

# Data Structures & Algorithms for Problem Solving (M20Temp3)

## Lecture # 15

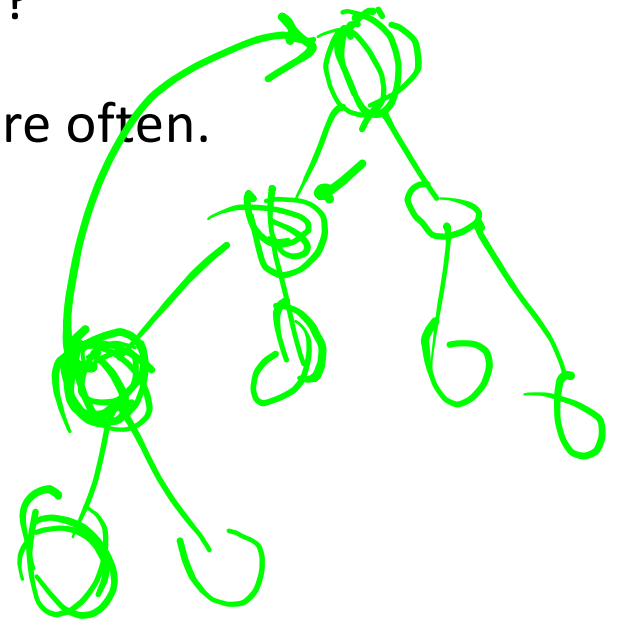
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# Another Way Towards Balanced Trees

- Should we even try to achieve balance at all times?
- In some settings, a few elements are accessed more often.
  - Do not know which are those elements.
  - Any example systems for dictionary search ?
- Does it suffice to keep these elements closer?



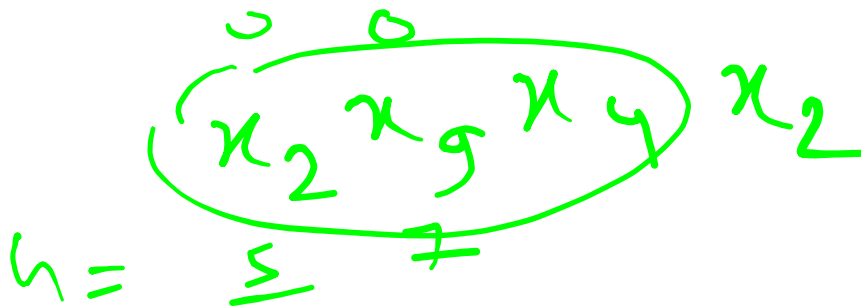
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# More on Search Trees

- Notice that a successful search operation can stop as soon as the element is found.
  - If the element is a leaf node, then search operation on that node takes the longest time.
  - **A successive search to the same node still takes the same amount of time.**
  - In some settings, a few elements are searched more often than the others.
    - should focus on optimizing these searches.
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# More on Search Trees

- One way to make future search operations on the same node is to bring that node (closer) to the root.
- This is what we will do. Called as **splaying**.
- The search tree using this technique is called as **splay tree**.



# Splay Trees

- In a splay tree, during every operation, including a search(), the current (search) tree is modified.
- The item searched is made as the root of the tree.
- During this process, other nodes also change their height.

