Multiresolution Analysis

Multiresolution Analysis

Many signals or images contain information at different scales or levels of detail (e.g., people vs buildings).

Analyzing information at the same scale will not be effective.

Use windows of different size (i.e., varying scale)

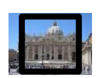
Multiresolution Analysis

Alternatively, use the same window size but analyze information at different resolutions:

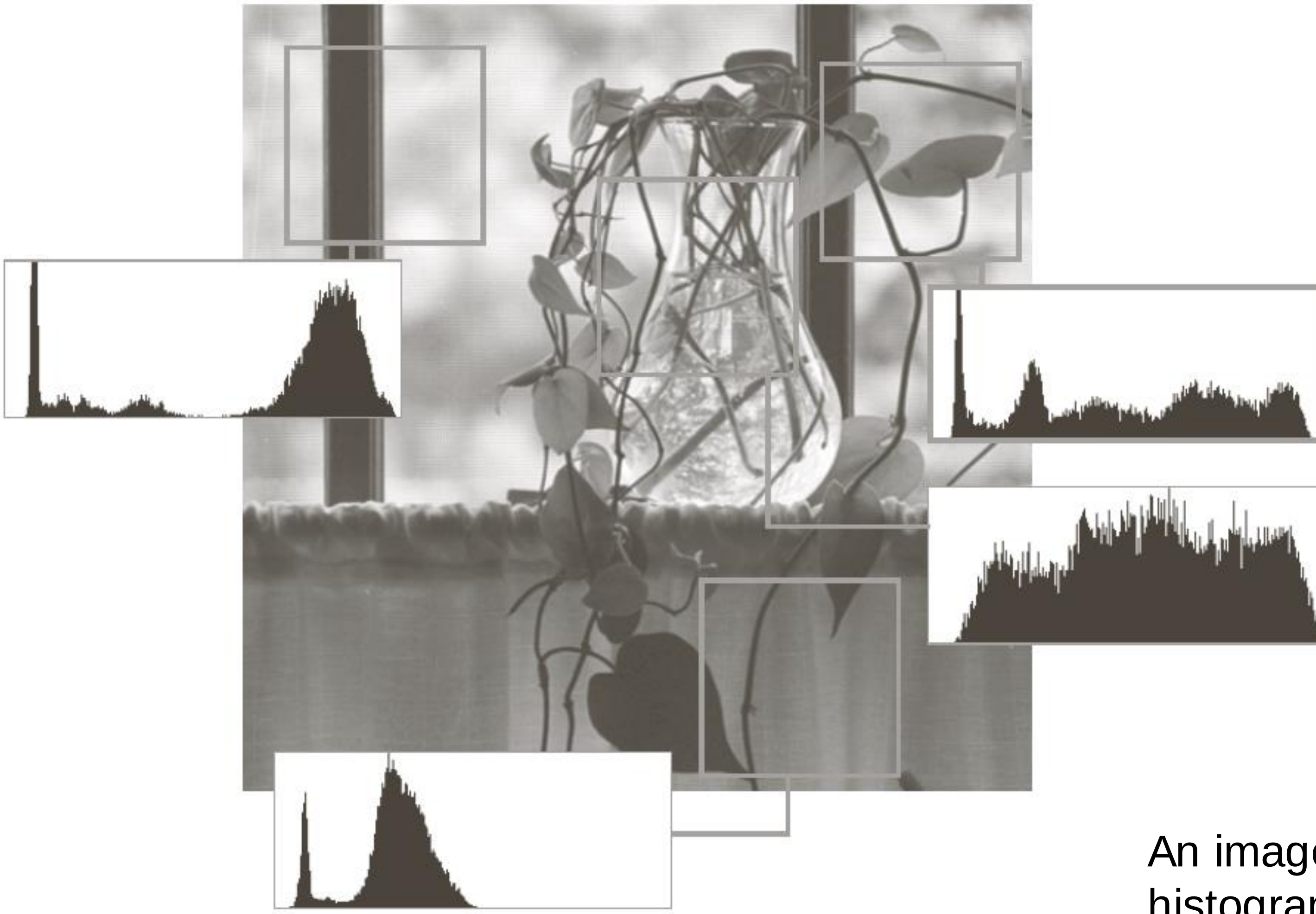
High resolution

Small size objects should be examined at a <u>high</u> resolution.

