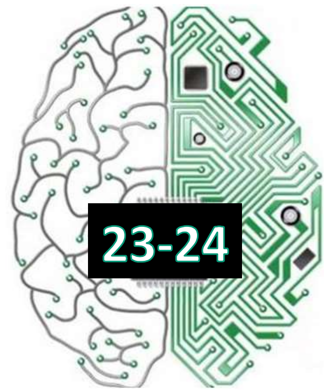
Open Elective Course [OE]
Course Code: CSO507
Winter 2023-24

Lecture#

Deep Learning

Unit-5: Sequence Modeling with Recurrent Neural Network (RNN)_Part-IV&V



Course Instructor:
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Vanishing/Exploding Gradient Problem

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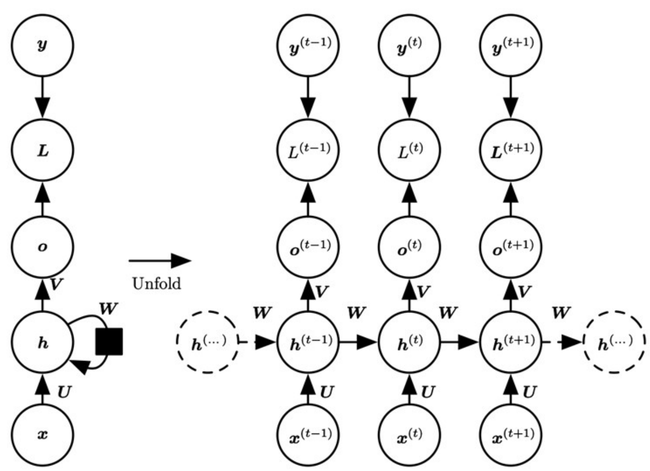
- Backpropagated errors multiply at each layer, resulting in exponential decay (if derivative is small) or growth (if derivative is large).
- Makes it very difficult train deep networks, or simple recurrent networks over many time steps.



Vanishing/Exploding Gradient Problem

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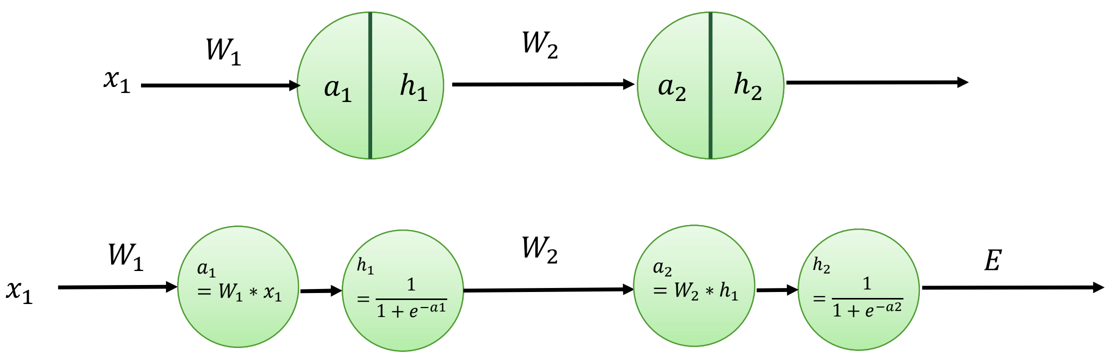

$$W_i = W_i - \eta \frac{\partial E}{\partial W_i}$$

$$\frac{\partial E}{\partial w_2} = \frac{\partial E}{\partial h_2} * \frac{\partial h_2}{\partial a_2} * \frac{\partial a_2}{\partial W_2}$$

$$\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial h_2} * \frac{\partial h_2}{\partial a_2} * \frac{\partial a_2}{\partial h_1} * \frac{\partial h_1}{\partial a_1} * \frac{\partial a_1}{\partial w_1}$$

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Activation Functions and Derivatives

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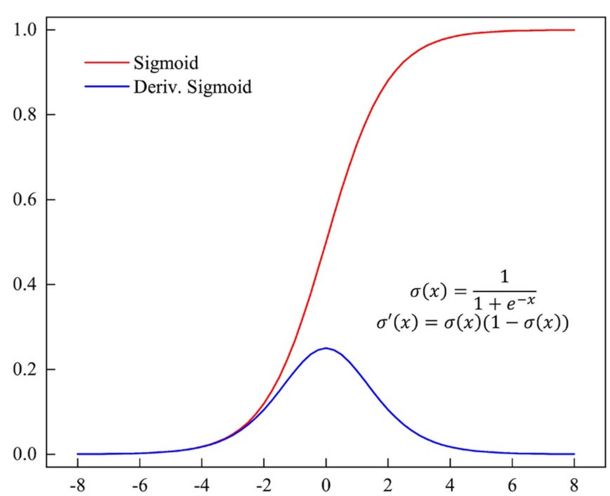

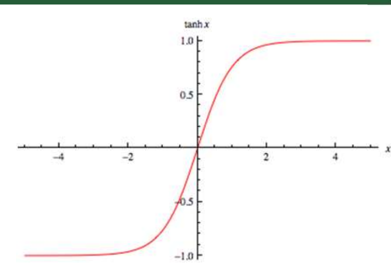

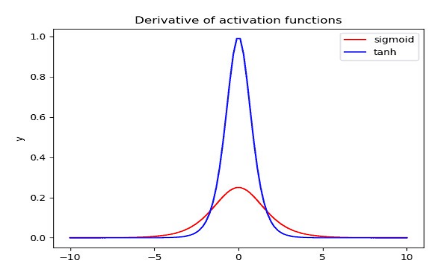

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Long Distance Dependencies

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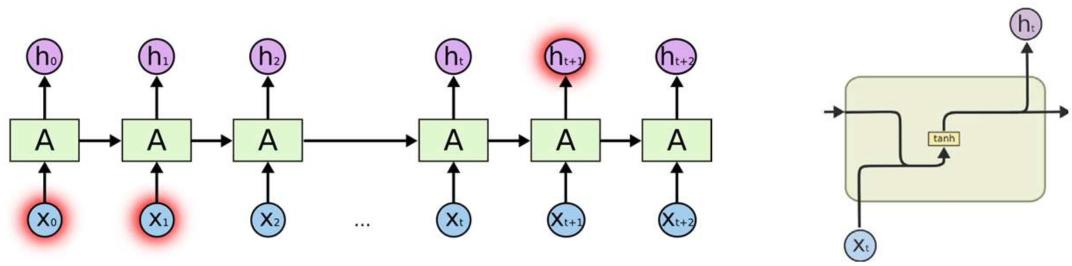
- It is very difficult to train simple recurrent networks (SRNs) to retain information over many time steps
- This makes it very difficult to learn SRNs that handle long-distance dependencies, such as subject-verb agreement.