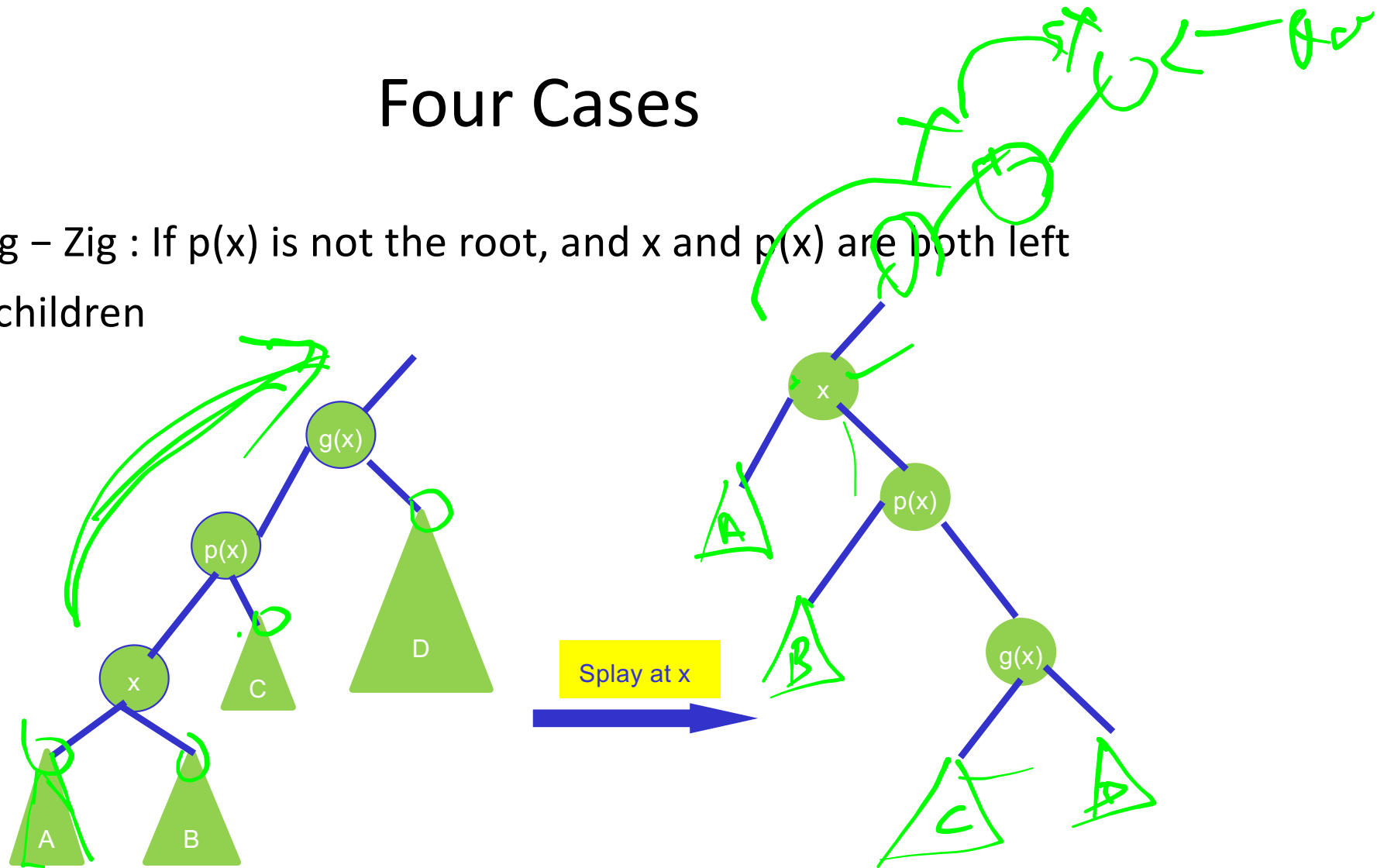


# Splay Trees Operation

- Let  $x$  be a node in the search tree.
  - To make  $x$  as the root, we use operations similar to that of rotations.
  - To splay a tree at node  $x$ , repeat the following splaying step until  $x$  is the root of the tree.
    - Let  $p(x)$  denotes the parent node of  $x$  and  $g(x)$  denotes