

Four State Model

In this model 4 states are defined in state-1 consists of 0 e-h pair, state-2 consists of 1 e-h pair, state-3 consists of 2 e-h pairs and the state-4 is the trap state. τ_1 is the rate of transition from state 2 to 1, τ_2 is the rate of transition from state 3 to 2, τ is rate of transition from state 4 to 1, and τ_a is rate of transition from 3 to 4. W_1 and W_2 are rates of transitions from state 1 to 2 and 2 to 3 respectively.

The following mathematical modelling is done to reproduce the results obtained in [Al. L. Efros and M. Rosen, Phys. Rev. Lett. 78, 1110](#) (1997)

The graphs obtained in the paper are

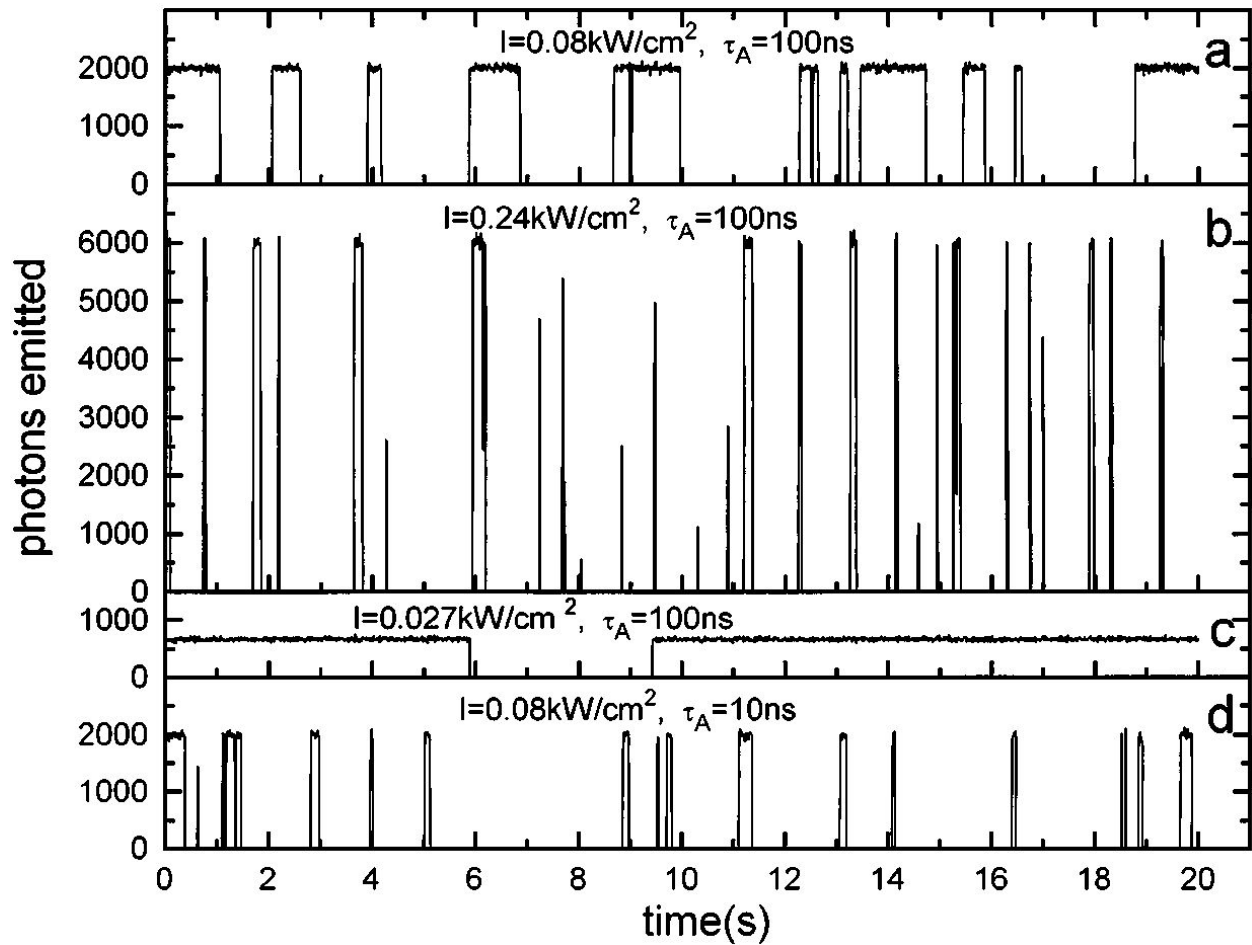


FIG. 1. The number of photons emitted in the simulation in 10 ms intervals by a single QD. We use t_1 4 ns, t_2 1 ns, t 0.8 s, s_1 s_2 1.0 3 10215 cm^2 , $h\nu$ 2.5 eV.

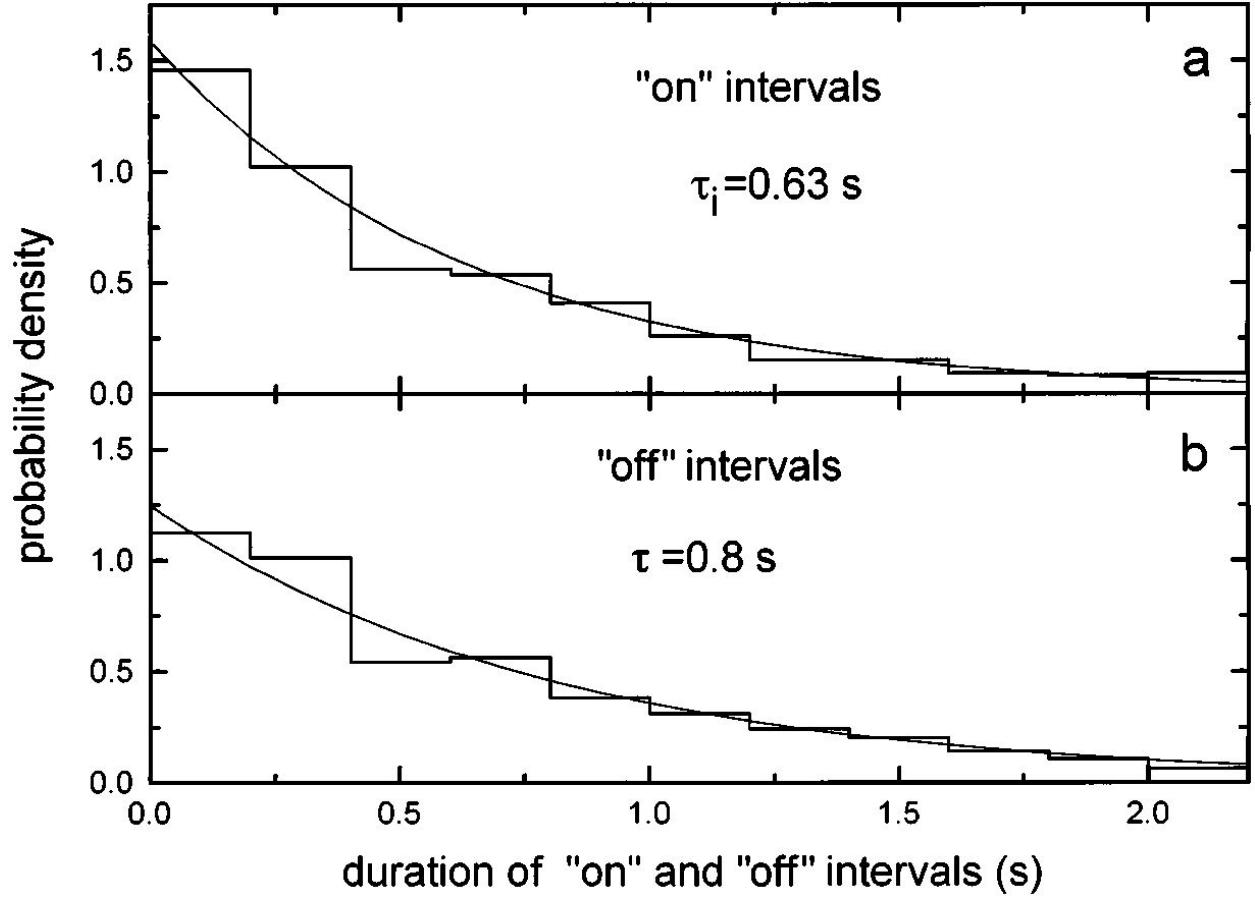


FIG. 2. Histogram of the (a) on and (b) off periods obtained in the Monte Carlo simulations of the photoluminescence random telegraph signal. We use time bins 0.2 s in width. The solid lines show the theoretical dependence described by Eq. (8). Both the simulations and the calculations used the same parameters as in Fig. 1(a).

