Optical Communication
ECL 402
4 Credit Course

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Contents of Lecture 2 to 5

Transmission Characteristics of Optical Fibers

- Fiber attenuation
- Fiber dispersion
- Group velocity
- Material dispersion
- Waveguide dispersion
- Chromatic dispersion compensation
- Polarization mode dispersion
- Polarization-maintaining fibers

Reading: Senior 3.1-3.4, 3.6, 3.8-3.13
Keiser 3.1 - 3.3

Transmission characteristics of optical fibers

- The transmission characteristics of most interest: **attenuation (*loss*)** and **bandwidth**.

- Now, *silica-based* glass fibers have losses about 0.2 dB/km (i.e. 95% launched power remains after 1 km of fiber transmission). This is essentially the *fundamental lower limit* for attenuation in silica-based glass fibers.

- **Fiber bandwidth** is limited by the *signal dispersion* within the fiber. Bandwidth determines the number of bits of information transmitted in a given time period. Now, fiber bandwidth has reached many 10's Gbit/s over many km's per wavelength channel.

Attenuation

- Signal attenuation within optical fibers is usually expressed in the logarithmic unit of the decibel.

The decibel, which is used for comparing two *power* levels, may be defined for a particular optical wavelength as the *ratio* of the output optical power P_o from the fiber to the input optical power P_i.

$$\text{Loss (dB)} = -10 \log_{10} (P_o/P_i) = 10 \log_{10} (P_i/P_o)$$

$$P_o \leq P_i$$

*In *electronics*, $\text{dB} = 20 \log_{10} (V_o/V_i)$

*The logarithmic unit has the advantage that the operations of *multiplication* (*and division*) reduce to *addition* (*and subtraction*).

In numerical values: $P_o/P_i = 10^{[-Loss(dB)/10]}$

The attenuation is usually expressed in decibels per unit length (i.e. dB/km):

$$\gamma L = -10 \log_{10} (P_o/P_i)$$

γ (dB/km): signal attenuation per unit length in decibels

L (km): fiber length

dBm

- dBm is a specific unit of power in decibels when the reference power is 1 mW:

$$\text{dBm} = 10 \log_{10} (\text{Power}/1\ \text{mW})$$

e.g. 1 mW = 0 dBm; 10 mW = 10 dBm; 100 mW = - 10 dBm

=> Loss (dB) = input power (dBm) - output power (dBm)

e.g. Input power = 1 mW (0 dBm), output power = 100 mW (-10 dBm)

$$\text{loss} = -10 \log_{10} (100\ \text{mW}/1\ \text{mW}) = 10\ \text{dB}$$

OR 0 dBm - (-10 dBm) = 10 dB

The dBm Unit

Example 3.2 As Sec. 1.3 describes, optical powers are commonly expressed in units of *dBm*, which is the decibel power level referred to 1 mW. Consider a 30-km long optical fiber that has an attenuation of 0.4 dB/km at 1310 nm. Suppose we want to find the optical output power P_{out} if 200 μW of optical power is launched into the fiber. We first express the input power in dBm units:

$$P_{in}(\text{dBm}) = 10\log\left[\frac{P_{in}(\text{W})}{1\,\text{mW}}\right]$$
$$= 10\log\left[\frac{200\times10^{-6}\,\text{W}}{1\times10^{-3}\,\text{W}}\right] = -7.0\ \text{dBm}$$

From Eq. (3.1*c*) with $P(0) = P_{in}$ and $P(z) = P_{out}$ the output power level (in dBm) at $z = 30$ km is

$$P_{out}(\text{dBm}) = 10\log\left[\frac{P_{out}(\text{W})}{1\,\text{mW}}\right]$$
$$= 10\log\left[\frac{P_{in}(\text{W})}{1\,\text{mW}}\right] - \alpha z$$
$$= -7.0\ \text{dBm} - (0.4\ \text{dB/km})\,(30\ \text{km})$$
$$= -19.0\ \text{dBm}$$

In unit of watts, the output power is

$$P(30\ \text{km}) = 10^{-19.0/10}(1\ \text{mW}) = 12.6\times10^{-3}\ \text{mW}$$
$$= 12.6\ \mu\text{W}$$

e.g. When the mean optical power launched into an 8 km length of fiber is 120 mW, the mean optical power at the output is 3 mW.

Determine:

(a) the overall signal attenuation (or loss) in decibels through the fiber assuming there are no *connectors* or *splices*

(b) the signal attenuation per kilometer for the fiber

(c) the overall signal attenuation for a 10 km optical link using the same fiber with *splices* (i.e. fiber connections) at 1 km intervals, each giving an attenuation of 1 dB

(d) the output/input power ratio in (c).

(a) signal attenuation = $-10 \log_{10}(P_o/P_i)$ = 16 dB

(b) 16 dB / 8 km = 2 dB/km

(c) the loss incurred along 10 km fiber = 20 dB.

With a total of 9 *splices* (i.e. fiber connections) along the link, each with an attenuation of 1 dB, the loss due to the splices is 9 dB.

=> the overall signal attenuation for the link = 20 + 9 dB = 29 dB.

(d) $P_o/P_i = 10^{(-29/10)} = 0.0013$

fiber attenuation mechanisms:

1. Material absorption
2. Scattering loss
3. Bending loss
4. Radiation loss (due to mode coupling)
5. Leaky modes

1. Material absorption losses in silica glass fibers

- Material absorption is a loss mechanism related to both *the material composition* and the *fabrication process* for the fiber. The optical power is lost as *heat* in the fiber.

- The light absorption can be *intrinsic* (due to the material components of the glass) or *extrinsic* (due to impurities introduced into the glass during fabrication).

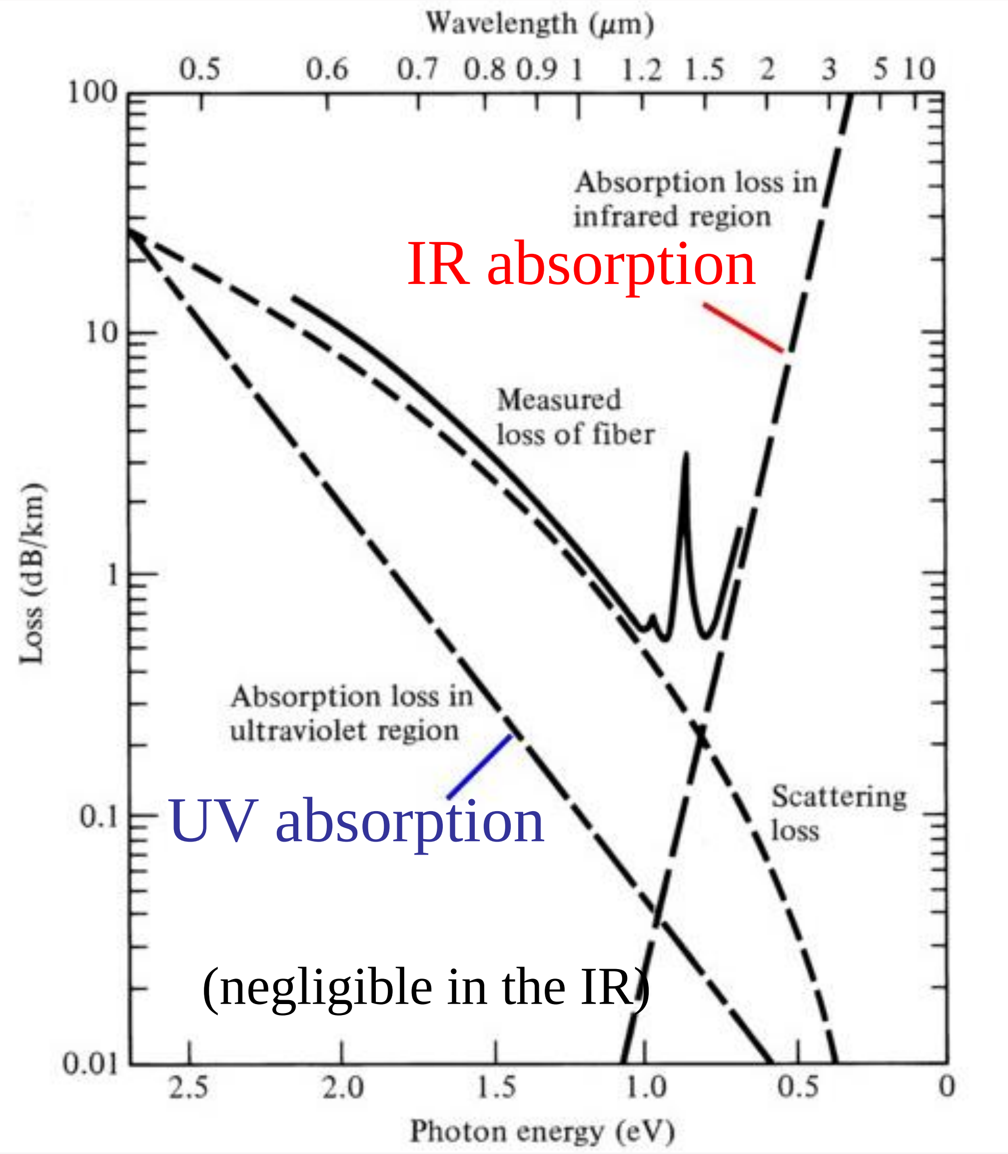
Intrinsic absorption

- Pure silica-based glass has *two* major intrinsic absorption mechanisms at optical wavelengths:

(1) a *fundamental UV absorption* edge, the peaks are centered in the *ultraviolet wavelength region*. This is due to the *electron transitions* within the glass molecules. The tail of this peak may extend into the the shorter wavelengths of the fiber transmission spectral window.

(2) A fundamental *infrared and far-infrared absorption edge*, due to *molecular vibrations* (such as Si-O). The tail of these absorption peaks may extend into the longer wavelengths of the fiber transmission spectral window.

Fundamental fiber attenuation characteristics



Extrinsic absorption

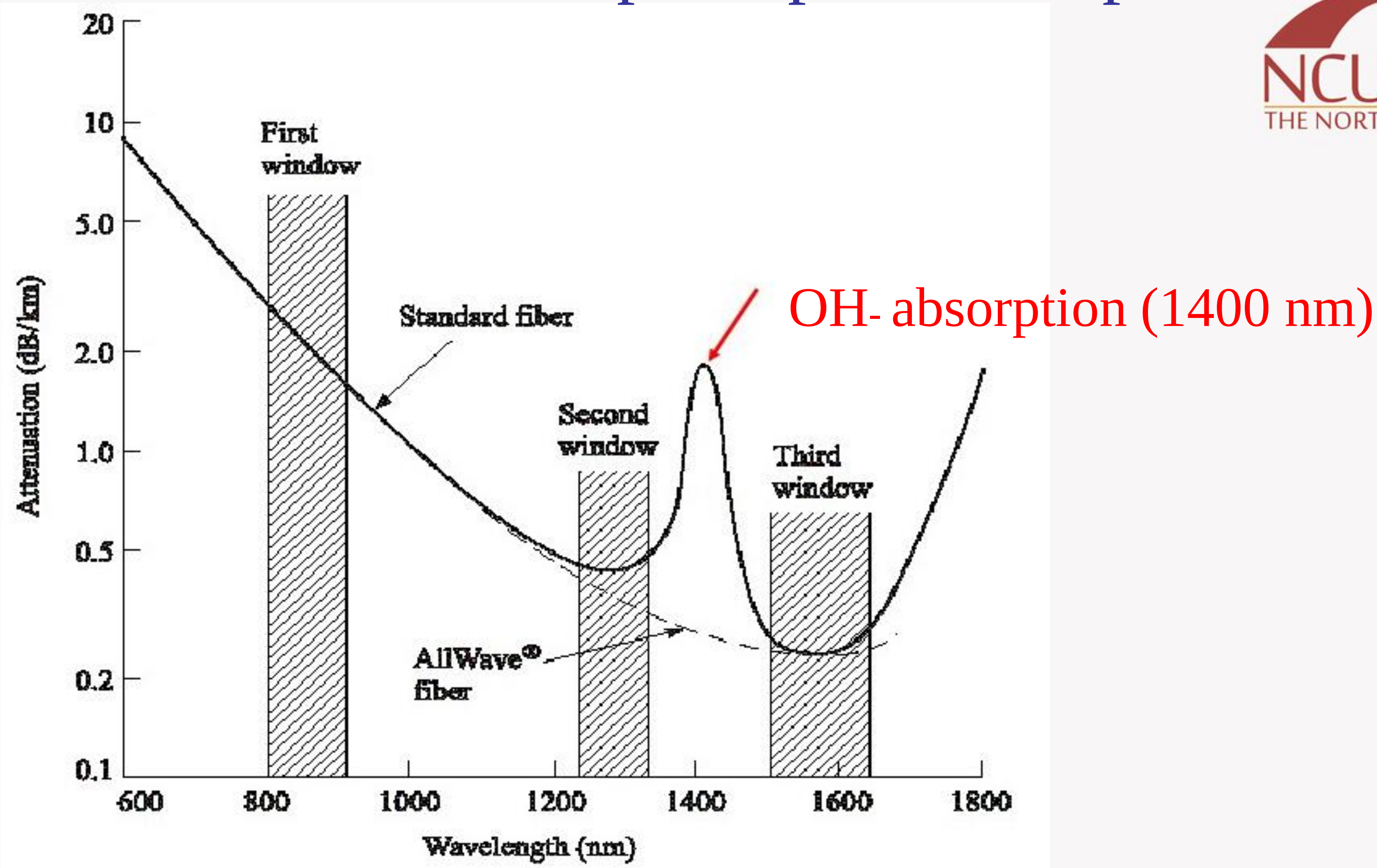
- Major extrinsic loss mechanism is caused by absorption due to water (*as the hydroxyl or OH- ions*) introduced in the glass fiber during *fiber pulling by means of oxyhydrogen flame*.