the parent node of  $p(x)$  i.e., grand-parent of  $x$ .
    - The following cases are used depending on whether  $x$  is a left child of  $p(x)$ , etc.
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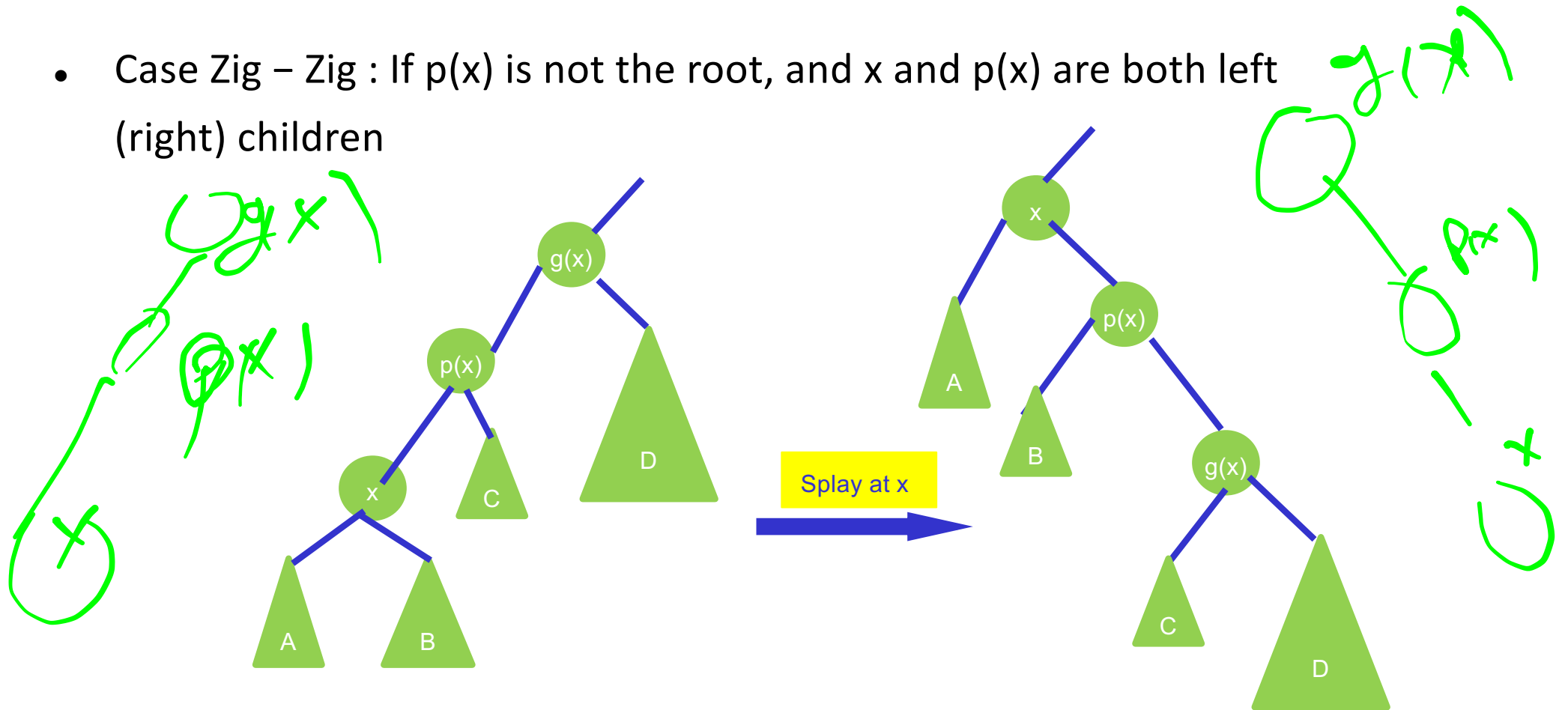
# Four Cases

- Case Zig – Zig : If  $p(x)$  is not the root, and  $x$  and  $p(x)$  are both left (right) children