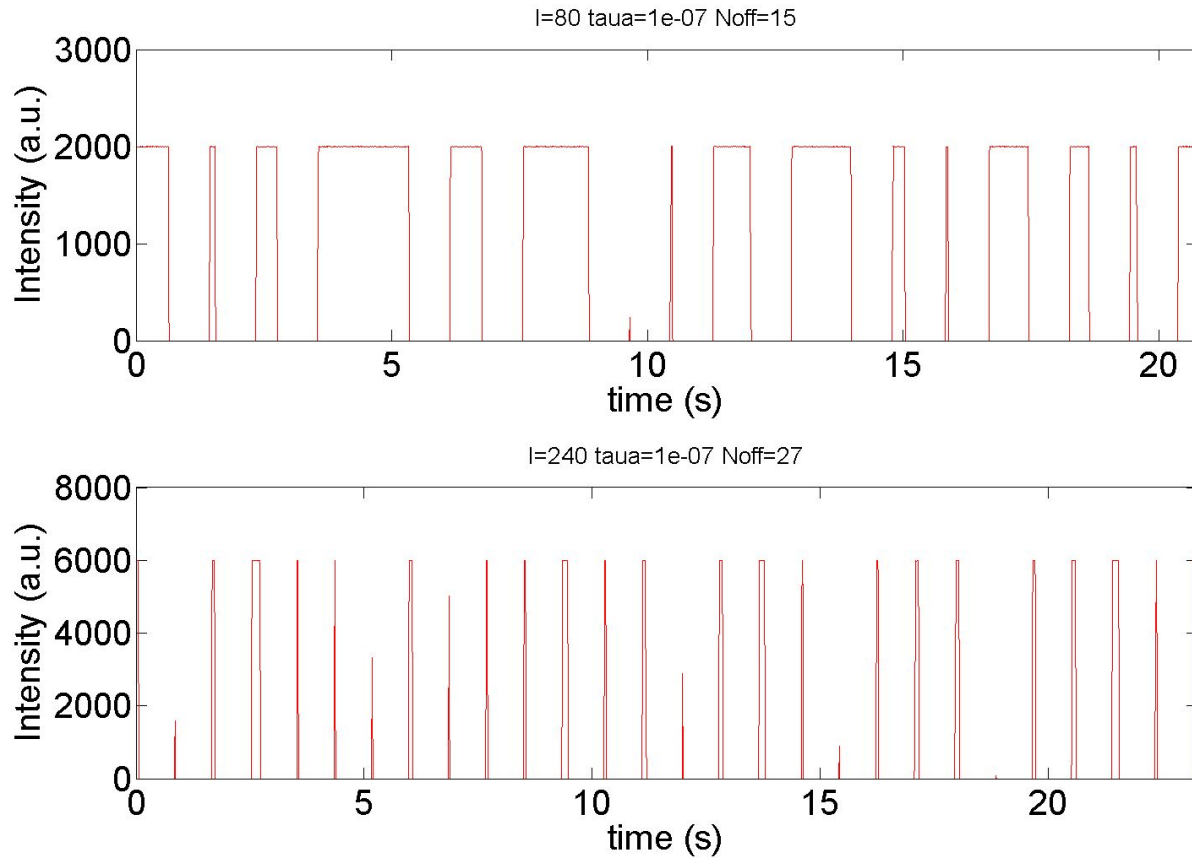
The following equations gives the probability of finding the electron in each state. $P_{0,1,2,+}$ is the probability of finding the electron in state 1,2,3,4 respectively.

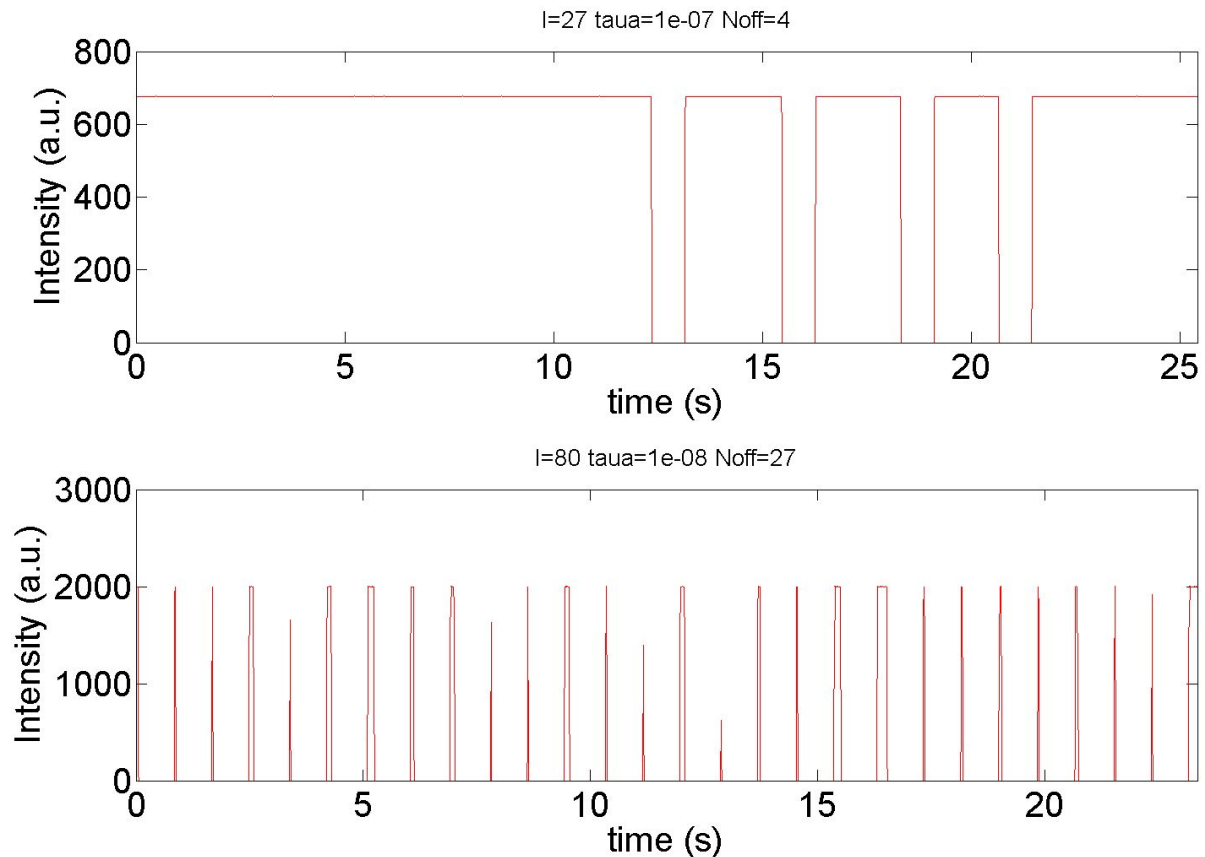
$$\begin{aligned}
 \dot{P}_0 &= -W_1 P_0 + \frac{P_1}{\tau_1}, \\
 \dot{P}_1 &= W_1 P_0 - \left(W_2 + \frac{1}{\tau_1} \right) P_1 + \frac{P_2}{\tau_2} + \frac{P_+}{\tau}, \\
 \dot{P}_2 &= W_2 P_1 - \left(\frac{1}{\tau_2} + \frac{1}{\tau_A} \right) P_2, \quad \dot{P}_+ = \frac{P_2}{\tau_A} - \frac{P_+}{\tau},
 \end{aligned} \tag{1}$$

Model-1

In this model the increment in time(t_{inc}) in each state is given by $1/\Sigma rates$.
The probability of electron for going in a particular state is $rate/\Sigma rates$.