Low resolution



Multiresolution Analysis



An image and its local histogram variations

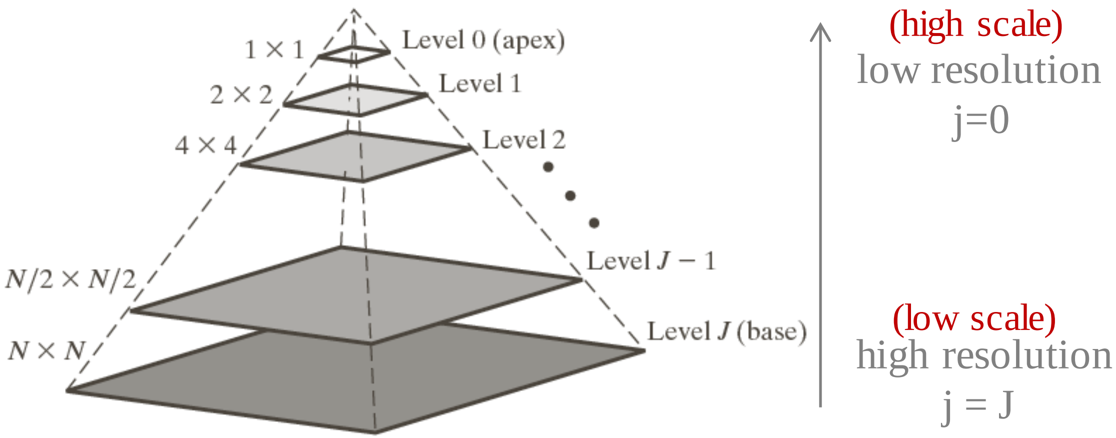
Multiresolution Analysis

Techniques for representing multiresolution information efficiently:

Pyramidal Coding

Subband Coding

Image Pyramid



A collection of decreasing resolution images arranged in the shape of a pyramid.

$J + 1$ levels where $J = logN$.

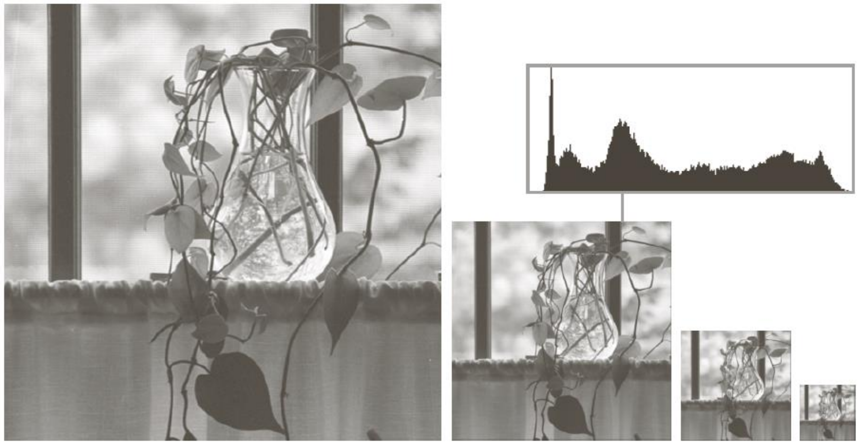
Usually, there are $P + 1$ levels where $P \leq J$.

Each level corresponds to a $2^j \times 2^j$ image, $0 \leq j \leq J$.

Base level: $2^J \times 2^J = 2^{logN} \times 2^{logN} = N \times N$.

<u>Example</u>: if N=256, there will be 8+1=9 levels.

