Application III – Operations on Polynomials

- Let us consider multiplication
- Can be done as repeated addition.
- So, multiply P1 with each term of P2.
- Add the resulting polynomials.

- Develop the pseudocode in class...

Application III – Operations on Polynomials

- We have P1 and P2 arranged in a linked list in decreasing order of exponents.
- We can scan these and add like terms.
  - Need to store the resulting term only if it has non-zero coefficient.
- The number of terms in the result polynomial P1+P2 need not be known in advance.
- We'll use as much space as there are terms in P1+P2.

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Application IV – Matrix Multiplication

- Consider another problem described as follows.
- The multiplication of two matrices A and B is understood as follows.
- For each i and j, $C[i,j] = \Sigma_k A[i,k].B[k,j]$.

Application IV – Matrix Multiplication

- If A and B are sparse, there are several issues in matrix multiplication if A, B, and C are stored as arrays.
  - Storage /Retrieval, Compatibility of indices
- Alternate storage models for sparse matrices exist.

Row	Col	Val
1	2	10
1	3	12
2	1	1
2	3	2

Row	Col	Val
1	1	2
1	2	5
2	2	1
[illegible]	1	[illegible]

A

0	10	12
1	0	2
0	0	0

B

2	5	0
0	1	0
3	0	0

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Application IV – Matrix Multiplication

$$\begin{bmatrix} 0 & 10 & 12 \\ 1 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 5 & 0 \\ 0 & 1 & 0 \\ 8 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 96 & 10 & 0 \\ 18 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Application IV – Matrix Multiplication

- To multiply A and B, get each row of A and each column of B multiply element-wise and sum to get one element of C.
- Not easy if sparse matrix are stored as sorted list. Can we do it efficiently ?

Row	Col	Val
1	2	10
1	3	12
2	1	1
2	3	2

Row	Col	Val
1	1	2
1	3	8
2	1	5
2	2	1

A

0	10	12
1	0	2
0	0	0

B^T

2	0	8
5	1	0
0	0	0

Matrix Multiplication – Column-Row Formulation

$$\underset{A}{\begin{bmatrix} 3 & 2 \\ 1 & 1 \\ 5 & 2 \end{bmatrix}} \times \underset{B}{\begin{bmatrix} 2 & 1 & 3 \\ 3 & 3 & 1 \end{bmatrix}} = \underset{C}{\begin{bmatrix} 12 & 9 & 11 \\ 5 & 4 & 4 \\ 16 & 11 & 17 \end{bmatrix}}$$

$$\begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix} * \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 3 & 9 \\ 2 & 1 & 3 \\ 10 & 5 & 15 \end{bmatrix}$$

$+$

$$\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} * \begin{bmatrix} 3 & \dots \end{bmatrix} = \begin{bmatrix} 6 & 6 & 2 \\ \dots & \dots & \dots \\ 6 & 6 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 9 & 11 \\ 5 & 4 & 4 \\ 16 & 11 & 17 \end{bmatrix}$$

Matrix Multiplication – Row-Row Formulation

$$\underbrace{\begin{bmatrix} 3 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix}}_{A} \times \underbrace{\begin{bmatrix} 2 & 1 & 3 \\ 3 & 3 & 1 \\ 2 & 2 & 1 \end{bmatrix}}_{B} = \underbrace{\begin{bmatrix} 12 & 9 & 11 \\ 9 & 8 & 6 \end{bmatrix}}_{C}$$

$$3 \times \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} + 2 \times \begin{bmatrix} 3 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 9 & 11 \end{bmatrix}$$

$$1 \times \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} + 1 \times \begin{bmatrix} 3 & 3 & 1 \end{bmatrix} + 2 \times \begin{bmatrix} 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 6 \end{bmatrix}$$

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