INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY
HYDERABAD

CVIT

Data Structures & Algorithms for Problem Solving (M20Temp3)

Lecture # 01

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Why DSAPS?

- Problem solving is essential to Computer Science.
- Think of some problems that we solve almost on a daily basis.
  - Go through contacts/messages that are already sorted
  - Find routes in a city from a point A to a point B
  - Find the best ticket given a certain constraints.
  - Find appropriate search keywords based on various criteria.

Why DSAPS?

- Central to the examples mentioned are **data structures** that increase the efficiency by at least an order of magnitude.
- As the size of the data gets larger, the importance of these data structures gets critical.
  - Consider the amount of data handled by Google Maps, or the Human Genome project, or the Collider project.
- The aims of this course
  - Understand these problems and the data structures/algorithms deployed.
  - Develop solutions using appropriate data structures.

About the course

- We will start by covering several fundamental data structures including
  - Stacks/linked-list/queues/lists
- Problem Solving and related Data Structures e.g., :
  - Search trees – especially AVL Trees and B-trees
  - Dynamic arrays and amortized data structures
  - String based problems and tries, suffix trees
  - Range querying via range trees
  - Randomization in computing via perfect hashing

About the course

- Will introduce practical motivations to each of the considered topics.
- Several problem solving sessions to fully understand the implications of using a data structure for problem solving.
- Emphasis also on correctness and efficiency.
- Elementary analysis
  - Online Laboratory sessions are therefore very important.

About the course - Material

- No prescribed textbook
- But you can refer to
  - “Data structures and algorithm analysis in C” by Mark Allen Weiss
  - “Introduction to Algorithms” by Thomas H. Cormen

About the course - Structure

- Weekly 3 lecture hours.
- 1 Laboratory sessions every week
  - about 4-5 problems to be solved in the session (TAs to assist).
  - Credits are given for lab performance
- Several homework assignments
  - About 5, one every two weeks.
  - Each set to have about 6-7 problems
- Strictly, no plagiarism Any detected case of plagiarism to be taken seriously.
- Actively use Moodle as platform for course content/news/assignments
- Email communication
  - Vineet Gandhi, vgandhi@iiit.ac.in
  - Avinash Sharma, asharma@iiit.ac.in
- Very important: Seek help early enough.

About the course – Grading Policy

- Assessment (Credit Distribution**)
  - Quiz (10%) + Exam (20%)
  - Assignments* (45%)
  - Lab Test (25%)

* If copying is detected, you will get 0 marks for the assignment

**This policy might slightly vary as course evolve over the semester

Organization (today's lecture)

1. Number Representation

UNDERSTAND BASICS

2. Number Operations

UNDERSTAND BASICS

3. Finding Greatest Common Divisor (GCD)

HOW TO FORMULATE ?

Number System

- A number system is a way to represent numbers.
- Several known number systems in practice today.
  - Hindu/Decimal/Arabic system
  - Roman system
  - Binary, octal, hexa-decimal.
  - Unary system
  - ...

Classifiable as

- positional
- non-positional

Number System

- Hindu/Decimal system
  - Numbers represented using the digits in {0, 1, ,..., 9}. E.g., 8,732,937,309
- Roman System
  - Numbers represented using the letters I, V, X, L, C, D, and M. E.g., X represents 10, L represents 50.
  - LX stands for 60, IV stands for 4, what is MMXIX?
  - MMMDDDCCCLLLXXXVVVIII – largest numbers without any overlines/subtractions. What is this number?
- Binary system
  - Numbers represented using the digits 0 and 1.
  - 10111 represents 23.

Number System

- Positional (aka value based) number systems associate a value to a digit based on the its position.
  - Example: Decimal, binary, ...
- Non-positional do not have such an association.
  - Example: Unary

Operation on Numbers

- Let us consider operations addition and multiplication.
- Hindu/Decimal system
  - Add digit wise
  - Carry of x from digit at position k to position k+1 equivalent to a value of $x.10^{k+1}$, $k \geq 0$.
  - Example: Adding 87 to 56 gives 143.
- Unary system
  - Probably, the first thing we learn.
  - To add two numbers x and y, create a number that contains the number of 1's in both x and y.
  - Example: Adding 1111 to 11111 results in 111111111.

Operation on Numbers

1	I	11	XI	50	L
2	II	12	XII	100	C
3	III	13	XIII	500	D
4	IV	14	XIV	1000	M
5	V	15	XV		
6	VI	16	XVI		
7	VII	17	XVII		
8	VIII	18	XVIII		
9	IX	19	XIX		
10	X	20	XX		

- Roman system
  - A bit complicated but possible.
  - Follow the following three steps:
    - Write the numbers side by side.
    - Arrange the letters in decreasing order of value.
    - Simplify.
  - Example: to add 32 and 67:
    - 32 = XXXII, 67 = LXVII.
    - XXXIILXVII
    - LXXXXVIIII – LXLIX – XCIX
    - Simplified as: XCIX
- Rules such as:
  - If there are 4I's, write it as IV.
  - If there are 4X's, write it as XL.
  - Write LXL as XC.
  - Similar rules apply.
- Careful when starting with numbers such as LXIV.
  - Can replace IV with IIII initially.

