Image Restoration

Noise Removal

Partially Adopted from Brian Mac Namee

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- ✓ Noise and Images
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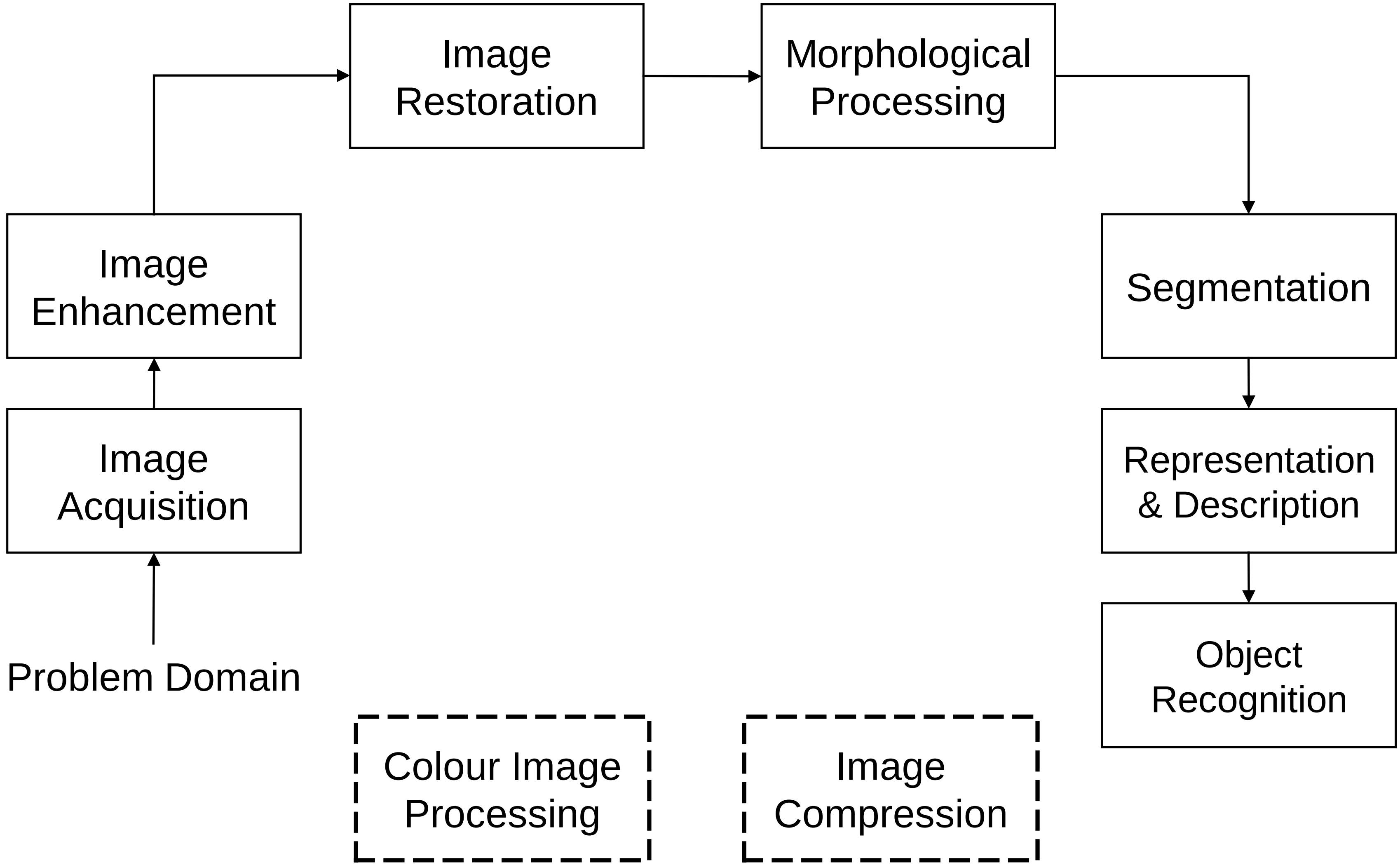
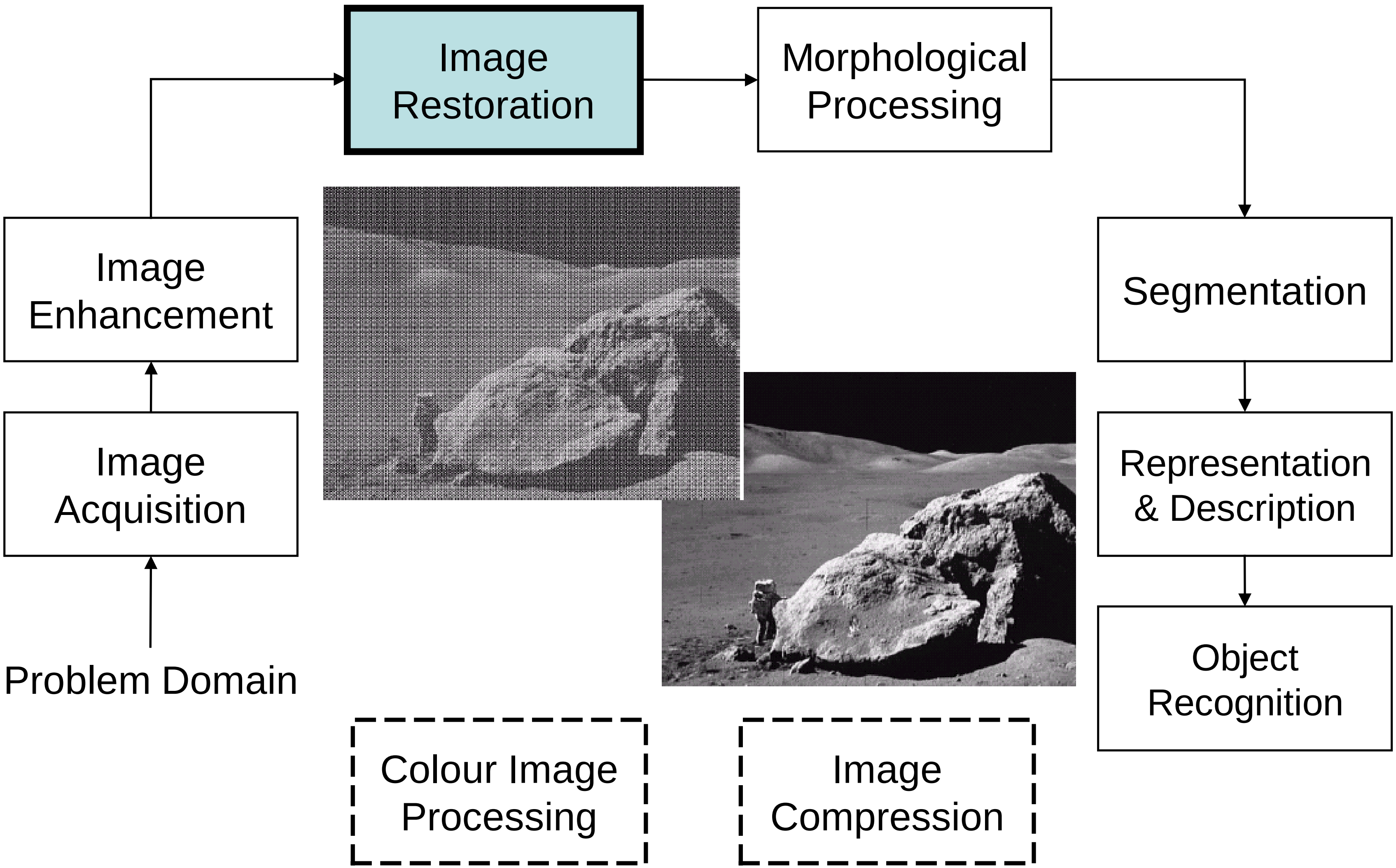
Phases of Digital Image Processing



Phases of Digital Image Processing : Image Restoration



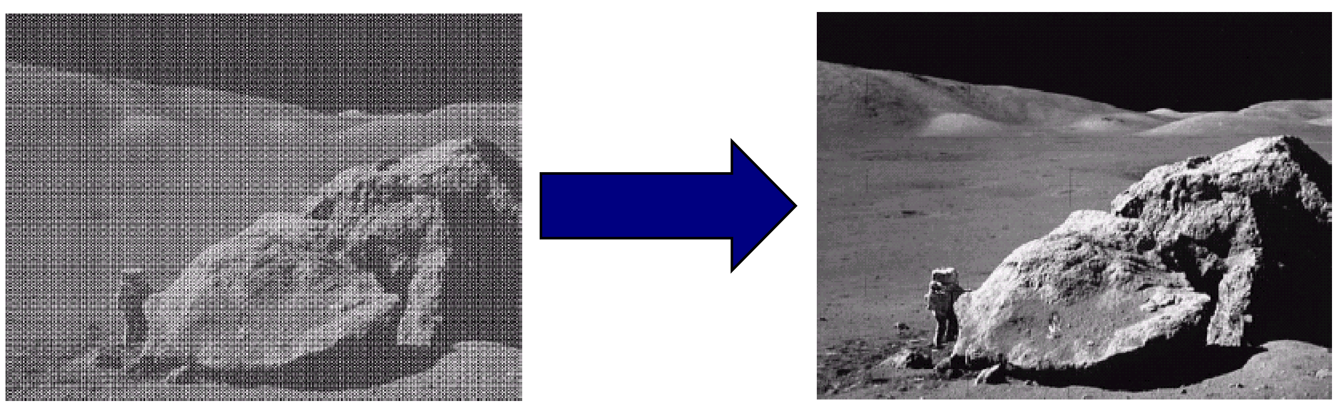
Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Image Restoration

Image restoration attempts to restore images that have been degraded.

