Advanced Problem Solving (CSE603)

Lecture # 05/06

Vineet Gandhi and Avinash Sharma
Center for Visual Information Technology (CVIT),
IIIT Hyderabad

Organization (today's lecture)

1. STACK

UNDERSTAND BASICS

2. Prefix Evaluation

HOW TO FORMULATE ?

3. Queue

UNDERSTAND BASICS

Motivation

- Think of developing a modern editor.
  - supports undo/redo among other things.
  - Suppose that currently words w1, w2, w3 are inserted in that order.
  - When we undo, which word has to be undone.
    - w3
  - Next undo should remove w2.
  - So, the order of undo's should be the reverse order of insertion.

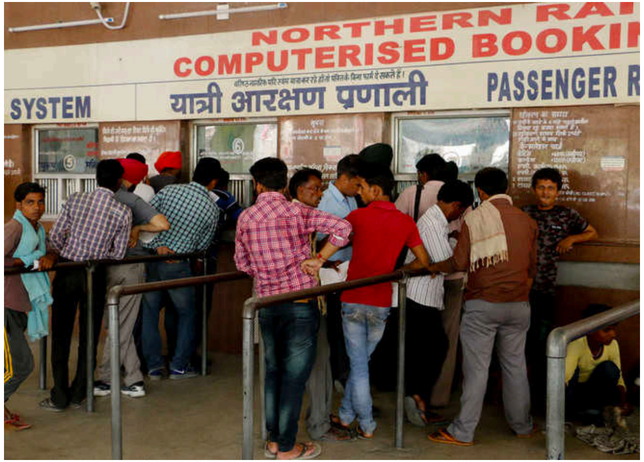
Motivation

- Imagine books piled on top of each other.
- To access a book in the pile, one may need to remove all the books on top of the book we need.

- Similarly, in some cafeterias, plates are piled.
  - The plate we take is the one that is placed the last on the pile.
  - see our dining hall plates.

Motivation

- Consider another kind of examples such as the following.
- The line at the serving station in our dining halls
- At a ticket booking counter



Motivation

- All these examples suggest that there is a particular order in accessing data items.
  - Last In First Out (LIFO), or
  - First In First Out (FIFO)
- Turns out that these orders has several other applications too.
- This lecture, we will formalize these orders and study their important applications in computing.

The Stack ADT

- We can say that some of the above examples are connected by:
  - a stack of words to be deleted/inserted
  - a stack of books to be removed/repiled
  - a stack of plates
- The common theme is the stack
- This stack can be formalized as an ADT.

The Stack ADT

- We have the following common(fundamental) operations.
- create() -- creates an empty stack
- push(item) – push an item onto the stack.
- pop() -- remove one item from the stack, from the top
- size() -- return the size of the stack.
- Other like Top(), IsEmpty() etc.

The Stack ADT

- One can implement a stack in several ways.
- We will start with using an array.
  - Only limitation is that we need to specify the maximum size to which the stack can grow.
  - Let us assume for now that this is not a problem.
  - the parameter n refers to the maximum size of the stack.

Stack Implementation

```
function create(S)
    //depends on the language..
    //so left unspecified for now
end-function.
```

```
function push(item)
Begin
    top = top + 1;
    S[top] = item;
end
```

Stack Implementation

```
function pop()
begin
    return S[top--];
end
```

```
function size()
begin
return top;
end
```

One Small Problem

- Suppose you create a stack of 10 elements.
- The stack already has 10 elements
- You issue another push() operation.
- What should happen?
  - Need some error-handling.
  - Modified code looks as follows.