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Gated RNNs

Long Short-Term Memory (LSTM)
Gated Recurrent Unit (GRU)

Gated RNNs

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- The most effective sequence models used in practical applications are called gated RNNs.

- These include the long short-term memory (LSTM) and networks based on the gated recurrent unit (GRU)
  - Create paths through time that have derivatives that neither vanish nor explode
  - *Accumulate* information such as evidence for a particular feature or category,
  - *Forget* the old state and start over.

Long Short Term Memory

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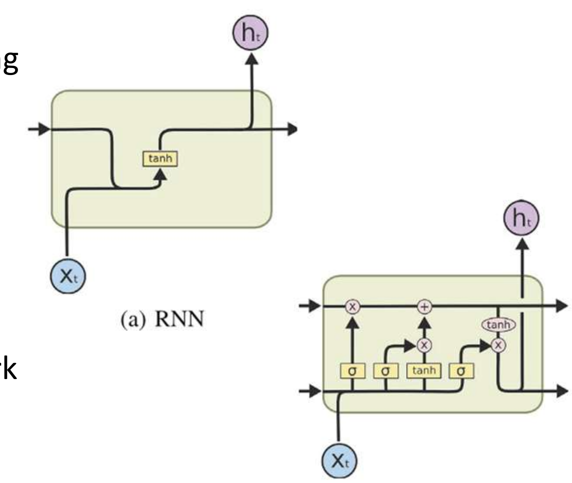
- LSTM networks, add additional gating units in each memory cell.
  - Forget gate
  - Input gate
  - Output gate

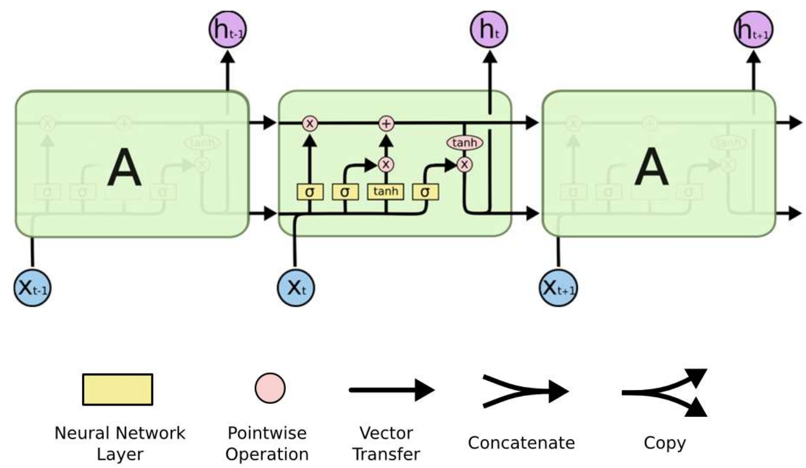
- Prevents vanishing/exploding gradient problem and allows network to retain state information over longer periods of time.



(a) RNN

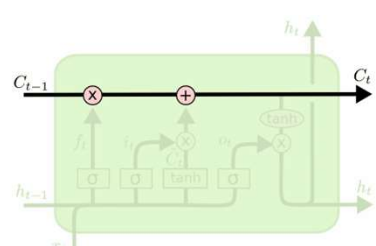
(b) LSTM

LSTM Network Architecture



Cell State

- Maintains a vector C_t that is the same dimensionality as the hidden state, h_t
- Information can be added or deleted from this state vector via the forget and input gates.



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Cell State Example

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- Want to remember person & number of a subject noun so that it can be checked to agree with the person & number of verb when it is eventually encountered.
- Forget gate will remove existing information of a prior subject when a new one is encountered.
- Input gate "adds" in the information for the new subject.

Forget Gate

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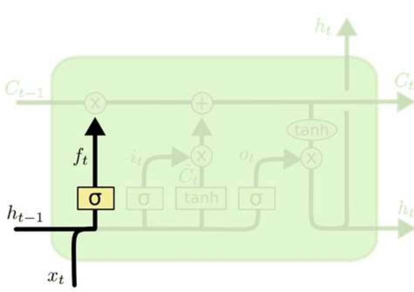
- Forget gate computes a 0-1 value using a logistic sigmoid output function from the input, x_t, and the current hidden state, h_t:
- Multiplicatively combined with cell state, "forgetting" information where the gate outputs something close to 0.



$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \ + \ b_f\right)$$

Input Gate

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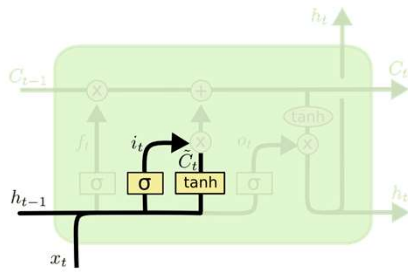
- First, determine which entries in the cell state to update by computing 0-1 sigmoid output.
- Then determine what amount to add/subtract from these entries by computing a tanh output (valued –1 to 1) function of the input and hidden state.



$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \ + \ b_i\right)$$
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \ + \ b_C)$$

Updating the Cell State

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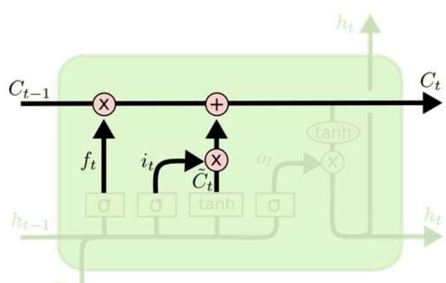
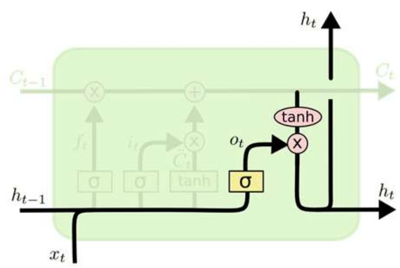
- Cell state is updated by using component-wise vector multiply to "forget" and vector addition to "input" new information.



