

$$\therefore R(\alpha_i | w_j) = \lambda_s (1 - P(w_i | x)) \rightarrow \textcircled{4}$$

Similarly for  $i = c+1$

$$R(\alpha_i | w_j) = \lambda_{ij} \mathbb{I} = \lambda_r \rightarrow \textcircled{5}$$

To ~~minimize~~  $R(\alpha_i | w_j)$ ,  $[1 - P(w_i | x)]$  needs to be really small. That means we need to maximize  $P(w_i | x)$

So the minimum risk is achieved by taking:

$$R(\alpha_i | x) \leq R(\alpha_{c+1} | x)$$

$$\therefore \lambda_s (1 - P(w_i | x)) \leq \lambda_r$$

$$\therefore 1 - P(w_i | x) \leq \frac{\lambda_r}{\lambda_s}$$

$$\therefore \boxed{P(w_i | x) \geq 1 - \frac{\lambda_r}{\lambda_s}}$$

Also, If  $R(\alpha_i | x) \leq R(\alpha_{c+1} | x)$  &  $\lambda = 0$  for  $i = j$ ,

$$\text{then } \boxed{P(w_i | x) \geq P(w_j | x)}$$

→ CASE 1 :  $\lambda_r = 0$

For case where Rejection Loss is 0 :-

$$1 - \frac{\lambda_r}{\lambda_s} \leq P(w_i | x)$$

$$1 - \frac{0}{\lambda_s} \leq P(w_i | x)$$

$$\Rightarrow P(w_i | x) \geq 1$$

i.e. we take  $i$  as  $c+1$  ; or, that we reject the class ~~at~~ Rejection leads to 0 Loss and higher probability.

→ CASE 2 :  $\lambda_r > \lambda_s$

When  $\lambda_r > \lambda_s$ , then  $\lambda_r / \lambda_s > 1$

$$\Rightarrow 1 - \frac{\lambda_r}{\lambda_s} \text{ is negative } \therefore P(w_i | x) \geq (\text{negative})$$

i.e. Rejection Loss is greater than substitution loss, so we don't reject.