Results:





Code:

```
clear;
clc;
close all;
tic
tau1=4e-9; %for transition from state 2 to 1
tau2=1e-9; %for transition from state 3 to 2
tau=0.8; %for transition from state 4 to 1
taua=100e-9; %for transition from state 3 to 4
s1=1e-15; %cross section of NC
s2=1e-15; %cross section of NC
h=2.5*1.6e-19;
I=0.08e3; %intensity
W1=s1*I/h; %rate of transition from state 1 to 2
W2=s2*I/h; %rate of transition from state 2 to 3
bin_size=10e-3;
trap=0; %count total number of trap states(3 to 4)
on=0; %count total number of on states(2 to 1)
```

```

i=3;    %index for array
n=800000;%total number of iterations
t(1)=0; %time array
next(1)=1;%stores the sequence of states with initial state as 1
on(1)=0;
[next(2), tinc(1),tr,on1]=state(next(1), W1,W2,tau1,tau2,taua,tau);
%next state is determined
on(2)=on1;
t(2)=t(1)+tinc(1);
while i<=n
    [next(i),
tinc(i-1),tr,on1]=state(next(i-1),W1,W2,tau1,tau2,taua,tau);
%iteratively next state is determined
    trap=trap+tr;
    on(i)=on1;
    t(i)=t(i-1)+tinc(i-1);
    i=i+1;
end
n2=fix(max(t)/bin_size);    %number of bins formed
binranges=0:bin_size:max(t);    %stores the various bins
[bincounts, ind]=histc(t,binranges);% bincounts stores the number of
indices of t array in each bin
sr=1;
for y=1:n2 %binning method
    on2(y)= sum(on(sr:bincounts(y)+sr));
    sr=bincounts(y)+1;
end
t1 = 0:bin_size:(n2-1)*bin_size;
c = [[0,0,0];[0.7,0.7,0.7];[1,0,0];[0,1,0];[0,0,1]];
line_style = {'-','--','-.-','-'};
plot(t1,on2,'Color', [c(3,:)],'LineStyle',line_style{1});
xlim([0, max(t)]);

%h_legend = legend('3 level','Location','NorthEast');
title(['I=',num2str(I), ' taua=',num2str(taua), '
Noff=',num2str(trap)], 'fontsize',16);
% set(h_legend,'fontsize',16, 'box', 'off');
xlabel('time (s)','fontsize',24);

```

```

ylabel('Intensity (a.u.)','fontsize',24);
set(gca, 'FontSize',24);

figname_png = ['Intensity and time for 4 state model 1.png'];
figname = ['Intensity and time for 4 state model 4'];
width = 30;
height = 10;
set(gcf, 'PaperPositionMode', 'manual');
set(gcf, 'PaperSize', [width height]);
set(gcf, 'PaperPosition', [0 0 width height]);
set(gca,'position',[0.1 0.19 .85 .7]);% specify these as the
fraction of the total.. between 0 and 1

print('-dpng','-r125',figname_png);
j=1;
for k=1:length(on2)
    if(on2(k)>mean(on2))
        on22(k)=1;
    else
        on22(k)=0;
    end
end
for i=1:length(t1)-1
    if(on22(i)-on22(i+1)>0)
        flag(j)=0;
        cr(j)=t1(i);
        j=j+1;
    end
    if(on22(i)-on22(i+1)<0)
        flag(j)=1;
        cr(j)=t1(i);
        j=j+1;
    end
end
end
b=1;
c=2;
ton(1)=cr(1);
for a=1:length(cr)-1

```

```

    if(flag(a)==0 && flag(a+1)==1)
        toff(b)=cr(a+1)-cr(a);
        b=b+1;
    end
    if(flag(a)==1 && flag(a+1)==0)
        ton(c)=cr(a+1)-cr(a);
        c=c+1;
    end
end
binranges=0.1:0.2:max(ton);    %stores the various bins
figure();
hist(ton,binranges)% bincounts stores the number of indices of t
array in each bin
title('Ton','fontsize',16);
% set(h_legend,'fontsize',16, 'box', 'off');
xlabel('duration of interval','fontsize',24);
ylabel('Probabilty density','fontsize',24);
set(gca, 'FontSize',24);
figname_png1 = ['on histogram.png'];
figname1 = ['on histogram'];
width = 20;
height = 10;
set(gcf, 'PaperPositionMode', 'manual');
set(gcf, 'PaperSize', [width height]);
set(gcf, 'PaperPosition', [0 0 width height]);
set(gca,'position',[0.1 0.19 .85 .7]);% specify these as the fraction
of the total.. between 0 and 1
print('-dpng','-r125',figname_png1);
binranges=0.1:0.2:max(toff);    %stores the various bins
figure();
hist(toff,binranges)% bincounts stores the number of indices of t
array in each bin
title('Toff','fontsize',16);
% set(h_legend,'fontsize',16, 'box', 'off');
xlabel('duration of interval','fontsize',24);
ylabel('Probabilty density','fontsize',24);
set(gca, 'FontSize',24);
figname_png2 = ['off histogram.png'];

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```

figname2 = ['off histogram'];
width = 20;
height = 10;
set(gcf, 'PaperPositionMode', 'manual');
set(gcf, 'PaperSize', [width height]);
set(gcf, 'PaperPosition', [0 0 width height]);
set(gca, 'position', [0.1 0.19 .85 .7]); % specify these as the fraction
of the total.. between 0 and 1
print('-dpng', '-r125', figname_png2);
toc

```

Function: state

```

function [next, tinc, tr, on1] = state(st, W1, W2, tau1, tau2, taua, tau)
tr=0;
on1=0;
if st==1
    tinc=1/(W1);
    num=rand();
    if(num<W1*tinc)
        next=2;
    end
end
if st==2
    tinc=1/(W2+(1/tau1));
    num=rand();
    if(num<W2*tinc)
        next=3;
    else
        next=1;
        on1=1;
    end
end
if st==3
    tinc=1/((1/tau2)+(1/taua));
    num=rand();
    if(num<(1/tau2)*tinc)
        next=2;
    end
end