$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

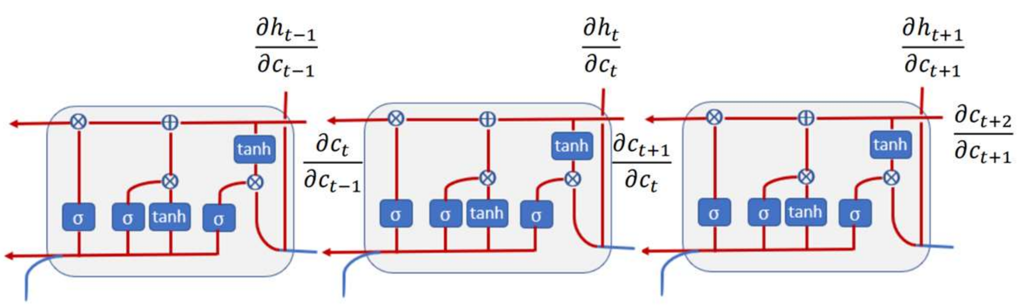
Output Gate

- Hidden state is updated based on a "filtered" version of the cell state, scaled to –1 to 1 using tanh.
- Output gate computes a sigmoid function of the input and current hidden state to determine which elements of the cell state to "output".



$$o_t = \sigma\left(W_o\ [h_{t-1}, x_t]\ +\ b_o\right)$$
$$h_t = o_t * \tanh\left(C_t\right)$$

Backpropagation in LSTM



$$\frac{\partial E}{\partial W} = \sum_{t=1}^{T} \frac{\partial E_t}{\partial W}$$

$$\frac{\partial E_t}{\partial W} = \frac{\partial E}{\partial h_t}\frac{\partial h_t}{\partial c_t}\frac{\partial c_t}{\partial c_{t-1}} \cdots \frac{\partial c_2}{\partial c_1}\frac{\partial c_1}{\partial W} = \frac{\partial E}{\partial h_t}\frac{\partial h_t}{\partial c_t}\left(\prod_{t=2}^{T}\frac{\partial c_t}{\partial c_{t-1}}\right)\frac{\partial c_1}{\partial W}$$

How does this contribute to handle vanishing gradient issue?

Backpropagation in Vanilla RNN

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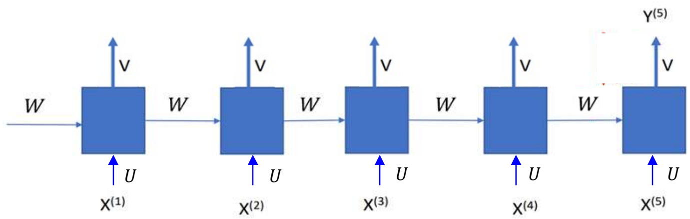
for $t = 1, \dots, \tau$:

$$a^{(t)} = b + Wh^{(t-1)} + Ux^{(t)}$$
$$h^{(t)} = \tanh\left(a^{(t)}\right)$$
$$y^{(t)} = c + V\,h^{(t)}$$



$$\frac{\partial E}{\partial W} = \sum_{t=1}^{T} \frac{\partial E_t}{\partial W}$$

Main cause of the vanishing gradient issue

$$\frac{\partial E_t}{\partial W} = \sum_{i=0}^{t-1} \frac{\partial E_t}{\partial y_t} \frac{\partial y_t}{\partial h_t} \boxed{\frac{\partial h_t}{\partial h_i}} \frac{\partial h_i}{\partial W}$$

$$\boxed{\frac{\partial h_t}{\partial h_i} = \frac{\partial h_t}{\partial h_{t-1}} \frac{\partial h_{t-1}}{\partial h_{t-2}} \cdots \frac{\partial h_{i+1}}{\partial h_i} = \prod_{k=i}^{t-1} \frac{\partial h_{k+1}}{\partial h_k}}$$

Backpropagation in Vanilla RNN

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for $t = 1, \dots, \tau$:

$$a^{(t)} = b + Wh^{(t-1)} + Ux^{(t)}$$
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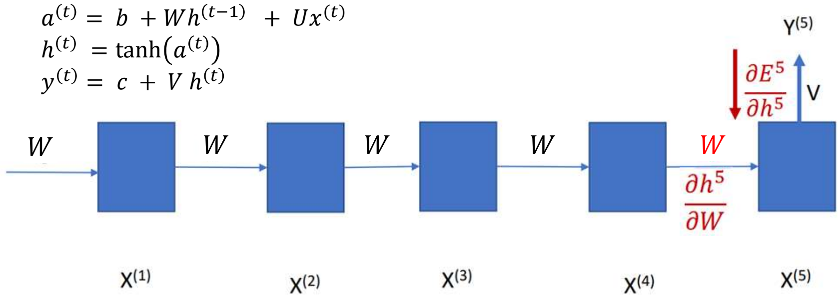

$$\frac{\partial E^5}{\partial W} = \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial W}$$

Backpropagation in Vanilla RNN

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for $t = 1, \ldots, \tau$:

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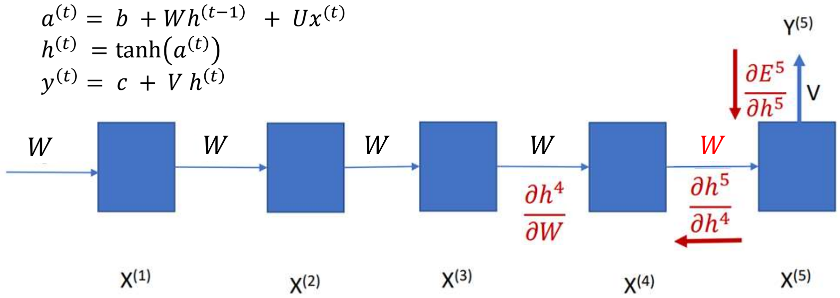

$$\frac{\partial E^5}{\partial W} = \frac{\partial E^5}{\partial y^5}\frac{\partial y^5}{\partial h^5}\frac{\partial h^5}{\partial W} + \frac{\partial E^5}{\partial y^5}\frac{\partial y^5}{\partial h^5}\frac{\partial h^5}{\partial h^4}\frac{\partial h^4}{\partial W}$$