- These OH- ions are bonded into the glass structure and have absorption peaks (due to *molecular vibrations*) at 1.38 μm.

- Since these OH- absorption peaks are sharply peaked, narrow spectral windows exist around 1.3 μm and 1.55 μm which are essentially unaffected by OH- absorption.

- The lowest attenuation for typical silica-based fibers occur at wavelength 1.55 μm at about 0.2 dB/km, approaching the *minimum possible attenuation* at this wavelength.

1400 nm OH- absorption peak and spectral windows



(Lucent 1998)

OFS AllWave fiber: example of a "low-water-peak" or "full spectrum" fiber. Prior to 2000 the fiber transmission bands were referred to as "windows."

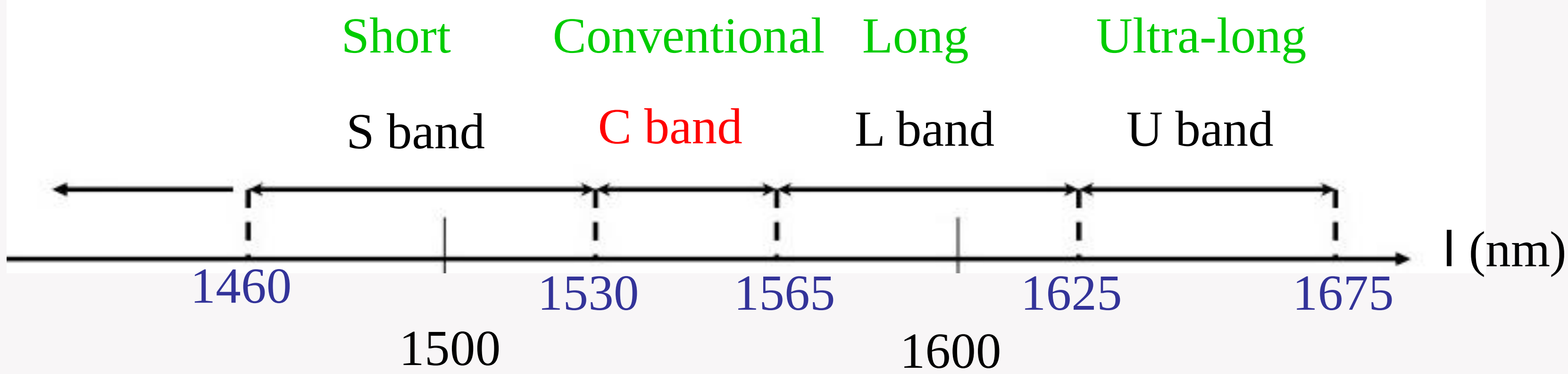
Three major spectral windows where fiber attenuation is low

The 1st window: 850 nm, attenuation 2 dB/km

The 2nd window: 1300 nm, attenuation 0.5 dB/km

The 3rd window: 1550 nm, attenuation 0.3 dB/km

1550 nm window is today's standard **long-haul** communication wavelengths.



Absorption Losses of Impurities

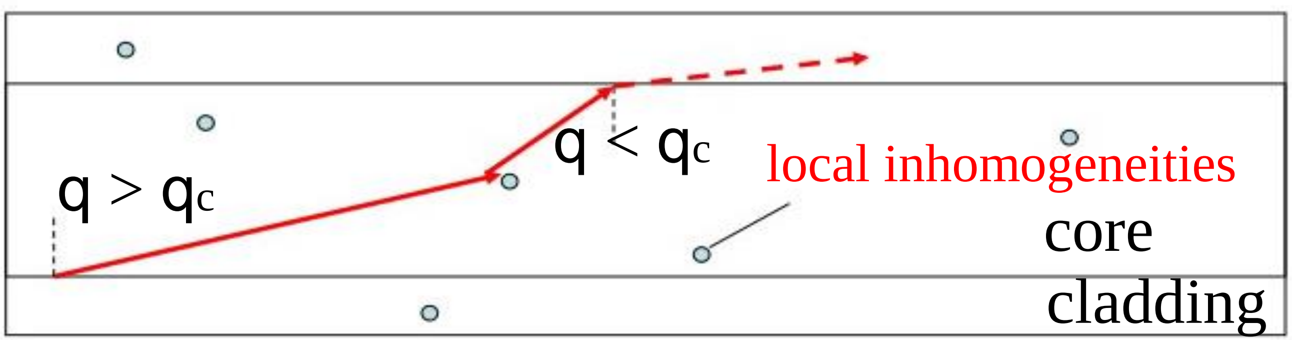
Table 3.1 *Examples of absorption loss in silica glass at different wavelengths due to 1 ppm of water-ions and various transition-metal impurities*

Impurity	*Loss due to 1 ppm of impurity (dB/km)*	*Absorption peak (nm)*
Iron: Fe^{2+}	0.68	1100
Iron: Fe^{3+}	0.15	400
Copper: Cu^{2+}	1.1	850
Chromium: Cr^{2+}	1.6	625
Vanadium: V^{4+}	2.7	725
Water: OH^-	1.0	950
Water: OH^-	2.0	1240
Water: OH^-	4.0	1380

2. Scattering loss

Scattering results in attenuation (*in the form of radiation*) as the scattered light may not continue to satisfy the total internal reflection in the fiber core.

One major type of scattering is known as *Rayleigh scattering*.



Silica glass is amorphous.

The scattered ray can escape by refraction according to Snell's Law.

- *Rayleigh scattering* results from *random inhomogeneities* that are small in size compared with the wavelength.

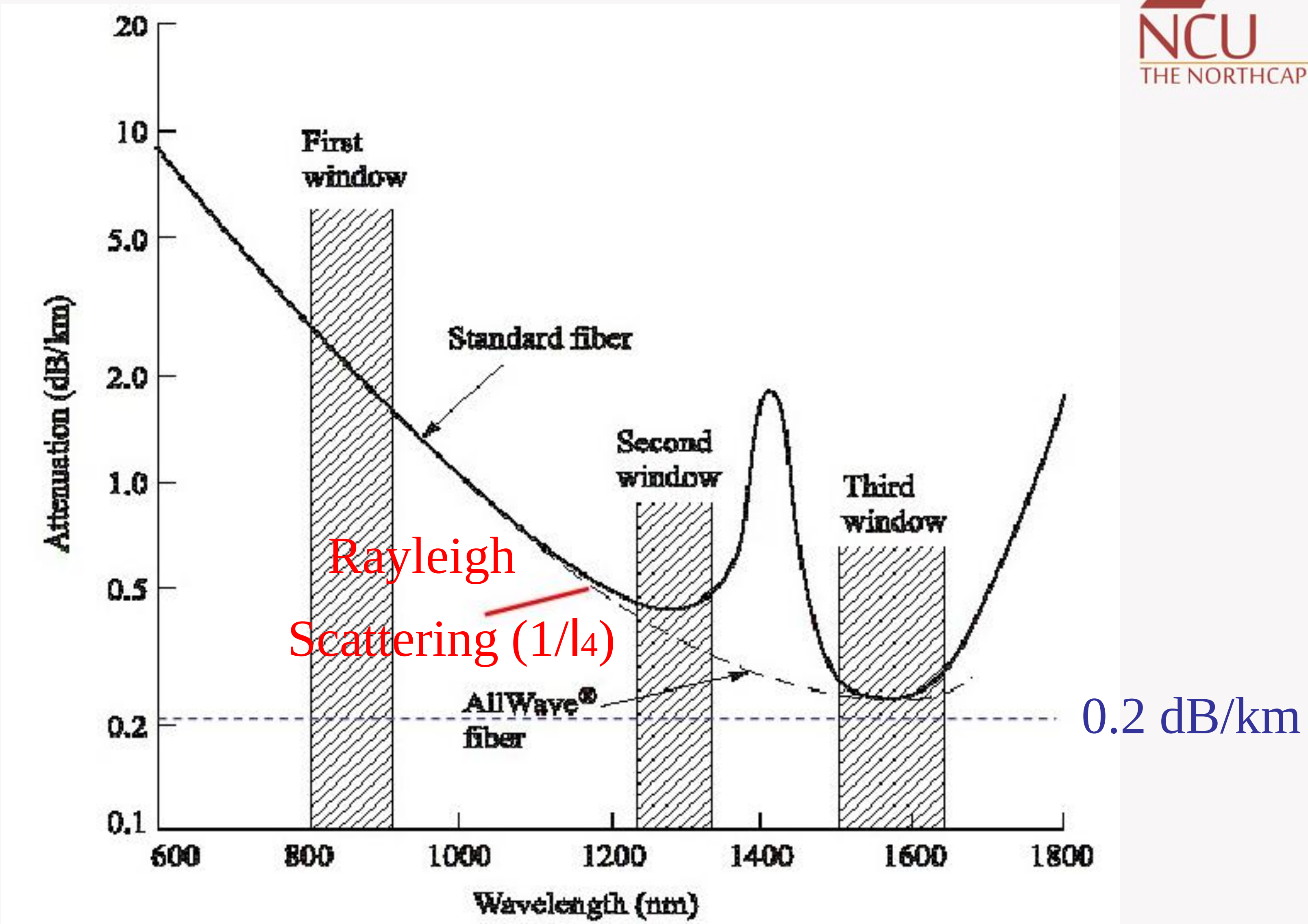
$\circ \ll \lambda$

- These inhomogeneities exist in the form of *refractive index fluctuations* which are frozen into the *amorphous* glass fiber upon fiber pulling. Such fluctuations *always exist and cannot be avoided* !

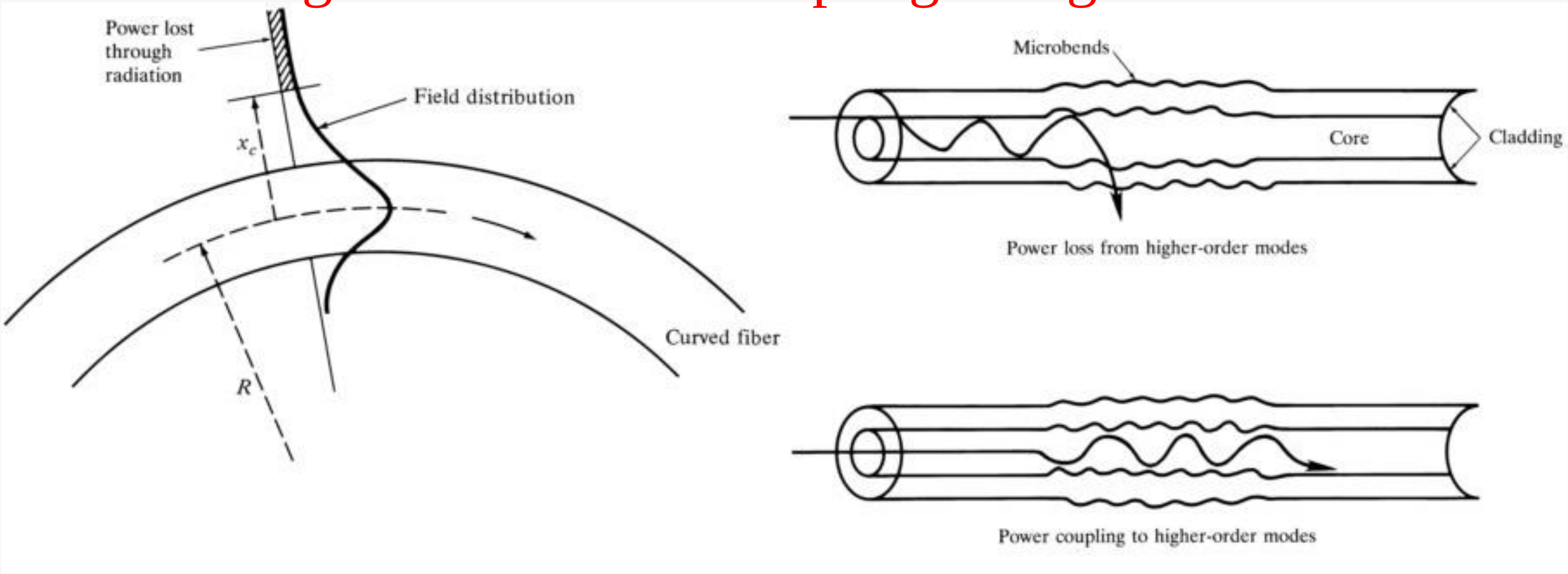
$$\textit{Rayleigh scattering} \text{ results in an attenuation (dB/km)} \propto 1/\lambda^4$$

Where else do we see Rayleigh scattering?