```



```

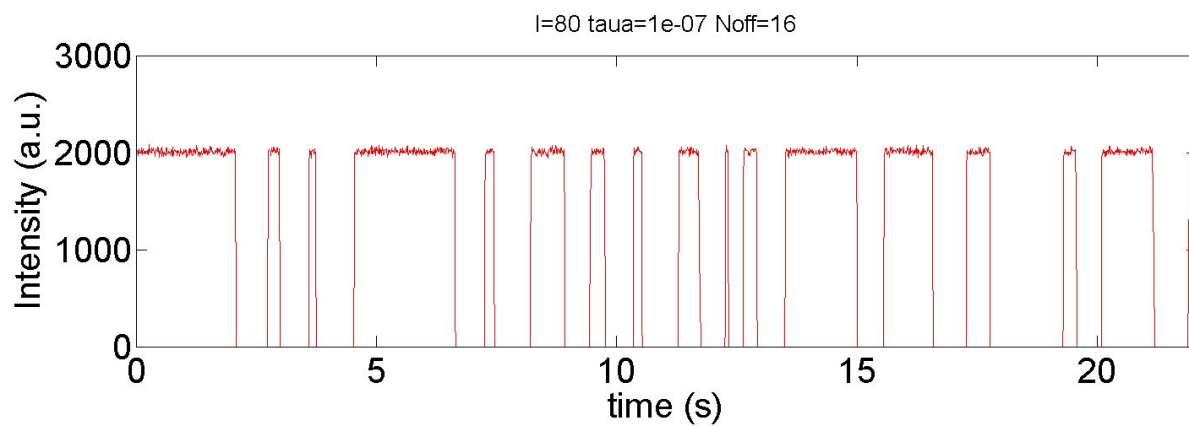
else
    next=4;
    tr=1;
end
end
if st==4
    tinc=1/(1/tau);
    num=rand();
    if(num<(1/tau)*tinc)
        next=2;
    end
end
end
end

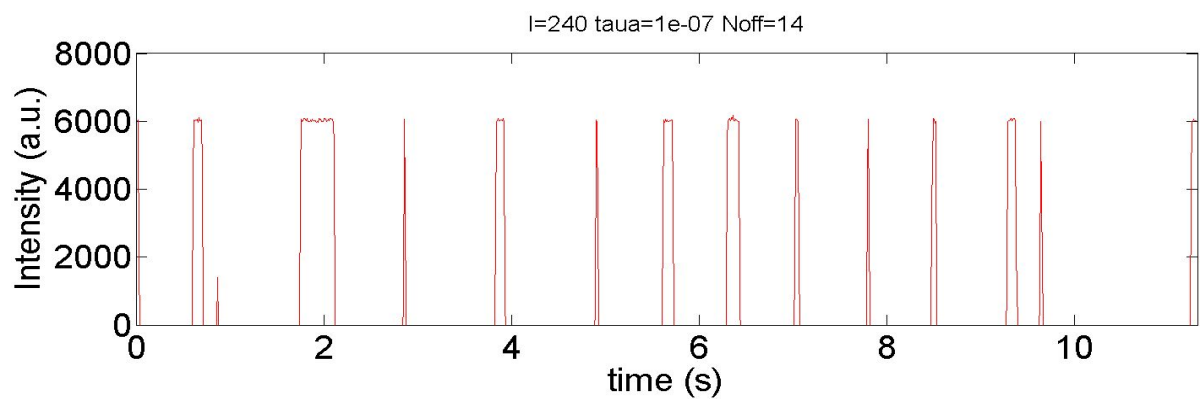
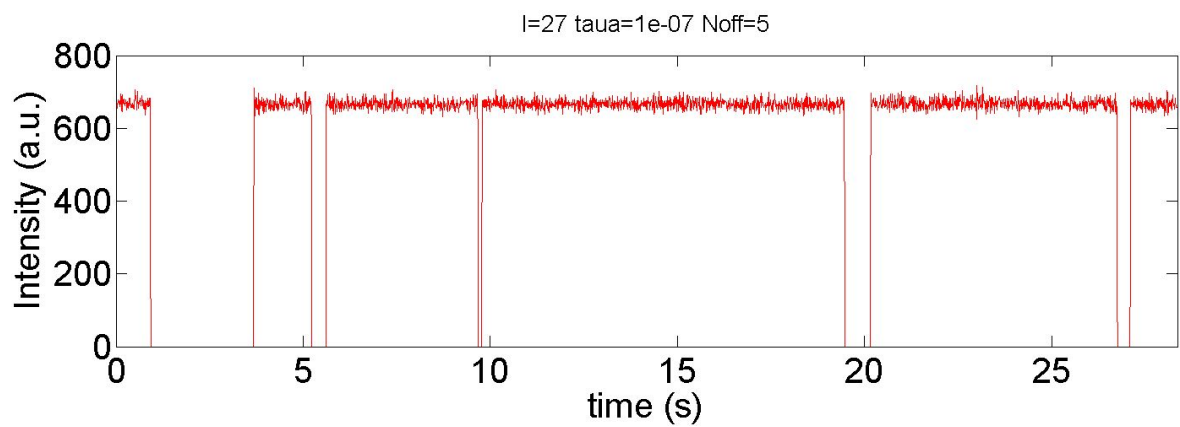
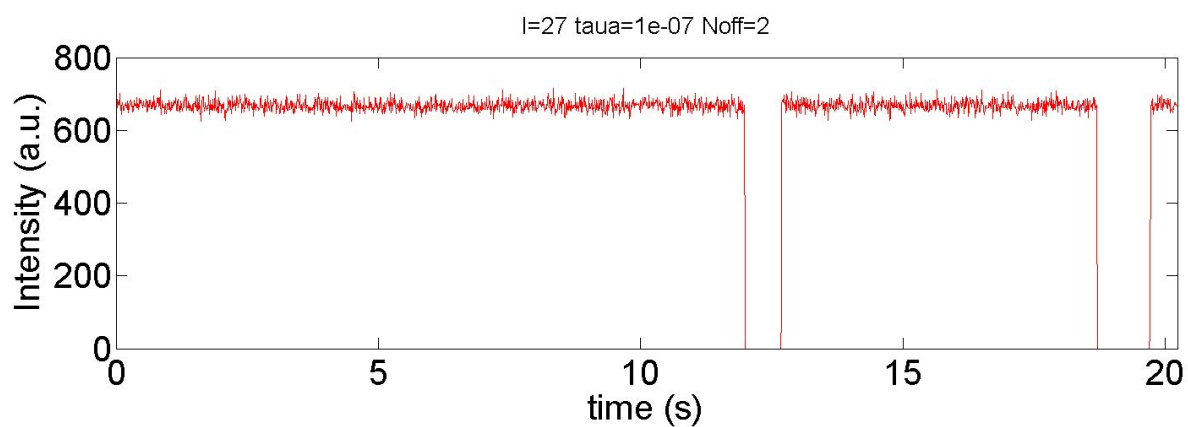
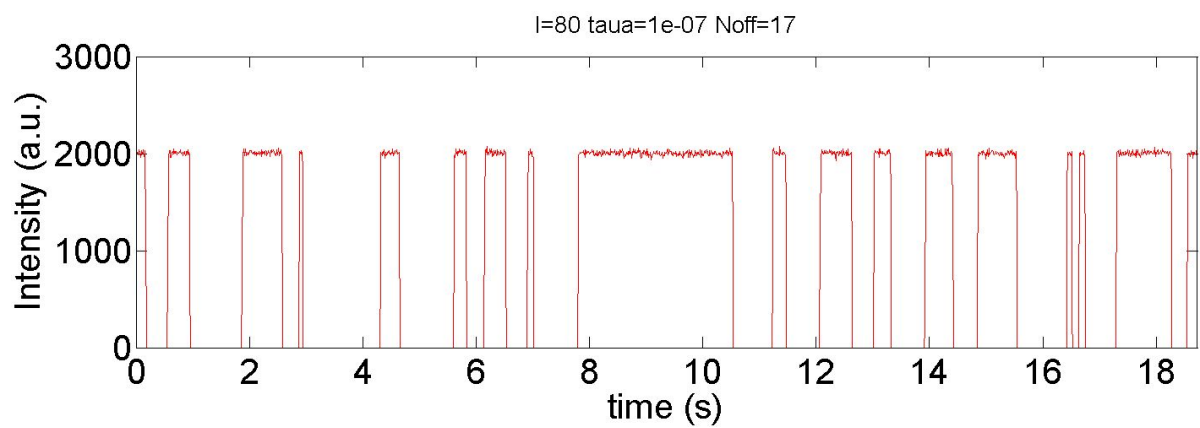
```