Backpropagation in Vanilla RNN

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for $t = 1, \ldots, \tau$:

$$a^{(t)} = b + Wh^{(t-1)} + Ux^{(t)}$$
$$h^{(t)} = \tanh\left(a^{(t)}\right)$$
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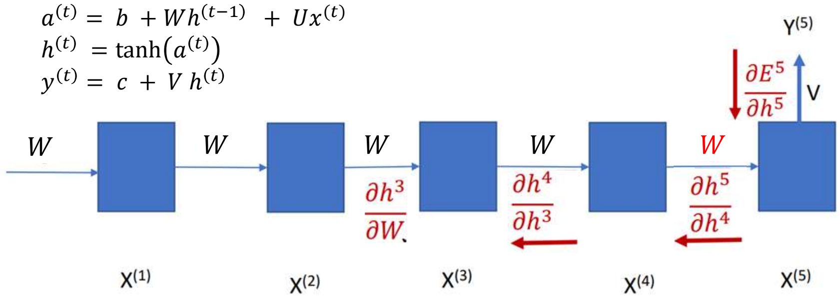

$$\frac{\partial E^5}{\partial W} = \frac{\partial E^5}{\partial y^5}\frac{\partial y^5}{\partial h^5}\frac{\partial h^5}{\partial W} + \frac{\partial E^5}{\partial y^5}\frac{\partial y^5}{\partial h^5}\frac{\partial h^5}{\partial h^4}\frac{\partial h^4}{\partial W} + \frac{\partial E^5}{\partial y^5}\frac{\partial y^5}{\partial h^5}\frac{\partial h^5}{\partial h^4}\frac{\partial h^4}{\partial h^3}\frac{\partial h^3}{\partial W}$$

Backpropagation in Vanilla RNN

for $t = 1, \dots, \tau$:

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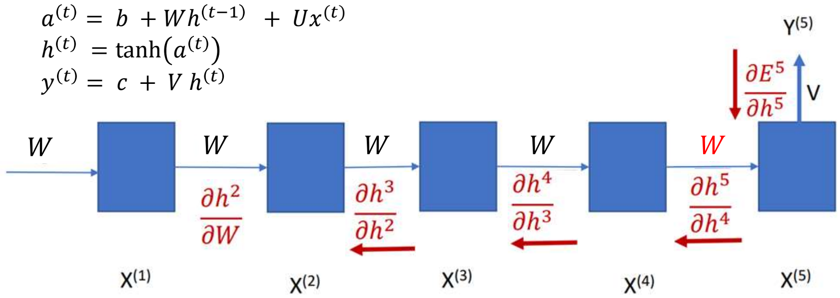

$$\frac{\partial E^5}{\partial W} = \frac{\partial E^5}{\partial y^5}\frac{\partial y^5}{\partial h^5}\frac{\partial h^5}{\partial W} + \frac{\partial E^5}{\partial y^5}\frac{\partial y^5}{\partial h^5}\frac{\partial h^5}{\partial h^4}\frac{\partial h^4}{\partial W} + \frac{\partial E^5}{\partial y^5}\frac{\partial y^5}{\partial h^5}\frac{\partial h^5}{\partial h^4}\frac{\partial h^4}{\partial h^3}\frac{\partial h^3}{\partial W} + \frac{\partial E^5}{\partial y^5}\frac{\partial y^5}{\partial h^5}\frac{\partial h^5}{\partial h^4}\frac{\partial h^4}{\partial h^3}\frac{\partial h^3}{\partial h^2}\frac{\partial h^2}{\partial W}$$

Backpropagation in Vanilla RNN

for $t = 1, \dots, \tau$:

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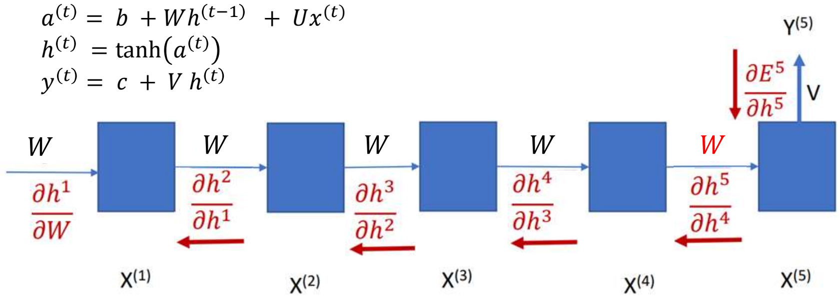

$$\frac{\partial E^5}{\partial W} = \frac{\partial E^5}{\partial y^5}\frac{\partial y^5}{\partial h^5}\frac{\partial h^5}{\partial W} + \frac{\partial E^5}{\partial y^5}\frac{\partial y^5}{\partial h^5}\frac{\partial h^5}{\partial h^4}\frac{\partial h^4}{\partial W} + \frac{\partial E^5}{\partial y^5}\frac{\partial y^5}{\partial h^5}\frac{\partial h^5}{\partial h^4}\frac{\partial h^4}{\partial h^3}\frac{\partial h^3}{\partial W} + \frac{\partial E^5}{\partial y^5}\frac{\partial y^5}{\partial h^5}\frac{\partial h^5}{\partial h^4}\frac{\partial h^4}{\partial h^3}\frac{\partial h^3}{\partial h^2}\frac{\partial h^2}{\partial W} + \frac{\partial E^5}{\partial y^5}\frac{\partial y^5}{\partial h^5}\frac{\partial h^5}{\partial h^4}\frac{\partial h^4}{\partial h^3}\frac{\partial h^3}{\partial h^2}\frac{\partial h^2}{\partial h^1}\frac{\partial h^1}{\partial W}$$

Backpropagation in Vanilla RNN

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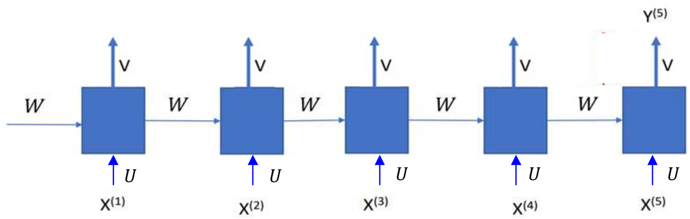

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$$y^{(t)} = c + V\,h^{(t)}$$

$$\frac{\partial E}{\partial W} = \sum_{t=1}^{T} \frac{\partial E_t}{\partial W}$$