Image Pyramid



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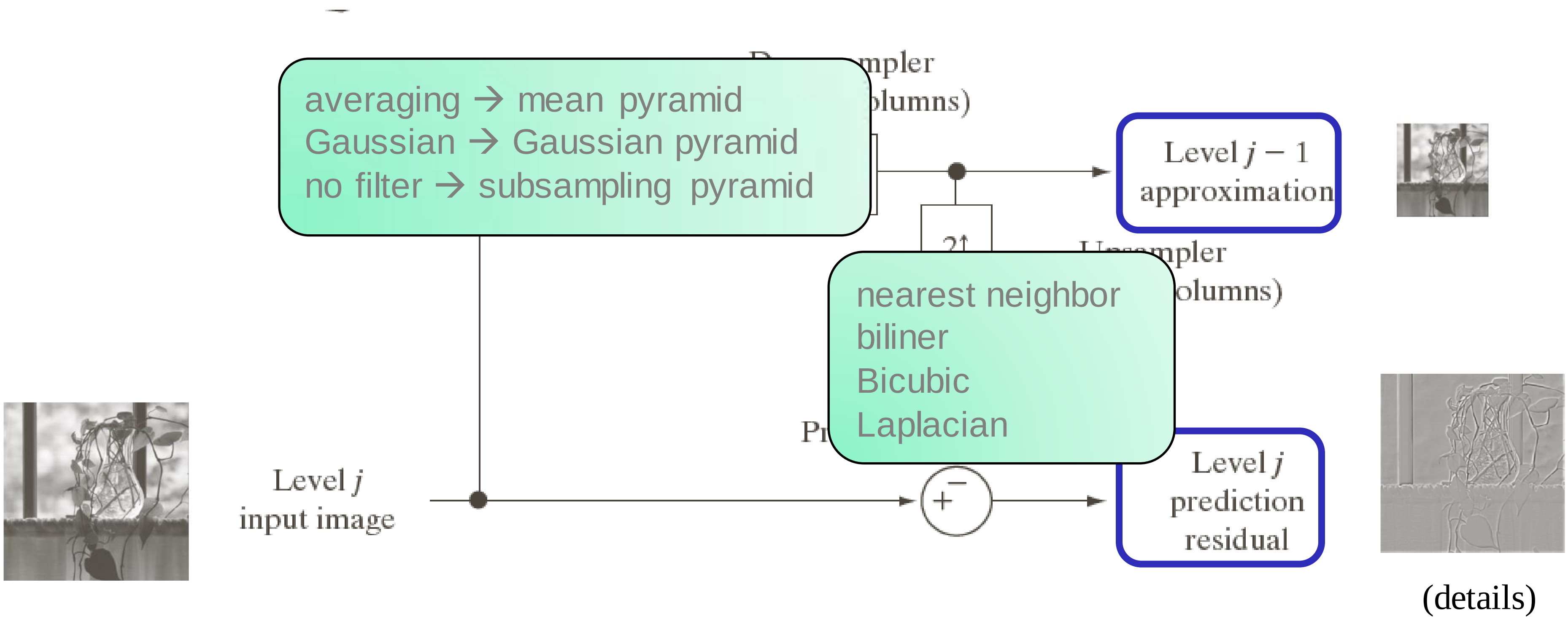
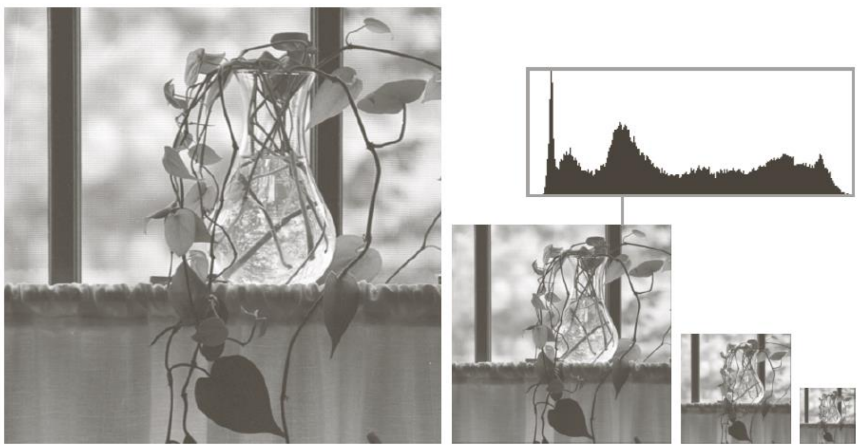
Pyramidal Coding

Two pyramids: approximation and prediction residual

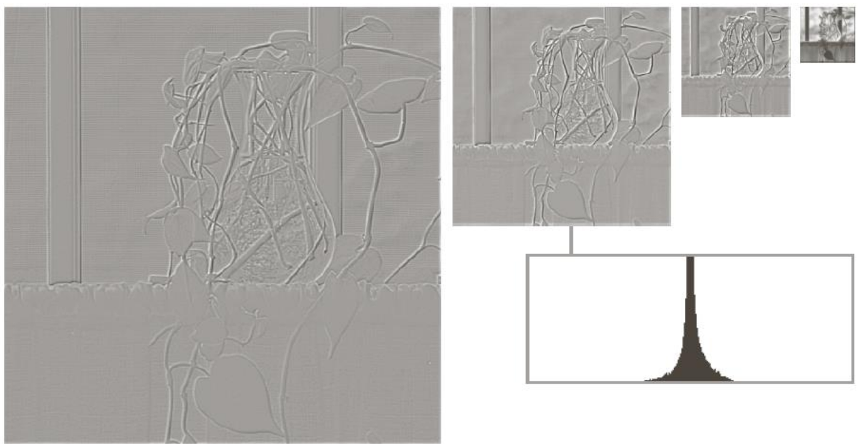


Pyramidal Coding

Approximation pyramid
(based on Gaussian filter)