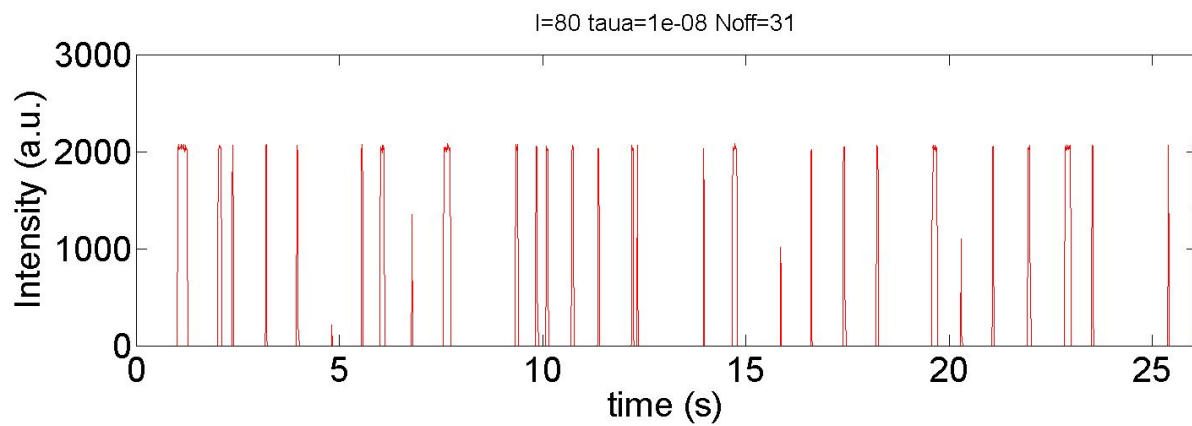
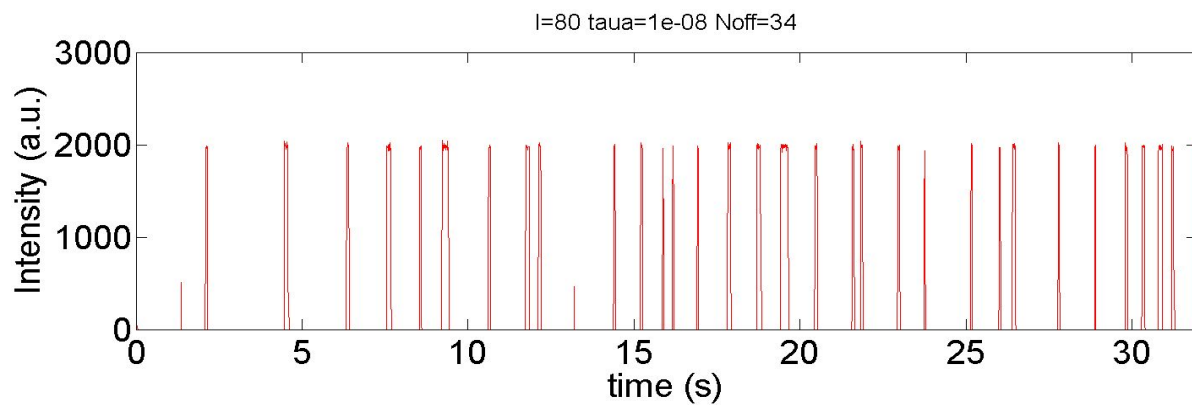
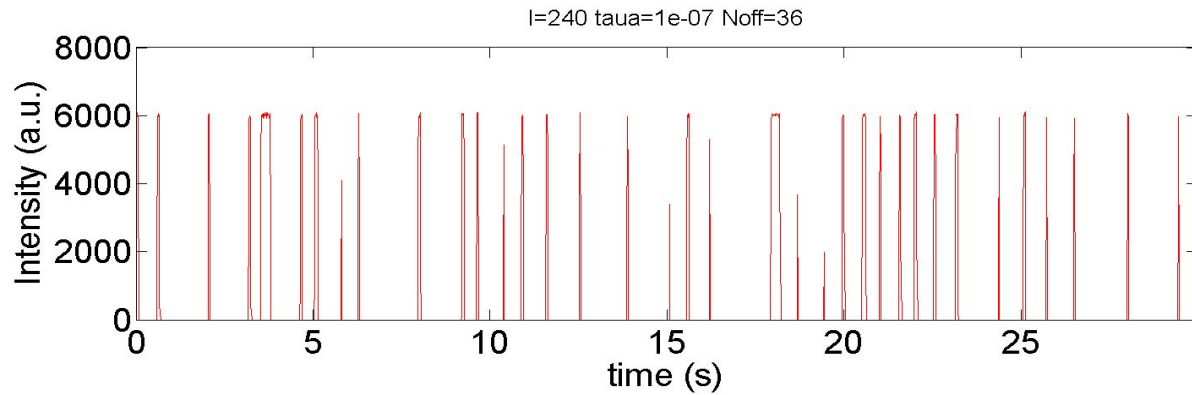
Model-2

In this model the increment in time(tinc) in each state is given by $f/\Sigma \text{rates}$ where f is a random number. The probability of electron for going in a particular state is $\text{rate} \cdot \text{tinc}$.

Results:







Code:

```
clear;
clc;
close all;
tic
tau1=4e-9; %for transition from state 2 to 1
tau2=1e-9; %for transition from state 3 to 2
tau=0.8; %for transition from state 4 to 1
```

```

taua=100e-9; %for transition from state 3 to 4
s1=1e-15;    %cross section of NC
s2=1e-15;    %cross section of NC
h=2.5*1.6e-19;
I=0.08e3;    %intensity
W1=s1*I/h;   %rate of transition from state 1 to 2
W2=s2*I/h;   %rate of transition from state 2 to 3
bin_size=10e-3;
trap=0; %count total number of trap states(3 to 4)
on=0;    %count total number of on states(2 to 1)
i=2;    %index for array
n=5e8;%total number of iterations
t(1)=0; %time array
next(1)=1;%stores the sequence of states with initial state as 1
on(1)=0;

while i<=n
    [next(i),
    tinc(i-1),tr,on1]=state_n(next(i-1),W1,W2,tau1,tau2,taua,tau);
    %iteratively next state is determined
    trap=trap+tr;
    on(i)=on1;
    t(i)=t(i-1)+tinc(i-1);
    i=i+1;
end
n2=fix(max(t)/bin_size);          %number of bins formed
binranges=0:bin_size:max(t);      %stores the various bins
[bincounts, ind]=histc(t,binranges);% bincounts stores the number of
indices of t array in each bin
sr=1;
for y=1:n2 %binning method
    on2(y)= sum(on(sr:bincounts(y)+sr));
    sr=bincounts(y)+1;
end
t1 = 0:bin_size:(n2-1)*bin_size;
c = [[0,0,0];[0.7,0.7,0.7];[1,0,0];[0,1,0];[0,0,1]];
line_style = {'-','--','-.-','-'};
plot(t1,on2,'Color', [c(3,:)], 'LineStyle',line_style{1});

