# Introduction to **Information Retrieval**

Hinrich Schütze and Christina Lioma

Lecture 12: Language Models for IR

#### Overview

- 1 Recap
- 2 Language models
- 3 Language Models for IR
- 4 Discussion

#### Overview

- 1 Recap
- 2 Language models
- 3 Language Models for IR
- 4 Discussion

# Indexing anchor text

- Anchor text is often a better description of a page's content than the page itself.
- Anchor text can be weighted more highly than the text page.
- A Google bomb is a search with "bad" results due to maliciously manipulated anchor text.
  - [dangerous cult] on Google, Bing, Yahoo

# PageRank

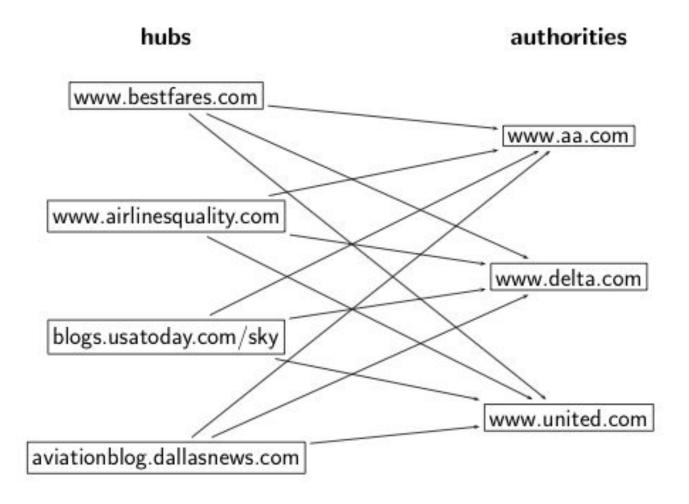
- Model: a web surfer doing a random walk on the web
- Formalization: Markov chain
- PageRank is the long-term visit rate of the random surfer or the steady-state distribution.
- Need teleportation to ensure well-defined PageRank
- Power method to compute PageRank.
  - PageRank is the principal left eigenvector of the transition probability matrix.

## Computing PageRank: Power method

|                             | $X_1$ $P_t(d_1)$ | $X_2$ $P_t(d_2)$ |                                  |                                  |                                 |
|-----------------------------|------------------|------------------|----------------------------------|----------------------------------|---------------------------------|
|                             |                  |                  | $P_{11} = 0.1$<br>$P_{21} = 0.3$ | $P_{12} = 0.9$<br>$P_{22} = 0.7$ |                                 |
| $t_{\scriptscriptstyle{0}}$ | 0                | 1                | 0.3                              | 0.7                              | = $\overrightarrow{x}$ P        |
| $t_{_{1}}$                  | 0.3              | 0.7              | 0.24                             | 0.76                             | $=\overrightarrow{x}P^2$        |
| $t_2$                       | 0.24             | 0.76             | 0.252                            | 0.748                            | $=\overrightarrow{x}P^3$        |
| $t_3$                       | 0.252            | 0.748            | 0.2496                           | 0.7504                           | $=\overrightarrow{x}P^4$        |
|                             |                  |                  |                                  | •                                |                                 |
| $t_{_{\infty}}$             | 0.25             | 0.75             | 0.25                             | 0.75                             | $=\overrightarrow{XP}^{\infty}$ |

PageRank vector =  $\overrightarrow{\pi}$  =  $(\pi_1, \pi_2)$  = (0.25, 0.75)  $P_{t}(d_1) = P_{t-1}(d_1) \cdot P_{11} + P_{t-1}(d_2) \cdot P_{21}$  $P_{t}(d_2) = P_{t-1}(d_1) \cdot P_{12} + P_{t-1}(d_2) \cdot P_{22}$ 