Main cause of the vanishing gradient issue

$$\frac{\partial E_t}{\partial W} = \sum_{i=0}^{t-1} \frac{\partial E_t}{\partial y_t} \frac{\partial y_t}{\partial h_t} \boxed{\frac{\partial h_t}{\partial h_i}} \frac{\partial h_i}{\partial W}$$

$$\boxed{\frac{\partial h_t}{\partial h_i} = \frac{\partial h_t}{\partial h_{t-1}} \frac{\partial h_{t-1}}{\partial h_{t-2}} \cdots \frac{\partial h_{i+1}}{\partial h_i} = \prod_{k=i}^{t-1} \frac{\partial h_{k+1}}{\partial h_k}}$$

Backpropagation in LSTM

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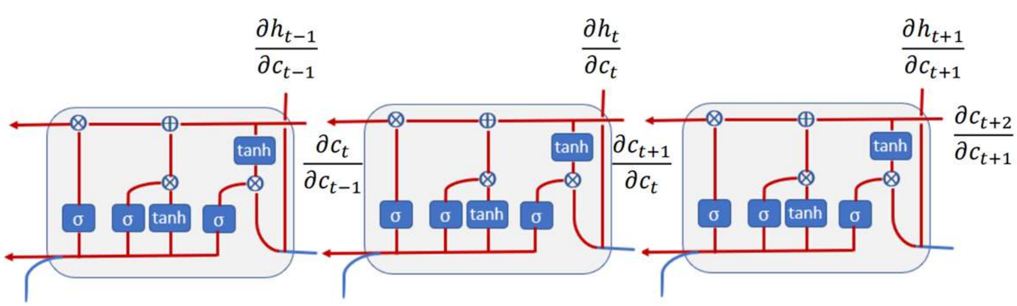

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How does this contribute to handle vanishing gradient issue?

Backpropagation in LSTM

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$$c_t = c_{t-1} \otimes f_t \oplus i_t \otimes \widetilde{c_t}$$

$$\frac{\partial c_t}{\partial c_{t-1}} = \frac{\partial}{\partial c_{t-1}}(c_{t-1} \otimes f_t \oplus i_t \otimes \widetilde{c_t})$$

$$= \frac{\partial}{\partial c_{t-1}}[c_{t-1} \otimes f_t] \oplus [i_t \otimes \widetilde{c_t}]$$

$$= \frac{\partial c_{t-1}}{\partial c_{t-1}} f_t + \frac{\partial f_t}{\partial c_{t-1}} c_{t-1} + \frac{\partial \widetilde{c_t}}{\partial c_{t-1}} i_t + \frac{\partial i_t}{\partial c_{t-1}} \widetilde{c_t}$$

$$= f_t + \frac{\partial f_t}{\partial c_{t-1}} c_{t-1} + \frac{\partial \widetilde{c_t}}{\partial c_{t-1}} i_t + \frac{\partial i_t}{\partial c_{t-1}} \widetilde{c_t}$$

Backpropagation in LSTM

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$$c_t = c_{t-1} \otimes f_t \oplus i_t \otimes \widetilde{c_t}$$

$$\frac{\partial c_t}{\partial c_{t-1}} = \frac{\partial}{\partial c_{t-1}}(c_{t-1} \otimes f_t \oplus i_t \otimes \widetilde{c_t})$$

$$= \frac{\partial}{\partial c_{t-1}}[c_{t-1} \otimes f_t] \oplus [i_t \otimes \widetilde{c_t}]$$

$$= \frac{\partial c_{t-1}}{\partial c_{t-1}} f_t + \frac{\partial f_t}{\partial c_{t-1}} c_{t-1} + \frac{\partial \widetilde{c_t}}{\partial c_{t-1}} i_t + \frac{\partial i_t}{\partial c_{t-1}} \widetilde{c_t}$$

$$= f_t + \frac{\partial f_t}{\partial c_{t-1}} c_{t-1} + \frac{\partial \widetilde{c_t}}{\partial c_{t-1}} i_t + \frac{\partial i_t}{\partial c_{t-1}} \widetilde{c_t}$$

$p \quad q \quad r \quad s$

Backpropagation in LSTM

Plus respective bias

$$p = \quad f_t$$

$$q = \frac{\partial f_t}{\partial c_{t-1}} c_{t-1} = \sigma'(W_f \cdot [h_{t-1}, x_t]) \cdot W_f \cdot o_{t-1} \otimes tanh'(c_{t-1}) \cdot c_{t-1}$$

$$r = \frac{\partial \tilde{c_t}}{\partial c_{t-1}} i_t = tanh'(W_c \cdot [h_{t-1}, x_t]) \cdot W_c \cdot o_{t-1} \otimes tanh'(c_{t-1}) \cdot i_t$$

$$s = \frac{\partial i_t}{\partial c_{t-1}} \tilde{c_t} = \sigma'(W_i \cdot [h_{t-1}, x_t]) \cdot W_i \cdot o_{t-1} \otimes tanh'(c_{t-1}) \cdot \tilde{c}_t$$

Backpropagation in LSTM

$$c_t = c_{t-1} \otimes f_t \oplus i_t \otimes \tilde{c_t}$$

$$\frac{\partial c_t}{\partial c_{t-1}} = \frac{\partial}{\partial c_{t-1}}(c_{t-1} \otimes f_t \oplus i_t \otimes \tilde{c_t})$$

$$= \frac{\partial}{\partial c_{t-1}}[c_{t-1} \otimes f_t] \oplus [i_t \otimes \tilde{c_t}]$$

$$= \frac{\partial c_{t-1}}{\partial c_{t-1}} f_t + \frac{\partial f_t}{\partial c_{t-1}} c_{t-1} + \frac{\partial \tilde{c_t}}{\partial c_{t-1}} i_t + \frac{\partial i_t}{\partial c_{t-1}} \tilde{c_t}$$

$$= f_t + \frac{\partial f_t}{\partial c_{t-1}} c_{t-1} + \frac{\partial \tilde{c_t}}{\partial c_{t-1}} i_t + \frac{\partial i_t}{\partial c_{t-1}} \tilde{c_t}$$

$p \quad q \quad r \quad s$

$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$$

$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$

$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

$$o_t = \sigma(W_o [h_{t-1}, x_t] + b_o)$$

$$h_t = tanh(c_t) \otimes o_t$$

$$\frac{\partial E_t}{\partial W} = \frac{\partial E}{\partial h_t} \frac{\partial h_t}{\partial c_t} \left(\prod_{t=2}^{T} (p + q + r + s) \right) \frac{\partial c_1}{\partial W}$$

