Assignment 3: Kalman Filter

You are given the task of develop a surface to air missile launcher which can track and fire missiles at the approaching enemy aircrafts. The enemy aircraft can be modelled as a discrete dynamic system with the state S_t at time t given by $S_t = \begin{bmatrix} v \\ x \end{bmatrix}$ where v is the 3-D velocity vector at the time instant t and x is the 3-D position vector at time instant t.

Assume that the 3-D acceleration vector a_t of the target is known and considered as a control signal u_t of the dynamic system. That is, $u_t = a_t$.
Thus, the equations to deduce the next corresponding values of the state are given by

$$\begin{aligned} v_{t+1} - v_t &= a_t \times \Delta t \\ x_{t+1} - x_t &= v_t \times \Delta t \end{aligned}$$

The time gap between the successive state estimates Δt is 1 milliseconds.
The measurements (RADAR readings) are available every T milliseconds. This implies that after obtaining a reading from the RADAR, the launcher needs to predict its target's location exactly T times before it receives another reading.
Assume that the RADAR readings are affected by Gaussian noise η characterised by a diagonal covariance matrix.

Experiments:
Track the target starting from an initial state to 50 steps. The measurements for the velocity vector and the position vector are affected by Gaussian noise η. For each time step, compute the error (Euclidean Distance) between the true position and the estimated position. The propagation of the true state can be done following the known system dynamics and the known initial state. However, the RADAR-based tracking system on the missile launcher makes use of Gaussian noise affected measurements for estimating the state. The tracking system uses a Kalman Filter for recursive state estimation.

Repeat your experiments with different levels of noise and different acceleration control. You may consider variations for the acceleration such as constant acceleration or time dependent acceleration. Also experiment with different values of T.