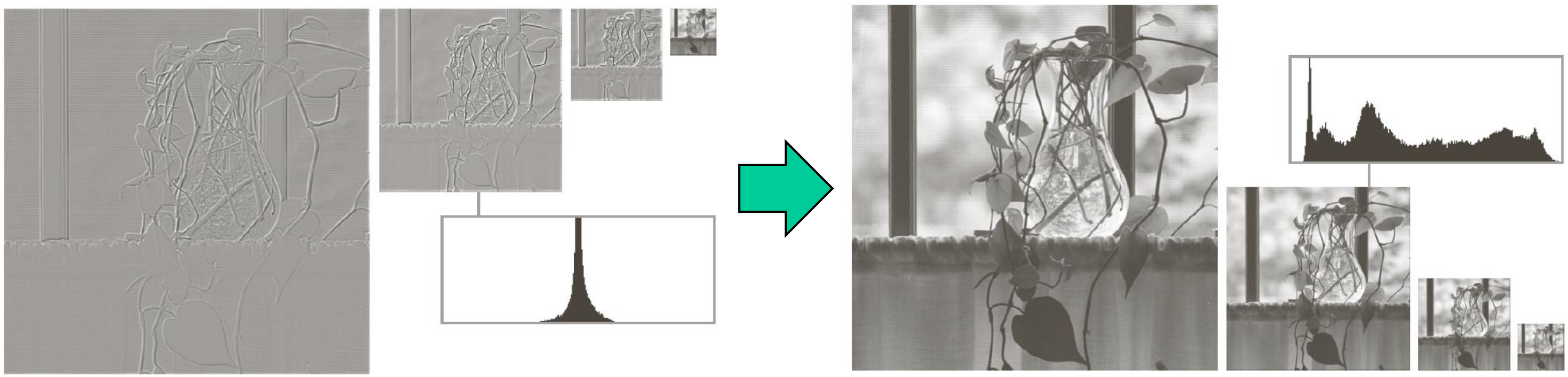
Prediction residual pyramid
(based on Laplacian filter)



<u>Note</u>: The last level is the same
as that of the approximation Pyramid.

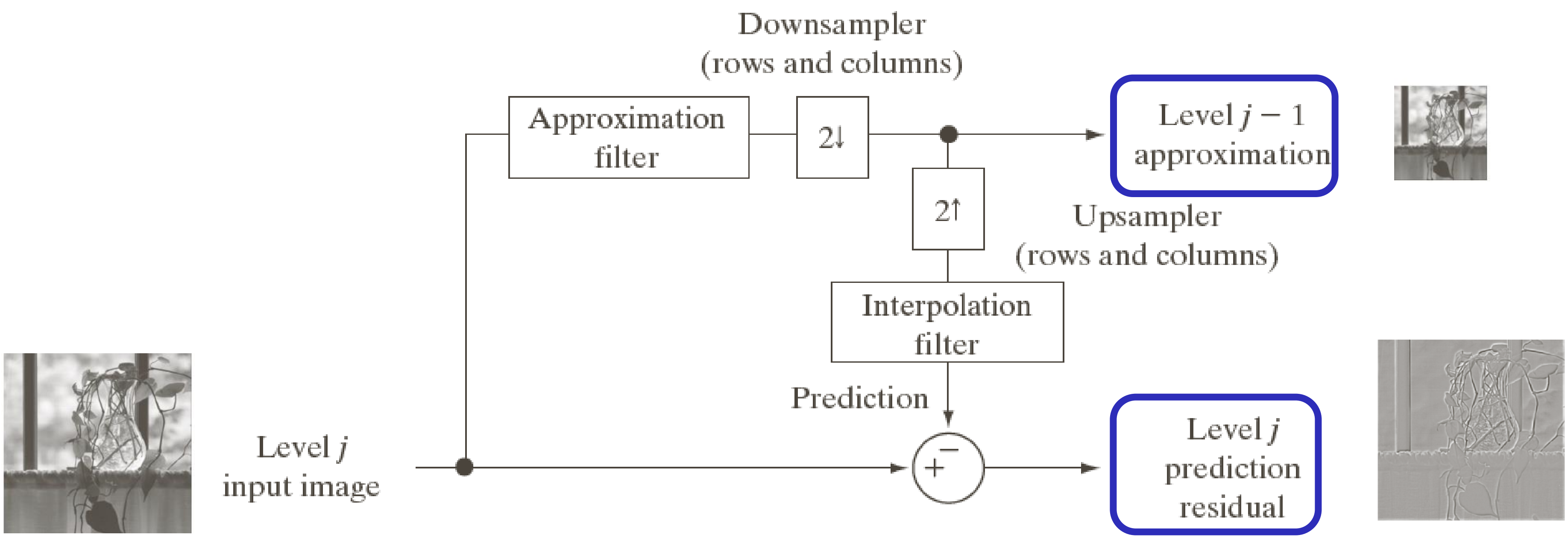
Pyramidal Coding

In the absence of quantization errors, the approximation pyramid can be re-constructed from the prediction residual pyramid.



