

# Application III – Operations on Polynomials

- Let us consider multiplication
- Can be done as repeated addition.
- So, multiply P1 with each term of P2.
- Add the resulting polynomials.
- Develop the pseudocode in class...



# Application III – Operations on Polynomials

- Let us consider multiplication
  - Can be done as repeated addition.
  - So, multiply P1 with each term of P2.
  - Add the resulting polynomials.
- ,
- Develop the pseudocode in class...

## Application IV – Matrix Multiplication

- Consider another problem described as follows.
- The multiplication of two matrices A and B is understood as follows.
- For each  $i$  and  $j$ ,  $C[i,j] = \sum_k A[i,k].B[k,j]$ .



## Application IV – Matrix Multiplication

- Consider another problem described as follows.
- The multiplication of two matrices A and B is understood as follows.
- For each i and j,  $C[i,j] = \sum_k A[i,k].B[k,j]$ .

$$A = \begin{bmatrix} & \overset{2,1}{\cancel{5}} & \overset{2,2}{\cancel{1}} & \overset{2,3}{\cancel{0}} \\ \cancel{2,1} & \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \end{bmatrix} \end{bmatrix}$$
$$B = \begin{bmatrix} 2 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}_{3 \times 2}^{2 \times 3}$$

$$C = A \cdot B$$

mxn    mn    nxk  
YXC

$$\left[ \begin{array}{c} 8 \\ \vdots \\ 22 \end{array} \right] \in C_{2 \times 2}^{2 \times 3}$$

# Application IV – Matrix Multiplication

- If A and B are sparse, there are several issues in matrix multiplication if A, B, and C are stored as arrays.
  - Storage /Retrieval, Compatibility of indices



0	10	12
1	0	2
0	0	0

A

2	5	0
0	1	0

B



Request control

# Application IV – Matrix Multiplication

If A and B are sparse, there are several issues in matrix multiplication if A, B, and C are stored as arrays.

- Storage /Retrieval, Compatibility of indices

0	10	12
1	0	2
0	0	0

A

2	5	0
0	1	0
8	0	0

B

# Application IV – Matrix Multiplication

- If A and B are sparse, there are several issues in matrix multiplication if A, B, and C are stored as arrays.
  - Storage /Retrieval, Compatibility of indices
- Alternate storage models for sparse matrices exist.

Row	Col	Val
-----	-----	-----

1	2	10
---	---	----

1	3	12
---	---	----

2	1	1
---	---	---

2	3	2
---	---	---

Row	Col	Val
-----	-----	-----

1	1	2
---	---	---

1	2	5
---	---	---

2	2	1
---	---	---

3	1	8
---	---	---

0	10	12
1	0	2
0	0	0

A

2	5	0
0	1	0
8	0	0

B

# Application IV – Matrix Multiplication

$$\begin{matrix} 0 & 10 & 12 \\ 1 & 0 & 2 \\ 0 & 0 & 0 \end{matrix} \times \begin{matrix} 2 & 5 & 0 \\ 0 & 1 & 0 \\ 8 & 0 & 0 \end{matrix} = \begin{matrix} 96 & 10 & 0 \\ 18 & 5 & 0 \\ 0 & 0 & 0 \end{matrix}$$

# Application IV – Matrix Multiplication

- To multiply A and B, get each row of A and each column of B multiply element-wise and sum to get one element of C.
- Not easy if sparse matrix are stored as sorted list. Can we do it efficiently ?

0	10	12
1	0	2
0	0	0

A

2	5	0
0	1	0

B



## Matrix Multiplication – Column-Row Formulation

1	1
5	2

×

B

2	1	3
3	3	1

=

C

12	9	11
5	4	4
16	11	17

3
1
5

\*

=

6	3	9
2	1	3
10	5	15

## Matrix Multiplication – Column-Row Formulation

**A**

3	2
1	1
5	2

×

**B**

2	1	3
3	3	1

=

**C**

12	9	11
5	4	4
16	11	17

3
1
5
2
1
2

\*

2	1	3
---	---	---

=

6	3	9
2	1	3
10	5	15

+

=

12	9	11
5	4	4
16	11	17

3
1
5
2
1
2

\*

3	3	1
---	---	---

=

6	6	2
3	3	1
6	6	2

+18

SA

SS

AA

SG

PR

DS

2020/8/21 10:14

# Matrix Multiplication – Row-Row Formulation

A	3	2	0
	1	1	2

$\times$

B	2	1	3
	3	3	1
	2	2	1

=

C	12	9	11
	9	8	6

$$\begin{array}{r} 3 \quad \times \quad 2 \quad 1 \quad 3 \\ + \quad 1 \quad \times \quad 2 \quad 1 \quad 3 \\ \hline 12 \quad 9 \quad 11 \end{array}$$
$$\begin{array}{r} 3 \quad \times \quad 2 \quad 1 \quad 3 \\ + \quad 1 \quad \times \quad 3 \quad 3 \quad 1 \\ \hline \end{array}$$

+118

SS

AA

SG

PR

DS

2020/8/21 10:16

⚠ Recording has started. This meeting is being recorded. By joining, you are giving consent for this meeting to be recorded. [Priv](#)

## Matrix Multiplication – Row-Row Formulation

$$\begin{matrix} A \\ \begin{array}{|c|c|c|} \hline 3 & 2 & 0 \\ \hline 1 & 1 & 2 \\ \hline \end{array} \end{matrix} \times \begin{matrix} B \\ \begin{array}{|c|c|c|} \hline 2 & 1 & 3 \\ \hline 3 & 3 & 1 \\ \hline 2 & 2 & 1 \\ \hline \end{array} \end{matrix} = \begin{matrix} C \\ \begin{array}{|c|c|c|} \hline 12 & 9 & 11 \\ \hline 9 & 8 & 6 \\ \hline \end{array} \end{matrix}$$

$$\begin{matrix} 3 & \times & 2 & 1 & 3 \\ + & & 2 & \times & 3 & 3 & 1 \end{matrix} = \begin{matrix} 12 & 9 & 11 \end{matrix}$$

$$\begin{matrix} 1 & \times & 2 & 1 & 3 \\ + & & 1 & \times & 3 & 3 & 1 \end{matrix} + \begin{matrix} 2 & \times & 2 & 2 & 1 \end{matrix} = \begin{matrix} 9 & 8 & 6 \end{matrix}$$

# Application IV – Matrix Multiplication

- To multiply A and B, get each row of A and each column of B multiply element-wise and sum to get one element of C.
- Not easy if sparse matrix are stored as sorted list. Can we do it efficiently ?

Row	Col	Val
1	2	10
1	3	12
2	1	1
2	3	2

Row	Col	Val
1	1	2
1	3	8
2	1	5
	1	

0	10	12
1	0	2
0	0	0

2	0	8
5	1	0
0	0	0

A

 $B^T$ 

Request control