HOMEWORK 2

1.] Given :-

$$\lambda(\alpha_i | \omega_j) = \begin{cases} 0 & i = j \quad i,j = 1, \dots c \\ \lambda_r & i = c+1 \\ \lambda_s & \text{otherwise} \end{cases} \rightarrow (1)$$

Problem :- Prove minimum risk is obtained if we decide ω_i, if $P(\omega_i | x) \geq P(\omega_j | x)$ for all j and $P(\omega_i | x) \geq 1 - \lambda_r / \lambda_s$

Solution :- Formula for Risk of a decision for class i given that the label is j is as follows:

$$R(\alpha_i | \omega_j) = \sum_{j=1}^{c} \lambda_{ij} \times p(\omega_j | x) \rightarrow (2)$$

→ Here λ_{ij} is the loss of picking i when the label is j.

$$\Rightarrow R(\alpha_i | \omega_j) = \lambda_{ii} \times p(\omega_i | x) + \sum_{j=1 \neq i}^{c} \lambda_{ij} \times p(\omega_j | x) \rightarrow (3)$$

Here, λ_{ii} is the loss when $j = i \Rightarrow 0$.

$$\therefore R(\alpha_i | \omega_j) = \sum_{j=1 \neq i}^{c} \lambda_{ij} \times p(\omega_j | x)$$

$$= \sum_{j=1 \neq i}^{c} \lambda_s \, p(\omega_j | x)$$

Now, $p(\omega_j | x) = 1 - P(\omega_i | x)$ {For all $j = 1$ to c & $j \neq i$

$\therefore \; R(\alpha_i | w_j) = \lambda_s (1 - P(w_i | x)) \rightarrow (4)$

Similarly for $i = c+1$

$$R(\alpha_i | w_j) = \lambda_{ij} = \lambda_r \rightarrow (5)$$

To ~~maximize~~ minimize $R(\alpha_i | w_j)$, $[1 - P(w_i | x)]$ needs to be really small. That means we need to maximize $P(w_i | x)$

So the minimum risk is achieved by taking:

$$R(\alpha_i | x) \leq R(\alpha_{c+1} | x)$$

$$\therefore \; \lambda_s (1 - P(w_i | x)) \leq \lambda_r$$

$$\therefore \; 1 - P(w_i | x) \leq \frac{\lambda_r}{\lambda_s}$$

$$\therefore \; \boxed{P(w_i | x) \geq 1 - \frac{\lambda_r}{\lambda_s}}$$

Also, If $R(\alpha_i | x) \leq R(\alpha_{c+1}, x)$, & $\lambda = 0$ for $i = j$,

then $\boxed{P(w_i | x) \geq P(w_j | x)}$

→ CASE 1 : $\lambda_r = 0$

For case where Rejection Loss is 0 :-

$$1 - \frac{\lambda_r}{\lambda_s} \le P(\omega_i | x)$$

$$1 - \frac{0}{\lambda_s} \le P(\omega_i | x)$$

$$\Rightarrow P(\omega_i | x) \ge 1$$

i.e. we take i as C+1 ; or, that we reject the class ~~as~~ Rejection leads to 0 Loss and higher probability.

→ CASE 2 : $\lambda_r > \lambda_s$

When $\lambda_r > \lambda_s$, then $\lambda_r / \lambda_s > 1$

$\Rightarrow 1 - \frac{\lambda_r}{\lambda_s}$ is negative $\therefore P(\omega_i | x) \ge$ (negative)

i.e. Rejection Loss is greater than substitution loss, so we don't reject.

2.] Question 2 is solved on Jupyter Notebook and the code along with explanation is uploaded on my github repository here:

https://github.com/rishabhgks/EECE_5644/blob/master/eece5644_hw2/EECE5644_Homework2_Q2.ipynb

3.] Question 3 is solved on Jupyter Notebook and the code along with explanation is uploaded on my github repository here:

https://github.com/rishabhgks/EECE_5644/blob/master/eece5644_hw2/EECE5644_Homework2_Q3.ipynb