HOMEWORK 2

1.] Given :-

$$\lambda(\alpha_i | \omega_j) = \begin{cases} 0 & i=j \quad\quad i,j = 1,\dots c \\ \lambda_r & i = c+1 \\ \lambda_s & \text{otherwise} \end{cases} \quad \rightarrow (1)$$

Problem :- Prove minimum risk is obtained if we decide ω_i, if $P(\omega_i|x) \geq P(\omega_j|x)$ for all j and $P(\omega_i|x) \geq 1 - \lambda_r/\lambda_s$

Solution :- Formula for Risk of a decision for class i given that the label is j is as follows:

$$R(\alpha_i|\omega_j) = \sum_{j=1}^{c} \lambda_{ij} \times p(\omega_j|x) \quad \rightarrow (2)$$

→ Here λ_{ij} is the loss of picking i when the label is j.

$$\Rightarrow R(\alpha_i|\omega_j) = \lambda_{ii} \times p(\omega_i|x) + \sum_{j=1 \neq i}^{c} \lambda_{ij} \times p(\omega_j|x) \quad \rightarrow (3)$$

Here, λ_{ii} is the loss when $j = i \Rightarrow 0$.

$$\therefore R(\alpha_i|\omega_j) = \sum_{j=1 \neq i}^{c} \lambda_{ij} \times p(\omega_j|x)$$

$$= \sum_{j=1 \neq i}^{c} \lambda_s \, p(\omega_j|x)$$

Now, $p(\omega_j|x) = 1 - P(\omega_i|x)$ {For all $j = 1$ to c & $j \neq i$