Additive update!
Helps in handling vanishing gradient issue

LSTM Training

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- Trainable with backprop derivatives such as:
  - Stochastic gradient descent with momentum
  - ADAM optimizer

- Each cell has many parameters (W_f, W_i, W_c, W_o)
  - Generally **requires lots of training data.**
  - Requires **lots of compute time that exploits GPU clusters**

Sequence to Sequence Transduction (Mapping)

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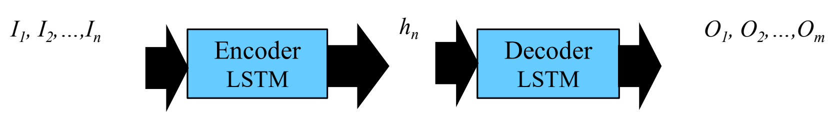
- Encoder/Decoder framework maps one sequence to a "deep vector" then another LSTM maps this vector to an output sequence.



- Train model "end to end" on I/O pairs of sequences.

Bi-directional LSTM (Bi-LSTM)

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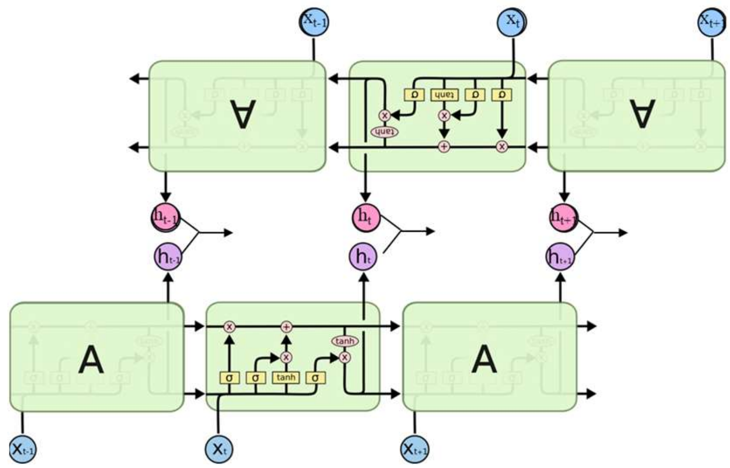

General Problems Solved with LSTMs

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- **Sequence labeling**
  - Train with supervised output at each time step computed using a single or multilayer network that maps the hidden state (h_t) to an output vector (O_t).
- **Language modeling**
  - Train to predict next input ($O_t = I_{t+1}$)
- **Sequence (e.g. text) classification**
  - Train a single or multilayer network that maps the final hidden state (h_n) to an output vector (O).

Successful Applications of LSTMs

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- **Speech recognition:** Language and acoustic modeling
- **Sequence labeling**
  - POS Tagging
  - NER
  - Phrase Chunking
- **Neural syntactic and semantic parsing**
- **Image captioning:** CNN output vector to sequence
- **Sequence to Sequence**
  - Machine Translation
  - Video Captioning (input sequence of CNN frame outputs)

Gated Recurrent Unit (GRU)

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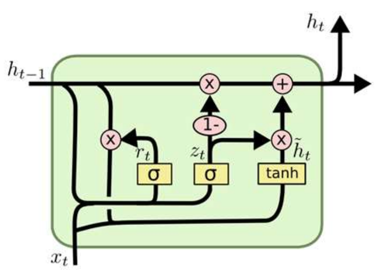
- Alternative RNN to LSTM that uses fewer gates
  - Update gate
  - Reset gate
  - Eliminates cell state vector



$$z_t = \sigma\left(W_z \cdot [h_{t-1}, x_t]\right)$$
$$r_t = \sigma\left(W_r \cdot [h_{t-1}, x_t]\right)$$
$$\tilde{h}_t = \tanh\left(W \cdot [r_t * h_{t-1}, x_t]\right)$$
$$h_t = (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t$$

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GRU vs. LSTM

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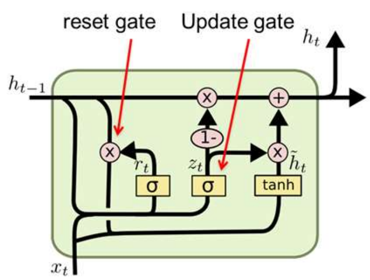
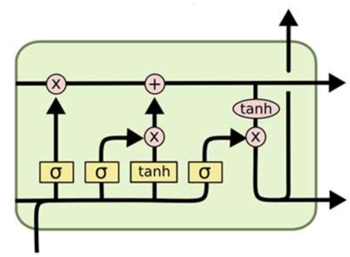
- GRU has significantly fewer parameters and trains faster.
- Experimental results comparing the two are still inconclusive, many problems they perform the same, but each has problems on which they work better.





Conclusions

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- By adding “gates” to an RNN, we can prevent the vanishing/exploding gradient problem.
- Trained LSTMs/GRUs can retain state information longer and handle long-distance dependencies.
- Recent impressive results on a range of challenging NLP problems.

RNN Encoder-Decoder with **Attention Mechanism**

Attention

- For many applications, it helps to add “attention” to RNNs.
- Allows network to learn to attend to different parts of the input at different time steps, shifting its attention to focus on different aspects during its processing.
- Used in image captioning to focus on different parts of an image when generating different parts of the output sentence.
- In Machine Translation, allows focusing attention on different parts of the source sentence when generating different parts of the translation.

Attention in Language Processing

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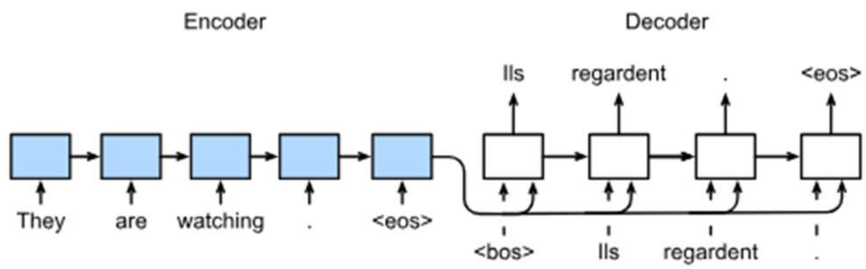
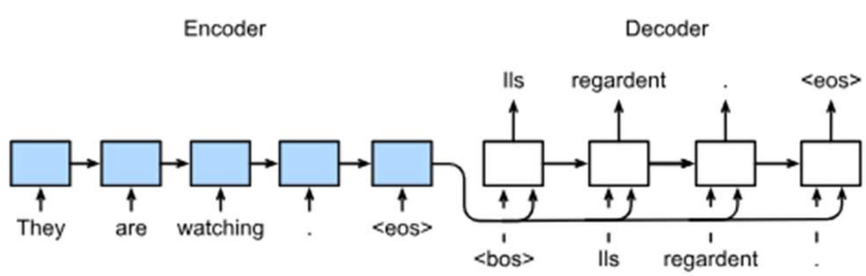
How are you doing? → Aap kaise Hain?



