CMSC 828T
Vision, Planning and Control in Aerial Robotics
Homework 0: Wright Brothers!
Due on: 11:59:59PM on September 5^{th}, 2017

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First of all, welcome to CMSC 828T! The course website is up and running and can be found here: https://cmsc828t.cs.umd.edu/. The course announcements will be made through Piazza and the link can be found here: https://piazza.com/umd/fall2017/cmsc828t. Please add yourself to piazza to discuss among your peers and ask questions. Please do not email us regarding the class (use piazza private posts if necessary).

Any questions you might have regarding this homework should be asked via piazza. It is highly recommended to discuss your questions between other students and you try to help other students. However, please DO NOT give answers via piazza or any other means.

As you all know, officially the *Wright Brothers* were the first to flight. They provided the foundation for flight that is common today. This homework will make sure that you come equipped with the right fundamental skillset, both in terms of theory and coding expertise to cruise through this course comfortably.

This homework will prepare you for basic math, specifically linear algebra that is the foundation to this course.

1 Refresh your Math

1. Can three vectors in the xy plane have $u \cdot v < 0$, $v \cdot w < 0$ and $u \cdot w < 0$?
2. Let $c \in R$. Suppose that A is an $n \times n$ matrix and that the sum of the entries in each column of A is c. Prove that c is an eigenvalue of A.
   Hint: Consider the sum of the row vectors of the matrix $A - cI$.
3. If $A(t)$ is a continuously-differentiable $n \times n$ matrix function that is invertible at each

t, show that

$$\frac{d}{dt}A^{-1}(t) = -A^{-1}(t)\ \dot{A}(t)\ A^{-1}(t)$$

4. If λ is an eigenvalue of A and X is the corresponding eigenvector, then prove that $\lambda - s$ is an eigenvalue of $A - sI$ for any scalar s and X is the corresponding eigenvector.

5. For any two $n \times n$ matrices, say A and B:

   (a) are real-symmetric matrices, both AB and BA always have the same eigenvalues. True or False?

   (b) matrix B is invertible, AB and BA always have the same eigenvalues. True or False?

   (c) matrix B is invertible, AB and BA always have the same eigenvectors. True or False?

   Give support for all your answers.

6. If rows of an $m \times n$ matrix A are linearly independent,

   (a) Is $Ax = b$ necessarily solvable?

   (b) If $Ax = b$ is solvable, is the solution necessarily unique?

7. Show that if A is a non-singular matrix, and λ is an eigenvalue of A, then $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}.

8. Create an 8×8 matrix H using the command `hilb(8)` in MATLAB. Generate a random vector x, and compute $Hx = b$. Add a tiny amount of noise to b. Then recover x from b by running the command $\hat{x} = H^{-1}b$. How accurate is the recovered x? Why did this happen? *You don't need to provide any code or console output, just describe what you did and what you got in a few sentences.*

9. Suppose we want to recover the solution to the system $Ax = b$. We don't know b exactly, but we have a noisy measurement vector $\hat{b}$. To do this, we could compute $\hat{x} = A^{-1}b$. Prove that