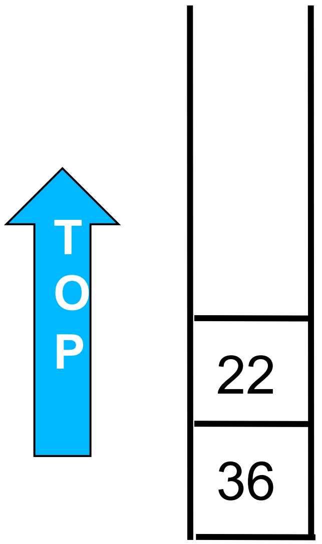
Push, Pop with Error Handling

```
function push(item)
begin
if top == n then
return “ERROR: STACK FULL”
else
S[top++] = item
end.
```

```
function pop()
begin
if (top == -1) then
return “ERROR: STACK EMPTY”
else
return S[top--]
end
```

Typical Convention

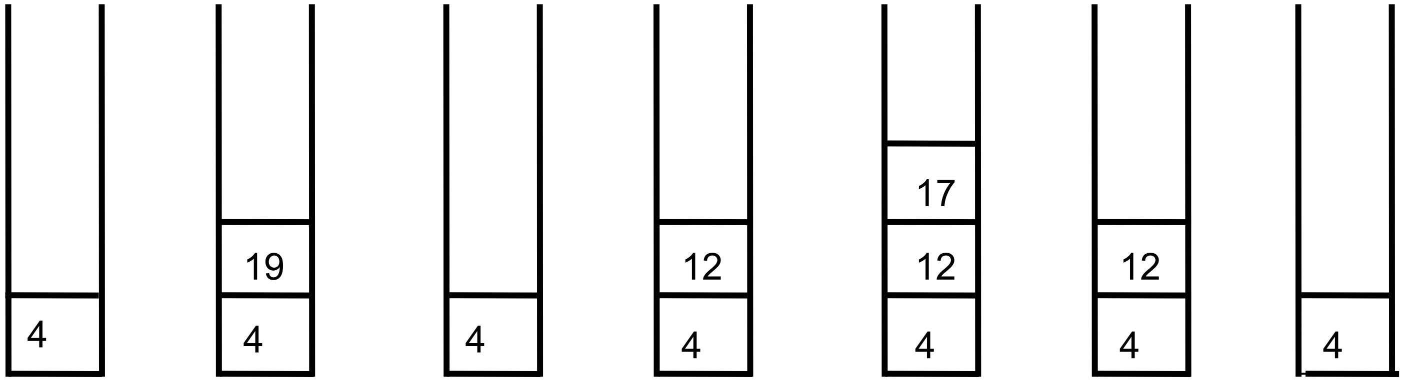
- When drawing stacks, a few standard conventions are as follows:
  - A stack is drawn as a box with one side open.
  - The stack is filled bottom up.
  - So, top of the stack is towards the North.



Example

- Consider an empty stack
- Consider the following sequence of operations
  - push(4), push(19), pop(), push(12), push(17), pop(), pop().
- Show the resulting stack.

Solution



- Consider an empty stack
- Consider the following sequence of operations
  - push(4), push(19), pop(), push(12), push(17), pop(), pop().
- Show the resulting stack.

Application of Stacks

- Balanced Parentheses
- Is counting closed and opened parenthesis enough?

Application of Stacks

- Balanced Parentheses

BALANCED EXPRESSION	UNBALANCED EXPRESSION
(a + b)	(a + b
[(c – d) * e]	[(c – d * e]
{ ( ) } [ ]	{ [( ]) }

Application of Stacks

- Balanced Parentheses

Application of Stacks

- Expression Evaluation
- Imagine pocket calculators or the Google calculator.
- One can type in an arithmetic expression and get the results.
  - Example: 4+2 = 6.
  - Another example: 6+3*2 = 12. And not 18. Why?
- For simplicity, let us consider only binary operators.

Expression Evaluation

- How do we evaluate expressions?
- There is a priority among operators.
  - Multiplication has higher priority than addition.
- When we automate expression evaluation, need to consider priority.
- To disambiguate, one also uses parentheses.
  - The previous example written as 6 + (3*2).
  - If we were thinking of a different result, we should have written (6+3) * 2 evaluating to 18.

Expression Evaluation (manual)

100 + 200 / 10 - 3 * 10

2^2^3

14-7+8

12+ 8/2*2 -6

12+ 8/2*(2 -6)

Expression Evaluation

- We evaluate expressions from left to right.
- All the while worrying about operator precedence.
- What is the difficulty?
- Consider a long expression.