Pyramidal Coding

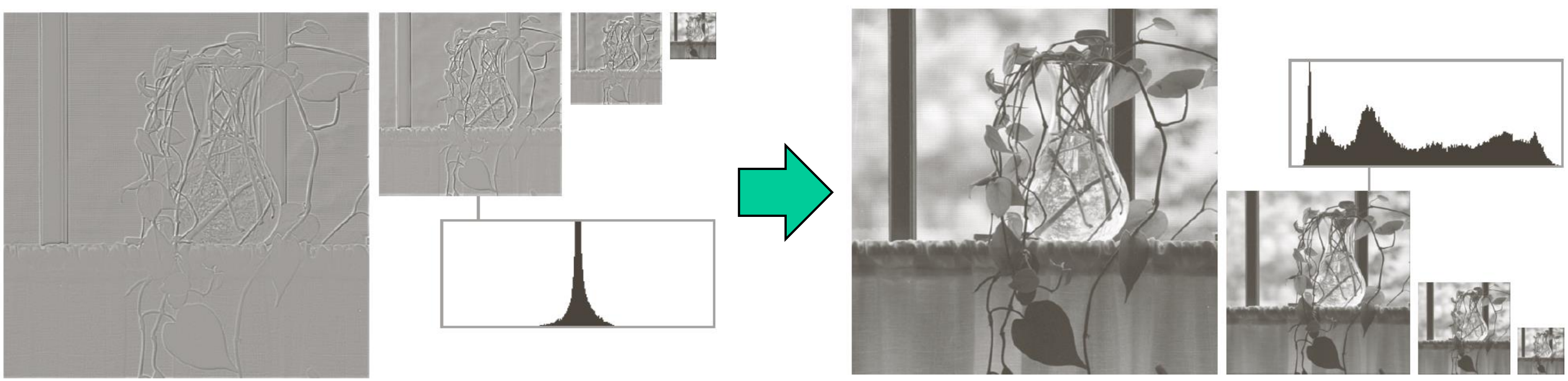
Just add the "details" or "residual error" back!



(details)

Pyramidal Coding

In the absence of quantization errors, the approximation pyramid can be re-constructed from the prediction residual pyramid.



- We need to keep the prediction residual pyramid only!
  - More efficient representation

Subband Coding

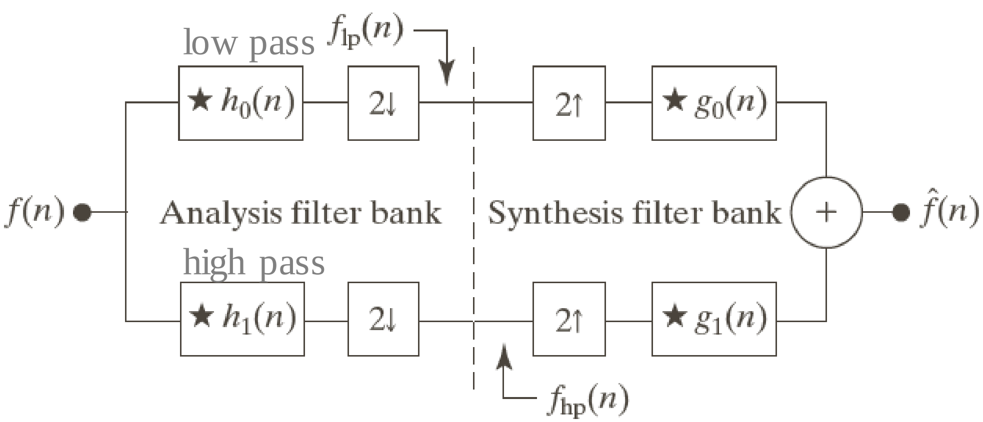
Decompose an image (or signal) into different frequency bands (analysis step).

Decomposed subbands can be re-assembled to reconstruct the original image without error (synthesis step).

Need to choose appropriate filters (i.e., “filter bank”).

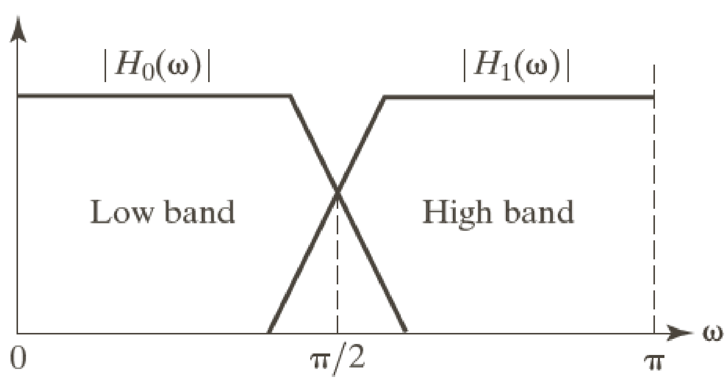
Subband Coding – 1D Example

2-band decomposition



$f_{lp}(n)$: approximation of f(n)

$f_{hp}(n)$: detail of f(n)