Rayleigh scattering is the dominant loss in today's fibers



Fiber bending loss and mode-coupling to higher-order modes



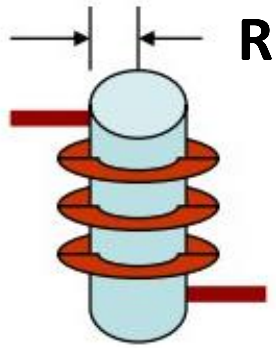
“macrobending”

(*how do we measure bending loss*?)

“microbending” - power coupling to higher-order modes that are more lossy.

Bending Losses in Fibers (1)

- Optical power escapes from tightly bent fibers
- Bending loss increases at longer wavelengths
  - Typical losses in 3 loops of standard 9-mm single-mode fiber (from: ***Lightwave***; Feb 2001; p. 156):
  - 2.6 dB at 1310 nm and 23.6 dB at 1550 nm for R = 1.15 cm
  - 0.1 dB at 1310 nm and 2.60 dB at 1550 nm for R = 1.80 cm
- Progressively tighter bends produce higher losses
- Bend-loss insensitive fibers have been developed and now are recommended
- Improper routing of fibers and incorrect storage of slack fiber can result in violations of bend radius rules



Test setup for checking bend loss: N fiber loops on a rod of radius R

Bending Losses in Fibers (2)

The total number of modes that can be supported by a curved fiber is less than in a straight fiber.

$$M_{eff} = M_{\infty}\left\{1 - \frac{\alpha+2}{2\alpha\Delta}\left[\frac{2a}{R} + \left(\frac{3}{2n_2kR}\right)^{2/3}\right]\right\}$$

Example 3.6 Consider a graded-index multimode fiber for which the index profile $\alpha = 2.0$, the core index $n_1 = 1.480$, the core-cladding index difference $\Delta = 0.01$, and the core radius $a = 25\ \mu m$. If the radius of curvature of the fiber is $R = 1.0$ cm, what percentage of the modes remain in the fiber at a 1300-nm wavelength?

Solution: From Eq. (3.7) the percentage of modes at a given curvature R is

$$\frac{M_{eff}}{M_{\infty}} = 1 - \frac{\alpha+2}{2\alpha\Delta}\left[\frac{2a}{R} + \left(\frac{3}{2n_2kR}\right)^{2/3}\right]$$

$$= 1 - \frac{1}{.01}\left[\frac{2(25)}{10000} + \left(\frac{3(1.3)}{2(1.465)2\pi(10000)}\right)^{2/3}\right]$$

$$= 0.42$$

Thus 42 percent of the modes remain in this fiber at a 1.0-cm bend radius.

Rayleigh scattering

$$\gamma_R = \frac{8\pi^3}{3\lambda^4} n^8 p^2 \beta_c K T_F \qquad (3.4)$$

where γ_R is the Rayleigh scattering coefficient, λ is the optical wavelength, n is the refractive index of the medium, p is the average photoelastic coefficient, β_c is the isothermal compressibility at a fictive temperature T_F, and K is Boltzmann's constant. The fictive temperature is defined as the temperature at which the glass can reach a state of thermal equilibrium and is closely related to the anneal temperature. Furthermore, the Rayleigh scattering coefficient is related to the transmission loss factor (transmissivity) of the fiber $\mathscr{L}$ following the relation [Ref. 12]:

$$\mathscr{L} = \exp(-\gamma_R L) \qquad (3.5)$$

where L is the length of the fiber. It is apparent from Eq. (3.4) that the fundamental component of Rayleigh scattering is strongly reduced by operating at the longest possible wavelength. This point is illustrated in Example 3.2.

Example 3.2

Silica has an estimated fictive temperature of 1400 K with an isothermal compressibility of $7 \times 10^{-11}\ m^2\ N^{-1}$ [Ref. 13]. The refractive index and the photoelastic coefficient for silica are 1.46 and 0.286 respectively [Ref. 13]. Determine the theoretical attenuation in decibels per kilometer due to the fundamental Rayleigh scattering in silica at optical wavelengths of 0.63, 1.00 and 1.30 μm. Boltzmann's constant is $1.381 \times 10^{-21}\ J\ K^{-1}$.

$$\gamma_R = \frac{8\pi^3 n^8 p^2 \beta_c K T_F}{3\lambda^4}$$

$$= \frac{248.15 \times 20.65 \times 0.082 \times 7 \times 10^{-11} \times 1.381 \times 10^{-23} \times 1400}{3 \times \lambda^4}$$

$$= \frac{1.895 \times 10^{-28}}{\lambda^4}\ \mathrm{m}^{-1}$$

At a wavelength of 0.63 μm:

$$\gamma_R = \frac{1.895 \times 10^{-28}}{0.158 \times 10^{-24}} = 1.199 \times 10^{-3}\ \mathrm{m}^{-1}$$

The transmission loss factor for 1 kilometer of fiber may be obtained using Eq. (3.5):

$$\mathcal{L}_{km} = \exp(-\gamma_R L) = \exp(-1.199 \times 10^{-3} \times 10^3)$$
$$= 0.301$$

The attenuation due to Rayleigh scattering in decibels per kilometer may be obtained from Eq. (3.1) where:

$$\text{Attenuation} = 10 \log_{10}(1/\mathcal{L}_{km}) = 10 \log_{10} 3.322$$
$$= 5.2\ \mathrm{dB\ km}^{-1}$$

At a wavelength of 1.0 μm:

$$\gamma_R = \frac{1.895 \times 10^{-28}}{10^{-24}} = 1.895 \times 10^{-4}\ \text{m}^{-1}$$

Using Eq. (3.5):

$$\mathscr{L}_{\text{km}} = \exp(-1.895 \times 10^{-4} \times 10^{3}) = \exp(-0.1895)$$
$$= 0.827$$

and Eq. (3.1):

$$\text{Attenuation} = 10 \log_{10} 1.209 = 0.8 \text{ dB km}^{-1}$$

At a wavelength of 1.30 μm:

$$\gamma_R = \frac{1.895 \times 10^{-28}}{2.856 \times 10^{-24}} = 0.664 \times 10^{-4}$$

Using Eq. (3.5):

$$\mathscr{L}_{\text{km}} = \exp(-0.664 \times 10^{-4} \times 10^{3}) = 0.936$$

and Eq. (3.1):

$$\text{Attenuation} = 10 \log_{10} 1.069 = 0.3 \text{ dB km}^{-1}$$

Mie scattering

Linear scattering may also occur at inhomogeneities which are comparable in size with the guided wavelength. These result from the nonperfect cylindrical structure of the waveguide and may be caused by fiber imperfections such as irregularities in the core–cladding interface, core–cladding refractive index differences along the fiber length, diameter fluctuations, strains and bubbles. When the scattering inhomogeneity size is greater than $\lambda/10$, the scattered intensity which has an angular dependence can be very large.

The scattering created by such inhomogeneities is mainly in the forward direction and is called Mie scattering. Depending upon the fiber material, design and manufacture, Mie scattering can cause significant losses. The inhomogeneities may be reduced by:

(a) removing imperfections due to the glass manufacturing process;

(b) carefully controlled extrusion and coating of the fiber;

(c) increasing the fiber guidance by increasing the relative refractive index difference.

By these means it is possible to reduce Mie scattering to insignificant levels.

Stimulated Brillouin scattering

Stimulated Brillouin scattering (SBS) may be regarded as the modulation of light through thermal molecular vibrations within the fiber. The scattered light appears as upper and lower sidebands which are separated from the incident light by the modulation frequency. The incident photon in this scattering process produces a phonon* of acoustic frequency as well as a scattered photon. This produces an optical frequency shift which varies with the scattering angle because the frequency of the sound wave varies with acoustic wavelength. The frequency shift is a maximum in the backward direction, reducing to zero in the forward direction, making SBS a mainly backward process.

$$P_B = 4.4 \times 10^{-3} d^2 \lambda^2 \alpha_{dB} \nu \text{ watts} \tag{3.6}$$

where d and λ are the fiber core diameter and the operating wavelength, respectively, both measured in micrometers, α_{dB} is the fiber attenuation in decibels per kilometer and ν is the source bandwidth (i.e. injection laser) in gigahertz. The expression given in Eq. (3.6) allows the determination of the threshold optical power which must be launched into a single-mode optical fiber before SBS occurs (see Example 3.3).

Stimulated Raman scattering

Stimulated Raman scattering (SRS) is similar to SBS except that a high-frequency optical phonon rather than an acoustic phonon is generated in the scattering process. Also, SRS can occur in both the forward and backward directions in an optical fiber, and may have an optical power threshold of up to three orders of magnitude higher than the Brillouin threshold in a particular fiber.

Using the same criteria as those specified for the Brillouin scattering threshold given in Eq. (3.6), it may be shown [Ref. 16] that the threshold optical power for SRS P_R in a long single-mode fiber is given by:

$$P_R = 5.9 \times 10^{-2} d^2 \lambda \alpha_{dB} \text{ watts} \tag{3.7}$$

where d, λ and α_{dB} are as specified for Eq. (3.6).

Example 3.3

A long single-mode optical fiber has an attenuation of 0.5 dB km^{-1} when operating at a wavelength of 1.3 μm. The fiber core diameter is 6 μm and the laser source bandwidth is 600 MHz. Compare the threshold optical powers for stimulated Brillouin and Raman scattering within the fiber at the wavelength specified.

Solution: The threshold optical power for SBS is given by Eq. (3.6) as:

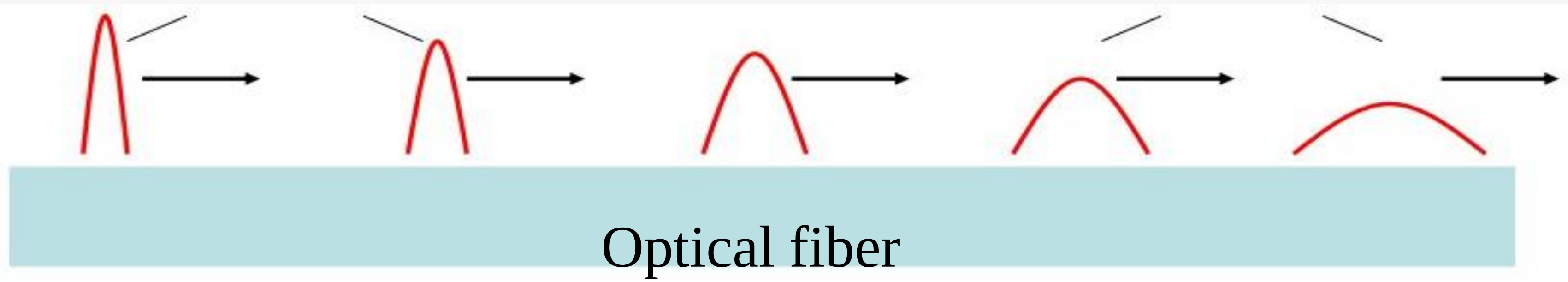
$$
\begin{aligned}
P_B &= 4.4 \times 10^{-3} d^2 \lambda^2 \alpha_{dB} \nu \\
&= 4.4 \times 10^{-3} \times 6^2 \times 1.3^2 \times 0.5 \times 0.6 \\
&= 80.3 \text{ mW}
\end{aligned}
$$

The threshold optical power for SRS may be obtained from Eq. (3.7), where:

$$
\begin{aligned}
P_R &= 5.9 \times 10^{-2} d^2 \lambda \alpha_{dB} \\
&= 5.9 \times 10^{-2} \times 6^2 \times 1.3 \times 0.5 \\
&= 1.38 \text{ W}
\end{aligned}
$$