```

```

xlim([0, max(t)]);

%h_legend = legend('3 level','Location','NorthEast');
title(['I=',num2str(I),' taua=',num2str(taua),'
Noff=',num2str(trap)],'fontsize',16);
%set(h_legend,'fontsize',16, 'box', 'off');
xlabel('time (s)','fontsize',24);
ylabel('Intensity (a.u.)','fontsize',24);
set(gca, 'FontSize',24);

filename_png = ['Intensity and time for 4 state model 8.png'];
filename = ['Intensity and time for 4 state model 8'];
width = 30;
height = 10;
set(gcf, 'PaperPositionMode', 'manual');
set(gcf, 'PaperSize', [width height]);
set(gcf, 'PaperPosition', [0 0 width height]);
set(gca,'position',[0.1 0.19 .85 .7]);% specify these as the
fraction of the total.. between 0 and 1

print('-dpng','-r125',filename_png);

on2=zeros(1,length(on2));
on2(on2>mean(on2))=1;
figure()
subplot(4,1,1)
plot(t1,on2,'o')
on23=on2(2:length(on2));
subplot(4,1,2)
plot(t1(2:length(t1)),on23,'o')
diff=on2(1:length(on2)-1)-on23;
subplot(4,1,3)
plot(t1(2:length(t1)),diff,'o')
A=find(diff==1);
B=find(diff==-1);
toff=t1(B)-t1(A);
ton=t1(A(2:length(A)))-t1(B(1:length(B)-1));
j=1;

```

```

binranges=0:0.2:max(ton);    %stores the various bins
figure();
histogram(ton,binranges,'Normalization','pdf')% bincounts stores the
number of indices of t array in each bin
    xlim([0, 4]);

title('Ton','fontsize',16);
%    set(h_legend,'fontsize',16, 'box', 'off');
xlabel('duration of interval','fontsize',24);
ylabel('Probabilty density','fontsize',24);
set(gca, 'FontSize',24);
figname_png1 = ['on histogram8.png'];
figname1 = ['on histogram8'];
width = 20;
height = 10;
set(gcf, 'PaperPositionMode', 'manual');
set(gcf, 'PaperSize', [width height]);
set(gcf, 'PaperPosition', [0 0 width height]);
set(gca,'position',[0.1 0.19 .85 .7]);% specify these as the fraction
of the total.. between 0 and 1
print('-dpng','-r125',figname_png1);
binranges=0:0.2:max(toff);    %stores the various bins
figure();
histogram(toff,binranges,'Normalization','pdf')% bincounts stores the
number of indices of t array in each bin
    xlim([0, 4]);

title('Toff','fontsize',16);
%    set(h_legend,'fontsize',16, 'box', 'off');
xlabel('duration of interval','fontsize',24);
ylabel('Probabilty density','fontsize',24);
set(gca, 'FontSize',24);
figname_png2 = ['off histogram8.png'];
figname2 = ['off histogram8'];
width = 20;
height = 10;
set(gcf, 'PaperPositionMode', 'manual');

```

```

set(gcf, 'PaperSize', [width height]);
set(gcf, 'PaperPosition', [0 0 width height]);
set(gca,'position',[0.1 0.19 .85 .7]);% specify these as the fraction
of the total.. between 0 and 1
print('-dpng','-r125',figname_png2);
toc

```

Function:state_n

```

function [next, tinc,tr,on1] =state_n(st, W1,W2,tau1,tau2,taua,tau)
tr=0;
on1=0;
f=rand();
if st==1
    tinc=f/(W1);
    num=rand();
    if(num<W1*tinc)
        next=2;
    else
        next=1;
    end
end
if st==2
    tinc=f/(W2+(1/tau1));
    num=rand();
    if(num<W2*tinc)
        next=3;
    elseif (W2*tinc<=num && num<(W2+(1/tau1))*tinc)
        next=1;
        on1=1;
    else
        next=2;
    end
end
if st==3
    tinc=f/((1/tau2)+(1/taua));
    num=rand();
    if(num<(1/tau2)*tinc)