# Four Cases

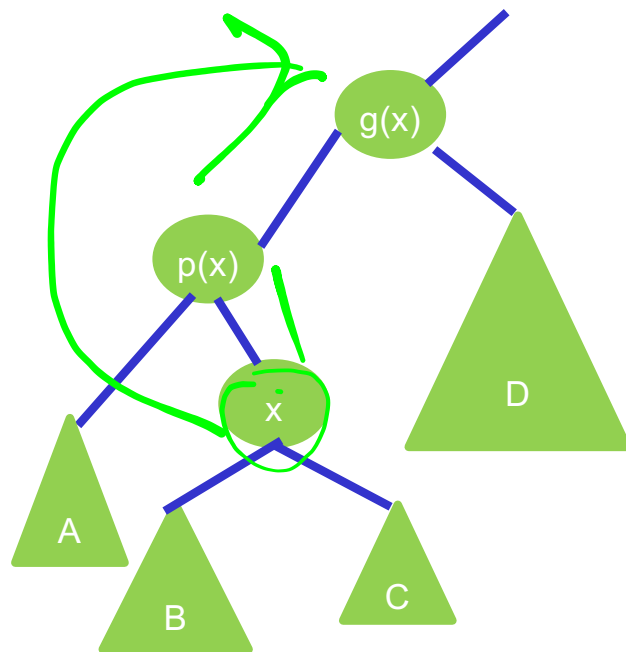
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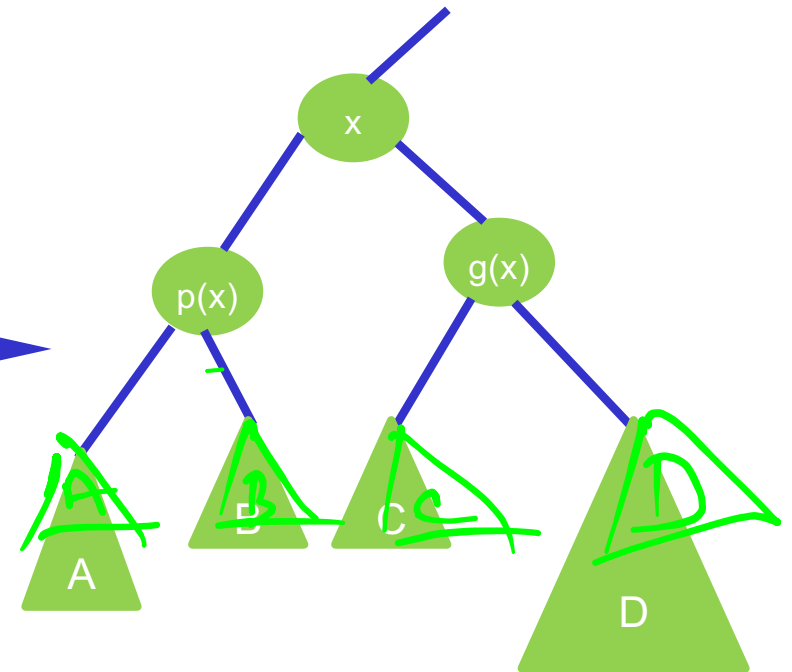


# Four Cases

- Case Zig – Zag - If  $p(x)$  is not the root, and  $x$  is right (left) child and  $p(x)$  is left (right) child.

