Lab 8 Solution

Problem 1

Minimize

$$f(x, y) = x^2 + y^2 + 2xy$$

subject to

$$x, y \in [0, 1]$$

```
# Load the nloptr library
library(nloptr)

# Problem 1: Minimize f(x, y) = x^2 + y^2 + 2xy with x, y in [0, 1]
obj_fun <- function(x) {
  return(x[1]^2 + x[2]^2 + 2 * x[1] * x[2])
}
```

The above code defines the multivariate function (objective function) that needs to be minimized.

```
res <- nloptr(
  x0 = c(0.5, 0.5), # initial values
  eval_f = obj_fun,
  lb = c(0, 0),
  ub = c(1, 1),
  opts = list("algorithm" = "NLOPT_GN_ISRES",
              "xtol_rel" = 1.0e-8, "maxeval" = 1000)
)
```

The variable **res** stores the required output obtained after minimization of the given function.

```r
res
```

```
Call:
nloptr(x0 = c(0.5, 0.5), eval_f = obj_fun, lb = c(0, 0), ub = c(1,
    1), opts = list(algorithm = "NLOPT_GN_ISRES", xtol_rel = 1e-08,
    maxeval = 1000))


Minimization using NLopt version 2.7.1

NLopt solver status: 5 ( NLOPT_MAXEVAL_REACHED: Optimization stopped because
maxeval (above) was reached. )

Number of Iterations....: 1000
Termination conditions:  xtol_rel: 1e-08    maxeval: 1000
Number of inequality constraints:  0
Number of equality constraints:    0
Current value of objective function:  1.92235014321056e-05
Current value of controls: 0.002379796 0.002004666
```

From the above, we can see that the minimum value of the function for relative tolerance 10^{-8} and maximum number of function evaluations as 1000 is quite close to the actual minimum value of the function i.e. 0. The function attains 0 at the extreme i.e. $(0, 0)$.

Problem 2

Minimize

$$f(x, y) = x^2 + y^2 - 2xy$$

subject to

$$x, y \in [0, 1] \text{ and } x + y = 1$$

```r
# Problem 2: Minimize f(x, y) = x^2 + y^2 - 2xy
# ..with x, y in [0, 1] and x + y = 1
obj_fun <- function(x) {
  return(x[1]^2 + x[2]^2 - 2 * x[1] * x[2])
}

constraint_fun <- function(x) {
  return(x[1] + x[2] - 1) # Equality constraint: x + y - 1 = 0
}
```

The above code defines the multivariate function (objective function) that needs to be minimized.

```r
res <- nloptr(
  x0 = c(0.5, 0.5),
  eval_f = obj_fun,
  lb = c(0, 0),
  ub = c(1, 1),
  eval_g_eq = constraint_fun,
  opts = list("algorithm" = "NLOPT_GN_ISRES",
              "xtol_rel" = 1.0e-8, "maxeval" = 1000)
)
```

The variable **res** stores the required output obtained after minimization of the given function.

```r
res
```

```
Call:
nloptr(x0 = c(0.5, 0.5), eval_f = obj_fun, lb = c(0, 0), ub = c(1,
    1), eval_g_eq = constraint_fun, opts = list(algorithm = "NLOPT_GN_ISRES",
    xtol_rel = 1e-08, maxeval = 1000))


Minimization using NLopt version 2.7.1

NLopt solver status: 5 ( NLOPT_MAXEVAL_REACHED: Optimization stopped because
maxeval (above) was reached. )

Number of Iterations....: 1000
Termination conditions:  xtol_rel: 1e-08    maxeval: 1000
Number of inequality constraints:  0
Number of equality constraints:    1
Current value of objective function:  0
Current value of controls: 0.5 0.5
```

From the above, we can see that the minimum value of the function for relative tolerance 10^{-8} and maximum number of function evaluations as 1000 is 0, which is attained at infinitely many points satisfying $x = y$. But, after the imposition of the equality constraint, we arrive at an unique point where the value of the objective function is minimized which is $(\frac{1}{2}, \frac{1}{2})$ and the value is 0.