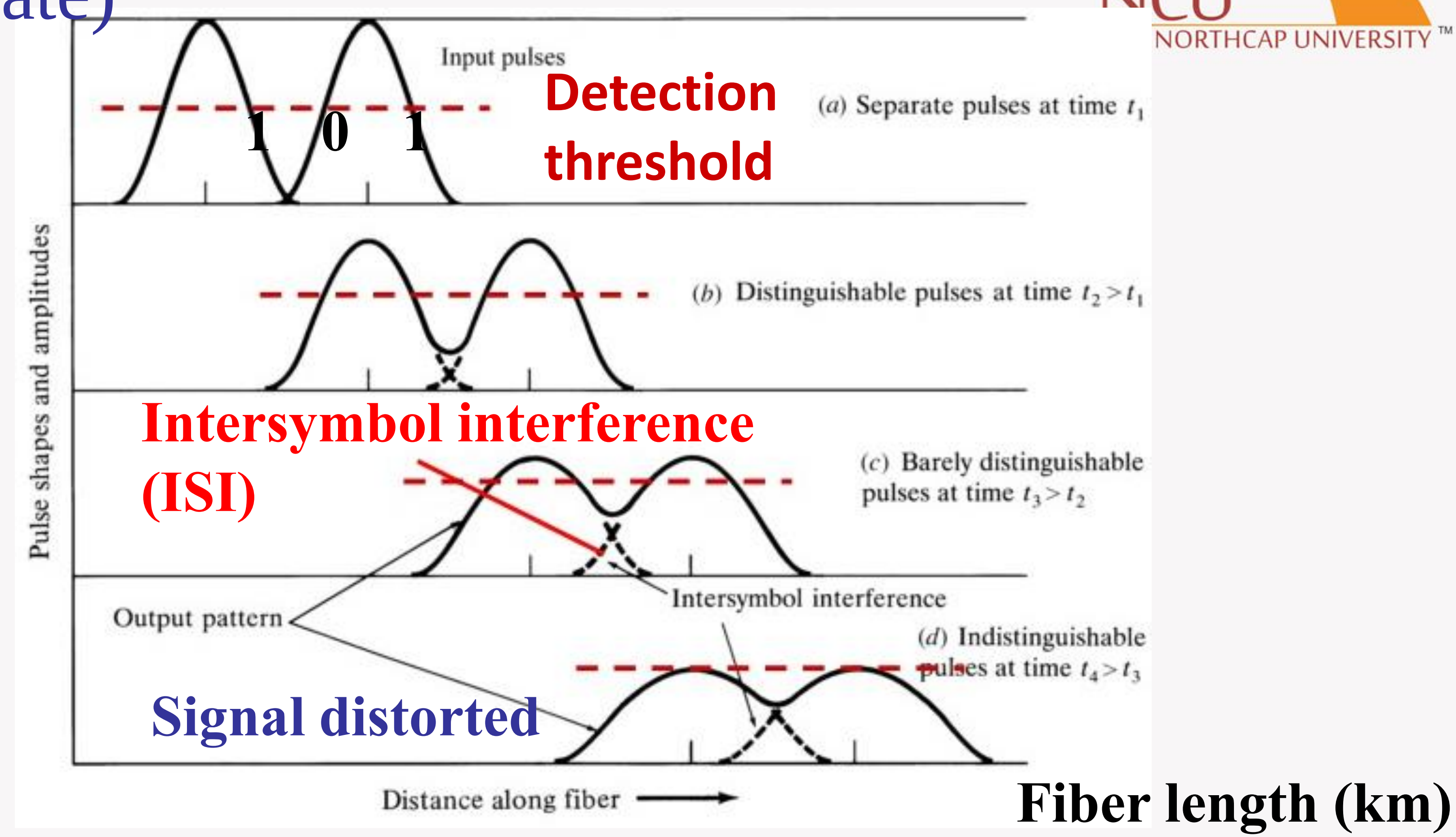
Fiber dispersion

- Fiber dispersion results in *optical pulse broadening* and hence *digital signal degradation*.



Dispersion mechanisms: 1. Modal (or *intermodal*) dispersion
2. Chromatic dispersion (CD)
3. Polarization mode dispersion (PMD)

Pulse broadening limits fiber bandwidth (data rate)

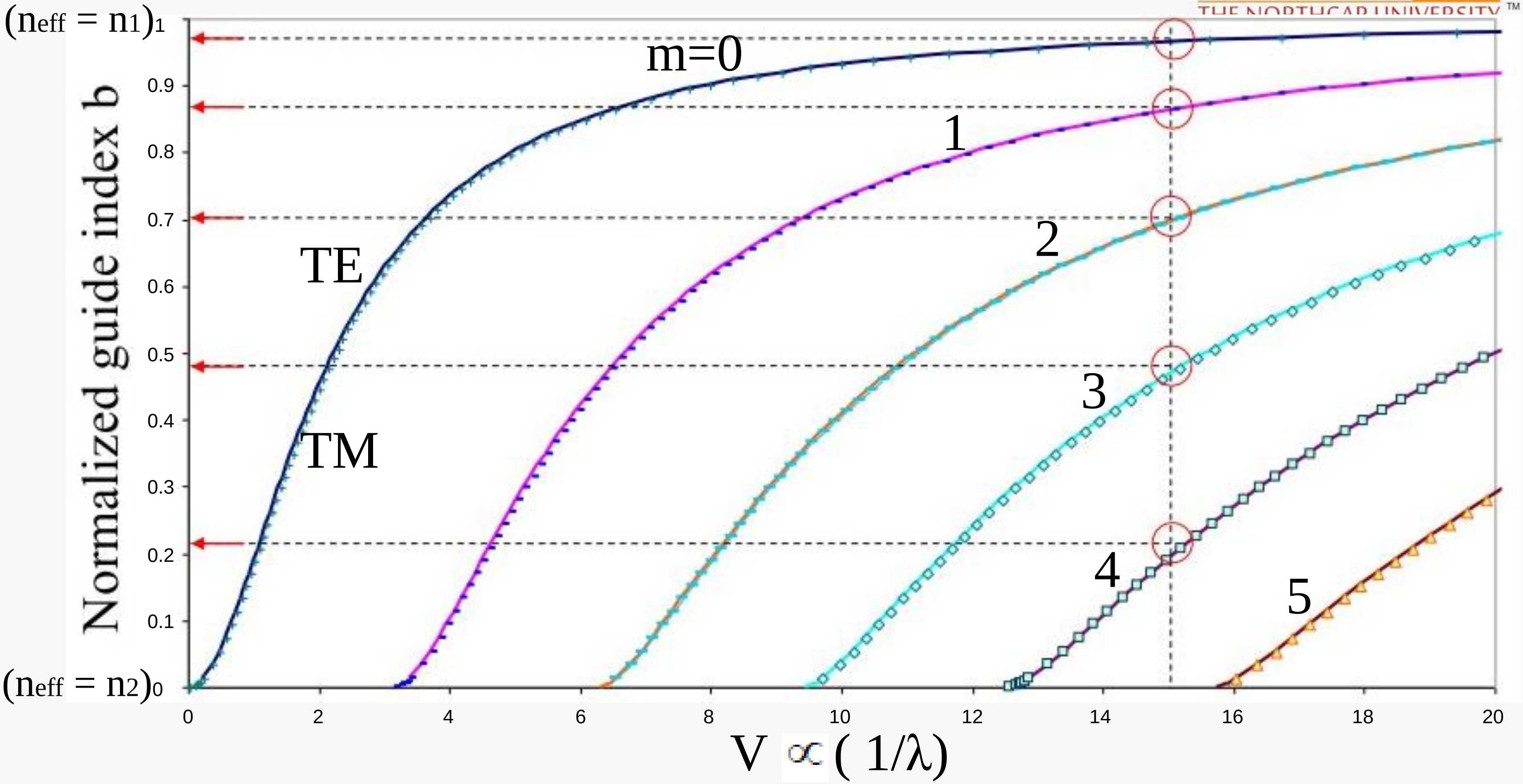


- An *increasing number of errors* may be encountered on the digital optical channel as the ISI becomes more pronounced.

1. Modal dispersion

- When numerous waveguide modes are propagating, they all travel with different net velocities with respect to the waveguide axis.
- An input waveform distorts during propagation because its energy is distributed among several modes, each traveling at a different speed.
- Parts of the wave arrive at the output before other parts, spreading out the waveform. This is thus known as multimode (modal) dispersion.
- Multimode dispersion does *not* depend on the source linewidth (even a *single* wavelength can be simultaneously carried by *multiple modes* in a waveguide).
- Multimode dispersion would *not* occur if the waveguide allows *only* one mode to propagate - the advantage of *single*-mode waveguides!

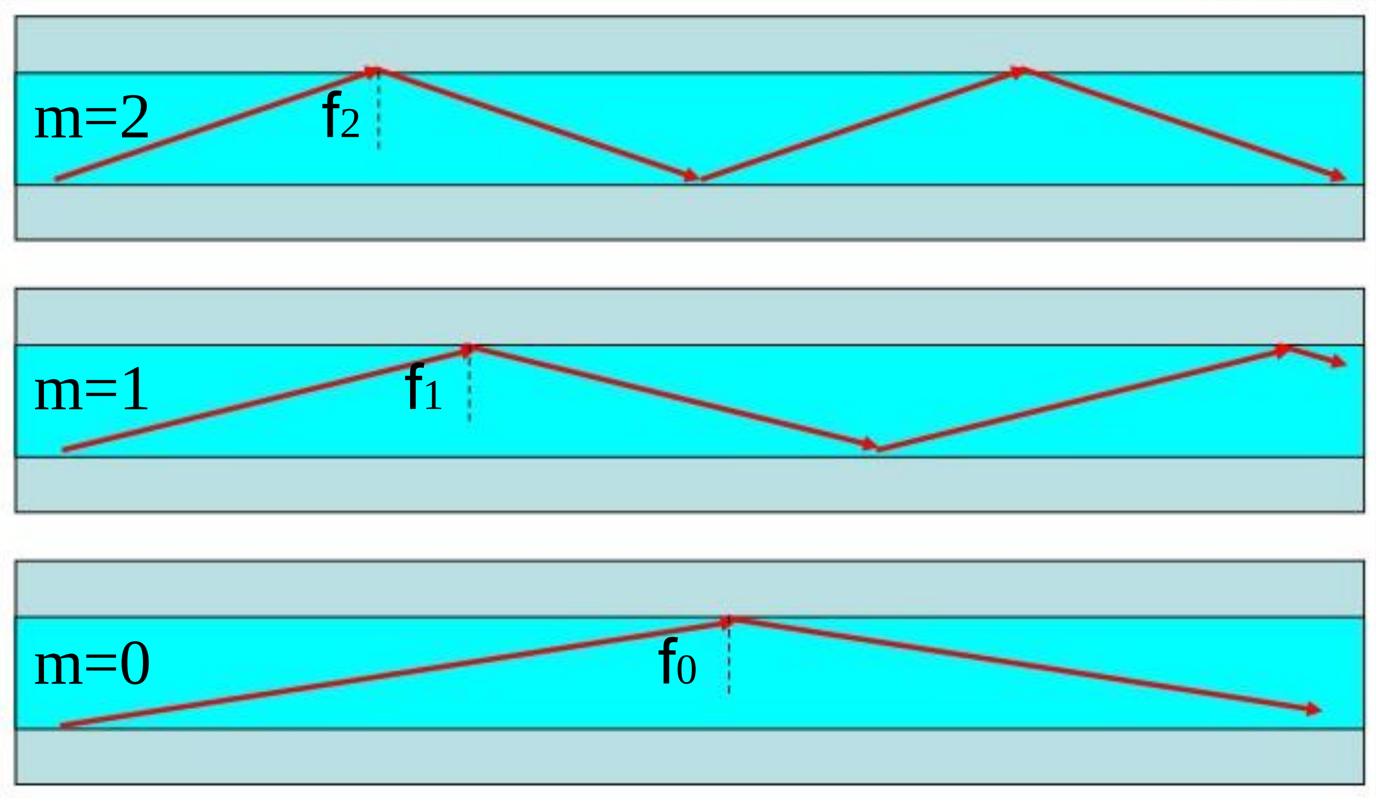
Modal dispersion as shown from the mode chart of a symmetric slab waveguide



- Phase velocity for mode m = $\omega/\beta_m = \omega/(n_{eff}(m)\ k_0)$

(note that m = 0 mode is the *slowest* mode)

Modal dispersion in multimode waveguides

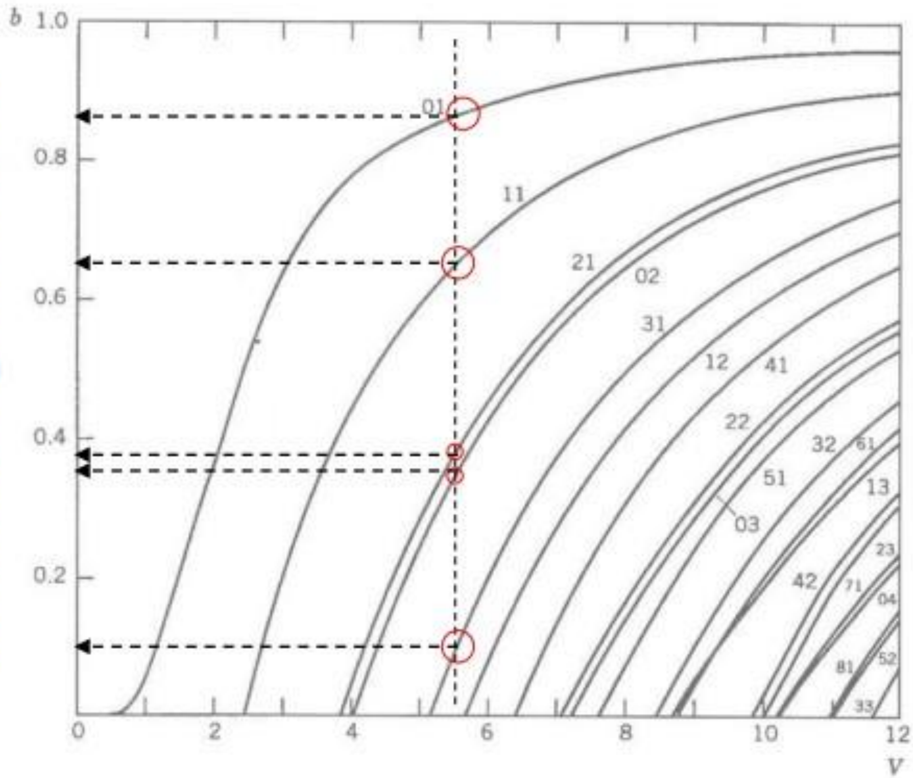


The carrier wave can propagate along all these different "zig-zag" ray paths of *different path lengths*.