```

```

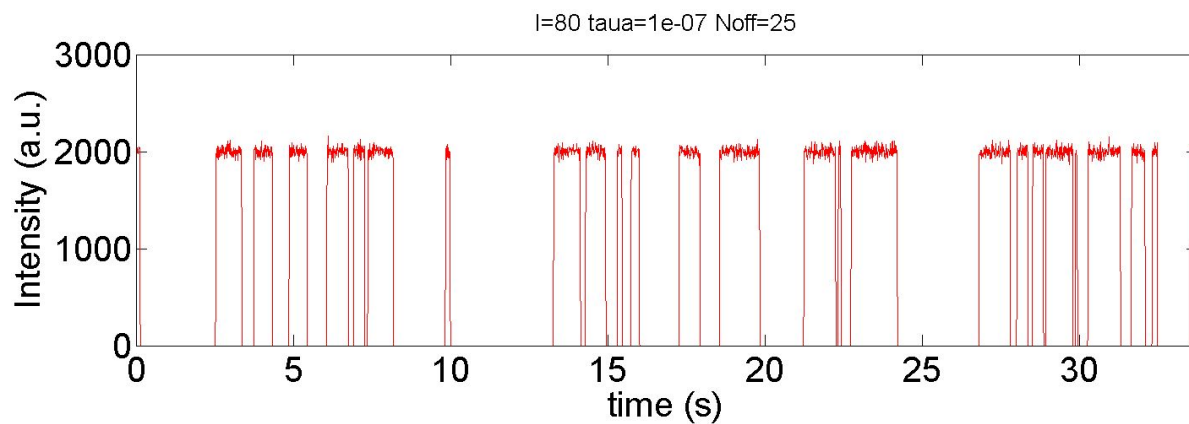
        next=2;
    elseif((1/tau2)*tinc<=num && num<((1/tau2)+(1/taua))*tinc)
        next=4;
        tr=1;
    else
        next=3;
    end
end
if st==4
    tinc=f/(1/tau);
    num=rand();
    if(num<(1/tau)*tinc)
        next=2;
    else
        next=4;
    end
end
end
end

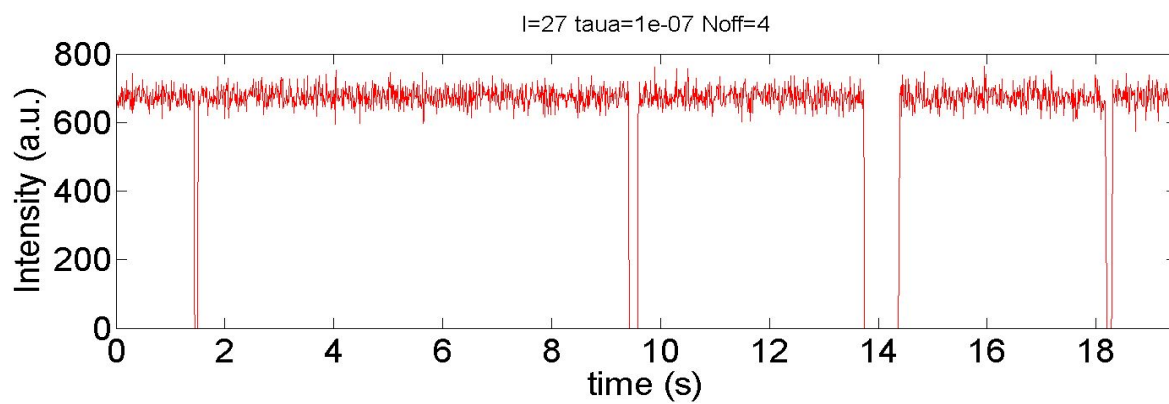
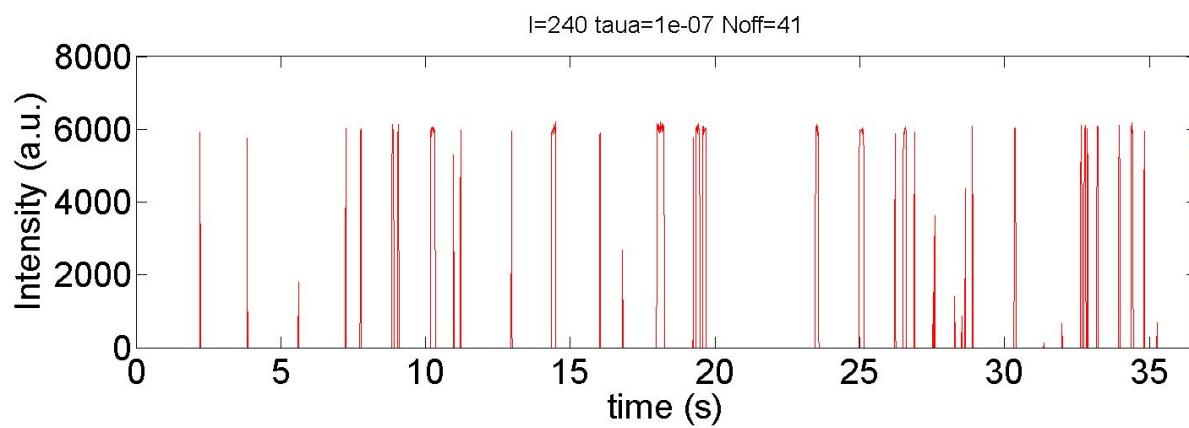
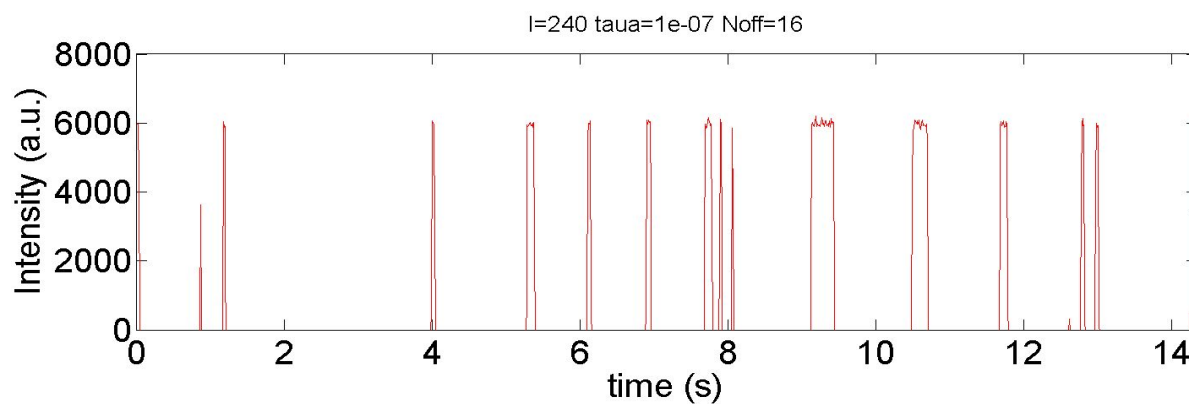
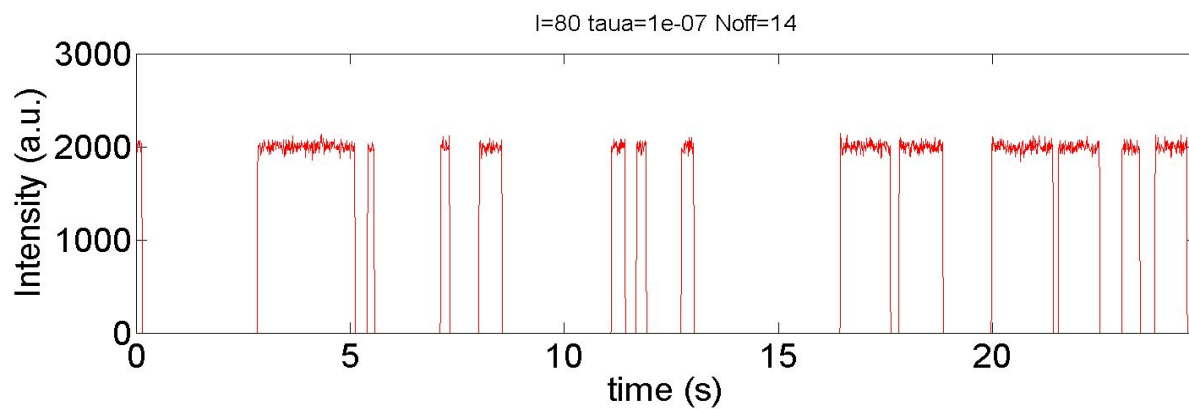
```

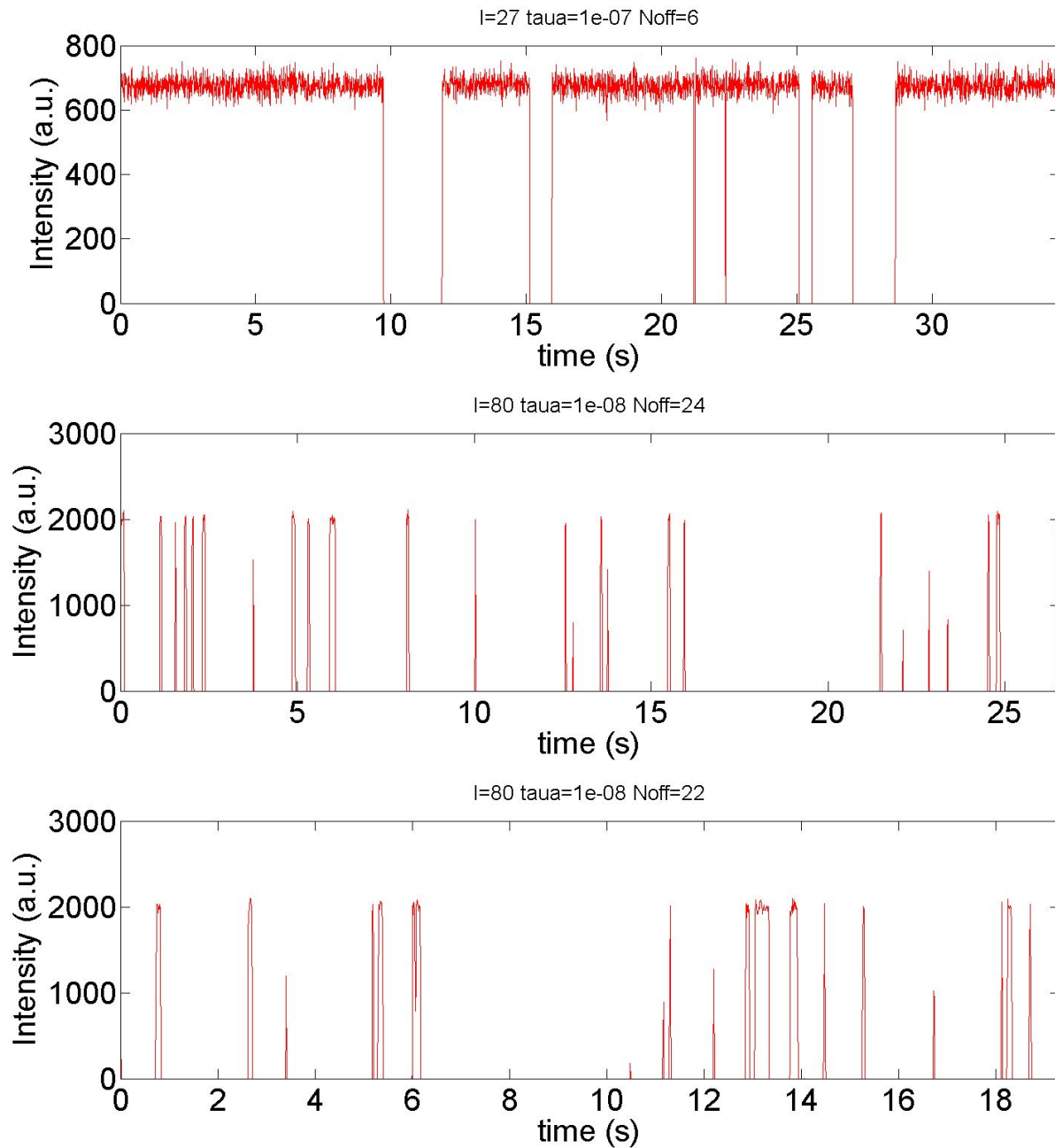
Model-3

In this model the increment in time(t_{inc}) in each state is given by $-\log(f)/\Sigma \text{rates}$ where f is a random number. The probability of electron for going in a particular state is $\text{rate}/\Sigma \text{rates}$.