Operation on Numbers

- Let us now consider multiplication.
- Typically, multiplication is achieved by repeated addition.
- Decimal system
  - Known approach.
- Roman system
  - How to multiply?
  - Much complicated, but is possible.

Multiplication in Roman Numbers

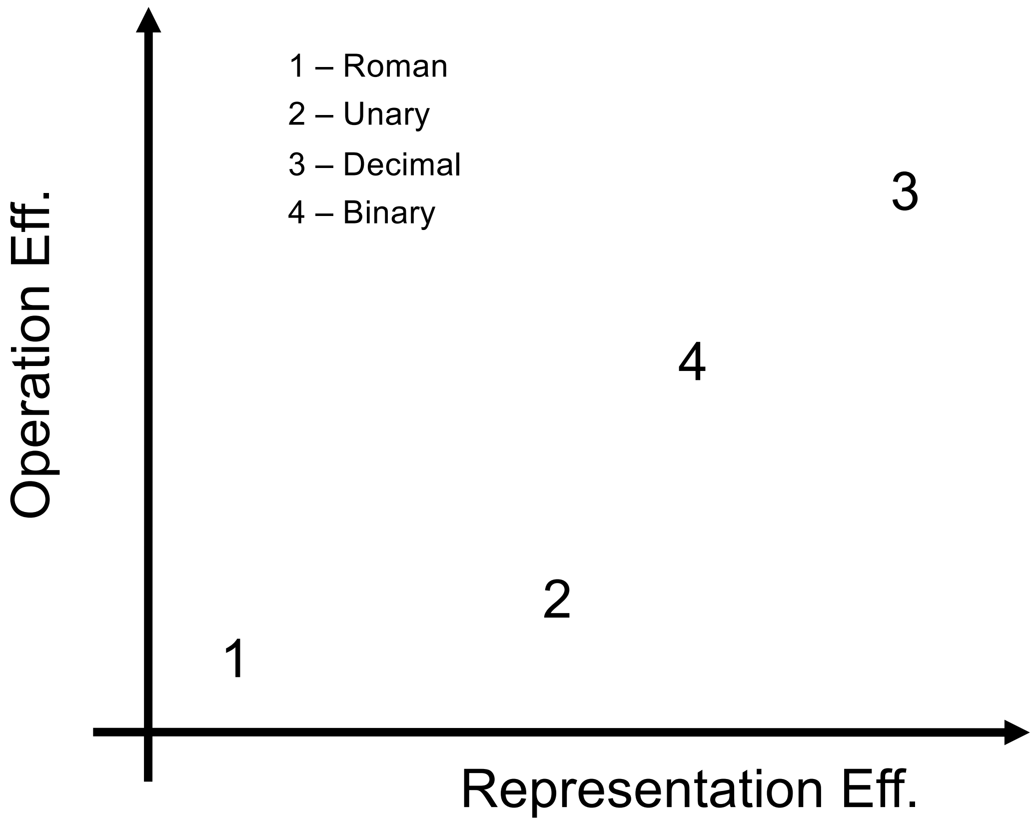
- Easy to imagine the following approach.
  - Multiplication is repeated addition
- Plus, think of a Roman number as the addition of 1000's + 100's + 50's + 10's + 5's + 1's.
- Multiply by each of these, and add as earlier.
- Example: LXII x XXXVII (62 x 37)
  - Multiply each of LXII by II. Meaning, make 2 copies of each symbol in LXII as LLXXIIII
  - Simplify using the rules of addition to CXXIV.
  - Now, multiply each of LXII by V. Start with LLLLLXXXXXIIIIIIIIII, simplify to CCCX.
  - Multiply LXII by XXX. That can be done in two ways. Either multiply by 3 followed by 10, or directly.

Multiplication in Roman Numbers

- Continuing the example,
  - Let us multiply by LXII by 3 as LLLXXXIIIIII and simplified as CLXXXVI.
  - Now, multiply CLXXXVI by 10 as CCCCCCCCCCLLLLLLLLLLXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXVVVVVVVVVVIIIIIIIIII and simplified as MDCCCLX.
  - Add all the constituents as CXXIV + CCCX + MDCCCLX =CDXXXIV + MDCCCLX = MMCCXCIV.
  - What is this number ?

Lesson Learnt

- Representation scheme for numbers influences the ease of performing operations.
- Roman system quite difficult to use.
- There are other such systems not in use today.



Further Operations

- Let us now fix the decimal system as the representation scheme.
- We will now focus on the efficiency of operations.
- Let us see further operations such as finding the GCD of two numbers.