#### HITS: Hubs and authorities



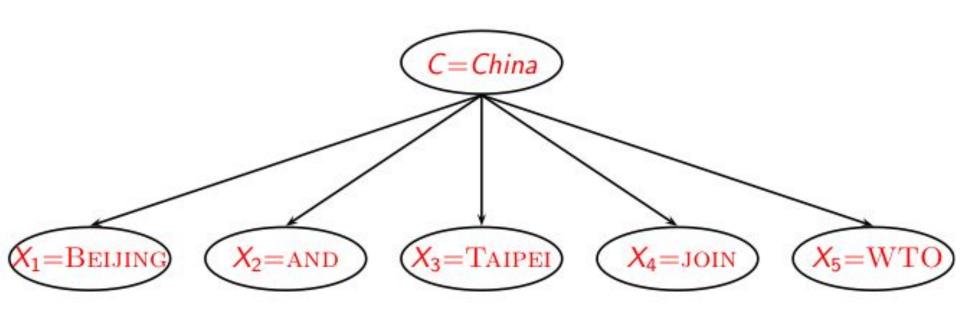
## HITS update rules

- A: link matrix
- $\vec{h}$ : vector of hub scores
- $\vec{a}$ : vector of authority scores
- HITS algorithm:
  - Compute  $\vec{h} = A\vec{a}$
  - Compute  $\vec{a} = A^T \vec{h}$
  - Iterate until convergence
  - Output (i) list of hubs ranked according to hub score and
    (ii) list of authorities ranked according to authority score

## Outline

- 1 Recap
- 2 Language models
- 3 Language Models for IR
- 4 Discussion

## Recall: Naive Bayes generative model



## Naive Bayes and LM generative models

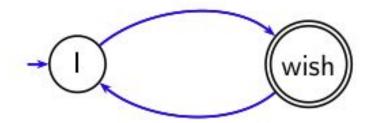
- We want to classify document d.
  - We want to classify a query q.
    - Classes: geographical regions like China, UK, Kenya.
      Each document in the collection is a different class.
- Assume that d was generated by the generative model.
  Assume that q was generated by a generative model.
- Key question: Which of the classes is most likely to have generated the document? Which document (=class) is most likely to have generated the query q?
  - Or: for which class do we have the most evidence? For which document (as the source of the query) do we have the most evidence?

## Using language models (LMs) for IR

- 1 LM = language model
- We view the document as a generative model that generates the query.
- What we need to do:
- 4 Define the precise generative model we want to use
- Estimate parameters (different parameters for each document's model)
- 6 Smooth to avoid zeros
- Apply to query and find document most likely to have generated the query
- Present most likely document(s) to user
- 9 Note that x y is pretty much what we did in Naive Bayes.

## What is a language model?

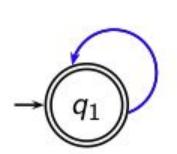
We can view a finite state automaton as a deterministic language



model.

I wish I wish I wish I wish . . . Cannot generate: "wish I wish" or "I wish I". Our basic model: each document was generated by a different automaton like this except that these automata are probabilistic.

## A probabilistic language model



| W    | $P(w q_1)$ | w     | $P(w q_1)$ |
|------|------------|-------|------------|
| STOP | 0.2        | toad  | 0.01       |
| the  | 0.2        | said  | 0.03       |
| a    | 0.1        | likes | 0.02       |
| frog | 0.01       | that  | 0.04       |
|      |            | ****  |            |

This is a one-state probabilistic finite-state automaton – a unigram language model – and the state emission distribution for its one state  $q_1$ . STOP is not a word, but a special symbol indicating that the automaton stops. frog said that toad likes frog STOP

 $P(\text{string}) = 0.01 \cdot 0.03 \cdot 0.04 \cdot 0.01 \cdot 0.02 \cdot 0.01 \cdot 0.02$ 

= 0.000000000048

## A different language model for each document

| language model of $d_1$ |        |       | language model of $d_2$ |      |        |       |        |
|-------------------------|--------|-------|-------------------------|------|--------|-------|--------|
| W                       | P(w .) | w     | P(w .)                  | W    | P(w .) | W     | P(w .) |
| STOP                    | .2     | toad  | .01                     | STOP | .2     | toad  | .02    |
| the                     | .2     | said  | .03                     | the  | .15    | said  | .03    |
| a                       | .1     | likes | .02                     | a    | .08    | likes | .02    |
| frog                    | .01    | that  | .04                     | frog | .01    | that  | .05    |
|                         |        |       |                         |      |        |       |        |

frog said that toad likes frog STOP  $P(\text{string}|M_{d1}) = 0.01 \cdot 0.03 \cdot 0.04 \cdot 0.01 \cdot 0.02 \cdot 0.01 \cdot 0.02 = 0.0000000000048 = 4.8 \cdot 10^{-12}$ 

$$P(\text{string}|M_{d2}) = 0.01 \cdot 0.03 \cdot 0.05 \cdot 0.02 \cdot 0.02 \cdot 0.01 \cdot 0.02 = 0.000000000120 = 12 \cdot 10^{-12}$$
  $P(\text{string}|M_{d1}) < P(\text{string}|M_{d2})$ 

Thus, document  $d_2$  is "more relevant" to the string "frog said that toad likes frog STOP" than  $d_1$  is.