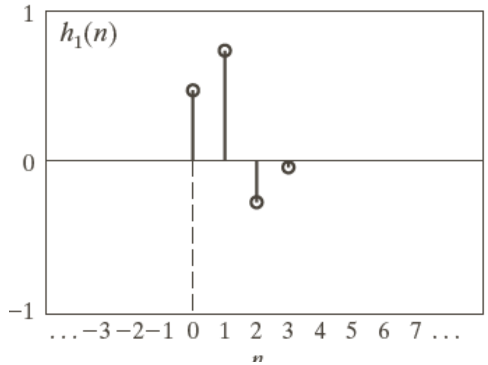
Its spectrum splitting properties

Subband Coding

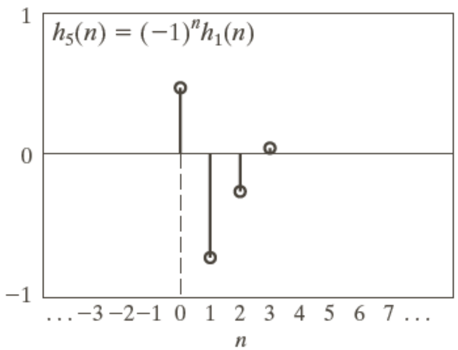
- It can be shown that for perfect reconstruction, the synthesis filters ($g_0(n)$ and $g_1(n)$) must be modulated versions of the analysis filters ($h_0(n)$, $h_1(n)$):

$$g_0(n) = (-1)^n h_1(n) \qquad\qquad g_0(n) = (-1)^{n+1} h_1(n)$$
$$g_1(n) = (-1)^{n+1} h_0(n) \quad \text{or} \quad g_1(n) = (-1)^n h_0(n)$$

original



modulated



2-D Separable WT

For images we use separable WT.

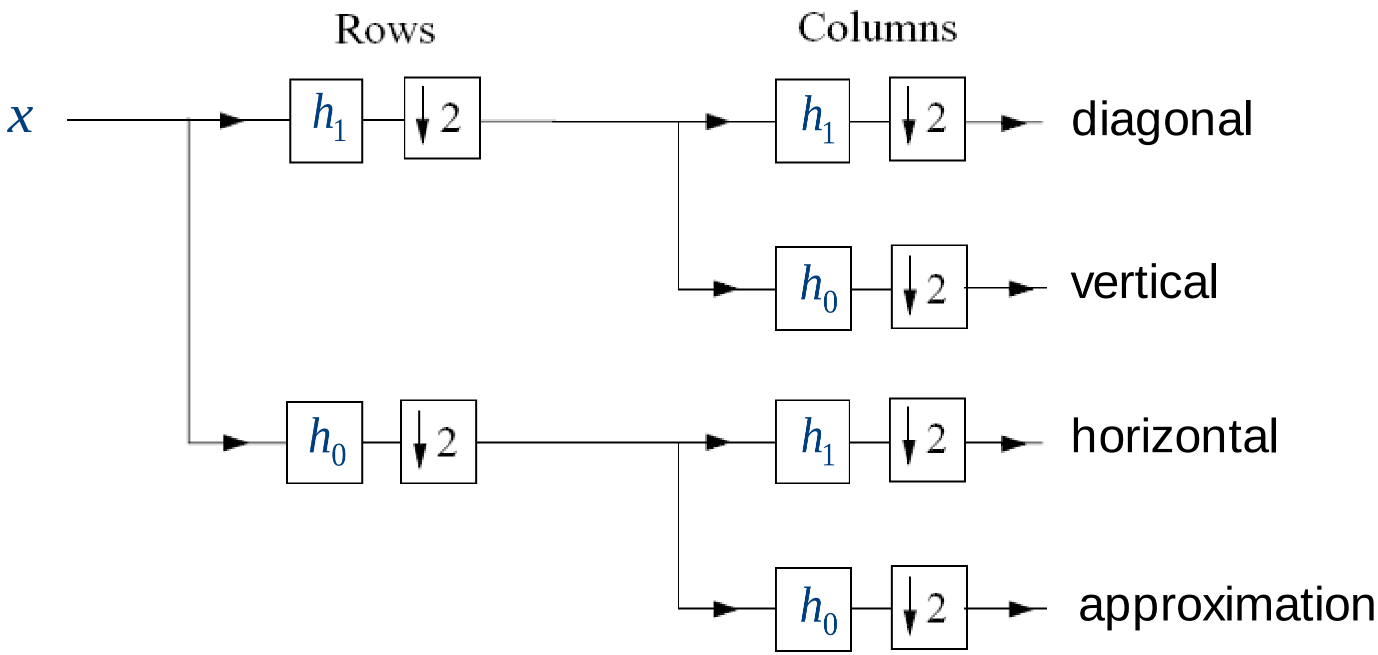
First we apply a 1-D filter bank to the rows of the image.

Then we apply same transformation to the columns of each channel of the result.