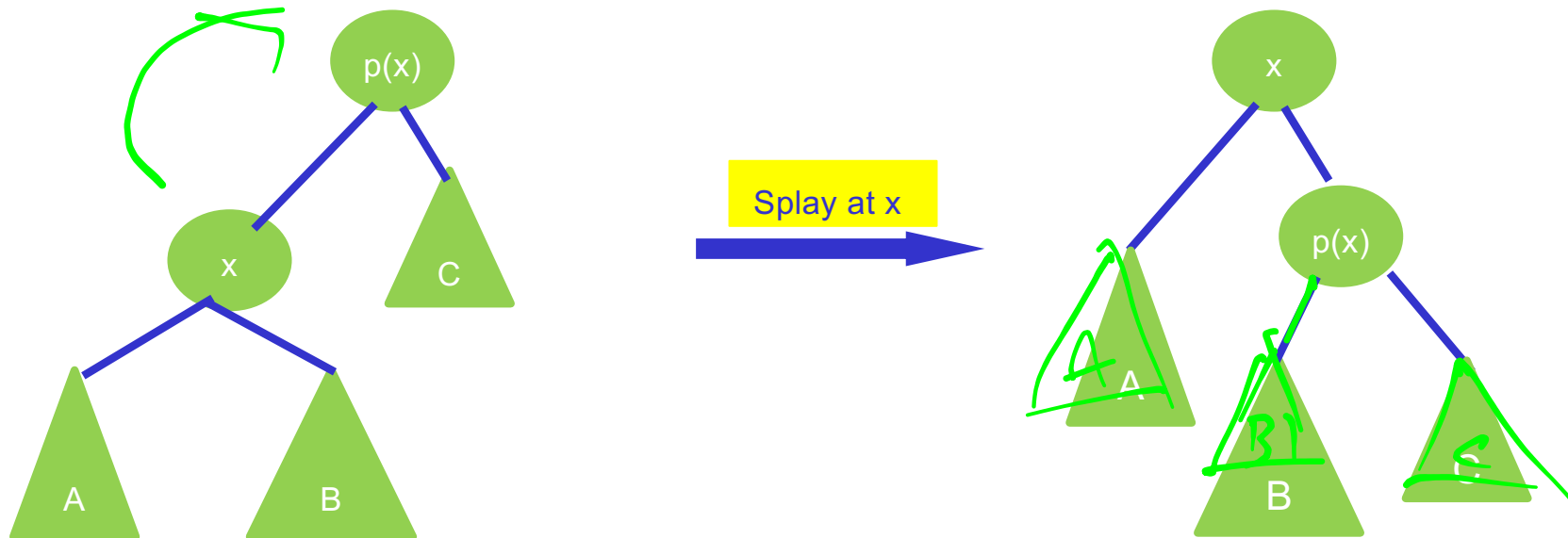


Splay at x



## Two More Cases

- What if  $p(x)$  is the root?  $g(x)$  is not defined.
- If  $x$  is the left child of  $p(x)$ , proceed as follows.



- The other case is easy to figure out.
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# Search(x) in a Splay Tree

- Proceed as search in a binary search tree.
- Once  $x$  is found, splay( $x$ ) till  $x$  is the root.
- Splay uses the above cases.



# Insert(x) & Delete(x)

## Insert

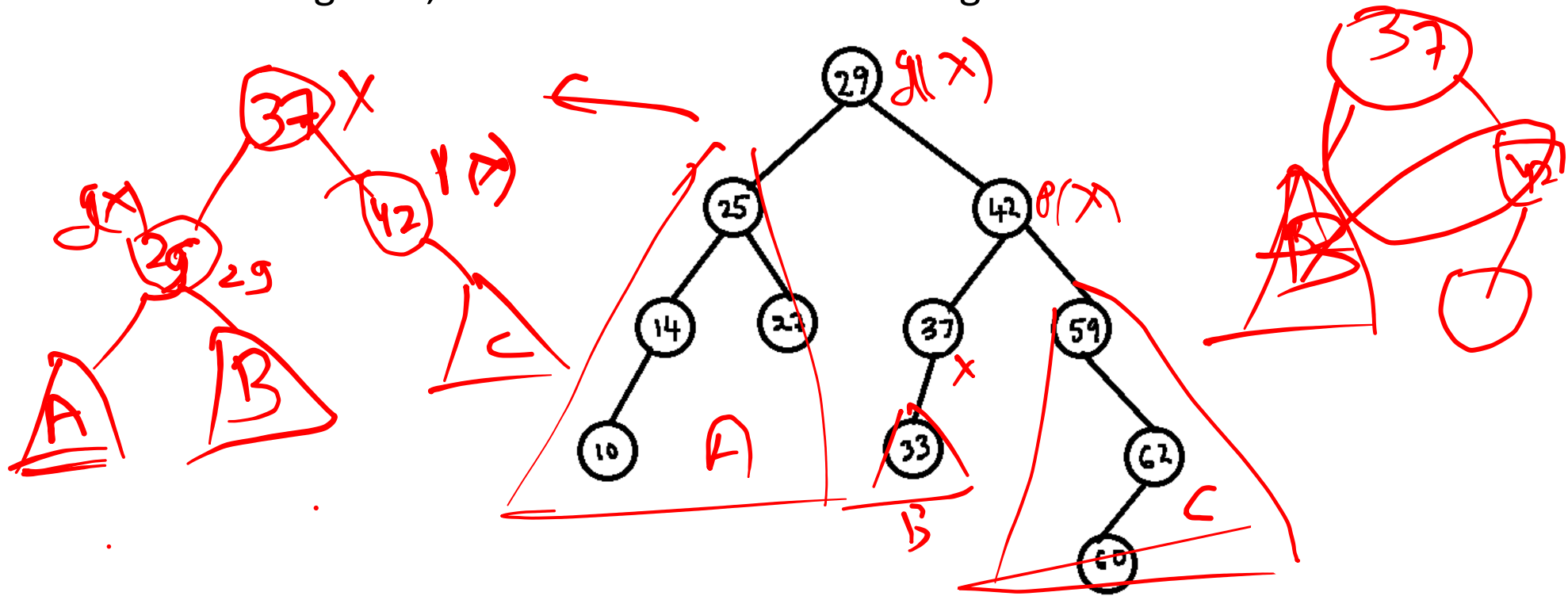
- Make x the root after inserting as in a binary search tree.