Modal dispersion as shown from the LP mode chart of a silica optical fiber

$(n_{eff} = n_1)$

Normalized guide index b



$(n_{eff} = n_2)$

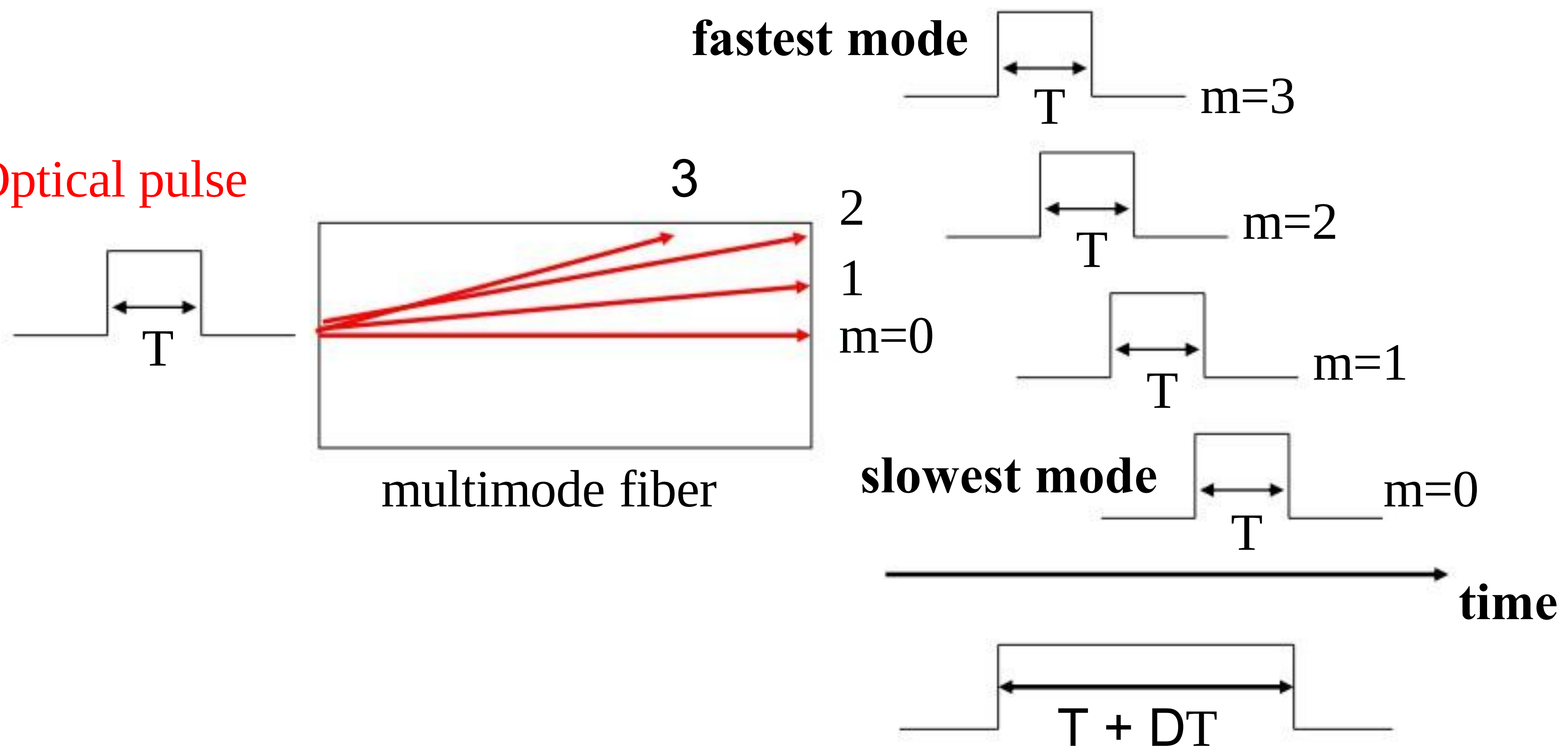
V (· 1/l)

- Phase velocity for LP mode = $w/b_{lm} = w/(n_{eff}(lm)\ k_0)$

(note that LP_{01} mode is the *slowest* mode)

Modal dispersion results in pulse broadening

Optical pulse

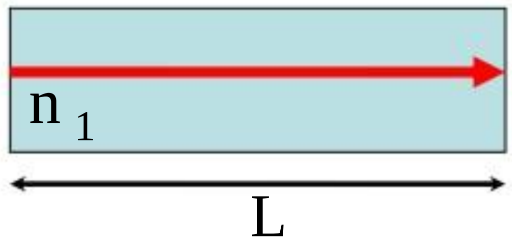


modal dispersion: different modes arrive at the receiver with different delays => pulse broadening

Estimated modal dispersion pulse broadening using phase velocity

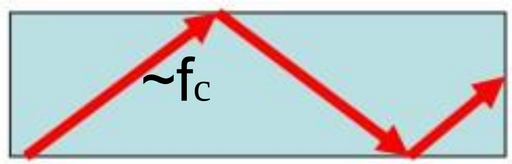
- A zero-order mode traveling near the waveguide axis needs time:

$t_0 = L/v_{m=0} \cdot \quad Ln_1/c$ $\qquad$ $(v_{m=0} \cdot \quad c/n_1)$



- The highest-order mode traveling near the critical angle needs time:

$t_m = L/v_m \cdot \quad Ln_2/c$ $\qquad$ $(v_m \cdot \quad c/n_2)$



=> the *pulse broadening* due to modal dispersion:

$$DT \cdot \quad t_0 - t_m \cdot \quad (L/c)\,(n_1 - n_2)$$

$$\cdot \quad (L/2cn_1)\,NA^2 \qquad (n_1 \sim n_2)$$

e.g. How much will a light pulse spread after traveling along 1 km of a step-index fiber whose NA = 0.275 and n_{core} = 1.487?

How does modal dispersion restricts fiber bit rate?

Suppose we transmit at a low bit rate of 10 Mb/s

=> Pulse duration = $1 / 10^7$ s = 100 ns

Using the above e.g., each pulse will spread up to ・ 100 ns (i.e. ・ pulse duration !) every km

・ The broadened pulses overlap! (**Intersymbol interference** (ISI))

*Modal dispersion limits the bit rate of a fiber-optic link to ~ 10 Mb/s.
(a coaxial cable supports this bit rate easily!)

Bit-rate distance product

• We can relate the pulse broadening DT to the *information-carrying capacity* of the fiber measured through the bit rate B.

• Although a precise relation between B and DT depends on many details, such as the pulse shape, it is intuitively clear that DT *should be less than the allocated bit time slot* given by 1/B.

• An *order-of-magnitude* estimate of the supported bit rate is obtained from the condition $B DT < 1$.

• ***Bit-rate distance product*** (limited by modal dispersion)

$$BL < 2c\, n_{core} / NA^2$$

This condition provides a rough estimate of a fundamental limitation of step-index multimode fibers.

(the *smaller is the NA, the larger is the bit-rate distance product*)

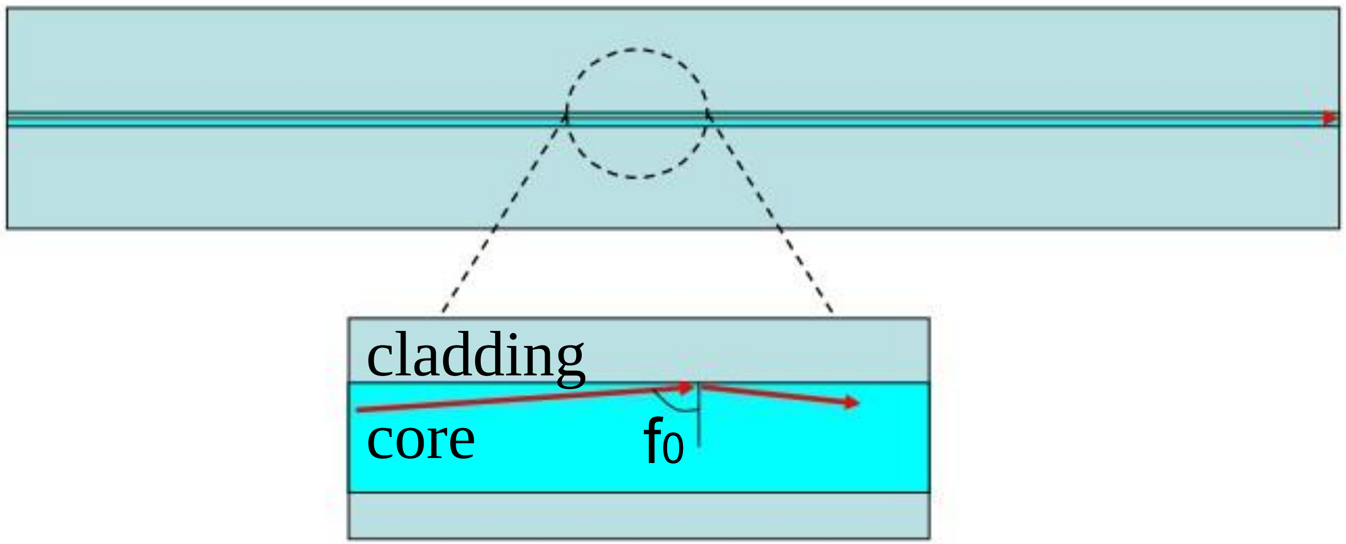
The capacity of optical communications systems is frequently measured in terms of the **bit rate-distance product**.

e.g. If a system is capable of transmitting 10 Mb/s over a distance of 1 km, it is said to have a *bit rate-distance* product of 10 (Mb/s)-km.

This may be suitable for some *local-area networks (LANs)*.

Note that the same system can transmit 100 Mb/s along 100 m, or 1 Gb/s along 10 m, or 10 Gb/s along 1 m, or 100 Gb/s along 10 cm, 1 Tb/s along 1 cm

Single-mode fiber eliminates modal dispersion

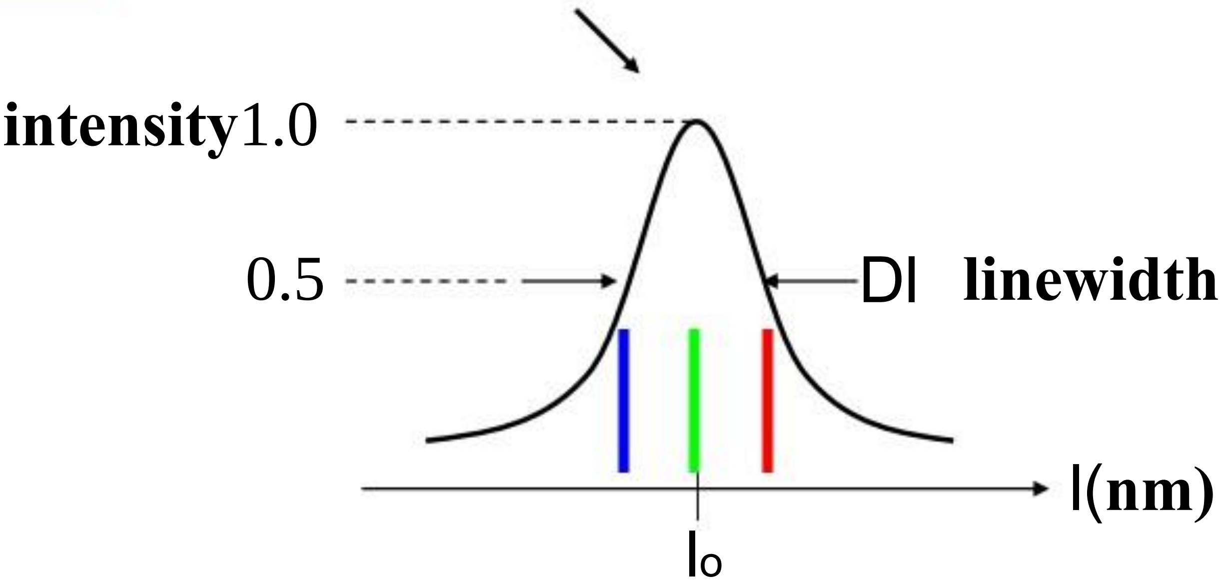


- The main advantage of *single*-mode fibers is to propagate *only one mode* so that *modal dispersion is absent*.