  2 + 3 * 8 * 2 * 2 + 1
- When we look at the first 2, we can hopefully remember that 2 is one of the operands.
- The next thing we see is the operator +. But what is the second operand for this operator.

Expression Evaluation

- This second operand may not be seen till the very end.
- Would it be helpful if we could associate the operands easily.
- But the way we write the expression, this is not easy.

Infix, prefix, postfix

a + b * c + d * e + f

a + b * c * d + e / f – g

Prefix Expression

- Given the above observations, we can write it as + 2 * 3 6.
- Another example: 3 + 4 + 2 * 6. The prefix is + 3 + 4 * 2 6.
- But can we write prefix expressions? We are used to writing infix expressions.
- Our next steps are as follows
  1. Given an infix expression, convert it into a prefix expressions.
  2. Evaluate a prefix expression.

Our Next Steps

- We have two problems. Of these let us consider the second problem first.
- The problem is to evaluate a given prefix expression.
- Our solution closely resembles how we do a manual calculation.

Evaluating a Prefix Expression

- Some observation(s)
  - The operator precedes the operands.
  - Therefore, the operands are usually pushed to the right of the prefix expression.
  - This suggests that we should evaluate from right to left.
- This helps us in devising an algorithm.
- Imagine that the prefix expression is stored in an array.
  - one operator/operand at an index.

Evaluating a Prefix Expression

- Can we use a stack?
- How can it be used?
- What should we store in the stack?

Evaluating a Prefix Expression

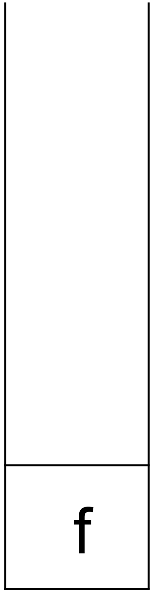
- The above suggests the following approach.
- Start from the right side.
- For every operand, push it onto the stack.
- For every operator, evaluate the operator by taking the top two elements of the stack.
  - place the result on top of the stack.

Example to Evaluate a Prefix Expression

- Consider the expression + * + a b + c d + e f.
- Show the contents of the stack and the output at every step.

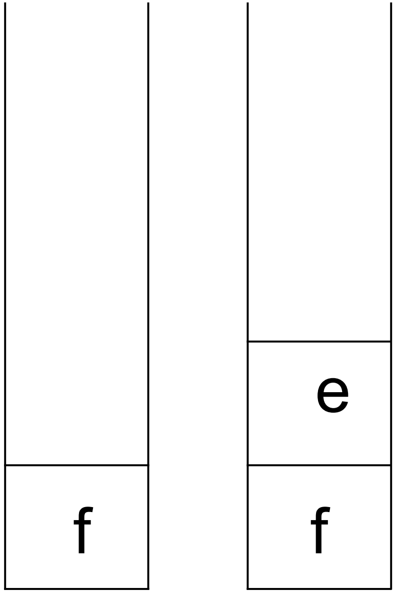
Example

\+ * + a b + c d + e f.



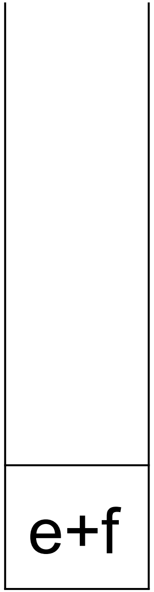
Example

\+ * + a b + c d + e f.



Example

\+ * + a b + c d + e f.



Example

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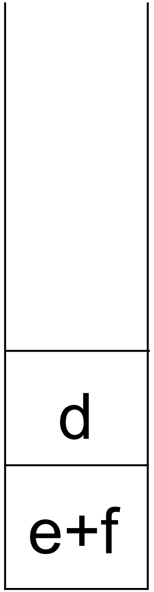
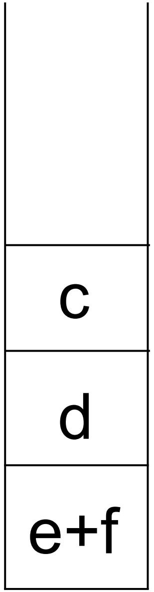


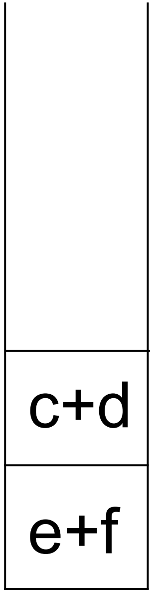
Example

+ * + a b + c d + e f.