15

## Outline

- 1 Recap
- 2 Language models
- 3 Language Models for IR
- 4 Discussion

## Using language models in IR

- Each document is treated as (the basis for) a language model.
- Given a query q
- Rank documents based on P(d|q)

$$P(d|q) = \frac{P(q|d)P(d)}{P(q)}$$

- P(q) is the same for all documents, so ignore
- P(d) is the prior often treated as the same for all d
  - But we can give a prior to "high-quality" documents, e.g., those with high PageRank.
- $P(q \mid d)$  is the probability of q given d.
- So to rank documents according to relevance to q, ranking according to P(q|d) and P(d|q) is equivalent.

#### Where we are

- In the LM approach to IR, we attempt to model the query generation process.
- Then we rank documents by the probability that a query would be observed as a random sample from the respective document model.
- That is, we rank according to P(q | d).
- Next: how do we compute  $P(q \mid d)$ ?

# How to compute P(q | d)

 We will make the same conditional independence assumption as for Naive Bayes.

$$P(q|M_d) = P(\langle t_1, \ldots, t_{|q|} \rangle | M_d) = \prod_{1 \leq k \leq |q|} P(t_k | M_d)$$

 $(|q|: length ofr q; t_k: the token occurring at position k in q)$ 

This is equivalent to:

$$P(q|M_d) = \prod_{\text{distinct term } t \text{ in } q} P(t|M_d)^{\text{tf}_{t,q}}$$

- $tf_{t,q}$ : term frequency (# occurrences) of t in q
- Multinomial model (omitting constant factor)

#### Parameter estimation

- Missing piece: Where do the parameters  $P(t|M_d)$ . come from?
- Start with maximum likelihood estimates (as we did for Naive Bayes)  $\hat{P}(t|M_d) = \frac{\mathrm{tf}_{t,d}}{|d|}$

 $(|d|: length of d; tf_{t,d}: # occurrences of t in d)$ 

- As in Naive Bayes, we have a problem with zeros.
- A single t with  $P(t|M_d) = 0$  will make  $P(q|M_d) = \prod P(t|M_d)$  zero.
- We would give a single term "veto power".
- For example, for query [Michael Jackson top hits] a document about "top songs" (but not using the word "hits") would have  $P(t|M_d) = 0$ . That's bad.
- We need to smooth the estimates to avoid zeros.

# Smoothing

- Key intuition: A nonoccurring term is possible (even though it didn't occur), . . .
- . . . but no more likely than would be expected by chance in the collection.
- Notation:  $M_c$ : the collection model;  $cf_t$ : the number of occurrences of t in the collection;  $T = \sum_t cf_t$ : the total number of tokens in the collection.

$$\hat{P}(t|M_d) = \frac{\mathrm{tf}_{t,d}}{|d|}$$

• We will use  $\hat{P}(t|M_c)$  to "smooth" P(t|d) away from zero.

#### Mixture model

- $P(t|d) = \lambda P(t|M_d) + (1-\lambda)P(t|M_c)$
- Mixes the probability from the document with the general collection frequency of the word.
- High value of  $\lambda$ : "conjunctive-like" search tends to retrieve documents containing all query words.
- Low value of  $\lambda$ : more disjunctive, suitable for long queries
- Correctly setting  $\lambda$  is very important for good performance.

# Mixture model: Summary

$$P(q|d) \propto \prod_{1 \leq k \leq |q|} (\lambda P(t_k|M_d) + (1-\lambda)P(t_k|M_c))$$

- What we model: The user has a document in mind and generates the query from this document.
- The equation represents the probability that the document that the user had in mind was in fact this one.