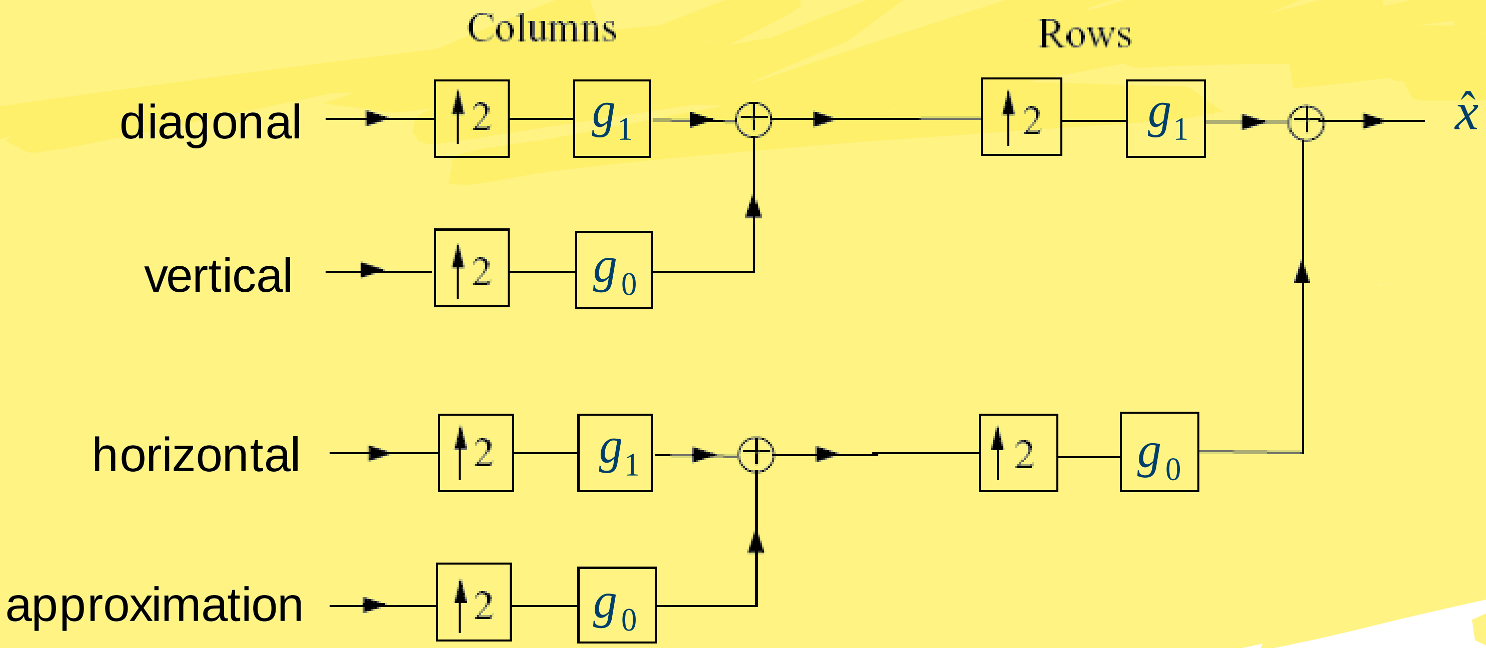
Therefore, we obtain 3 highpass channels corresponding to vertical, horizontal, and diagonal, and one approximation image.

We can iterate the above procedure on the lowpass channel.