Example

\+ * + a b + c d + e f.



Example

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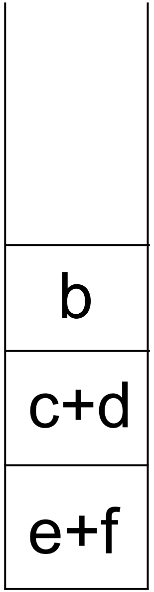
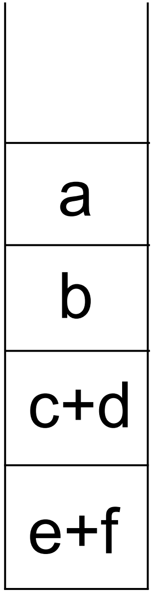


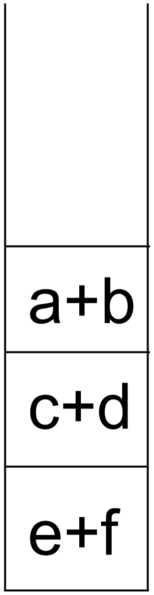
Example

+ * + a b + c d + e f.



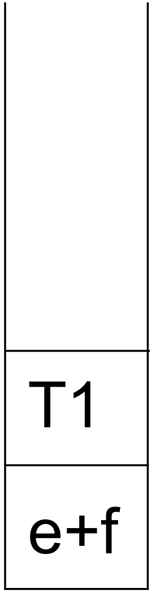
Example

+ * + a b + c d + e f.



Example

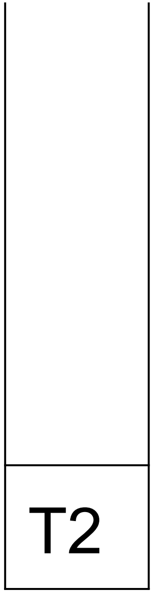
\+ * + a b + c d + e f.



$$T1 = (a+b) * (c+d)$$

Example

+ * + a b + c d + e f.



T1 = (a+b) * (c+d)

T2 = (T1) + (e+f)

Algorithm for Evaluating a Prefix Expression

```
Algorithm EvaluatePrefix(E)
begin
        Stack S;
        for i = n down to 1 do
        begin
        if E[i] is an operator, say o then
                operand2 = S.pop();
                operand1 = S.pop();
                value = operand1 o operand2;
                S.push(value);
        else
                S.push(E[i]);
        end-for
end-algorithm
```

- Here, n refers to the number of operators + the number of operands.
- The time taken for the above algorithm is linear in n.
  - There is only one for loop which looks at each element, either operand or operator, once.
- We will see an example next.

Reading Exercise

- We omitted a few details in our description.
- Some of them are:
  - How to handle unary operators?
  - How can this be extended to ternary operators?

- Another possibility is to use postfix expressions.
  - Also called as Reverse Polish Notation.
- They can be evaluated left to right with a stack.
- Try to arrive at the details.

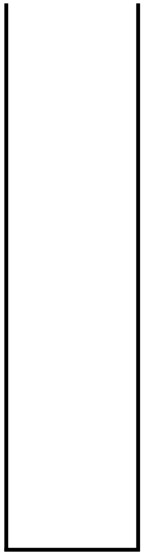
Back to The First Question

- Let us now consider how to convert a given infix expression to its prefix/postfix equivalent.
- The issues
  - Operands not easily known
  - There may be parentheses also in the expression.
  - Operators have precedence.