Attention in Language Processing

40

How are you doing? → Aap kaise hain?



Attention-based Model

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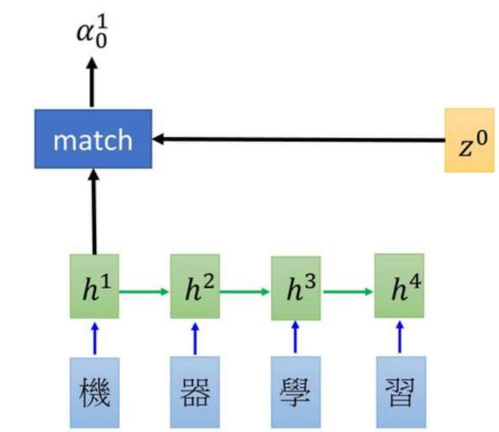
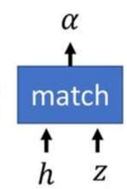


Jointly learned with other part of the network



What is match ?

Design by yourself

- Cosine similarity of z and h
- Small NN whose input is z and h, output a scalar
- $\alpha = h^T W z$

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Attention-based Model

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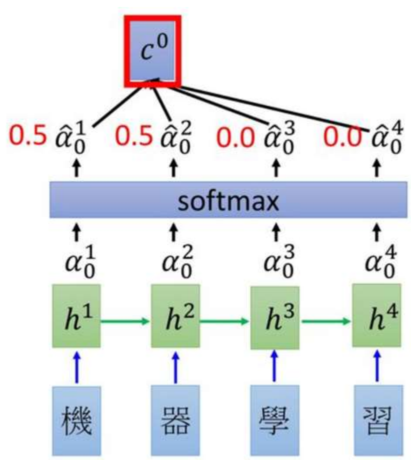

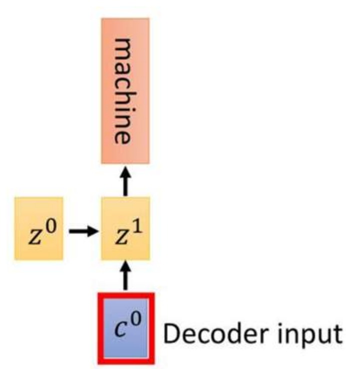

$$c^0 = \sum \hat{\alpha}_0^i h^i$$

$$= 0.5h^1 + 0.5h^2$$

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Attention-based Model

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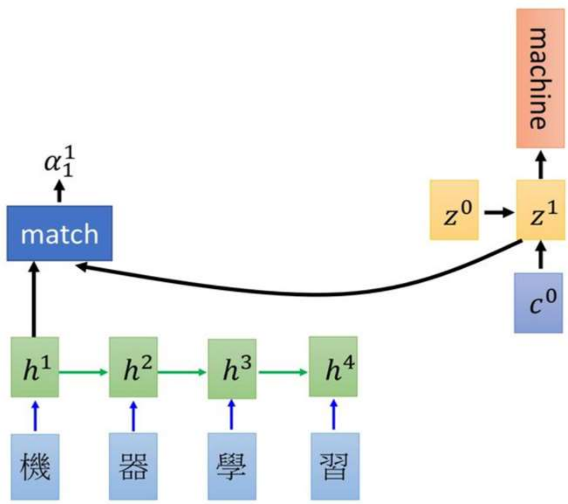

Attention-based Model

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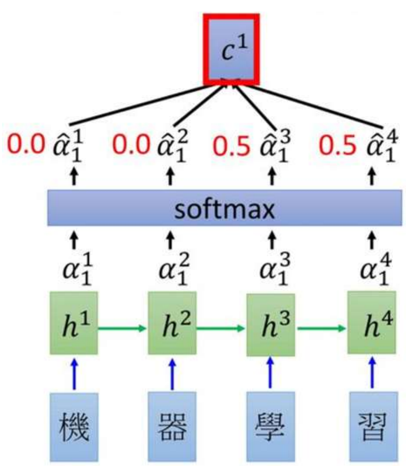

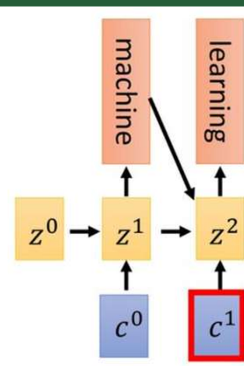

$$c^1 = \sum \hat{\alpha}_1^i h^i$$
$$= 0.5h^3 + 0.5h^4$$

Attention-based Model

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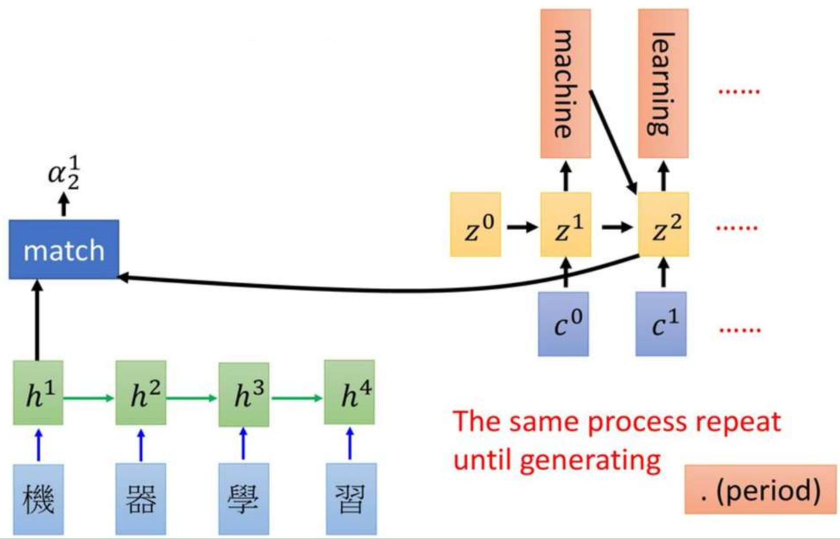