- However, *pulse broadening does not disappear altogether*. The *group velocity* associated with the fundamental mode is *frequency dependent* within the pulse *spectral linewidth* because of chromatic dispersion.

2. Chromatic dispersion

- Chromatic dispersion (CD) may occur in *all* types of optical fiber. The optical pulse broadening results from the *finite spectral linewidth of the optical source and the modulated carrier*.



*In the case of the semiconductor laser Dl corresponds to only a fraction of % of the centre wavelength lo. For LEDs, Dl is likely to be a significant percentage of lo.

Spectral linewidth

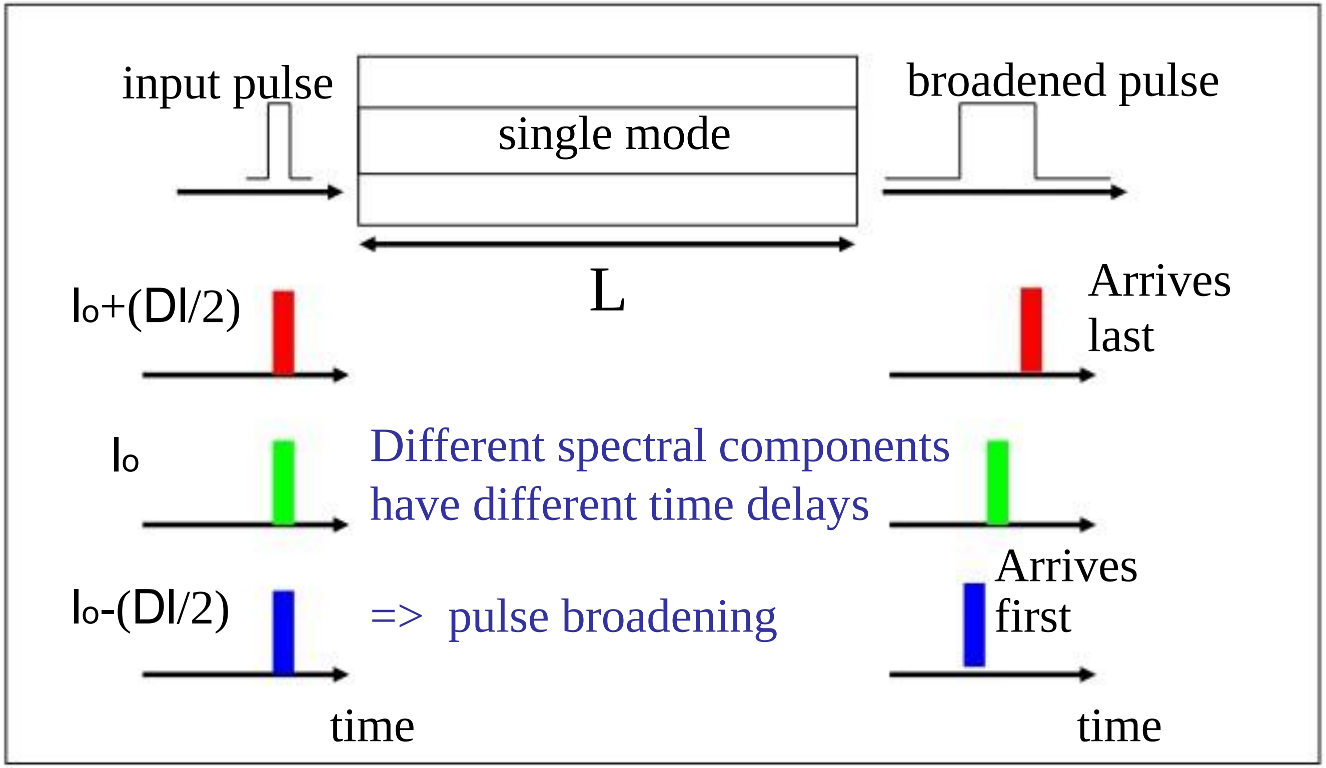
- Real sources emit over a range of wavelengths. This range is the *source linewidth* or *spectral width*.

- The smaller is the linewidth, the smaller is the spread in wavelengths or frequencies, the more *coherent* is the source.

- An *ideal* perfectly coherent source emits light at a *single* wavelength. It has *zero* linewidth and is *perfectly* monochromatic.

Light sources	**Linewidth (nm)**
Light-emitting diodes	20 nm - 100 nm
Semiconductor laser diodes	1 nm - 5 nm
Nd:YAG solid-state lasers	0.1 nm
HeNe gas lasers	0.002 nm

- Pulse broadening occurs because there may be *propagation delay differences* among the *spectral components* of the transmitted signal.



Chromatic dispersion (CD): Different spectral components of a *pulse* travel at different *group velocities*. This is known as *group velocity dispersion* (GVD).

Dispersion

For no overlapping of light pulses down on an optical fiber link the digital bit rate B_T must be less than the reciprocal of the broadened (through dispersion) pulse duration (2τ). Hence:

$$B_T \le \frac{1}{2\tau} \quad \text{pulse duration} \tag{3.10}$$

$$B_T(\max) \simeq \frac{0.2}{\sigma} \text{ bit s}^{-1}$$

rms width

$$B_T(\max) = 2B \quad \text{(Maximum bandwidth } B\text{)} \tag{3.12}$$

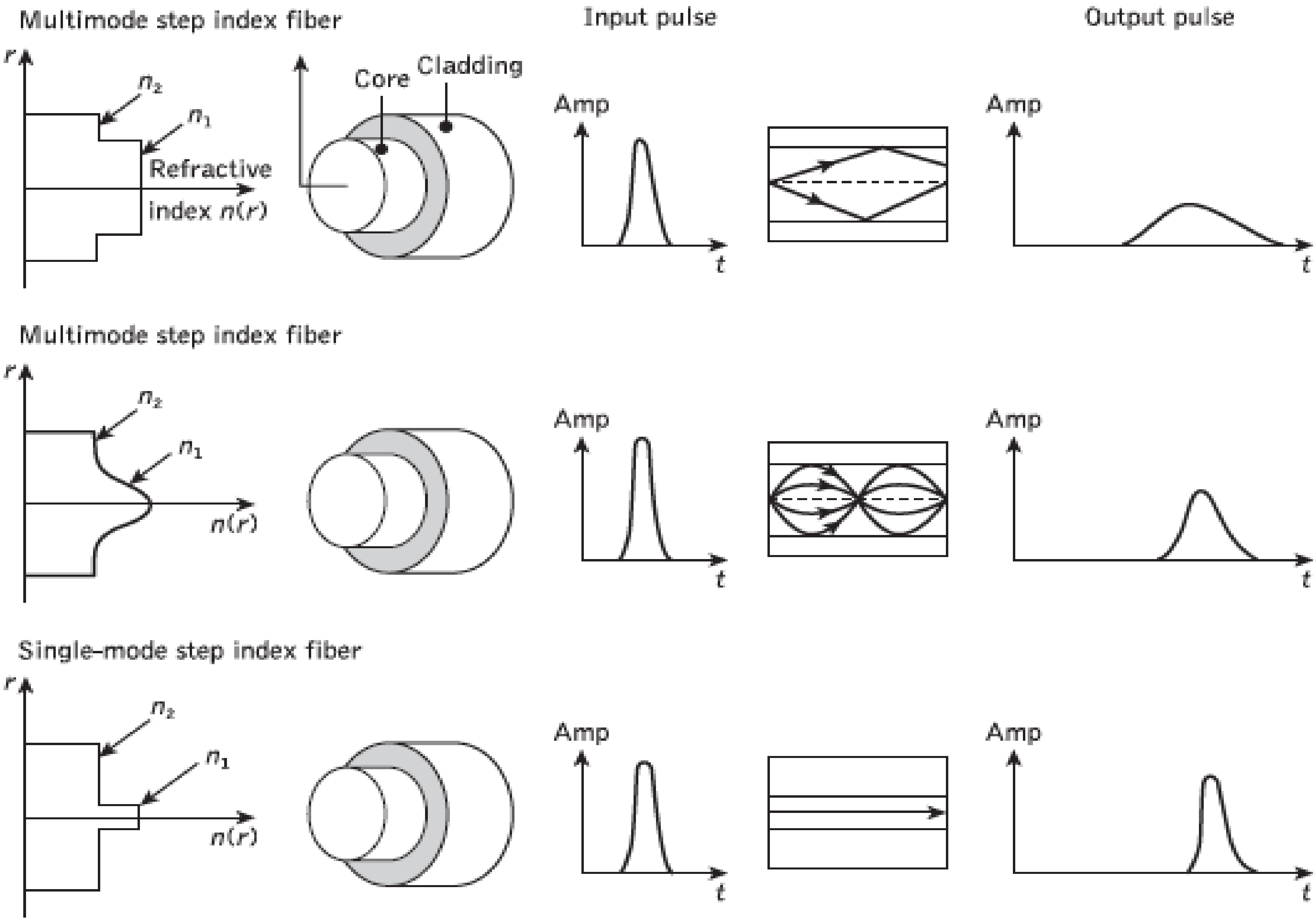


Figure 3.9 Schematic diagram showing a multimode step index fiber, multimode graded index fiber and single-mode step index fiber, and illustrating the pulse broadening due to intermodal dispersion in each fiber type

Example 3.5

A multimode graded index fiber exhibits total pulse broadening of 0.1 μs over a distance of 15 km. Estimate:

(a) the maximum possible bandwidth on the link assuming no intersymbol interference;

(b) the pulse dispersion per unit length;

(c) the bandwidth–length product for the fiber.

Solution: (a) The maximum possible optical bandwidth which is equivalent to the maximum possible bit rate (for return to zero pulses) assuming no ISI may be obtained from Eq. (3.10), where:

$$B_{opt} = B_T = \frac{1}{2\tau} = \frac{1}{0.2 \times 10^{-6}} = 5 \text{ MHz}$$

(b) The dispersion per unit length may be acquired simply by dividing the total dispersion by the total length of the fiber:

$$\text{Dispersion} = \frac{0.1 \times 10^{-6}}{15} = 6.67 \text{ ns km}^{-1}$$

(c) The bandwidth–length product may be obtained in two ways. Firstly by simply multiplying the maximum bandwidth for the fiber link by its length. Hence:

$$B_{opt}L = 5\ \text{MHz} \times 15\ \text{km} = 75\ \text{MHz km}$$

Alternatively, it may be obtained from the dispersion per unit length using Eq. (3.10) where:

$$B_{opt}L = \frac{1}{2 \times 6.67 \times 10^{-6}} = 75\ \text{MHz km}$$

3.9.1 Material dispersion

Pulse broadening due to material dispersion results from the different group velocities of the various spectral components launched into the fiber from the optical source. It occurs when the phase velocity of a plane wave propagating in the dielectric medium varies non-linearly with wavelength, and a material is said to exhibit material dispersion when the second differential of the refractive index with respect to wavelength is not zero (i.e. $d^2n/d\lambda^2 \neq 0$). The pulse spread due to material dispersion may be obtained by considering the group delay τ_g in the optical fiber which is the reciprocal of the group velocity υ_g defined by Eqs (2.37) and (2.40). Hence the group delay is given by:

$$\tau_g = \frac{d\beta}{d\omega} = \frac{1}{c}\left(n_1 - \lambda\frac{dn_1}{d\lambda}\right) \tag{3.13}$$

where n_1 is the refractive index of the core material. The pulse delay τ_m due to material dispersion in a fiber of length L is therefore:

$$\tau_m = \frac{L}{c}\left(n_1 - \lambda\frac{dn_1}{d\lambda}\right) \tag{3.14}$$

For a source with rms spectral width σ_λ and a mean wavelength λ, the rms pulse broadening due to material dispersion σ_m may be obtained from the expansion of Eq. (3.14) in a Taylor series about λ where:

$$\sigma_m = \sigma_\lambda\frac{d\tau_m}{d\lambda} + \sigma_\lambda\frac{2d^2\tau_m}{d\lambda^2} + \dots \tag{3.15}$$

As the first term in Eq. (3.15) usually dominates, especially for sources operating over the 0.8 to 0.9 μm wavelength range, then:

$$\sigma_m \simeq \sigma_\lambda \frac{d\tau_m}{d\lambda} \tag{3.16}$$

Hence the pulse spread may be evaluated by considering the dependence of τ_m on λ, where from Eq. (3.14):

$$\frac{d\tau_m}{d\lambda} = \frac{L\lambda}{c}\left[\frac{dn_1}{d\lambda} - \frac{d^2 n_1}{d\lambda^2} - \frac{dn_1}{d\lambda}\right]$$

$$= \frac{-L\lambda}{c}\frac{d^2 n_1}{d\lambda^2} \tag{3.17}$$

Therefore, substituting the expression obtained in Eq. (3.17) into Eq. (3.16), the rms pulse broadening due to material dispersion is given by:

$$\sigma_m \simeq \frac{\sigma_\lambda L}{c}\left|\lambda \frac{d^2 n_1}{d\lambda^2}\right| \tag{3.18}$$

The material dispersion for optical fibers is sometimes quoted as a value for $|\lambda^2(d^2n_1/d\lambda^2)|$ or simply $|d^2n_1/d\lambda^2|$.