Lets look at postfix first

infix-prefix

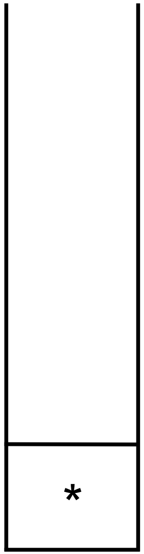
- Let us consider an expression of the form a + b + c * d + e * f.



f

infix-prefix

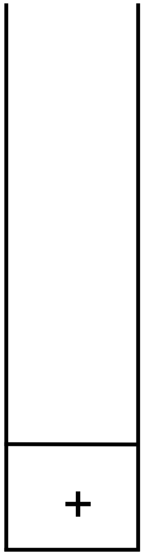
- Let us consider an expression of the form a + b + c * d + e * f.



f e

infix-prefix

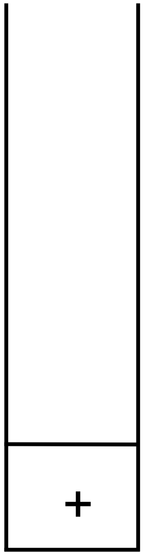
- Let us consider an expression of the form a + b + c * d + e * f.



f e *

infix-prefix

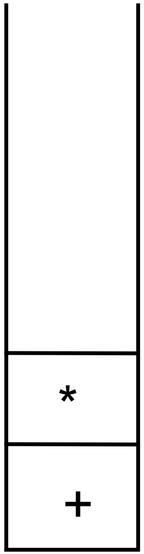
- Let us consider an expression of the form a + b + c * d + e * f.



f e * d

infix-prefix

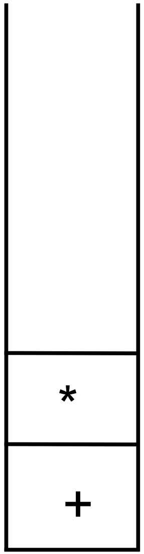
- Let us consider an expression of the form a + b + c * d + e * f.



f e * d

infix-prefix

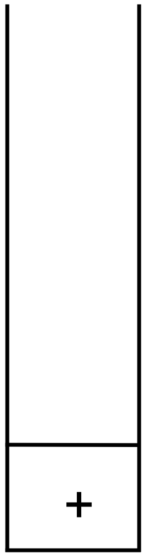
- Let us consider an expression of the form a + b + c * d + e * f.



f e * d c

infix-prefix

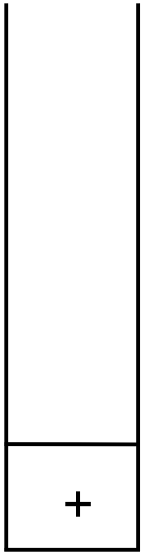
- Let us consider an expression of the form a + b + c * d + e * f.



f e * d c *

infix-prefix

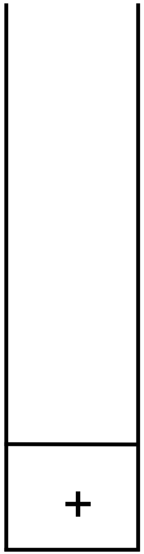
- Let us consider an expression of the form a + b + c * d + e * f.



f e * d c * +

infix-prefix

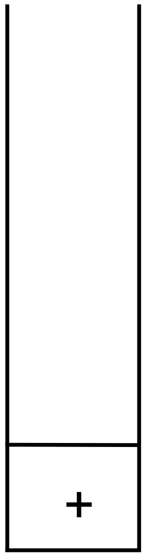
- Let us consider an expression of the form a + b + c * d + e * f.



f e * d c * + b

infix-prefix

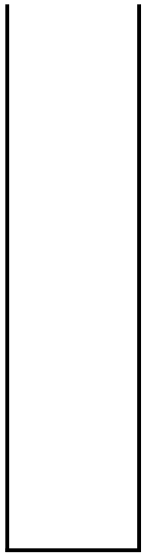
- Let us consider an expression of the form a + b + c * d + e * f.



f e * d c * + b +

infix-prefix

- Let us consider an expression of the form a + b + c * d + e * f.



f e * d c * + b + a +

Invert as:

+ a + b + * c d * e f

Reading Exercise

- Read or devise ways to handle parentheses.
  - Open parentheses indicates the start of a subexpression, closing parentheses indicates the end of the subexpression.
  - Important to keep track of these.

- Similarly, how to handle unary operators?