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Attention in Image Caption Generation

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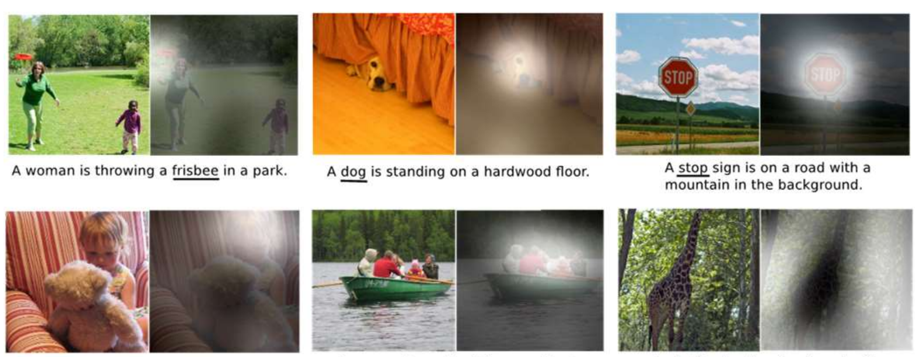

A woman is throwing a frisbee in a park.

A dog is standing on a hardwood floor.

A stop sign is on a road with a mountain in the background.

A little girl sitting on a bed with a teddy bear.

A group of people sitting on a boat in the water.

A giraffe standing in a forest with trees in the background.

Kelvin Xu, Jimmy Ba, Ryan Kiros, Kyunghyun Cho, Aaron Courville, Ruslan Salakhutdinov, Richard Zemel, Yoshua Bengio, "Show, Attend and Tell: Neural Image Caption Generation with Visual Attention", ICML, 2015

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Image caption generation using attention

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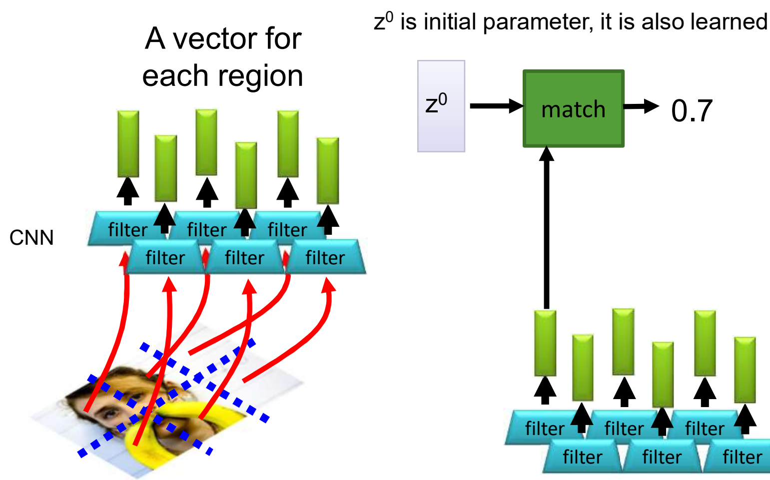

Image Caption Generation

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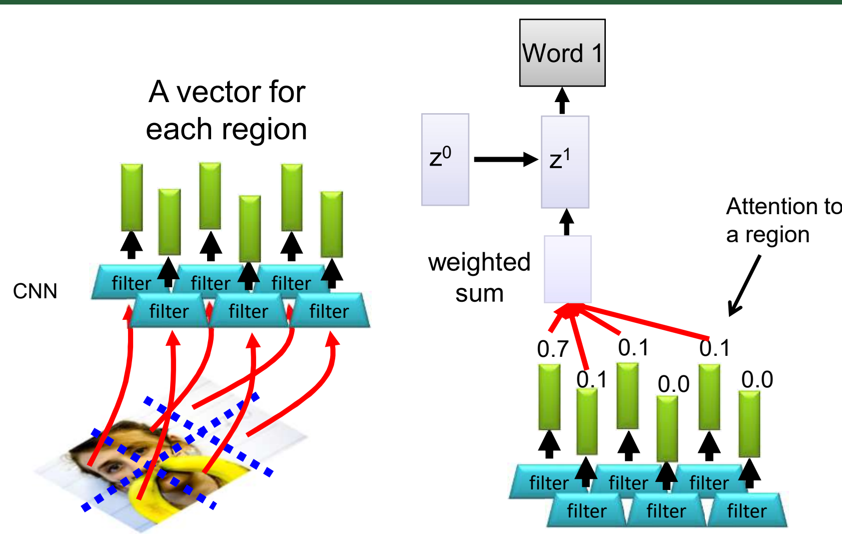

Image Caption Generation

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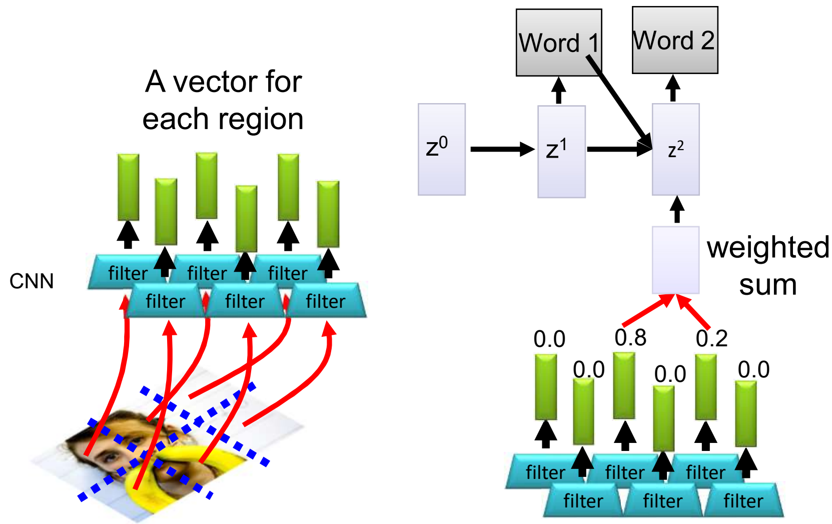

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Questions?