$$\frac{\|x - \hat{x}\|}{\|x\|} \leq \kappa \frac{\|b - \hat{b}\|}{\|b\|}$$

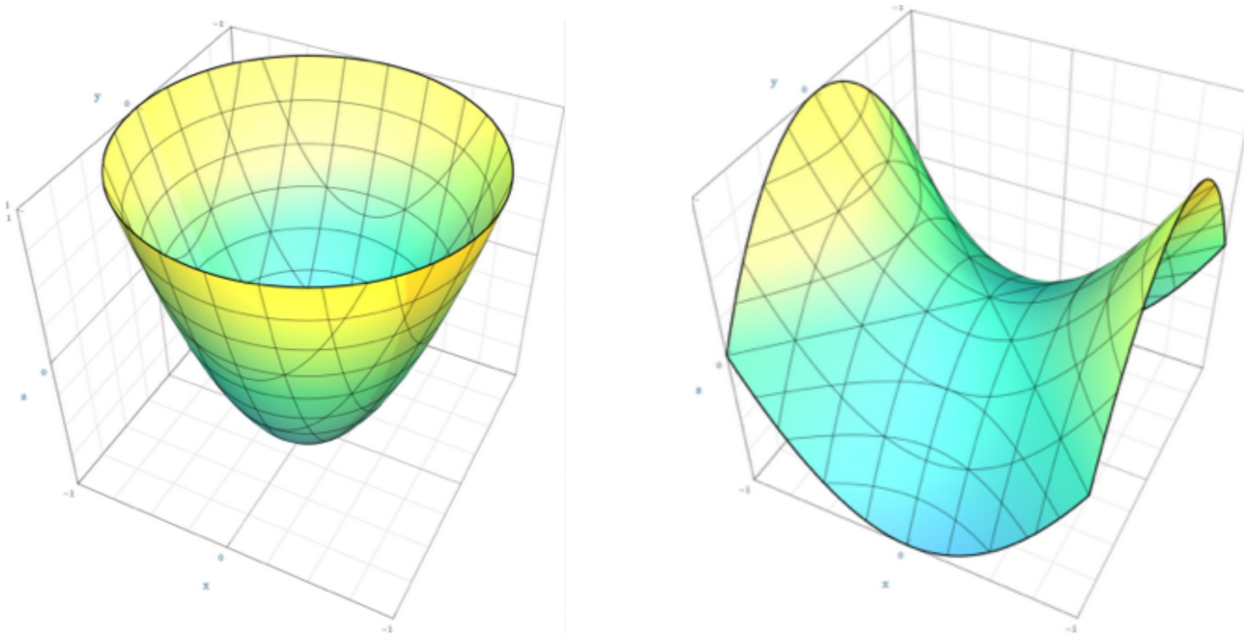
   Here κ is the condition number of A.
   Hint: Definition of condition number.

10. Consider the measurement model

$$y = Dx + \eta$$

   where $D \in \mathbb{R}^{m \times n}$ is a measurement matrix, and $\eta \in \mathbb{R}^{n \times 1}$ is a noise vector with distribution

$$\eta \sim \frac{1}{\sqrt{(2\pi)^m |\Sigma|}} e^{-\frac{1}{2}\eta^T \Sigma^{-1} \eta}$$



(a) (b)

Figure 1

for some covariance matrix Σ, where $|\Sigma|$ denotes the determinant of Σ. Suppose we have prior knowledge that each entry in x is draw from an I.I.D. Laplace distribution

$$x_i \sim \frac{1}{2b} e^{-\frac{|x|}{b}}$$

Derive the Negative Log-Likelihood (NLL) function for x given y. Write the complete NLL without throwing away any constants (although you may use Bayes rule, which implicitly throws away a normalization constant).

11. Prove that the shortest path between two points on a the surface of the sphere is the straight line (curve) on the sphere surface (in spherical co-ordinates), using the Euler-Lagrange equation.

12. Both the plots in Figs. 1(a) and 1(b) are derived from $f(x) = \frac{1}{2}x^T Hx + g^T x + c$

    (a) What are the constraints on H for (a) and for (b)?

    (b) Find the optimal value in (a) and (b), if they exist.

13. Suppose the random column vectors X, Y live in $\mathbb{R}^n$ and $\mathbb{R}^m$ respectively, and the vector (X, Y) in $\mathbb{R}^{n+m}$ has a multivariate normal distribution whose covariance is the symmetric positive-definite matrix

$$\Sigma = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$$

Here $A \in \mathbb{R}^{n \times n}$ is the covariance matrix of X, $C \in \mathbb{R}^{m \times m}$ is the covariance matrix of Y and $B \in \mathbb{R}^{n \times m}$ is the covariance matrix between X and Y. Prove that the conditional covariance of X given Y is the Schur complement of C in Σ. *Hint: Definitions of Schur complement, conditional covariance.*

2 Submission Guidelines

Submit your assignment named according the format `YourDirectoryName_hw0.zip` onto ELMS/Canvas (**Please compress it as .zip compressed file format. Other formats will not be acceptable.**) Your `DirectoryName` is the username to your UMD e-mail ID. If your email ID is `johnsmith@terpmail.umd.edu`, your `DirectoryName` is `johnsmith`. Your zip file should contain PDF document typeset in LaTeX. If your submission does not comply with the guidelines, you'll be given **ZERO** credit.

3 Collaboration Policy

You can discuss with any number of people. But the solution you turn in MUST be your own. Plagiarism is strictly prohibited. Plagiarism checker will be used to check your submission. Please make sure to **cite** any references from papers, websites, or any other student's work you might have referred.