## Example

- Collection:  $d_1$  and  $d_2$
- d<sub>1</sub>: Jackson was one of the most talented entertainers of all time
- $d_2$ : Michael Jackson anointed himself King of Pop
- Query q: Michael Jackson
- Use mixture model with  $\lambda = 1/2$
- $P(q|d_1) = [(0/11 + 1/18)/2] \cdot [(1/11 + 2/18)/2] \approx 0.003$
- $P(q|d_2) = [(1/7 + 1/18)/2] \cdot [(1/7 + 2/18)/2] \approx 0.013$
- Ranking:  $d_2 > d_1$

# Exercise: Compute ranking

- Collection:  $d_1$  and  $d_2$
- $d_1$ : Xerox reports a profit but revenue is down
- $d_2$ : Lucene narrows quarter loss but decreases further
- Query q: revenue down
- Use mixture model with  $\lambda = 1/2$
- $P(q|d_1) = [(1/8 + 2/16)/2] \cdot [(1/8 + 1/16)/2] = 1/8 \cdot 3/32 = 3/256$
- $P(q|d_2) = [(1/8 + 2/16)/2] \cdot [(0/8 + 1/16)/2] = 1/8 \cdot 1/32 = 1/256$
- Ranking:  $d_2 > d_1$

## Outline

- 1 Recap
- 2 Language models
- 3 Language Models for IR
- 4 Discussion

## LMs vs. Naive Bayes

- Different smoothing methods: mixture model vs. add-one
- We classify the query in LMs; we classify documents in text classification.
- Each document is a class in LMs vs. classes are human-defined in text classification
- The formal model is the same: multinomial model.
  - Actually: The way we presented Naive Bayes, it's not a true multinomial model, but it's equivalent.

# Vector space (tf-idf) vs. LM

|                  |        | precision |        | significant? |
|------------------|--------|-----------|--------|--------------|
| Rec.             | tf-idf | LM        | %chg   |              |
| 0.0              | 0.7439 | 0.7590    | +2.0   | -2.          |
| 0.1              | 0.4521 | 0.4910    | +8.6   |              |
| 0.2              | 0.3514 | 0.4045    | +15.1  | *            |
| 0.4              | 0.2093 | 0.2572    | +22.9  | *            |
| 0.6              | 0.1024 | 0.1405    | +37.1  | *            |
| 0.8              | 0.0160 | 0.0432    | +169.6 | *            |
| 1.0              | 0.0028 | 0.0050    | +76.9  |              |
| 11-point average | 0.1868 | 0.2233    | +19.6  | *            |

The language modeling approach always does better in these experiments . . . . but note that where the approach shows significant gains is at higher levels of recall.

# LMs vs. vector space model (1)

- LMs have some things in common with vector space models.
- Term frequency is directed in the model.
  - But it is not scaled in LMs.
- Probabilities are inherently "length-normalized".
  - Cosine normalization does something similar for vector space.
- Mixing document and collection frequencies has an effect similar to idf.
  - Terms rare in the general collection, but common in some documents will have a greater influence on the ranking.

# LMs vs. vector space model (2)

- LMs vs. vector space model: commonalities
  - Term frequency is directly in the model.
  - Probabilities are inherently "length-normalized".
  - Mixing document and collection frequencies has an effect similar to idf.
- LMs vs. vector space model: differences
  - LMs: based on probability theory
  - Vector space: based on similarity, a geometric/linear algebra notion
  - Collection frequency vs. document frequency
  - Details of term frequency, length normalization etc.

# Language models for IR: Assumptions

- Simplifying assumption: Queries and documents are objects of same type. Not true!
  - There are other LMs for IR that do not make this assumption.
  - The vector space model makes the same assumption.
- Simplifying assumption: Terms are conditionally independent.
  - Again, vector space model (and Naive Bayes) makes the same assumption.
- Cleaner statement of assumptions than vector space
- Thus, better theoretical foundation than vector space
  - ... but "pure" LMs perform much worse than "tuned" LMs.

#### Resources

- Chapter 12 of IR
- Resources at http://ifnlp.org/ir
  - Ponte and Croft's 1998 SIGIR paper (one of the first on LMs in IR)
  - Lemur toolkit (good support for LMs in IR)