- Identify the degradation process and attempt to reverse it.

Similar to image enhancement, but more objective.



Noise and Images

The sources of noise in digital images arise during image acquisition (digitization) and transmission.

Imaging sensors can be affected by environmental conditions during image acquisition and by the quality of the sensing elements.

Interference can be added to an image during transmission.

Noise Model

A noisy image is given in the spatial domain by:

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

where *h(x, y)* is the spatial representation of the degraded function, *f(x, y)* is the original image pixel, *η(x, y)* is the noise term, and *g(x, y)* is the resulting noisy pixel.

In frequency domain:

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

Noise Model

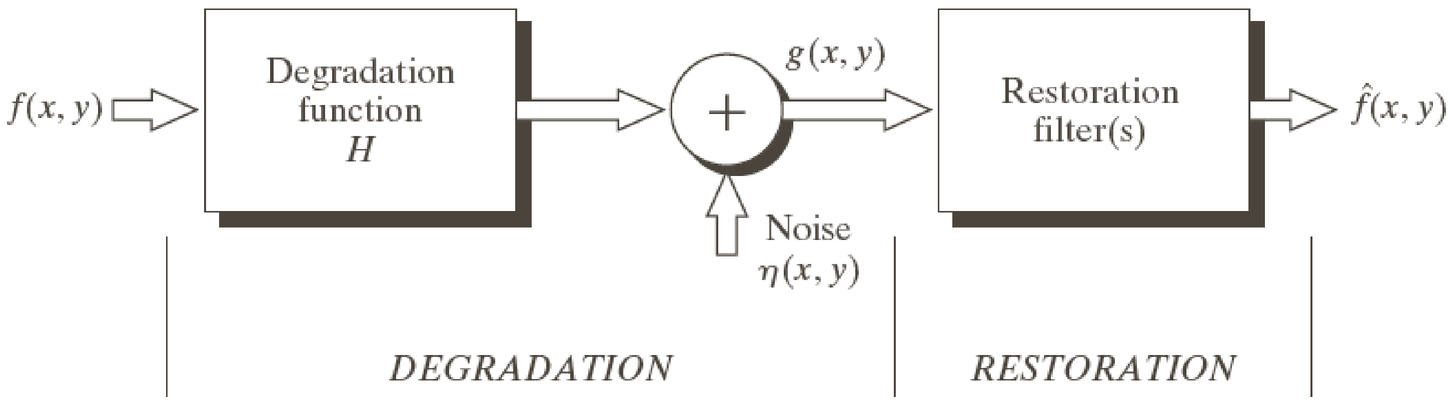


FIGURE 5.1 A model of the image degradation/restoration process.

We consider that H is the identity operator. We deal with <u>only degradation due to noise</u>.

Noise Model

A noisy image to be modelled as follows:

$$g(x, y) = f(x, y) + \eta(x, y)$$

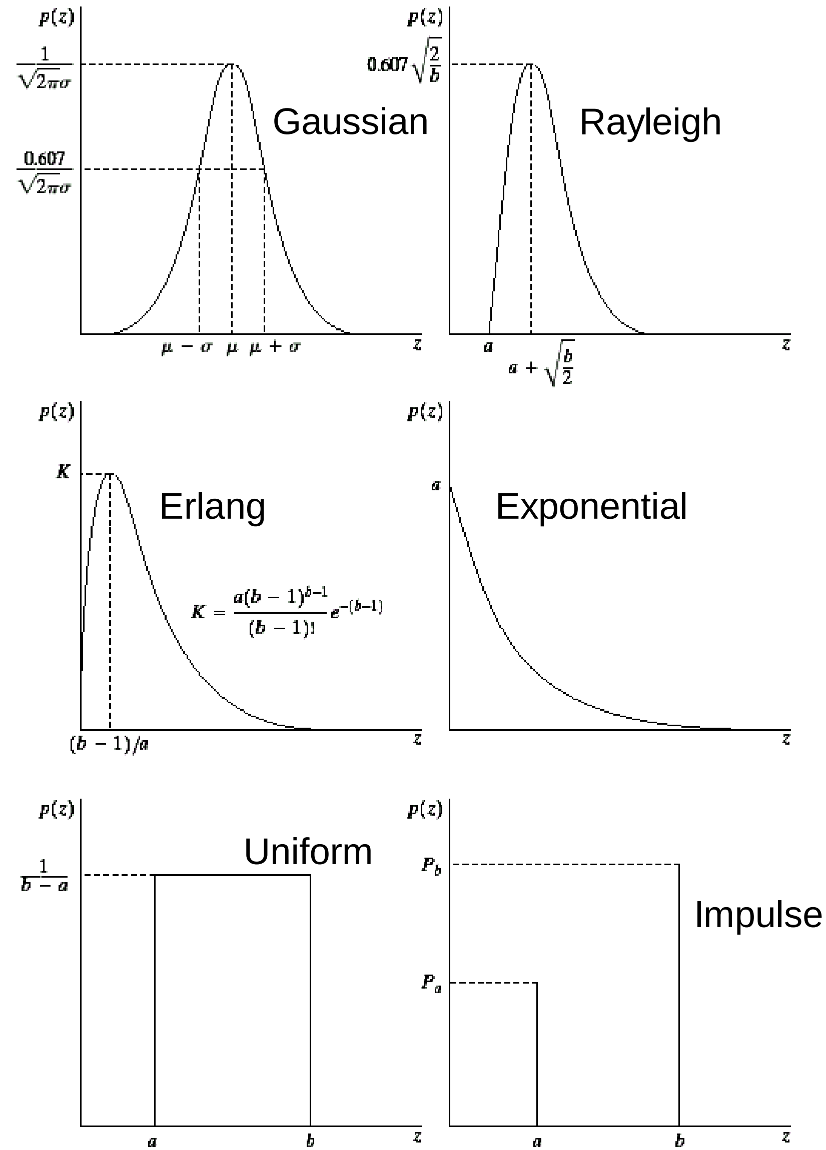
where $f(x, y)$ is the original image pixel, $\eta(x, y)$ is the noise term and $g(x, y)$ is the resulting noisy pixel.

If we can estimate the model of the noise in an image, then this will help us to figure out how to restore the image.

Noise Models

There are many different models for the image noise term $\eta(x, y)$:

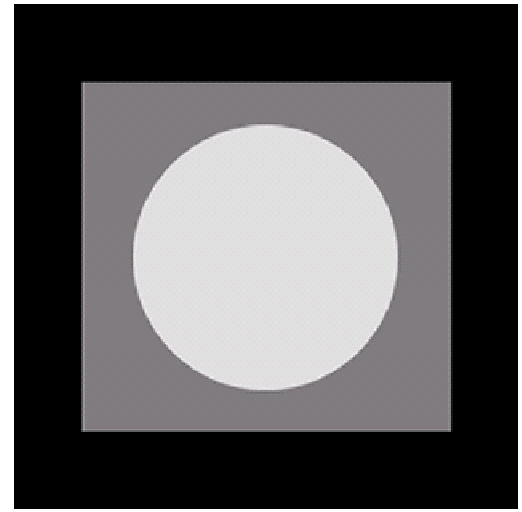
- Gaussian
- Rayleigh
- Erlang(Gamma)
- Exponential
- Uniform
- Impulse (*Salt-and-pepper* noise)



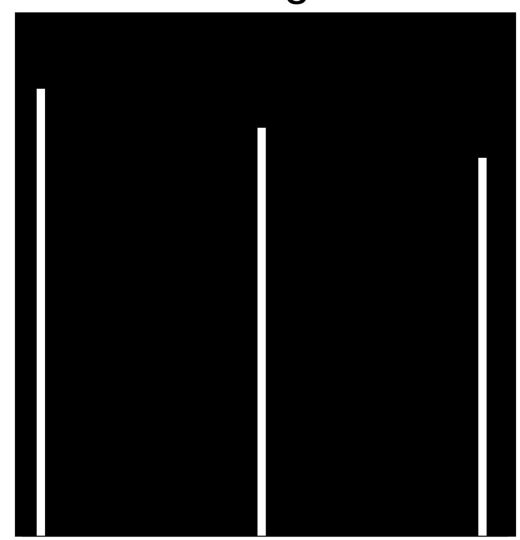
Noise Example

The test pattern to the right is ideal for demonstrating the addition of noise.

Notice the result of adding noise based on various models to this image.

