



CCE Proficiency – 2019

Study Assignment 3 - Basics of Machine Learning

3.3 ILLUSTRATION

Consider that for a particular problem there are five training sets as shown in Table 3.3.

Table 3.3 Training sets

S. no.	Inputs		Output
	I_1	I_2	O
1	0.4	-0.7	0.1
2	0.3	-0.5	0.05
3	0.6	0.1	0.3
4	0.2	0.4	0.25
5	0.1	-0.2	0.12

In this problem, there are two inputs and one output and already, the values lie between -1 to 1 and hence, there is no need to normalize the values. Assume two neurons in the hidden layer. The neural network architecture is shown in Fig. 3.16.

With the data of the first training set.

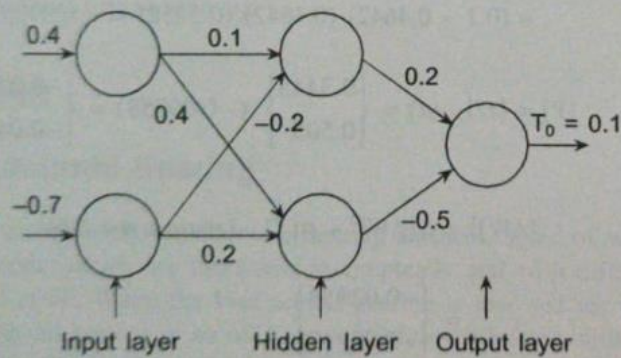


Fig. 3.16 MFNN architecture for the illustration.

Step 1: $\{O\}_I = \{I\}_I = \begin{Bmatrix} 0.4 \\ -0.7 \end{Bmatrix}$

Step 2: Initialize the weights as

$$[V]^0 = \begin{bmatrix} 0.1 & 0.4 \\ -0.2 & 0.2 \end{bmatrix}_{2 \times 2}; \quad [W]^0 = \begin{Bmatrix} 0.2 \\ -0.5 \end{Bmatrix}_{2 \times 1}$$

Step 3: Find $\{I\}_H = [V]^T \{O\}_I$ as

$$= \begin{bmatrix} 0.1 & -0.2 \\ 0.4 & 0.2 \end{bmatrix} \begin{Bmatrix} 0.4 \\ -0.7 \end{Bmatrix} = \begin{Bmatrix} 0.18 \\ 0.02 \end{Bmatrix}$$

Step 4: $\{O\}_H = \begin{Bmatrix} \frac{1}{1 + e^{-0.18}} \\ \frac{1}{1 + e^{-0.02}} \end{Bmatrix} = \begin{Bmatrix} 0.5448 \\ 0.505 \end{Bmatrix}$

Step 5: $\{I\}_O = [W]^T \{O\}_H = (0.2 - 0.5) \begin{Bmatrix} 0.5448 \\ 0.505 \end{Bmatrix} = -0.14354$

Step 6: $\{O\}_O = \left(\frac{1}{1 + e^{0.14354}} \right) = 0.4642$

Step 7: Error = $(T_O - O_O)^2 = (0.1 - 0.4642)^2 = 0.13264$

Step 8: Let us adjust the weights

First find $d = (T_O - O_O) (O_O)(1 - O_O)$
 $= (0.1 - 0.4642) (0.4642) (0.5358) = -0.09058$

$$[Y] = \{O\}_H \langle d \rangle = \begin{Bmatrix} 0.5448 \\ 0.505 \end{Bmatrix} \langle -0.09058 \rangle = \begin{Bmatrix} -0.0493 \\ -0.0457 \end{Bmatrix}$$

Step 9:

$$[\Delta W]^1 = \alpha [\Delta W]^0 + \eta [Y] \quad (\text{assume } \eta = 0.6)$$

$$= \begin{Bmatrix} -0.02958 \\ -0.02742 \end{Bmatrix}$$

Step 10: $\{e\} = [W] \{d\} = \begin{Bmatrix} 0.2 \\ -0.5 \end{Bmatrix} (-0.09058) = \begin{Bmatrix} -0.018116 \\ 0.04529 \end{Bmatrix}$

Step 11: $\{d^*\} = \begin{Bmatrix} (-0.018116) (0.5448) (1 - 0.5448) \\ (0.04529) (0.505) (1 - 0.505) \end{Bmatrix} = \begin{Bmatrix} -0.00449 \\ 0.01132 \end{Bmatrix}$

$$\begin{aligned}\text{Step 12: } [X] &= \{O\}_T \langle d^* \rangle = \begin{Bmatrix} 0.4 \\ -0.7 \end{Bmatrix} \langle -0.00449 \quad 0.01132 \rangle \\ &= \begin{bmatrix} -0.001796 & 0.004528 \\ 0.003143 & -0.007924 \end{bmatrix}\end{aligned}$$

$$\text{Step 13: } [\Delta V]^1 = \alpha [\Delta V]^0 + \eta [X] = \begin{bmatrix} -0.001077 & 0.002716 \\ 0.001885 & -0.004754 \end{bmatrix}$$

$$\begin{aligned}\text{Step 14: } [V]^1 &= \begin{bmatrix} 0.1 & 0.4 \\ -0.2 & 0.2 \end{bmatrix} + \begin{bmatrix} -0.001077 & 0.002716 \\ 0.001885 & -0.004754 \end{bmatrix} = \begin{bmatrix} 0.0989 & 0.04027 \\ -0.1981 & 0.19524 \end{bmatrix} \\ [W]^1 &= \begin{Bmatrix} 0.2 \\ -0.5 \end{Bmatrix} + \begin{Bmatrix} -0.02958 \\ -0.02742 \end{Bmatrix} = \begin{Bmatrix} 0.17042 \\ -0.52742 \end{Bmatrix}\end{aligned}$$

Step 15: With the updated weights $[V]$ and $[W]$, error is calculated again and next training set is taken and the error will be adjusted.

Step 16: Iterations are carried out till we get the error less than the tolerance.

Step 17: Once weights are adjusted the network is ready for inference.