Lets move to Queue

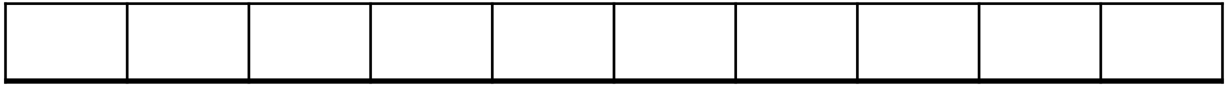
- Consider a different setting.
- Think of booking a ticket at a train reservation office.
  - When do you get your chance?
- Think of a traffic junction.
  - On a green light, which vehicle(s) go(es) first.?
- Think of airplanes waiting to take off.
  - Which one takes off first?

The Queue

- The fundamental operations for such a data structure are:
  - Create : create an empty queue
  - Enqueue : Insert an item into the queue
  - Dequeue : Delete an item from the queue.
  - size : return the number of elements in the queue.

The Queue

- Can use an array also to implement a queue.
- We will show how to implement the operations.
  - We will skip create() and size().
- We will store two counters : front and rear
- Insertions happen at the rear
- Deletions happen from the front.

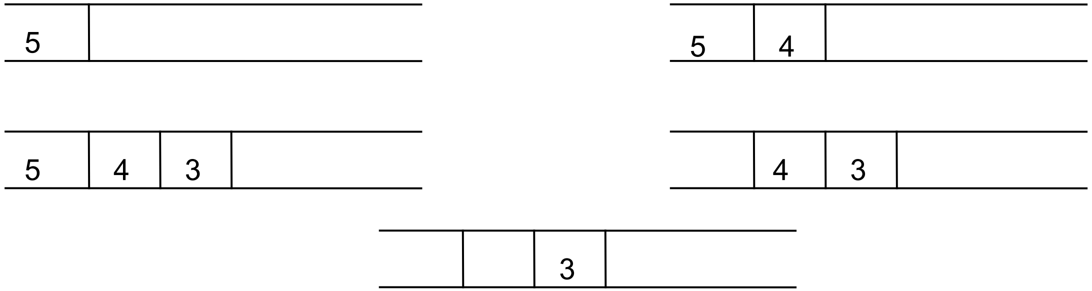


```
IsEmpty()
begin
if front==-1 && rear == -1
    return true;
else
    return false
end
```

```
Enqueue(x)
begin
if rear == MAXSIZE then
    return;
else if IsEmpty()
    front ← rear ← 0;
else
    rear = rear+1;
Queue[rear] = x;
end
```

```
Dequeue(x)
begin
if IsEmpty()
    return;
else if front==rear
    front ← rear ← -1;
else
    front = front +1;
end
```

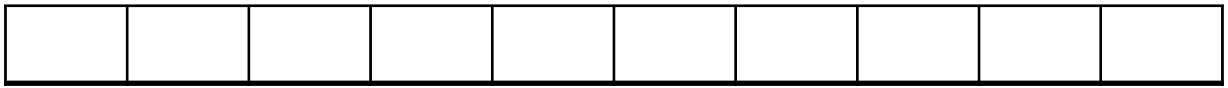
Queue Example

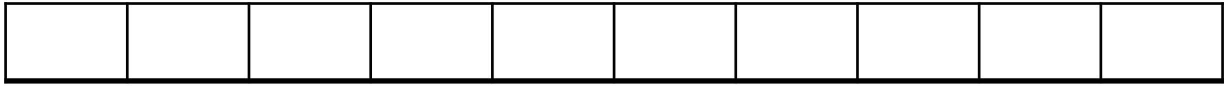


- Starting from an empty queue, consider the following operations.
  - Enqueue(5), Enqueue(4), Enqueue(3), Dequeue(), Dequeue()
- The result is shown in the figure above.