2-D Analysis Filter Bank

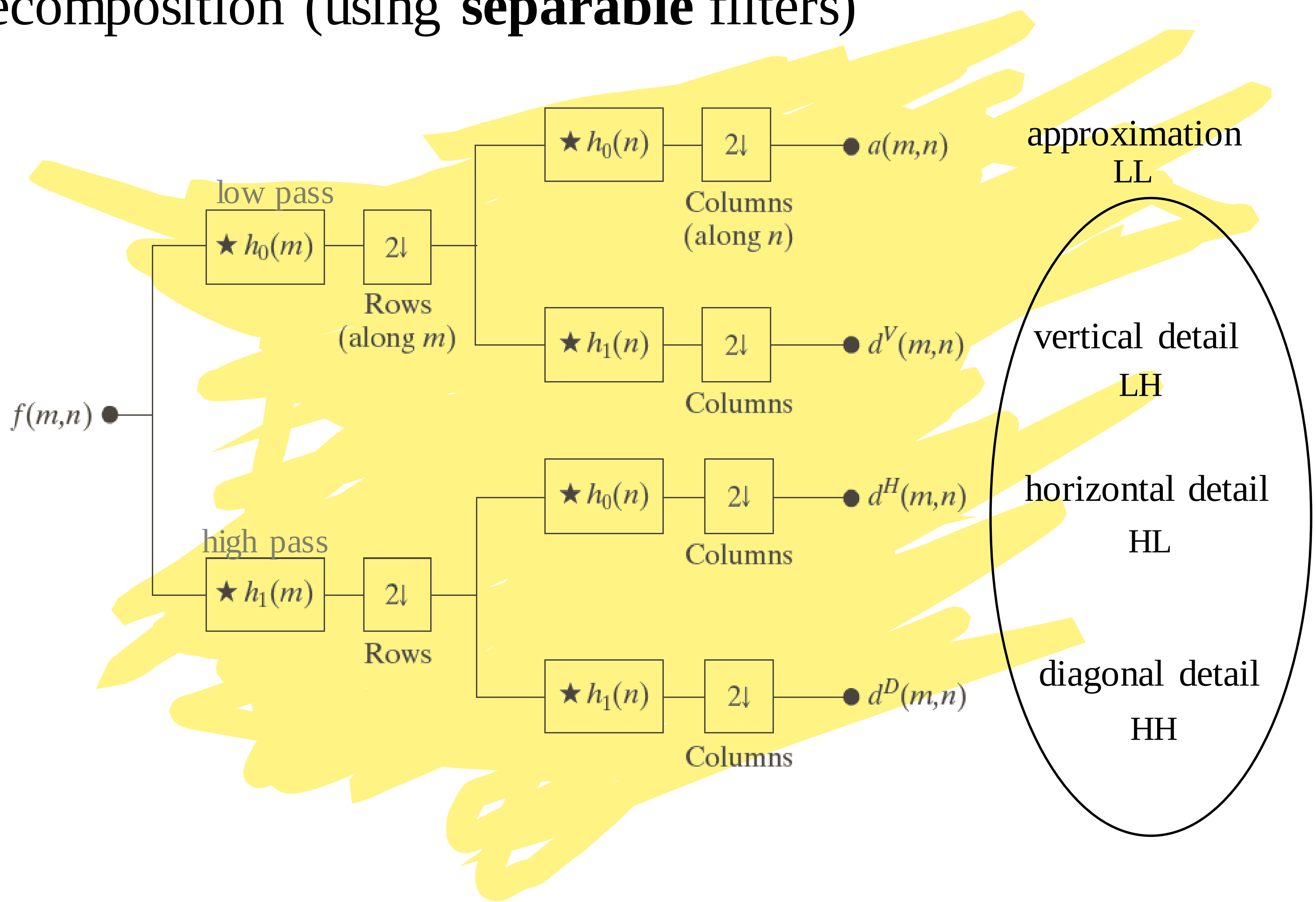


2-D Synthesis Filter Bank



Subband Coding – 2D Example

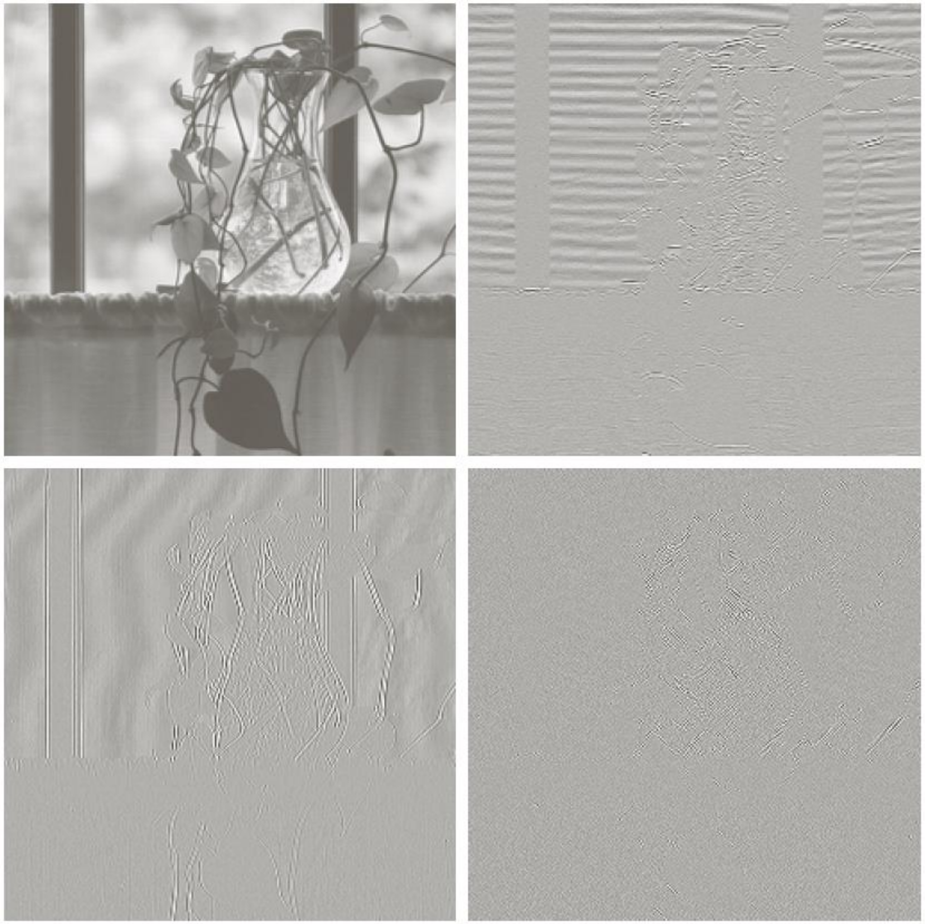
4-band decomposition (using **separable** filters)



Subband Coding

approximation

horizontal detail



vertical detail

diagonal detail

Summary

In this lecture we have begun looking at multiresolution analysis.

It is widely used to handle images containing information at different scales or levels of detail.