However, it may be given in terms of a material dispersion parameter M which is defined as:

$$M = \frac{1}{L}\frac{d\tau_m}{d\lambda} = \frac{\lambda}{c}\left|\frac{d^2 n_1}{d\lambda^2}\right| \tag{3.19}$$

and which is often expressed in units of ps nm^{-1} km^{-1}.

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Example 3.6

A glass fiber exhibits material dispersion given by $|\lambda^2(d^2n_1/d\lambda^2)|$ of 0.025. Determine the material dispersion parameter at a wavelength of 0.85 μm, and estimate the rms pulse broadening per kilometer for a good LED source with an rms spectral width of 20 nm at this wavelength.

Solution: The material dispersion parameter may be obtained from Eq. (3.19):

$$M = \frac{\lambda}{c}\left|\frac{d^2n_1}{d\lambda^2}\right| = \frac{1}{c\lambda}\left|\lambda^2\frac{d^2n_1}{d\lambda^2}\right|$$

$$= \frac{0.025}{2.998\times10^5\times850}\ \text{s nm}^{-1}\ \text{km}^{-1}$$

$$= 98.1\ \text{ps nm}^{-1}\ \text{km}^{-1}$$

The rms pulse broadening is given by Eq. (3.18) as:

$$\sigma_m \simeq \frac{\sigma_\lambda L}{c}\left|\lambda\frac{d^2n_1}{d\lambda^2}\right|$$

Therefore in terms of the material dispersion parameter M defined by Eq. (3.19):

$$\sigma_m \simeq \sigma_\lambda LM$$

Hence, the rms pulse broadening per kilometer due to material dispersion:

$$\sigma_m(1\ \text{km}) = 20\times1\times98.1\times10^{-12} = 1.96\ \text{ns km}^{-1}$$

Example 3.7

Estimate the rms pulse broadening per kilometer for the fiber in Example 3.6 when the optical source used is an injection laser with a relative spectral width σ_λ/λ of 0.0012 at a wavelength of 0.85 μm.

Solution: The rms spectral width may be obtained from the relative spectral width by:

$$\sigma_\lambda = 0.0012\lambda = 0.0012 \times 0.85 \times 10^{-6}$$
$$= 1.02 \text{ nm}$$

The rms pulse broadening in terms of the material dispersion parameter following Example 3.6 is given by:

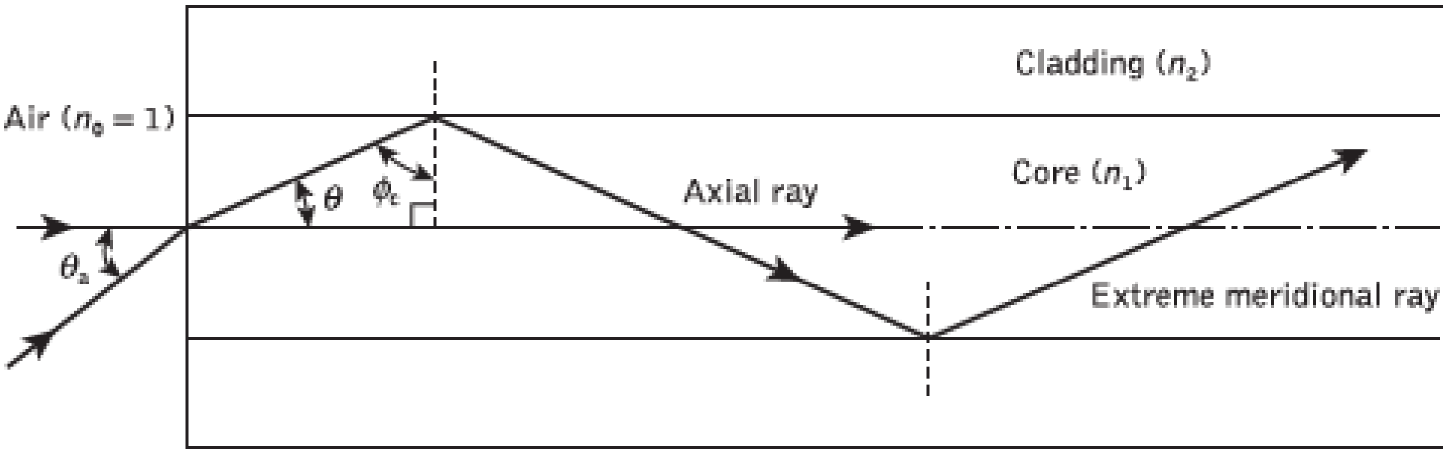
$$\sigma_m \simeq \sigma_\lambda LM$$

Therefore, the rms pulse broadening per kilometer due to material dispersion is:

$$\sigma_m \simeq 1.02 \times 1 \times 98.1 \times 10^{-12} = 0.10 \text{ ns km}^{-1}$$

Hence, in this example the rms pulse broadening is reduced by a factor of around 20 (i.e. equivalent to the reduced rms spectral width of the injection laser source) compared with that obtained with the LED source of Example 3.6.

Multimode step index fiber



the time taken for the axial ray to travel along a fiber of length L gives the minimum delay time T_{Min} and:

$$T_{\text{Min}} = \frac{\text{distance}}{\text{velocity}} = \frac{L}{(c/n_1)} = \frac{Ln_1}{c} \tag{3.20}$$

where n_1 is the refractive index of the core and c is the velocity of light in a vacuum.

The extreme meridional ray exhibits the maximum delay time T_{Max} where:

$$T_{\text{Max}} = \frac{L/\cos\theta}{c/n_1} = \frac{Ln_1}{c\cos\theta} \tag{3.21}$$

Using Snell's law of refraction at the core–cladding interface following Eq. (2.2):

$$\sin\phi_c = \frac{n_2}{n_1} = \cos\theta \tag{3.22}$$

where n_2 is the refractive index of the cladding. Furthermore, substituting into Eq. (3.21) for $\cos\theta$ gives:

$$T_{\mathrm{Max}} = \frac{Ln_1^2}{cn_2} \tag{3.23}$$

The delay difference δT_s between the extreme meridional ray and the axial ray may be obtained by subtracting Eq. (3.20) from Eq. (3.23). Hence:

$$\begin{aligned} \delta T_s = T_{\mathrm{Max}} - T_{\mathrm{Min}} &= \frac{Ln_1^2}{cn_2} - \frac{Ln_1}{c} \\ &= \frac{Ln_1^2}{cn_2}\left(\frac{n_1 - n_2}{n_1}\right) && (3.24) \\ &\simeq \frac{Ln_1^2\Delta}{cn_2} \quad \text{when } \Delta \ll 1 && (3.25) \end{aligned}$$

where Δ is the relative refractive index difference. However, when $\Delta \ll 1$, then from the definition given by Eq. (2.9), the relative refractive index difference may also be given approximately by:

$$\Delta \simeq \frac{n_1 - n_2}{n_2} \tag{3.26}$$

Hence rearranging Eq. (3.24):

$$\delta T_s = \frac{L n_1}{c}\left(\frac{n_1 - n_2}{n_2}\right) \simeq \frac{L n_1 \Delta}{c} \tag{3.27}$$

Also substituting for Δ from Eq. (2.10) gives:

$$\delta T_s \simeq \frac{L(NA)^2}{2 n_1 c} \tag{3.28}$$

$$\int_{-\infty}^{\infty} p_i(t)\, \mathrm{d}t = 1$$

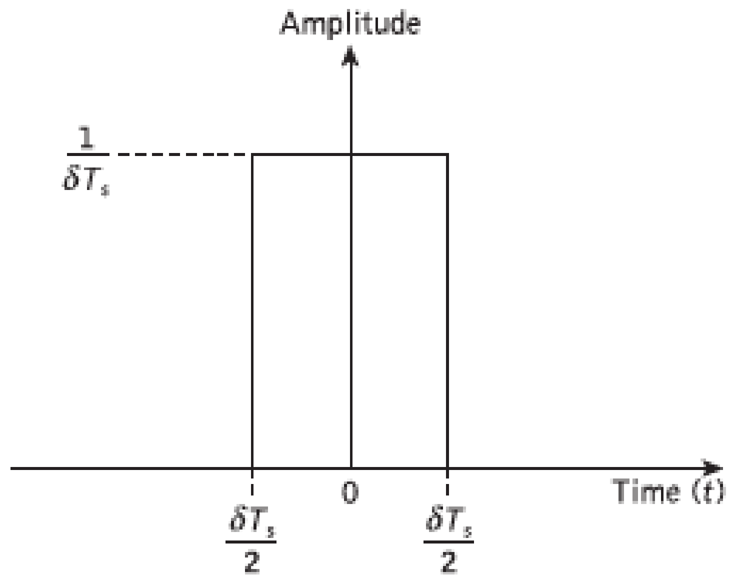


Figure 3.12 An illustration of the light input to the multimode step index fiber consisting of an ideal pulse or rectangular function with unit area

It may be noted that $p_i(t)$ has a constant amplitude of $1/\delta T_s$ over the range:

$$\frac{-\delta T_s}{2} \leq p(t) \leq \frac{\delta T_s}{2}$$

The rms pulse broadening at the fiber output due to intermodal dispersion for the multimode step index fiber σ_s (i.e. the standard deviation) may be given in terms of the variance σ_s^2 as (see Appendix C):

$$\sigma_s^2 = M_2 - M_1^2 \tag{3.30}$$

where M_1 is the first temporal moment which is equivalent to the mean value of the pulse and M_2, the second temporal moment, is equivalent to the mean square value of the pulse. Hence:

$$M_1 = \int_{-\infty}^{\infty} t p_i(t)\, \mathrm{d}t \tag{3.31}$$

and:

$$M_2 = \int_{-\infty}^{\infty} t^2 p_i(t)\, \mathrm{d}t \tag{3.32}$$

The mean value M_1 for the unit input pulse of Figure 3.12 is zero, and assuming this is maintained for the output pulse, then from Eqs (3.30) and (3.32):

$$\sigma_s^2 = M_2 = \int_{-\infty}^{\infty} t^2 p_i(t)\, \mathrm{d}t \tag{3.33}$$

Integrating over the limits of the input pulse (Figure 3.12) and substituting for $p_i(t)$ in Eq. (3.33) over this range gives:

$$\sigma_s^2 = \int_{-\delta T_s/2}^{\delta T_s/2} \frac{1}{\delta T_s} t^2 \, dt$$

$$= \frac{1}{\delta T_s}\left[\frac{t^3}{3}\right]_{-\delta T_s/2}^{\delta T_s/2} = \frac{1}{3}\left(\frac{\delta T_s}{2}\right)^2 \qquad (3.34)$$

Hence substituting from Eq. (3.27) for δT_s gives:

$$\sigma_s \simeq \frac{L n_1 \Delta}{2\sqrt{3}c} \simeq \frac{L(NA)^2}{4\sqrt{3}n_1 c} \qquad (3.35)$$

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Example 3.8

A 6 km optical link consists of multimode step index fiber with a core refractive index of 1.5 and a relative refractive index difference of 1%. Estimate:

(a) the delay difference between the slowest and fastest modes at the fiber output;

(b) the rms pulse broadening due to intermodal dispersion on the link;

(c) the maximum bit rate that may be obtained without substantial errors on the link assuming only intermodal dispersion;

(d) the bandwidth–length product corresponding to (c).