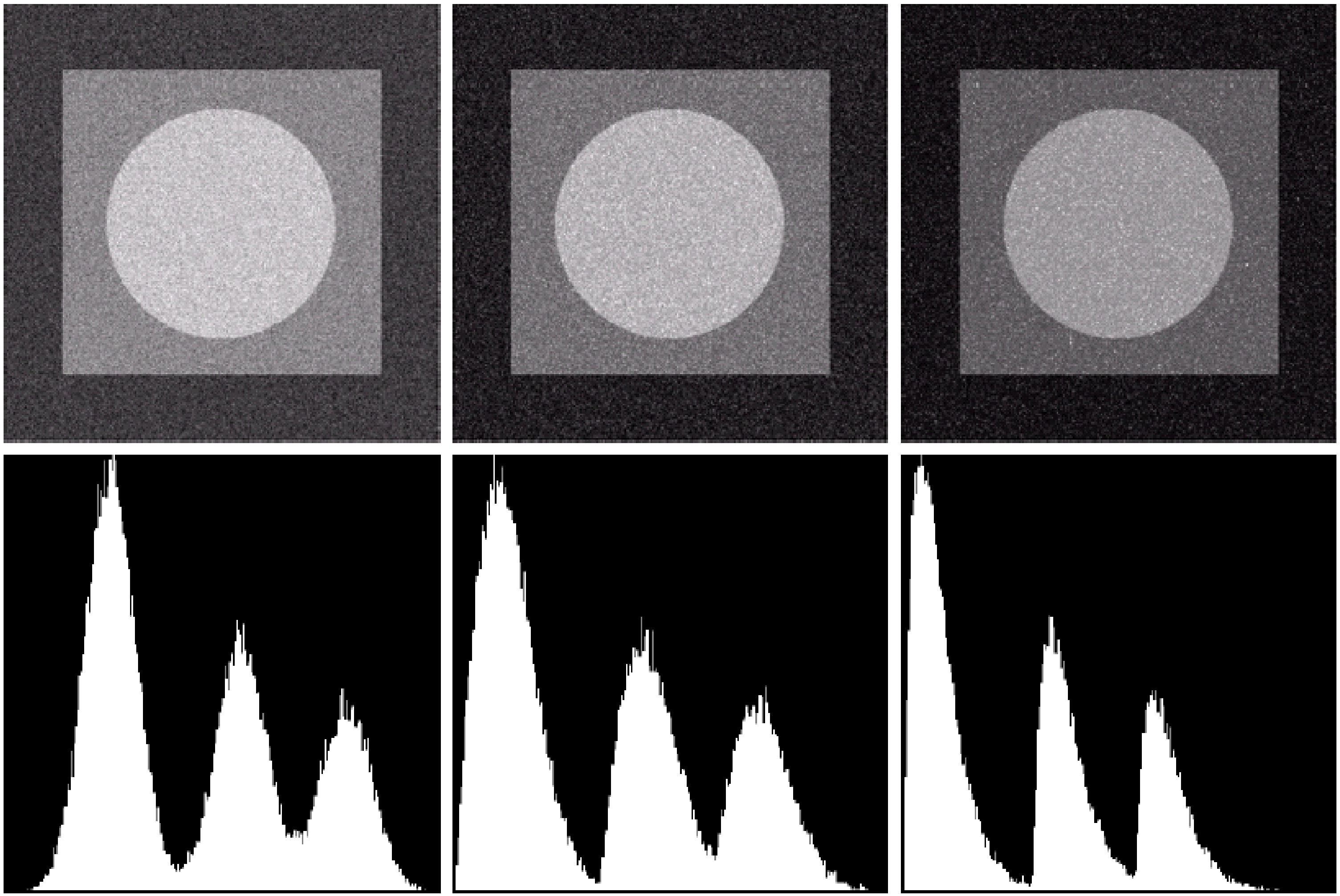


Image



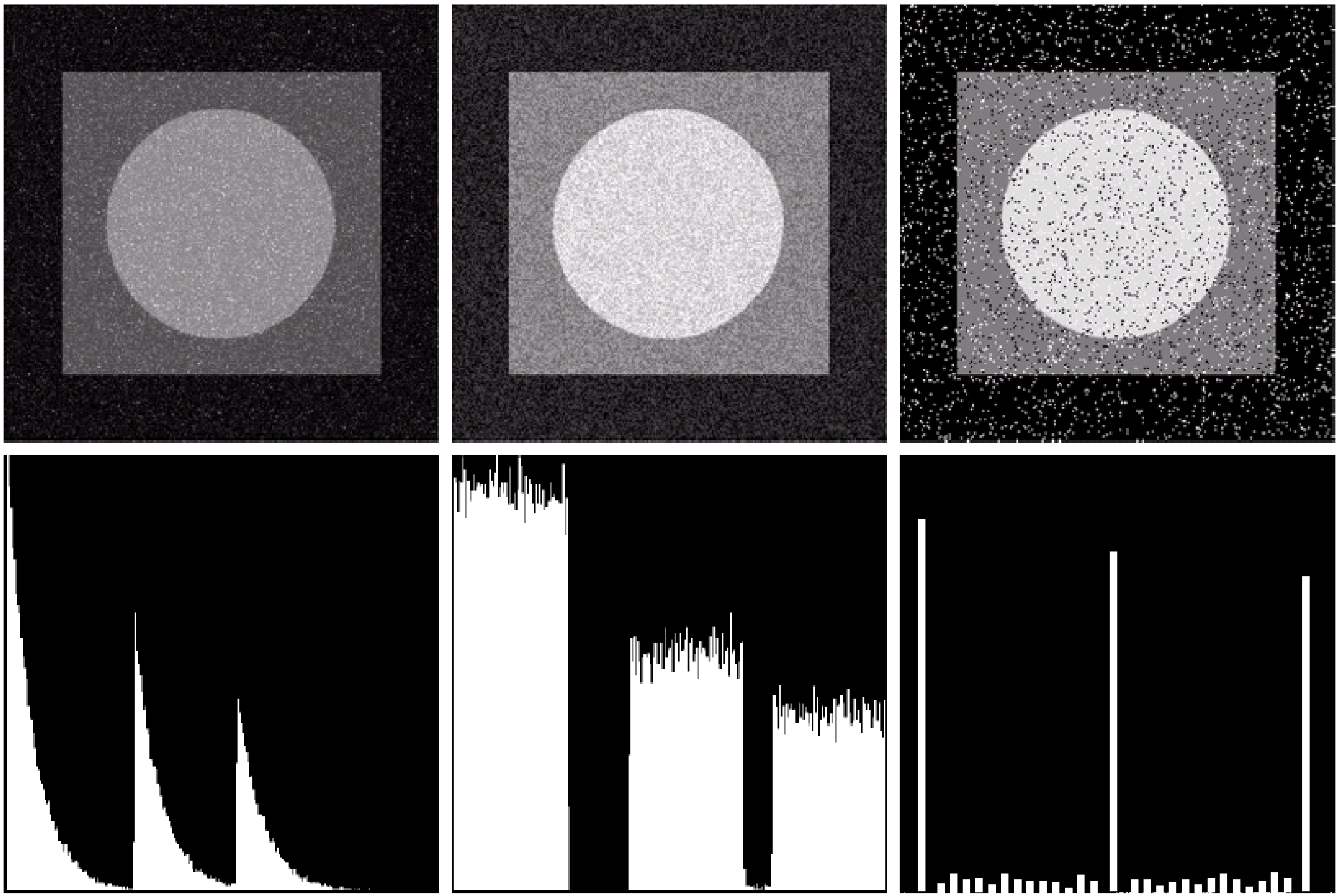
Histogram

Noise Example



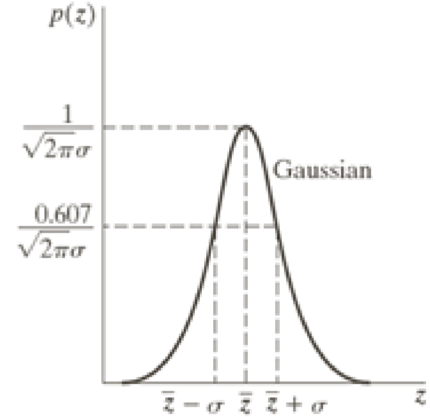
Gaussian Rayleigh Erlang

Noise Example



Exponential Uniform Impulse

Noise Model



Gaussian Noise:

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2/2\sigma^2}$$

Due to its mathematical tractability in both the spatial and frequency domains, Gaussian (also called *normal*) noise models are used frequently in practice.

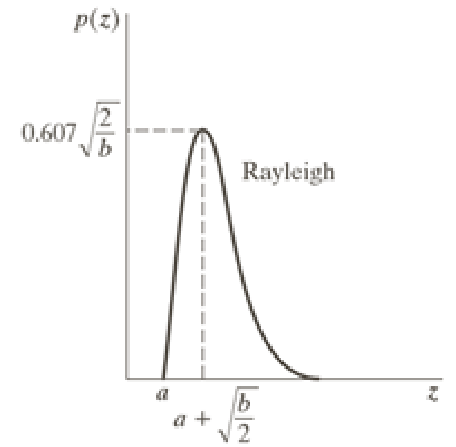
z = gray level,

μ = mean of the avg. value of z,

σ = standard deviation

- 70% of its values in the range $[(\mu - \sigma),(\mu + \sigma)]$, and about 95% will be in the range $[(\mu - 2\sigma),(\mu + 2\sigma)]$.

Noise Model



Rayleigh Noise:

$$p(z) = \begin{cases} \frac{2}{b}(z-a)\, e^{-(z-a)^2/b}, \text{for } z \geq a \\ 0 \qquad\qquad\qquad\qquad\quad, \text{for } z < a \end{cases}$$

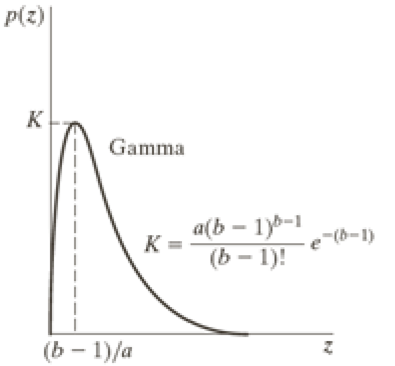
The displacement from the origin and the fact that the basic shape of this density is skewed to the right.

Mean: $\mu = a + \sqrt{\pi b/4},$

Variance: $\sigma^2 = \frac{b(4-\pi)}{4}$

The Rayleigh density can be quite useful for approximating skewed histogram.

Noise Model



Erlang (Gamma) Noise:

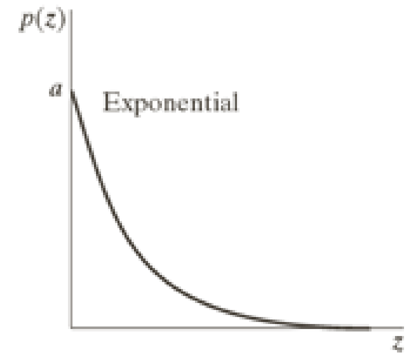
$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az}, & \text{for } z \geq 0 \\ 0, & \text{for } z < 0 \end{cases}$$

$a > 0$, b is a positive integer

Mean: $\mu = \frac{b}{a}$, Variance: $\sigma^2 = \frac{b}{a^2}$

- When the denominator is the gamma function Γ(b), then it is called Gamma noise.

Noise Model



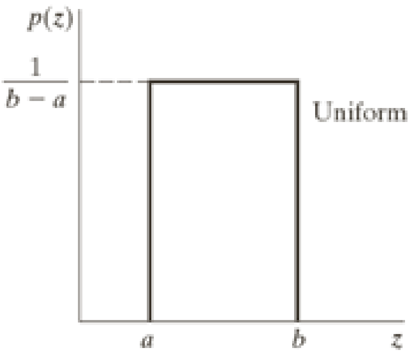
Exponential Noise:

$$p(z) = \begin{cases} ae^{-az}, \text{for } z \geq 0 \\ 0 \quad\quad\;\, , \text{for } z < 0 \end{cases}$$

$$a > 0, \mu = \frac{1}{a}, \sigma^2 = \frac{1}{a^2}$$

- This PDF is a special case of the Erlang PDF, with $b = 1$.

Noise Model

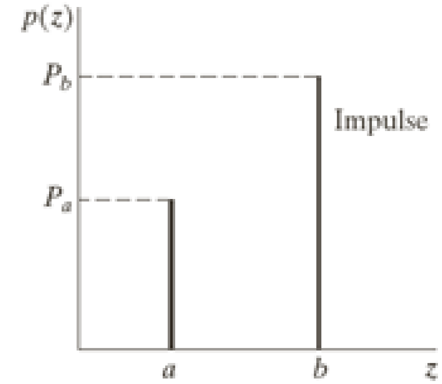


Uniform Noise:

$$p(z) = \begin{cases} \dfrac{1}{b-a}, \text{if } a \le z \le \mathrm{b} \\ 0 \quad , otherwise \end{cases}$$

$$\mu = \frac{a+b}{2}, \sigma^2 = \frac{(b-a)^2}{12}$$

Noise Model



Impulse (Salt-and-pepper) Noise:

$$p(z) = \begin{cases} P_a, \text{ for } z = a \\ P_b, \text{ for } z = b \\ 0, otherwise \end{cases}$$

- If $b > a$, gray-level 'b' will appear as a <u>light dot</u> in the image, level 'a' will appear like a <u>dark dot</u>.
- If either P_a or P_b is zero, then it is called <u>unipolar</u>.
- If $P_a \approx P_b$, then it is called salt-and-pepper noise.
  - Salt = white
  - Pepper = black

Noise Model

As a group, these PDFs provide useful tools for modeling a broad range of noise corruption situation found in practice.

Gaussian noise arises in a image due to factors such as electronic circuit noise and sensor noise due to poor illumination and/or high temperature.

Noise Model

The Rayleigh density is helpful in characterizing noise phenomena in range imaging.

The exponential and gamma densities find application in laser imaging.

Impulse noise is found in situation where quick transient, such as faulty switching take place during imaging.

Filtering to Remove Noise

We can use spatial filters of different kinds to remove different kinds of noise.

The *arithmetic mean* filter is calculated as follows:

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$

This is implemented as the simple smoothing filter.

Blurs the image to remove noise.

Different Kinds of Mean Filters

There are different kinds of mean filters all of which exhibit slightly different behaviour:

- Geometric Mean
- Harmonic Mean
- Contraharmonic Mean

Mean Filters

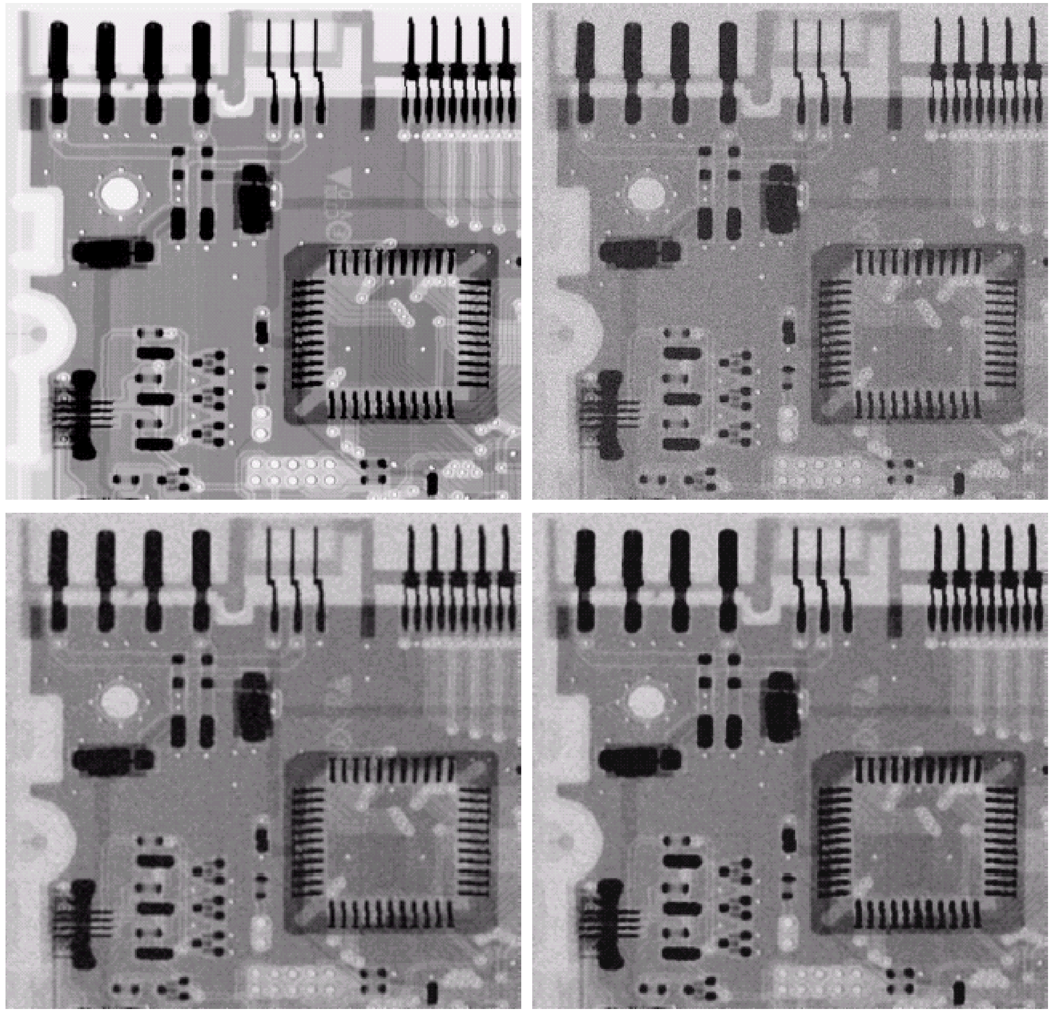
There are other variants on the mean which can give different performance.

Geometric Mean:

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

Achieves smoothing comparable to the arithmetic mean filter, but it tends to lose less image detail in the process.

Noise Removal Examples



Original Image

Image Corrupted by Gaussian Noise with zero Mean and Variance of 400

After a 3*3 Arithmetic Mean Filter

After a 3*3 Geometric Mean Filter

Geometric Mean filter did not blur as much as the Arithmetic Mean filter.

The connector fingers at the top of the image are much sharper for geometric mean filter output.

Mean Filters

Harmonic Mean:

$$\hat{f}(x,y) = \frac{mn}{\displaystyle\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

Works well for salt noise, but fails for pepper noise.

Also does well for other kinds of noise such as Gaussian noise.

Mean Filters

Contraharmonic Mean:

$$\hat{f}(x, y) = \frac{\displaystyle\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\displaystyle\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

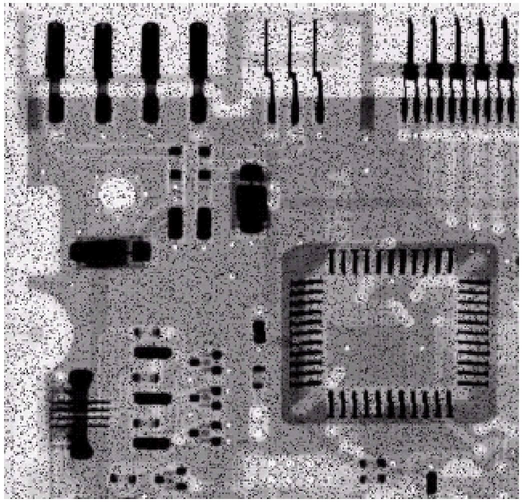
Q is the *order* of the filter and adjusting its value changes the filter's behaviour.

Positive values of Q eliminate pepper noise.

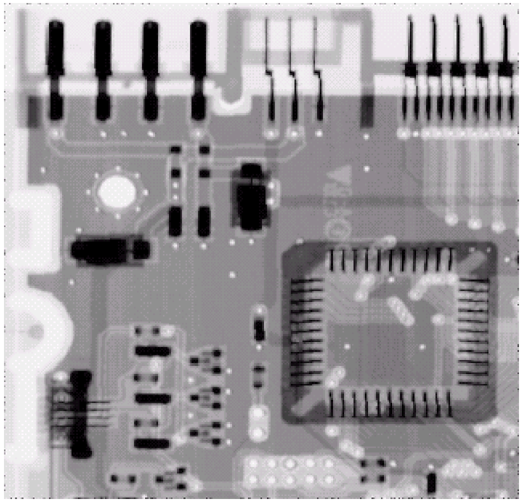
Negative values of Q eliminate salt noise.

Noise Removal Examples

Image Corrupted by Pepper Noise with Probability of 0.1



Result of Filtering with 3*3 Contraharmonic Q=1.5



Noise Removal Examples

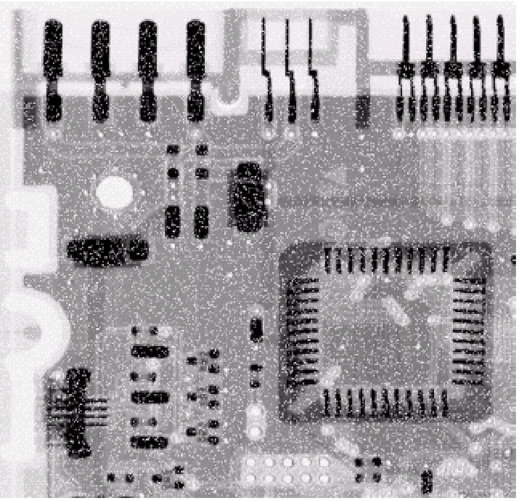
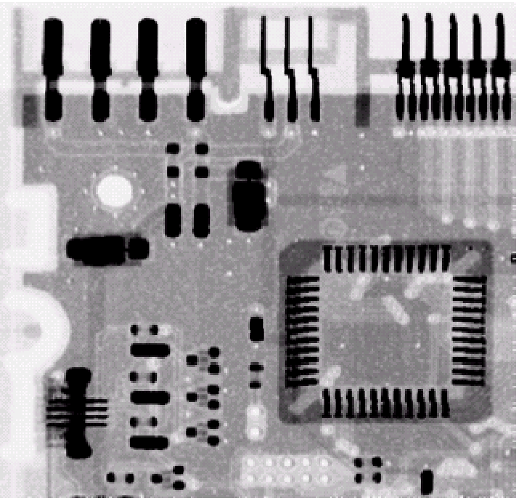


Image Corrupted By Salt Noise with Probability of 0.1

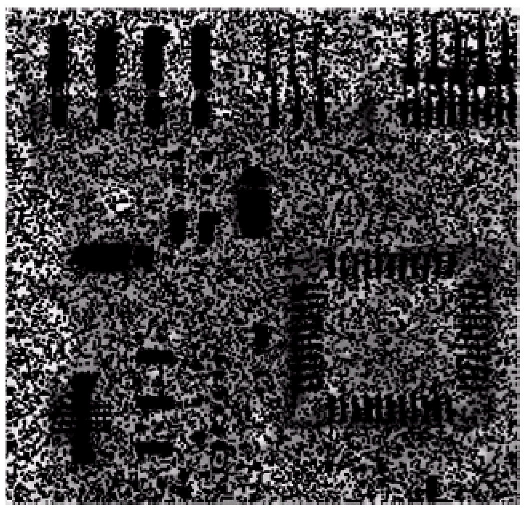


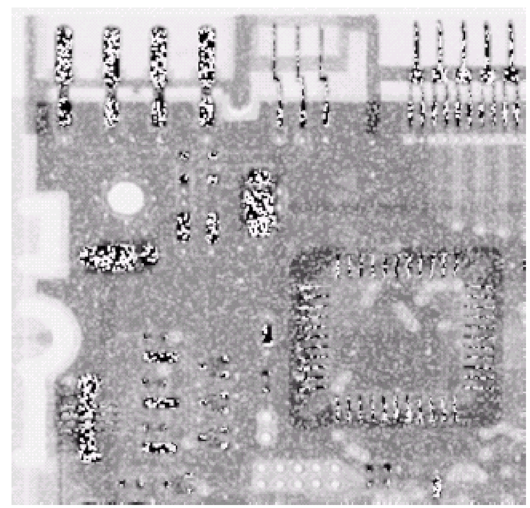
Result of Filtering with 3*3 Contraharmonic Q=-1.5

Drawbacks of Contraharmonic Filter

Choosing the wrong value for Q when using the contraharmonic filter can have drastic results.

Result of Filtering with 3*3 Contraharmonic Q=-1.5





Result of Filtering with 3*3 Contraharmonic Q=1.5

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Order Statistics Filters

Spatial filters that are based on ordering the pixel values that make up the neighbourhood operated on by the filter.

Useful spatial filters include:

- Median filter
- Max and min filter
- Midpoint filter
- Alpha trimmed mean filter

Median Filter

Median Filter: $$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

Excellent at noise removal, without the smoothing effects that can occur with other smoothing filters.

Particularly good when <u>salt and pepper noise</u> is present.

Max and Min Filter

Max Filter: $$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\}$$

Min Filter: $$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$

Max filter is good for <u>pepper noise</u> and min is good for <u>salt noise</u>.

Midpoint Filter

Midpoint Filter:

$$\hat{f}(x,y)=\frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\}+\min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

This filter combines order statistics and averaging.

Good for random Gaussian and uniform noise.

Alpha-Trimmed Mean Filter

Alpha-Trimmed Mean Filter:

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

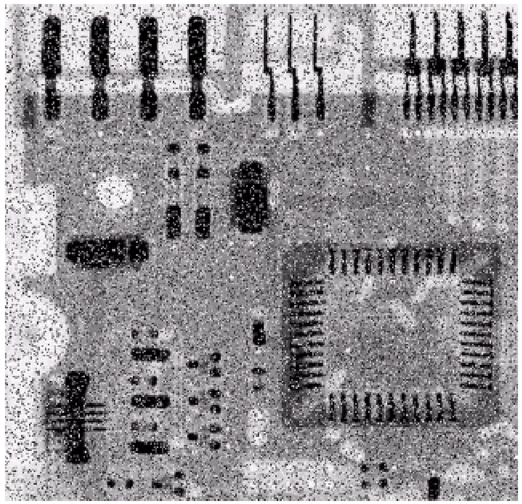
We can delete the $d/2$ lowest and $d/2$ highest grey levels.

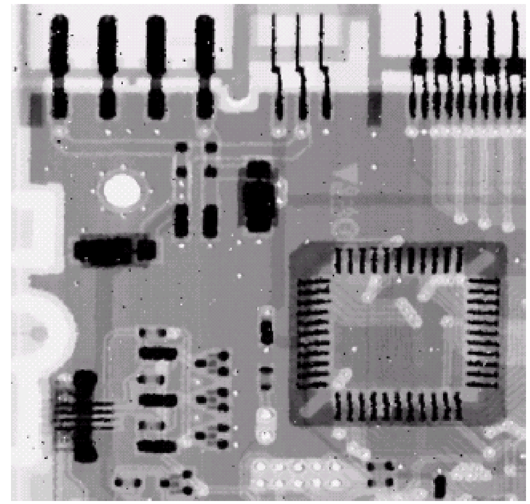
So, $g_r(s,\ t)$ represents the remaining $mn - d$ pixels.

In general, this filter is useful to handle multiple types of noise, such as a combination of salt-and-pepper and Gaussian noise.

Noise Removal Examples

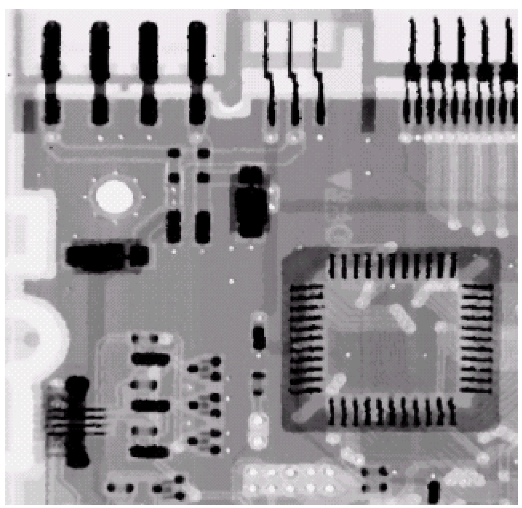
Image Corrupted by Salt and Pepper Noise

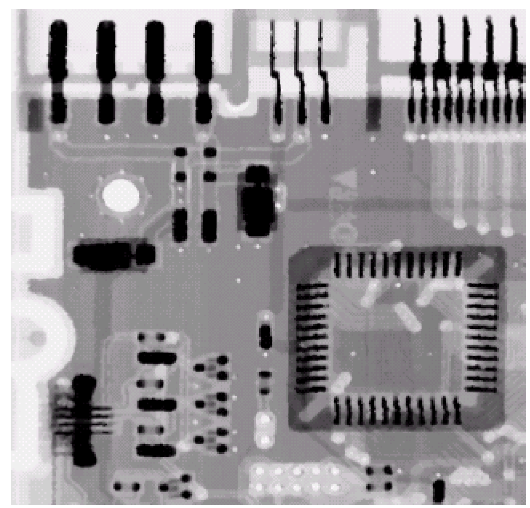




Result of 1 Pass with a 3*3 Median Filter

Result of 2 Passes with a 3*3 Median Filter

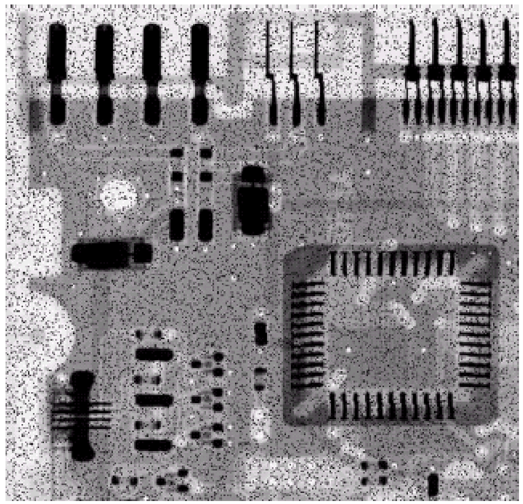




Result of 3 Passes with a 3*3 Median Filter

Noise Removal Examples

Image Corrupted by Pepper Noise



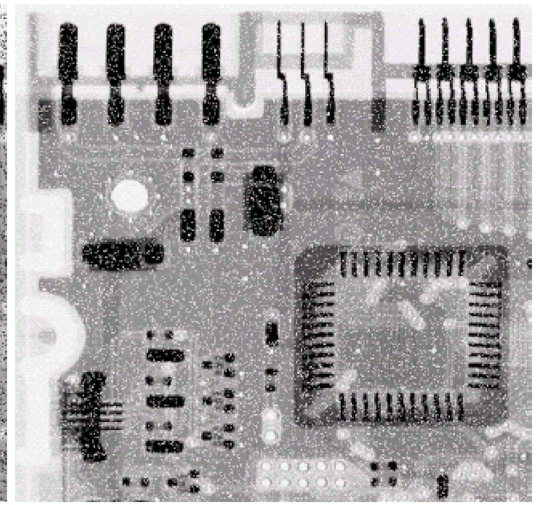
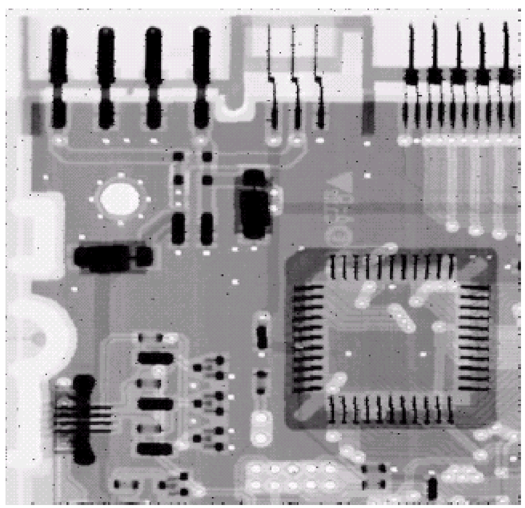
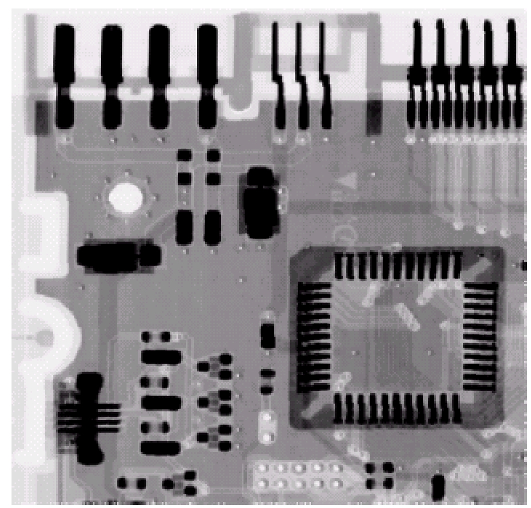


Image Corrupted by Salt Noise

Result of Filtering above with a 3*3 Max Filter





Result of Filtering above with a 3*3 Min Filter

Noise Removal Examples

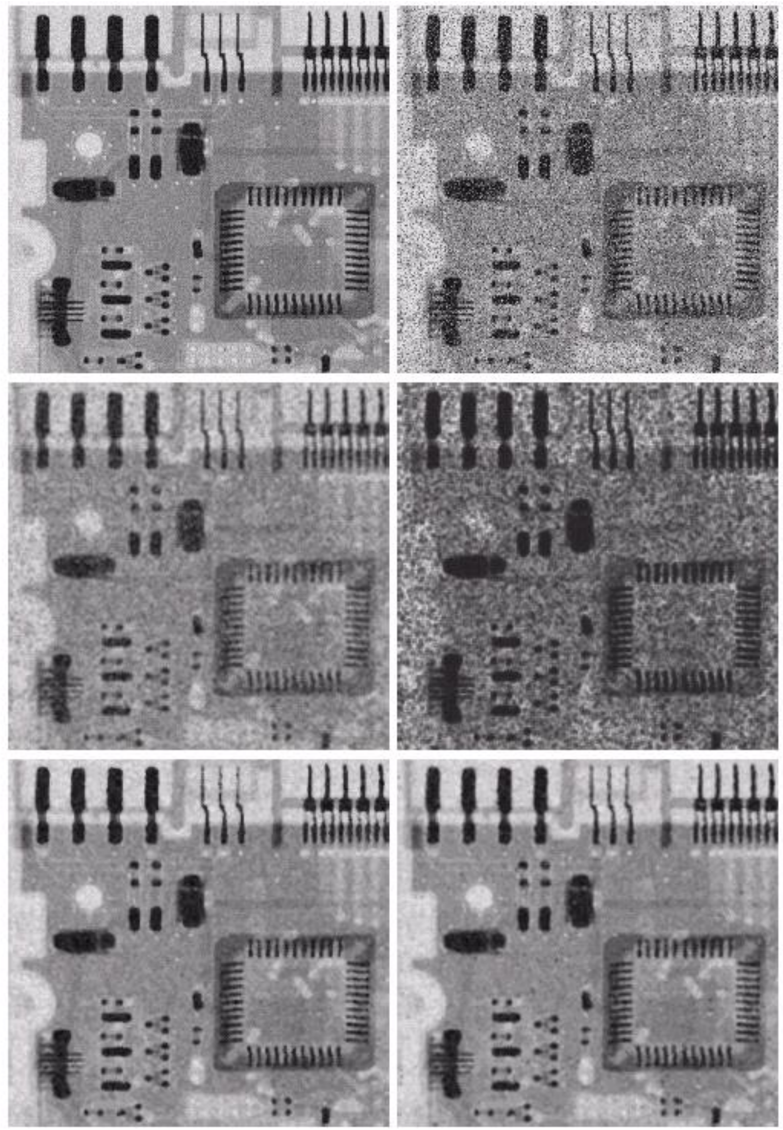


Image Corrupted by Uniform Noise

Image Further Corrupted by Salt and Pepper Noise

Filtered by 5*5 Arithmetic Mean Filter

Filtered by 5*5 Geometric Mean Filter

Filtered by 5*5 Median Filter

Filtered by 5*5 Alpha-Trimmed Mean Filter (d=5)

Adaptive Filters

The filters discussed so far are applied to an entire image without any regard for how image characteristics vary from one point to another.

The behaviour of adaptive filters changes depending on the characteristics of the image inside the filter region.

Adaptive Median Filtering

The median filter performs relatively well on impulse noise as long as the spatial density of the impulse noise is not large.

The adaptive median filter can handle much more spatially dense impulse noise, and also performs some smoothing for non-impulse noise.

The key insight in the adaptive median filter is that the filter size changes depending on the characteristics of the image.

Adaptive Median Filtering

The adaptive median filter has three purposes:

- Remove impulse noise
- Provide smoothing of other noise
- Reduce distortion (such as excessive thinning or thickening of object boundaries.)

Adaptive Median Filtering

Filtering looks at each original pixel image in turn and generates a new filtered pixel.

The notations used in this filter are:

z_{min} = Minimum grey level in S_{xy}

z_{max} = Maximum grey level in S_{xy}

z_{med} = Median of grey levels in S_{xy}

z_{xy} = Gray level at coordinates (x, y)

S_{max} = Maximum allowed size of S_{xy}

Adaptive Median Filtering

Level A: $A1 = z_{med} - z_{min}$

$A2 = z_{med} - z_{max}$

If $A1 > 0$ **AND** $A2 < 0$, **Go to** Level B

Else increase the window size

If window size $\leq S_{max}$ repeat Level A

Else output z_{xy}

Level B: $B1 = z_{xy} - z_{min}$

$B2 = z_{xy} - z_{max}$

If $B1 > 0$ **AND** $B2 < 0$, output z_{xy}

Else output z_{med}

Adaptive Filtering Example

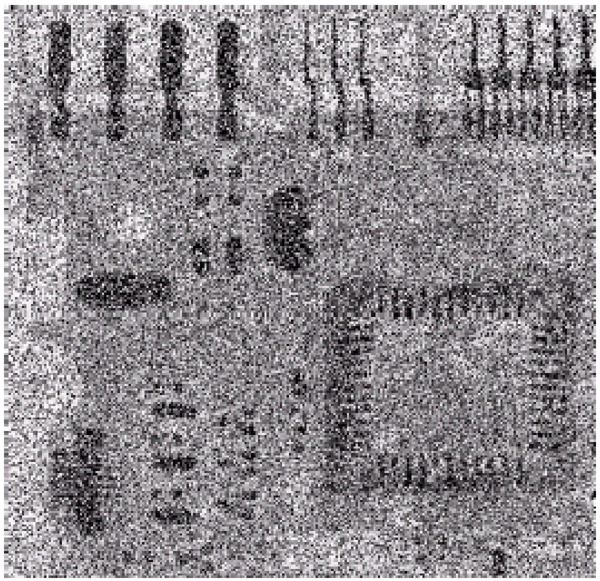
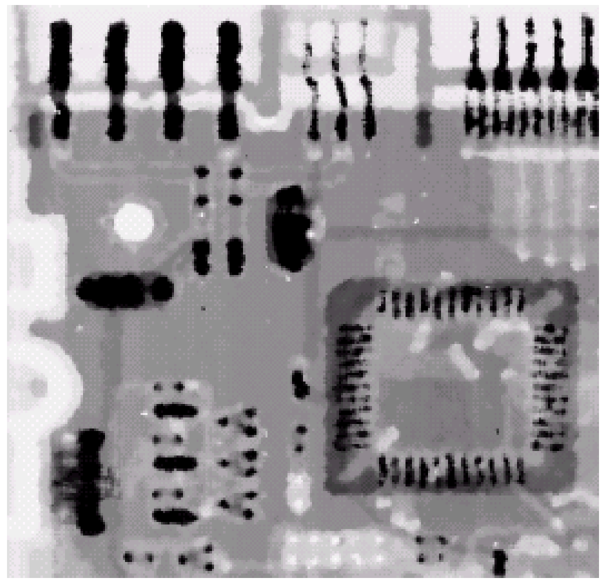
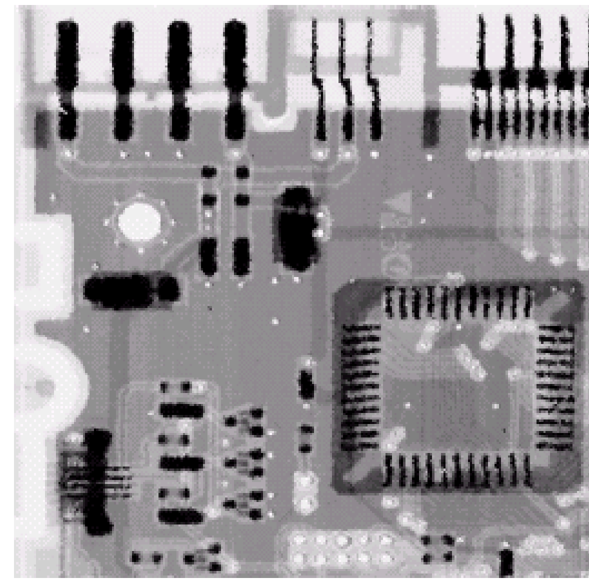


Image corrupted by salt and pepper noise with probabilities $P_a = P_b = 0.25$



Result of filtering with a 7 * 7 median filter



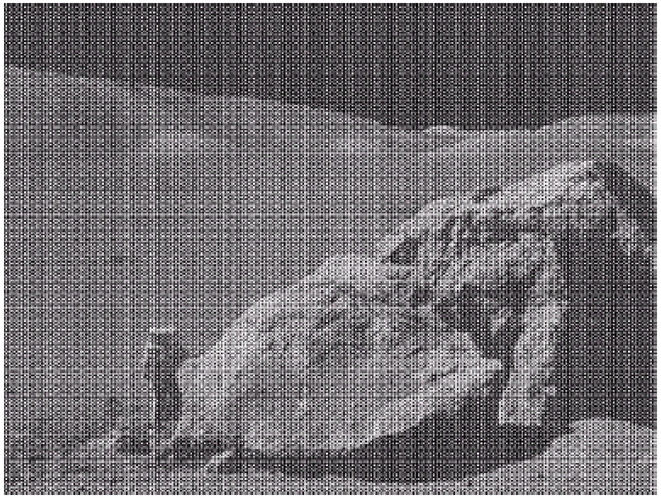
Result of adaptive median filtering of size 7*7

Periodic Noise

Generally arises due to electrical or electromagnetic interference.

Gives rise to regular noise patterns in an image.

<u>Frequency domain techniques</u> in the Fourier domain are most effective at removing periodic noise.



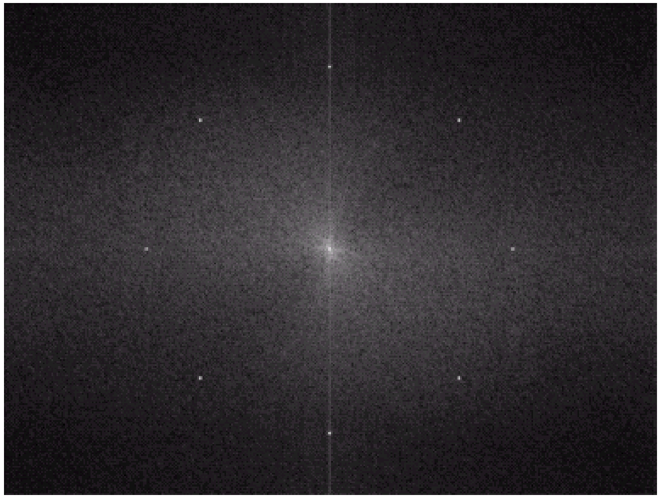


Image corrupted by sinusoidal noise and Fourier spectrum of the corrupted image.

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Filters is Frequency Domain

- Bandreject Filters
- Bandpass Filters
- Notch Filters

Bandreject Filters

Removing periodic noise from an image involves removing a particular band (range) of frequencies about the origin of the Fourier transform of that image.

An ideal bandreject filter is given as follows:

$$H(u,v)=\begin{cases}1 & if\ D(u,v)<D_0-\dfrac{W}{2}\\ 0 & if\ D_0-\dfrac{W}{2}\le D(u,v)\le D_0+\dfrac{W}{2}\\ 1 & if\ D(u,v)>D_0+\dfrac{W}{2}\end{cases}$$

$D(u,v)$ is the distance from the origin of the centered frequency rectangle, W is the width of the band, and D_0 is its radial center.

Bandreject Filters

Butterworth:

$$H(u,v) = \frac{1}{1 + \left[\frac{D(u,v)W}{D^2(u,v) - \ D_0{}^2}\right]^{2n}}$$

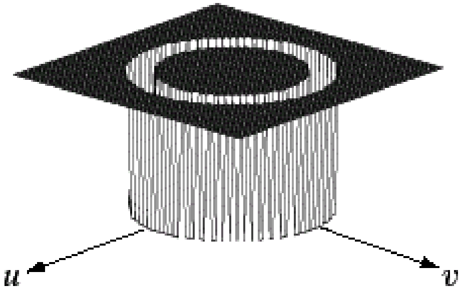
Gaussian:

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D^2(u,v) - D_0{}^2}{D(u,v)W}\right]^2}$$

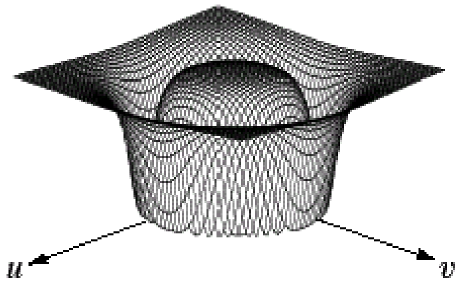
- Their filter is generally used when the location of the noise component in the frequency domain is approximately known.

Bandreject Filters

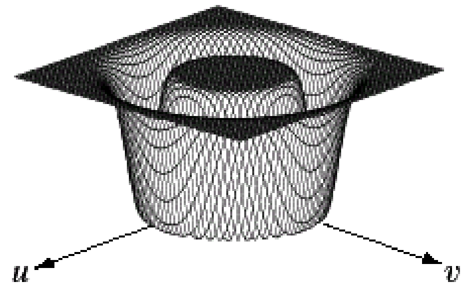
The perspective plots of ideal band reject filter is shown below, along with Butterworth and Gaussian versions of the filter.



Ideal Bandreject Filter



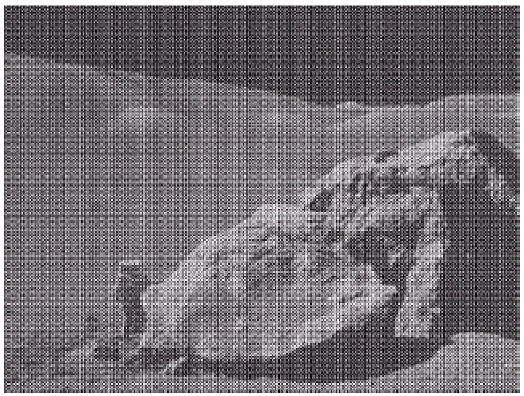
Butterworth Bandreject Filter (of order 1)



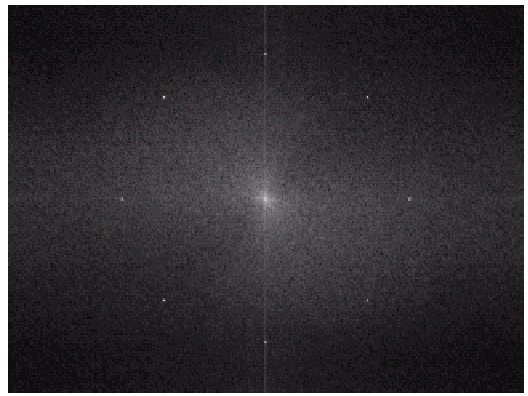
Gaussian Bandreject Filter

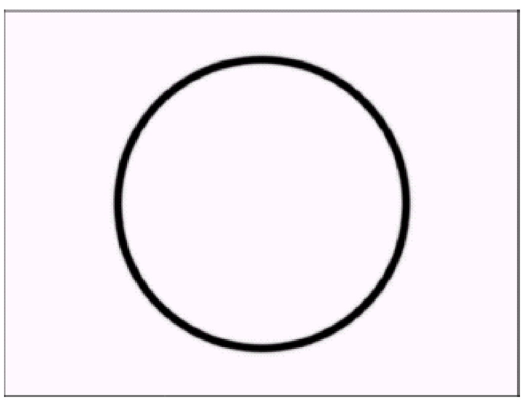
Bandreject Filter Example

Image corrupted by sinusoidal noise of various frequencies



Fourier spectrum of corrupted image





Butterworth bandreject filter of order 4

Filtered image

The noise components are easily seen as symmetric pairs of bright dots in the FT.

The noise components lie on an approximate circle about origin of the transform.

So, a circularly symmetric bandreject filter is a good choice.

The improvement is quite evident. Even small details and textures were restored effectively.

This would not be possible by a direct spatial filtering approach.

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

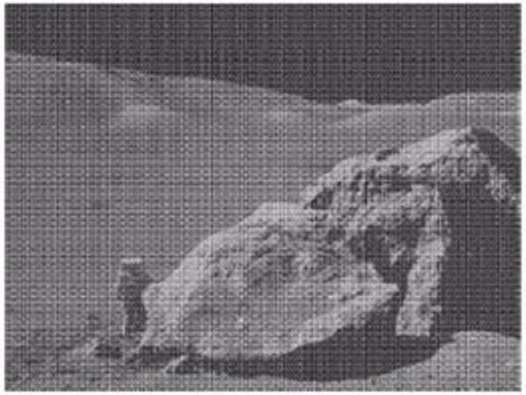
Bandpass Filter

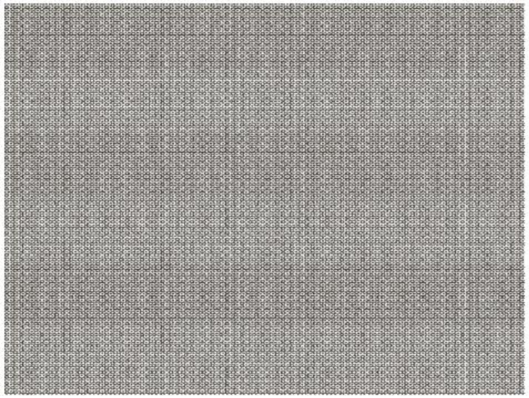
$$H_{bp}(u,v) = 1 - H_{br}(u,v)$$

- Straight forward use of this filter is not a common procedure because it generally removes too much image detail.

Bandpass Filter

- Bandpass filtering is quite useful in isolating the effect on an image of selected frequency bands.





Noise pattern of the image (left) obtained by bandpass filtering.

Bandpass Filter

- This image is generated by (1) using bandpass filter equation to obtain the bandpass filter corresponding to the bandreject filter used in that example; and (2) taking the inverese transform of the bandpass-filtered transform.

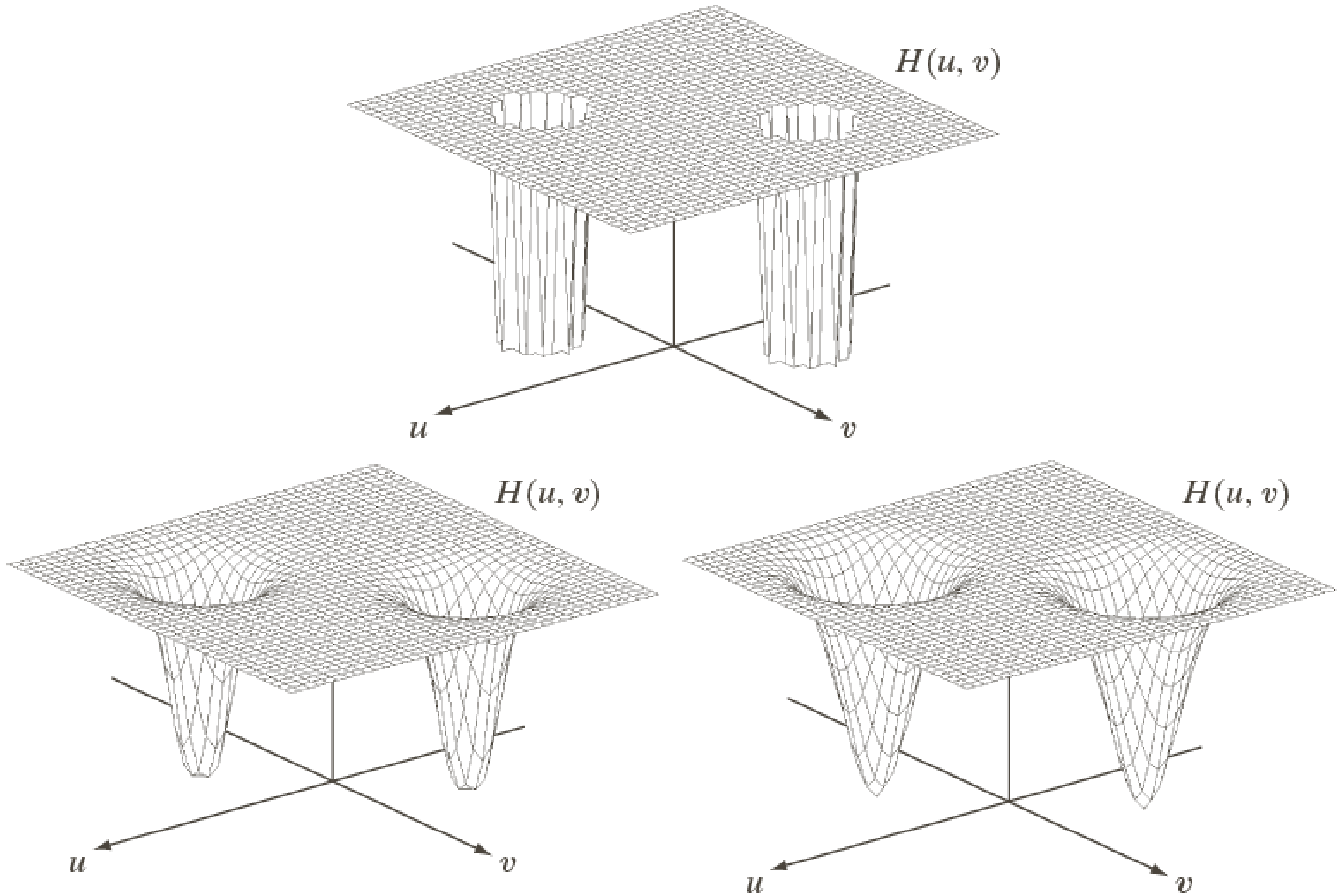
- This example shows that bandpass filtering helps to isolate the noise pattern.

Notch Filter

A filter that rejects (or passes) frequencies within a very narrow band of frequencies.

A filter that rejects (or passes) frequencies in predefined neighbourhoods about a center frequency.

Notch Reject Filter



a
b c

FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.

Notch Filter

Due to symmetry in Fourier transform, notch filters must appear in symmetric pairs about the origin in order to obtain meaningful results.

The one exception to this rule is if the notch filter is located at the origin, in which case it appears by itself.

Although we show only one pair for illustration purposes, the number of pairs of notch filters that can be implemented is arbitrary. The shape of the notch areas also can be arbitrary (e.g., rectangular).

Notch Reject Filter

Ideal:

$$H(u,v) = \begin{cases} 0 \; if \; D_1(u,v) \le D_0 \; or \; D_2(u,v) \le D_0 \\ 1 \qquad\qquad\qquad\qquad\qquad\qquad otherwise \end{cases}$$

$$D_1(u,v) = \left[\left(u - \frac{M}{2} - u_0\right)^2 + \left(v - \frac{N}{2} - v_0\right)^2\right]^{1/2}$$

$$D_2(u,v) = \left[\left(u - \frac{M}{2} + u_0\right)^2 + \left(v - \frac{N}{2} + v_0\right)^2\right]^{1/2}$$

(u_0, v_0): center and by symmetry $(-u_0, -v_0)$

Notch Reject Filter

- As usual, the assumption is that the center of the frequency rectangle has been shifted to the point $(\frac{M}{2}, \frac{N}{2})$, accordingly to the filtering procedure outlined in filtering in frequency domain.

- Therefore, the values of (u_0, v_0) are with respect to the shifted center.

Notch Reject Filter

Butterworth:

$$H(u,v) = \frac{1}{1 + \left[\frac{{D_0}^2}{D_1(u,v)D_2(u,v)}\right]^n}$$

Gaussian:

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D_1(u,v)D_2(u,v)}{{D_0}^2}\right]}$$

- A notch filter rejects (or passes) frequencies in predefined neighbourhoods about a center frequency.

Summary

- Image restoration is generally used for noise removal.
- Restoration is slightly more objective than enhancement.
- Spatial domain techniques are particularly useful for removing random noise.
- Frequency domain techniques are particularly useful for removing periodic noise.