## Delete

- Delete x as in a binary search tree.
  - If y is the node physically deleted, then make the parent of y,  $p(y)$ , as the root., i.e.,  $\text{splay}(p(y))$
  - This is a bit artificial, but required for analysis to go through.
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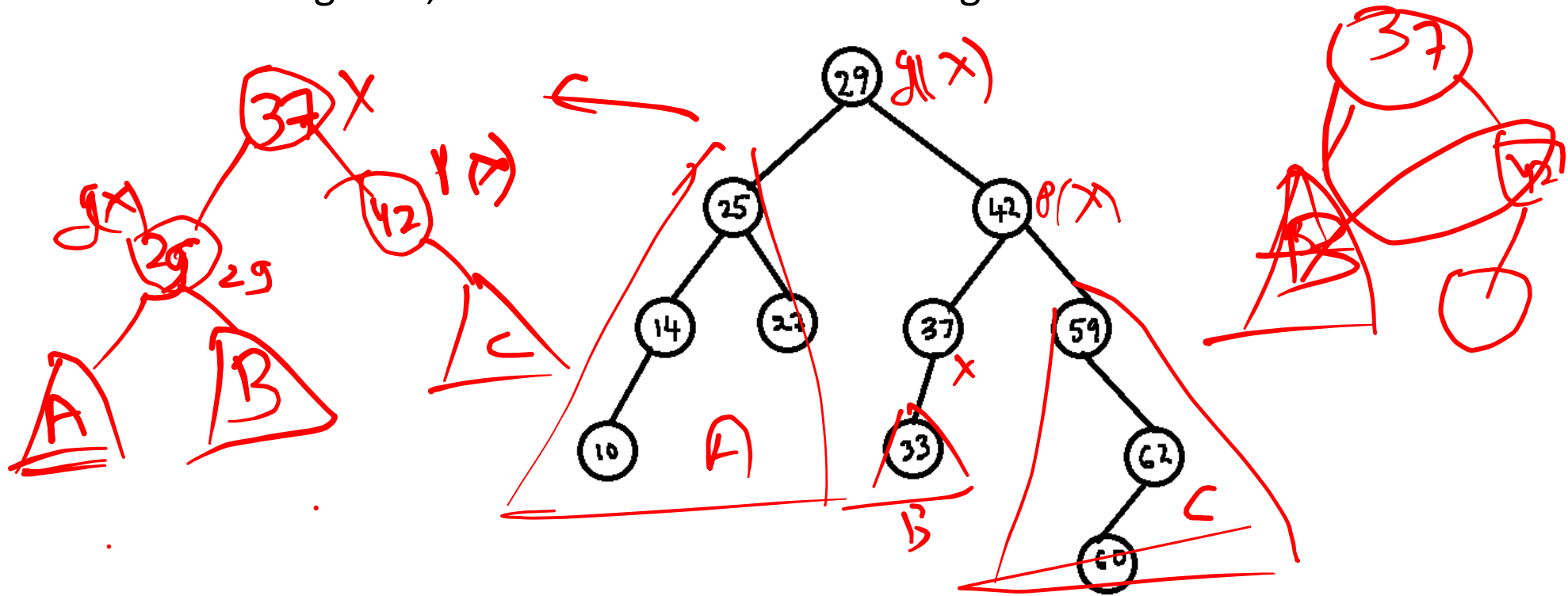
# Practice Problem on Splay Tree

- Consider the following splay tree. Splay the nodes 37, then 14 in the resulting tree, and then 29 in the resulting tree.



# Practice Problem on Splay Tree

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# Analysis

- Analyzing the splay tree is a bit tough at this stage.
  - Here are a few results:
  - Any sequence of  $m$  operations on a splay tree can be completed in time  $O((m+n) \log n)$ .
  - <sup>2</sup>Topic for advanced classes.
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**Thank You**