Solution: (a) The delay difference is given by Eq. (3.27) as:

$$\delta T_s \simeq \frac{Ln_1\Delta}{c} = \frac{6 \times 10^3 \times 1.5 \times 0.01}{2.998 \times 10^8}$$

$$= 300 \text{ ns}$$

(b) The rms pulse broadening due to intermodal dispersion may be obtained from Eq. (3.35) where:

$$\sigma_s = \frac{Ln_1\Delta}{2\sqrt{3}c} = \frac{1}{2\sqrt{3}}\,\frac{6\times 10^3 \times 1.5 \times 0.01}{2.998\times 10^8}$$

$$= 86.7 \text{ ns}$$

(c) The maximum bit rate may be estimated in two ways. Firstly, to get an idea of the maximum bit rate when assuming no pulse overlap, Eq. (3.10) may be used where:

$$B_T(\max) = \frac{1}{2\tau} = \frac{1}{2\delta T_s} = \frac{1}{600\times 10^{-9}}$$

$$= 1.7 \text{ Mbit s}^{-1}$$

Alternatively an improved estimate may be obtained using the calculated rms pulse broadening in Eq. (3.11) where:

$$B_T(\max) = \frac{0.2}{\sigma_s} = \frac{0.2}{86.7\times 10^{-9}}$$

$$= 2.3 \text{ Mbit s}^{-1}$$

(d) Using the most accurate estimate of the maximum bit rate from (c), and assuming return to zero pulses, the bandwidth–length product is:

$$B_{opt} \times L = 2.3 \text{ MHz} \times 6 \text{ km} = 13.8 \text{ MHz km}$$

Multimode graded index fiber

$$\sigma_g = \frac{Ln_1\Delta^2}{20\sqrt{3}c}$$

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Example 3.9

Compare the rms pulse broadening per kilometer due to intermodal dispersion for the multimode step index fiber of Example 3.8 with the corresponding rms pulse broadening for an optimum near-parabolic profile graded index fiber with the same core axis refractive index and relative refractive index difference.

Solution: In Example 3.8, σ_s over 6 km of fiber is 86.7 ns. Hence the rms pulse broadening per kilometer for the multimode step index fiber is:

$$\frac{\sigma_s(1\text{ km})}{L} = \frac{86.7}{6} = 14.4\text{ ns km}^{-1}$$

Using Eq. (3.42), the rms pulse broadening per kilometer for the corresponding graded index fiber is:

$$\sigma_g(1\text{ km}) = \frac{Ln_1\Delta^2}{20\sqrt{3}c} = \frac{10^3 \times 1.5 \times (0.01)^2}{20\sqrt{3} \times 2.998 \times 10^8}$$

$$= 14.4\text{ ps km}^{-1}$$

Example 3.10

A multimode step index fiber has a numerical aperture of 0.3 and a core refractive index of 1.45. The material dispersion parameter for the fiber is 250 ps nm^{-1} km^{-1} which makes material dispersion the totally dominating chromatic dispersion mechanism. Estimate (a) the total rms pulse broadening per kilometer when the fiber is used with an LED source of rms spectral width 50 nm and (b) the corresponding bandwidth–length product for the fiber.

Solution: (a) The rms pulse broadening per kilometer due to material dispersion may be obtained from Eq. (3.18), where:

$$\sigma_m(1\ \text{km}) \simeq \frac{\sigma_\lambda L \lambda}{c} \left| \frac{d^2 n_1}{d\lambda^2} \right| = \sigma_\lambda L M = 50 \times 1 \times 250\ \text{ps km}^{-1}$$
$$= 12.5\ \text{ns km}^{-1}$$

The rms pulse broadening per kilometer due to intermodal dispersion for the step index fiber is given by Eq. (3.35) as:

$$\sigma_s(1\ \text{km}) \simeq \frac{L(NA)^2}{4\sqrt{3}n_1c} = \frac{10^3 \times 0.09}{4\sqrt{3} \times 1.45 \times 2.998 \times 10^8}$$

$$= 29.9\ \text{ns km}^{-1}$$

The total rms pulse broadening per kilometer may be obtained using Eq. (3.43), where $\sigma_c \simeq \sigma_m$ as the waveguide dispersion is negligible and $\sigma_n = \sigma_s$ for the multimode step index fiber. Hence:

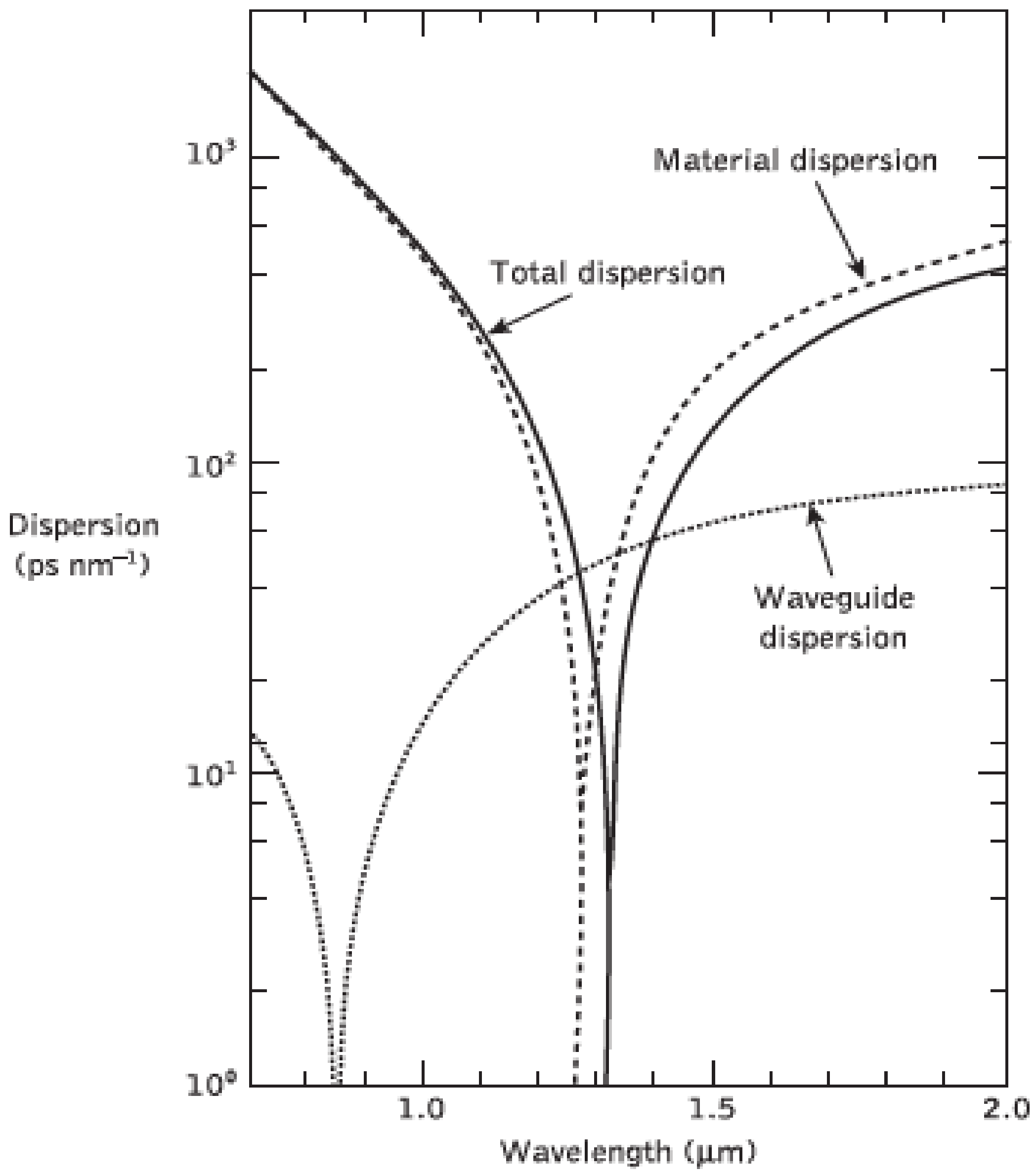
$$\sigma_T = (\sigma_m^2 + \sigma_s^2)^{\frac{1}{2}} = (12.5^2 + 29.9^2)^{\frac{1}{2}}$$

$$= 32.4\ \text{ns km}^{-1}$$

(b) The bandwidth–length product may be estimated from the relationship given in Eq. (3.11) where:

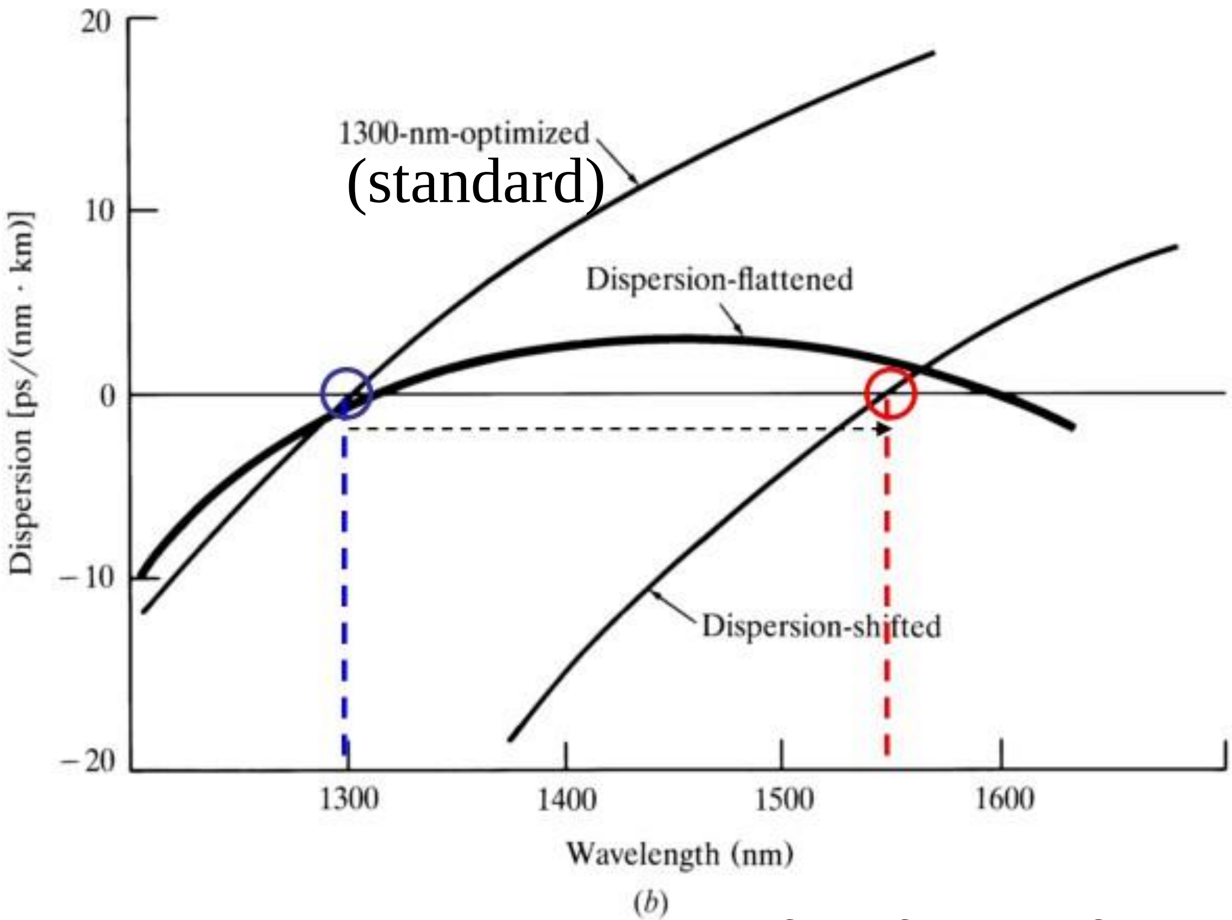
$$B_{opt} \times L = \frac{0.2}{\sigma_T} = \frac{0.2}{32.4 \times 10^{-9}}$$

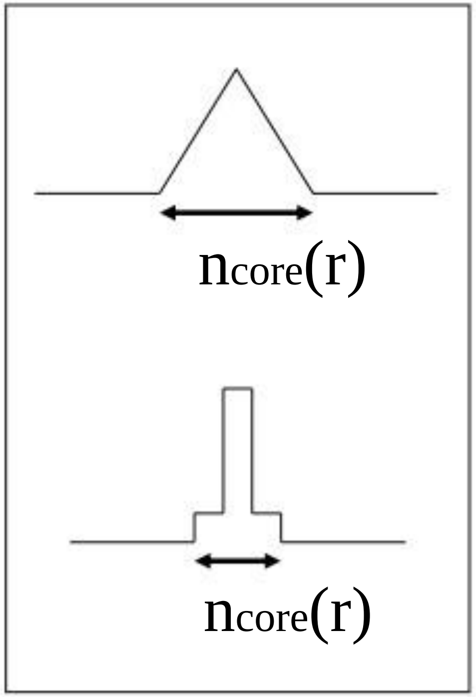
$$= 6.2\ \text{MHz km}$$



Material dispersion
Total dispersion
Waveguide
dispersion
Dispersion
(ps nm^{-1})
10^3
10^2
10^1
10^0
1.0
1.5
2.0
Wavelength (µm)

Dispersion-shifted and flattened fibers





- The design of dispersion-modified fibers often involves the use of multiple cladding layers and a tailoring of the refractive index profile.

Any Questions ???

Thank You !!