Results:







Code:

```
clear;
clc;
close all;
tic
tau1=4e-9; %for transition from state 2 to 1
tau2=1e-9; %for transition from state 3 to 2
```

```

tau=0.24;    %for transition from state 4 to 1
taua=10e-9; %for transition from state 3 to 4
s1=1e-15;   %cross section of NC
s2=1e-15;   %cross section of NC
h=2.5*1.6e-19;
I=0.08e3;   %intensity
W1=s1*I/h;  %rate of transition from state 1 to 2
W2=s2*I/h;  %rate of transition from state 2 to 3
bin_size=10e-3;
trap=0; %count total number of trap states(3 to 4)
on=0;  %count total number of on states(2 to 1)
i=2;   %index for array
n=2000000;%total number of iterations
t(1)=0; %time array
next(1)=1;%stores the sequence of states with initial state as 1
on(1)=0;
%[next(2), tinc(1),tr,on1]=state_n(next(1),
W1,W2,tau1,tau2,taua,tau); %next state is determined
%on(2)=on1;
%t(2)=t(1)+tinc(1);
while i<=n
    [next(i),
tinc(i-1),tr,on1,f,num]=state_n1(next(i-1),W1,W2,tau1,tau2,taua,tau);
%iteratively next state is determined
    trap=trap+tr;
    on(i)=on1;
    t(i)=t(i-1)+tinc(i-1);

    F(i-1)= f;
    N(i-1) = num;
    i=i+1;
end
n2=fix(max(t)/bin_size);          %number of bins formed
binranges=0:bin_size:max(t);     %stores the various bins
[bincounts, ind]=histc(t,binranges);% bincounts stores the number of
indices of t array in each bin
sr=1;
for y=1:n2 %binning method

```

```

    on2(y)= sum(on(sr:bincounts(y)+sr));
    sr=bincounts(y)+1;
end
t1 = 0:bin_size:(n2-1)*bin_size;
c = [[0,0,0];[0.7,0.7,0.7];[1,0,0];[0,1,0];[0,0,1]];
line_style = {'-','--','-.-','-'};
    plot(t1,on2,'Color', [c(3,:)],'LineStyle',line_style{1});
    xlim([0, max(t)]);
    %h_legend = legend('3 level','Location','NorthEast');
    title(['I=',num2str(I),' taua=',num2str(taua),'
Noff=',num2str(trap)],'fontsize',16);
%    set(h_legend,'fontsize',16, 'box', 'off');
    xlabel('time (s)','fontsize',24);
    ylabel('Intensity (a.u.)','fontsize',24);
    set(gca, 'FontSize',24);

    figname_png = ['Intensity and time for 4 state model 7.png'];
    figname = ['Intensity and time for 4 state model 7'];
    width = 30;
    height = 10;
    set(gcf, 'PaperPositionMode', 'manual');
    set(gcf, 'PaperSize', [width height]);
    set(gcf, 'PaperPosition', [0 0 width height]);
    set(gca,'position',[0.1 0.19 .85 .7]);% specify these as the
fraction of the total.. between 0 and 1

    print('-dpng','-r125',figname_png);
j=1;
for k=1:length(on2)
    if(on2(k)>mean(on2))
        on22(k)=1;
    else
        on22(k)=0;
    end
end
for i=1:length(t1)-1
    if(on22(i)-on22(i+1)>0)
        flag(j)=0;

```

```

        cr(j)=t1(i);
        j=j+1;
    end
    if(on22(i)-on22(i+1)<0)
        flag(j)=1;
        cr(j)=t1(i);
        j=j+1;
    end
end
b=1;
c=2;
ton(1)=cr(1);
for a=1:length(cr)-1
    if(flag(a)==0 && flag(a+1)==1)
        toff(b)=cr(a+1)-cr(a);
        b=b+1;
    end
    if(flag(a)==1 && flag(a+1)==0)
        ton(c)=cr(a+1)-cr(a);
        c=c+1;
    end
end
binranges=0:0.2:max(ton);    %stores the various bins
figure();
hist(ton,binranges)% bincounts stores the number of indices of t
array in each bin
title('Ton','fontsize',16);
% set(h_legend,'fontsize',16, 'box', 'off');
xlabel('duration of interval','fontsize',24);
ylabel('Probabililty density','fontsize',24);
set(gca, 'FontSize',24);
figname_png1 = ['on histogram.png'];
figname1 = ['on histogram'];
width = 30;
height = 10;
set(gcf, 'PaperPositionMode', 'manual');
set(gcf, 'PaperSize', [width height]);
set(gcf, 'PaperPosition', [0 0 width height]);

```

```

set(gca,'position',[0.1 0.19 .85 .7]);% specify these as the fraction
of the total.. between 0 and 1
print('-dpng','-r125',figname_png1);
binranges=0:0.2:max(toff);    %stores the various bins
figure();
hist(toff,binranges)% bincounts stores the number of indices of t
array in each bin
title('Toff','fontsize',16);
%   set(h_legend,'fontsize',16, 'box', 'off');
xlabel('duration of interval','fontsize',24);
ylabel('Probabilty density','fontsize',24);
set(gca, 'FontSize',24);
figname_png2 = ['off histogram.png'];
figname2 = ['off histogram'];
width = 30;
height = 10;
set(gcf, 'PaperPositionMode', 'manual');
set(gcf, 'PaperSize', [width height]);
set(gcf, 'PaperPosition', [0 0 width height]);
set(gca,'position',[0.1 0.19 .85 .7]);% specify these as the fraction
of the total.. between 0 and 1
print('-dpng','-r125',figname_png2);toc

```

Function: state_n1

```

function [next, tinc,tr,on1,f,num] =state_n1(st,
W1,W2,tau1,tau2,taua,tau)
tr=0;
on1=0;
f=rand();
if st==1
    tinc=-log(f)/(W1);
    norm=1/W1;
    num=rand();
    if(num<W1*norm)
        next=2;
    else
        next=1;
    end
end

```

```

end
if st==2
    tinc=-log(f)/(W2+(1/tau1));
    norm=1/(W2+(1/tau1));
    num=rand();
    if(num<W2*norm)
        next=3;
    elseif (W2*norm<=num && num<(W2+(1/tau1))*norm)
        next=1;
        on1=1;
    else
        next=2;
    end
end
if st==3
    tinc=-log(f)/((1/tau2)+(1/taua));
    norm=1/((1/tau2)+(1/taua));
    num=rand();
    if(num<(1/tau2)*norm)
        next=2;
    elseif((1/tau2)*norm<=num && num<((1/tau2)+(1/taua))*norm)
        next=4;
        tr=1;
    else
        next=3;
    end
end
if st==4
    tinc=-log(f)/(1/tau);
    norm=1/(1/tau);
    num=rand();
    if(num<(1/tau)*norm)
        next=2;
    else
        next=4;
    end
end
end
end

```

Plots of Probability Distribution of 'on' and 'off' time using the second model:

