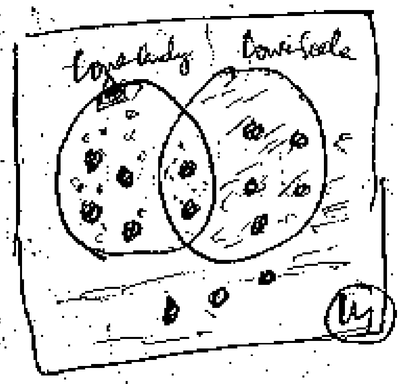
Conditional Probability

Soda \ Candy	Loves Candy	Doesn't love candy	
Love Soda	2 $\frac{2}{14}$	5 $\frac{5}{14}$	$2+5=\frac{7}{14}$
Doesn't love Soda	4 $\frac{4}{14}$	3 $\frac{3}{14}$	$4+3=\frac{7}{14}$
	$\frac{2+4}{14}$	$\frac{5+3}{14}$	



Contingency Table

$$P(LC \text{ and } S) = \frac{2}{14} = 0.14$$

$$P(LC \text{ and } NS) = \frac{4}{14} = 0.29$$
(Lcandy)

$$P(NC \text{ and } LS) = \frac{5}{14} = 0.36$$
only LS

$$P(NC \text{ and } LS) = \frac{3}{14} = 0.21$$

We can also determine the Probability that someone loves Soda, regardless of how they feel about Candy.

$$2+5 = \frac{7}{14}$$

~~What~~ Knowing that the next person we meet will love Soda. What is the probability that the next person also loves Candy?

In other words, what is the Probability that someone loves Candy and Soda given that we know they love Soda?

$P(\text{LC and LS} \mid \text{loves soda}) =$

This is called the Conditional Probability.

$$P(\text{LC and LS}) = \frac{2}{14} = 0.14$$

different

$$P(\text{LC and LS} \mid \text{LS}) = \frac{2}{2+5} = 0.29$$

In this case, knowing that they loved soda increased the probability that they would love candy.

- Now, we have probability that someone does not love candy and loves soda

$$P(\text{NC and LS} \mid \text{LS}) = \frac{5}{2+5} = \frac{5}{7} = 0.71$$

$$= \frac{\frac{5}{14}}{\frac{2+5}{14}} = 0.71$$

We got the same probability that we got before.

No need divide with the total population.

$$\frac{\frac{5}{14} \; P(\text{NC and LS})}{\frac{2+5}{14} \; P(\text{LS})}$$

$$P(NC \text{ and } LS \mid LS) = \frac{P(NC \text{ and } LS)}{P(LS)}$$

Conditional Probability is the Probability that something will happen, scaled by whatever knowledge we already have about the event.

Baye's Theorem

$$P(NC \,\&\, LS \mid LS) = \frac{P(NC \,\&\, LS)}{P(LS)} = 0.71$$

Standard notation — Redundant

$$P(NC \mid LS) = \frac{P(NC \,\&\, LS)}{P(LS)}$$

$$P(NC \,\&\, LS \mid LS) = \frac{P(NC \,\&\, LS)}{P(LS)} = 0.71$$

different knowledge

$$P(NC \,\&\, LS \mid NC) = \frac{P(NC \,\&\, LS)}{P(NC)} = 0.63$$

$$P(NC \,\&\, LS \mid LS) \times P(LS) = \frac{P(NC \,\&\, LS)}{\cancel{P(LS)}} \times \cancel{P(LS)}$$

$$P(NC \,\&\, LS \mid NC) \times P(NC) = \frac{P(NC \,\&\, LS)}{\cancel{P(NC)}} \times P(\ldots$$

$$P(NC \& LS \mid LS) \times P(LS) = P(NC \& LS \mid NC) \times P(NC)$$

$$P(NC \& LS \mid LS) = \frac{P(NC \& LS \mid NC) \times P(NC)}{P(LS)}$$

$$P(NC \& LS \mid NC) = \frac{P(NC \& LS \mid LS) \times P(LS)}{P(NC)}$$

~~Derivation~~ derived Bayes Theorem.

A = doesn't have candy (NC)

B = loses soda (LS)

$$P(A \& B \mid B) = \frac{P(A \& B \mid A) \times P(A)}{P(B)}$$

$$P(A \& B \mid A) = \frac{P(A \& B \mid B) \times P(B)}{P(A)}$$