Greatest Common Divisor (GCD)

- Given two positive numbers, x and y, the largest number that divides both x and y is called the greatest common divisor of x and y. Denoted gcd(x,y).
- Several approaches exist to find the gcd.
- Approach 1 : List all the divisors of both x and y. Find the common divisors, and the largest among the common divisors.
- Example for Approach 1: x = 24, y = 42,
  - divisors of 24 are {1, 2, 3, 4, 6, 8, 12, 24}.
  - divisors of 42 are {1, 2, 3, 6, 7, 14, 21, 42}.
  - Common divisors are {1, 2, 3, 6}. Hence, gcd(24, 42) = 6.

Are There Other Representation Formats?

- Yes, recall the fundamental theorem of arithmetic.
- Any number can be expressed uniquely as a product of primes.
- So, a product of primes representation is also possible.
- Not easy to add though.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

GCD – Approach II

- Use the fundamental theorem of arithmetic, write x and y as:
  - $x = p_1^{a1} . p_2^{a2} \dots . p_k^{ak}$, $y = p_1^{b1} . p_2^{b2} \dots . p_r^{br}$
  - It holds that $gcd(x,y) = p_1^{\min\{a1,b1\}} . p_2^{\min\{a2,b2\}} \dots p_r^{\min\{ar,br\}}$.
- Example Approach II, let x = 24, y = 42.
  - $x = 2^3.3$, $y = 2.3.7$
  - $gcd(x,y) = 2.3 = 6$.

Which approach is better?

- Both are actually bad from a computational point of view.
- Both require a number to be factorized.
  - a computationally difficult task.
- For fairly large numbers, both approaches require a lot of computation.
  - Try for yourself by writing a simple program. Check what is the number at which things start getting real slow.
- Is there a better approach?
  - Indeed there is, given by the Greek mathematician Euclid.
  - Celebrated as a breakthrough.

Euclid's algorithm for GCD

- Based on the following lemma.
- Lemma : Let x, y be two positive integers. Let q and r be integers such that $x = y.q + r$. Then, $gcd(x,y) = gcd(y, r)$.
  - Argue that the common divisors of x and y are also common divisors of y and r.
  - Let d divide both x and y. Then, d divides $x – yq = r$.
  - The converse also applies in a similar fashion.
- The above lemma suggests the following algorithms for gcd.
  - Apply the above lemma repeatedly till the remainder is 0.
  - Let r_1, r_2, ..., be the remainders.

Euclid's algorithm for GCD

- Let r_2, r_3, ..., be the remainders with $r_0 = x$ and $r_1 = y$.

- We have that:

$$r_0 = r_1 q_1 + r_2$$
$$r_1 = r_2 q_2 + r_3$$
$$r_2 = r_3 q_3 + r_4$$

and so on, till $r_{n-1} = r_n\, q_n + 0$

- By the result of the above lemma, it also holds that:

$$\begin{aligned} \gcd(r_0, r_1) &= \gcd(r_1, r_2) \\ &= \gcd(r_2, r_3) \\ &= \dots \\ &= \gcd(r_{n-1}, r_n) \\ &= \gcd(r_n, 0) = r_n \end{aligned}$$

- Notice that r_n is the last nonzero remainder in the process.

Euclid's algorithm for GCD

```
Algorithm GCD-Euclid(a,b)
    x := a, y := b;
    while (y is not  0)
        r := x mod y;x := y;y := r;
    end-while
End-Algorithm.
```

- Example, x = 42 and y = 24.
- Iteration 1: r = 18, x = 24; y = 18
- Iteration 2: r = 6, x = 18, y = 6
- Iteration 3: r = 0.

Euclid's algorithm for GCD

- Why is this efficient?
- It can be shown that given numbers x and y, the algorithm requires only about log min{x,y} iterations.
  - Compared to about sqrt{x} for Approach I.
  - Why does approach 1 takes sqrt{x} iterations?
- There is indeed a difference for large numbers.
- The example suggests that also efficient ways to perform operations are of interest.

Tentative Course Plan

#	Topic	Assignments
01	Introduction	
02	Linked List	
03	Stack and Queue	
04	Recursion	Assign #1
05	Intro to Trees	
06	Binary Search Trees	
07	AVL Trees	
08	R&B Trees, Splay Trees	Assign #2
09	B+ Trees, Heaps	
10	Hashing	
11	Searching in Higher Dimensions	
12	Range Trees	

#	Topic	Assignments
13	Trie	Assign #3
14	Suffix Tree	
15	Graph Traversal (BFS)	
16	Shortest Path on Graphs	
17	Graph Traversal (DFS)	Assign #4
18	Minimum Spanning Trees	
19	Searching in Integer Data	
20	Van Emde Boas tree	
21	Amortized Analysis	Assign #5
22	Sorting	
23	Algorithm Design	