actual minimum value of the function i.e.  $-1$ . The function attains its minimum value at  $(0, -1)$ .

### Problem 3

Minimize

$$f(x_1, \dots, x_4) = x_1^2 + x_2^2 + x_3^2 + x_4^2$$

subject to

$$x_1 + x_2 + x_3 + x_4 = 1 \text{ and } x_4 \geq A.$$

Consider three cases:

1.  $A < 1/4$
2.  $A = 1/4$
3.  $A > 1/4$

```
# Objective Function
func <- function(x){
  sum(x^2)
}
```

The above code defines the objective function that needs to be minimized.

```
# Equality Constraints:
```

```
eval_g_eq <- function(x){  
  sum(x) - 1  
}
```

The above code defines the equality constraints subject to which the objective function is to be minimized.

```
solve_p3 <- function(A) {  
  eval_g_ineq <- function(x) {  
    A - x[4]  
  }  
  
  res <- nloptr(  
    x0 = c(rep(A / 3, 3), 1 - A), # reasonable start  
    eval_f = func,  
    eval_g_ineq = eval_g_ineq,  
    lb = rep(-2, 4),  
    ub = rep(10, 4),  
    eval_g_eq = eval_g_eq,  
    opts = list(  
      "algorithm" = "NLOPT_GN_ISRES",  
      "xtol_rel" = 1e-8, "maxeval" = 1e8  
    )  
  )  
  return(res)  
}
```

The above code minimizes the objective function subject to different values of the variable  $A$ .

```
# Case (i)  $A < 1/4$   
res_p3_case1 <- solve_p3(A = 0.2)  
print(res_p3_case1)
```

Call:

```
nloptr(x0 = c(rep(A/3, 3), 1 - A), eval_f = func, lb = rep(-2,  
  4), ub = rep(10, 4), eval_g_ineq = eval_g_ineq, eval_g_eq = eval_g_eq,  
  opts = list(algorithm = "NLOPT_GN_ISRES", xtol_rel = 1e-08,
```

```
maxeval = 1e+08))
```

Minimization using NLOpt version 2.7.1

NLOpt solver status: 4 ( NLOPT\_XTOL\_REACHED: Optimization stopped because  
xtol\_rel or xtol\_abs (above) was reached. )

Number of Iterations.....: 83298

Termination conditions: xtol\_rel: 1e-08 maxeval: 1e+08

Number of inequality constraints: 1

Number of equality constraints: 1

Optimal value of objective function: 0.25000041702258

Optimal value of controls: 0.2495092 0.2504014 0.2499556 0.2501338

```
# Case (ii) A = 1/4  
res_p3_case2 <- solve_p3(A = 0.25)  
print(res_p3_case2)
```

Call:

```
nloptr(x0 = c(rep(A/3, 3), 1 - A), eval_f = func, lb = rep(-2,  
4), ub = rep(10, 4), eval_g_ineq = eval_g_ineq, eval_g_eq = eval_g_eq,  
opts = list(algorithm = "NLOPT_GN_ISRES", xtol_rel = 1e-08,  
maxeval = 1e+08))
```

Minimization using NLOpt version 2.7.1

NLOpt solver status: 4 ( NLOPT\_XTOL\_REACHED: Optimization stopped because  
xtol\_rel or xtol\_abs (above) was reached. )

Number of Iterations.....: 88612

Termination conditions: xtol\_rel: 1e-08 maxeval: 1e+08

Number of inequality constraints: 1

Number of equality constraints: 1

Optimal value of objective function: 0.250000144645659

Optimal value of controls: 0.2499964 0.2499764 0.2497421 0.250285

```
# Case (iii) A > 1/4  
res_p3_case3 <- solve_p3(A = 0.3)  
print(res_p3_case3)
```

Call:

```
nloptr(x0 = c(rep(A/3, 3), 1 - A), eval_f = func, lb = rep(-2,
  4), ub = rep(10, 4), eval_g_ineq = eval_g_ineq, eval_g_eq = eval_g_eq,
  opts = list(algorithm = "NLOPT_GN_ISRES", xtol_rel = 1e-08,
    maxeval = 1e+08))
```

Minimization using NLOpt version 2.7.1

NLOpt solver status: 4 ( NLOPT\_XTOL\_REACHED: Optimization stopped because  
xtol\_rel or xtol\_abs (above) was reached. )

Number of Iterations.....: 71635

Termination conditions: xtol\_rel: 1e-08 maxeval: 1e+08

Number of inequality constraints: 1

Number of equality constraints: 1

Optimal value of objective function: 0.253337826612802

Optimal value of controls: 0.2319622 0.2331029 0.234935 0.3

From the above results we can see that the objective function has a minimum value of  $\frac{1}{4}$  irrespective of the value of the variable  $A$ , subject to the given constraints.