Other Variations of the Queue

- To save space, a circular queue is also proposed.
- Operations that update front and rear have to be based on modulo arithmetic.
- The circular queue is declared full when (rear+1)%N == front





```
IsEmpty()
begin
if front==-1 && rear == -1
    return true;
else
    return false
end
```

```
Enqueue(x)
begin
if (rear+1)%N+1 == front then
    return;
else if IsEmpty()
    front ← rear ← 0;
else
    rear = (rear+1)%N;
Queue[rear] = x;
end
```

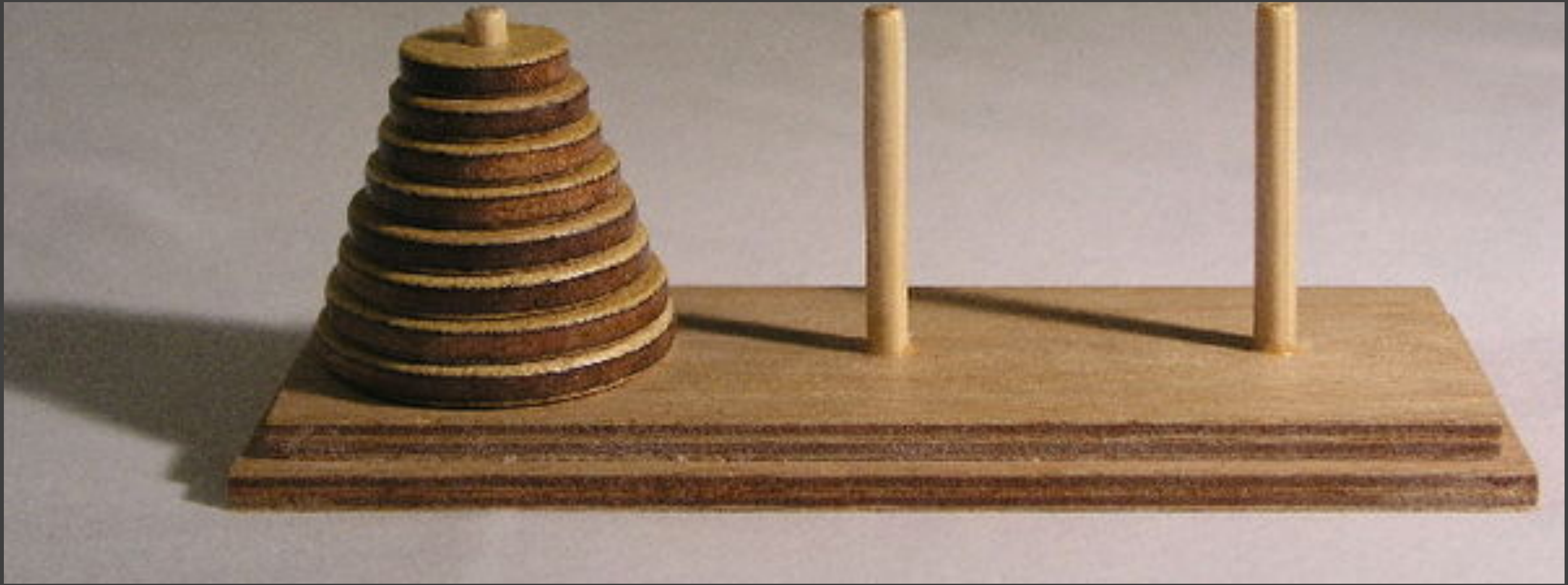
```
Dequeue(x)
begin
if IsEmpty()
    return;
else if front==rear
    front ← rear ← -1;
else
    front = (front +1)%N;
end
```

A Sample Application with Stack and Queue

- A palindrome is a string that reads the same forwards and backwards, ignoring non-alphabetic characters.
- Examples:
  - Malayalam
  - Wonton? not now
  - Madam, i'm Adam
- Problem: Given a string, determine if it is a palindrome.
  - May not know the length of the string apriori.

A Sample Application with Stack and Queue

- Need to compare the first character with the last character.
- So, store the characters in a stack and a queue also.
- Once notified of the end of the string, compare the top of the stack with the front of the queue.
  - Continue until both the stack and the queue are empty.



Towers of Hanoi

- The objective of the puzzle is to move the entire stack to another rod, obeying the following simple rules:
- Only one disk can be moved at a time.
- Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack or on an empty rod.
- No larger disk may be placed on top of a smaller disk.

Towers of Hanoi