Splines

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A Simulation using Splines

We will perform a simulation study to compare the performance of several different spline methods. Consider the following settings:

- Training data $n = 30$: Generate x from $[-1, 1]$ uniformly, and then generate $y = \sin(\pi x) + \epsilon$, where ϵ's are iid standard normal

```
#data generation
n <- 30
x <- runif(n,-1, 1)
y <- sin(pi * x) + rnorm(n, 0, 1)
data_train <- data.frame(cbind(x, y))
```

- Consider several different spline methods:
  - Write your own code (you cannot use `bs()` or similar functions) to implement a continuous piecewise linear spline fitting. Choose knots at $(-0.5, 0, 0.5)$

```
#continuous piecewise linear spline function which
#returns the spline matrix that can be used for linear fitting
my_linear_spline <- function(x, knots) {
  #initialise spline matrix
  n = length(knots)
  spline_mat <- matrix(0, nrow = length(x), ncol = n + 1)
  spline_mat[, 1] <- x
  #creating spline matrix
  for (i in 1:length(x)) {
    if (x[i] > knots[1]) {
      spline_mat[i, 2] <- x[i] - knots[1]
    }
    if (x[i] > knots[2]) {
      spline_mat[i, 3] <- x[i] - knots[2]
    }
    if (x[i] > knots[3]) {
      spline_mat[i, 4] <- x[i] - knots[3]
    }
  }
  data <- data.frame(cbind(spline_mat, y))
  return(data)
}

#fitting the linear
```

```r
knots <- c(-0.5, 0, 0.5)
linear_spline_data <- my_linear_spline(x, knots)
linear_spline <- lm(y ~ . , data = linear_spline_data)
```

- Use existing functions to implement a quadratic spline 2 knots. Choose your own knots.

```r
#using spline library for quadratic spline
library(splines)
knots <- c(-0.5, 0.5)
quad_spline <-
  lm(y ~ bs(x, knots = knots, degree = 2), data = data_train)
```

- Use existing functions to implement a natural cubic spline with 3 knots. Choose your own knots.

```r
#Fitting the natural cubic spline using ns library
knots <- c(-0.5, 0, 0.5)
nat_cubic_spline <-
  lm(y ~ ns(x, knots = knots), data = data_train)
```

- Use existing functions to implement a smoothing spline. Use the built-in ordinary leave-one-out cross-validation to select the best tuning parameter.

```r
#Fitting the smooth spline using splines library
library(splines)
smooth_spline <- smooth.spline(x, y, cv = TRUE)
```

- After fitting these models, evaluate their performances by comparing the fitted functions with the true function value on an equispaced grid of 1000 points on $[-1, 1]$. Use the squared distance as the metric.

```r
#generating test data
x_test <- seq(from = -1,
              to = 1,
              length.out = 1000)
y_test <- sin(pi * x_test)
data_test <- data.frame(cbind("x" = x_test, "y" = y_test))

#Prediction using linear spline
linear_spline_data <- my_linear_spline(x_test, knots)
y_hat_linear <- predict(linear_spline, newdata = linear_spline_data)

#Prediction using quadratic spline
y_hat_quad <- predict(quad_spline, newdata = data_test)

#Prediction using NCS
y_hat_nat_cubic_spline <-
  predict(nat_cubic_spline, newdata = data_test)

#Prediction using smooth spline
y_hat_smooth_spline <- predict(smooth_spline, x_test)$y
```

```r
#Computing squared error for each
error_linear <- sum((y_test - y_hat_linear) ^ 2)
error_quad <- sum((y_test - y_hat_quad) ^ 2)
error_nat_cubic_spline <- sum((y_test - y_hat_nat_cubic_spline) ^ 2)
error_smooth_spline <- sum((y_test - y_hat_smooth_spline) ^ 2)

#Results
error_linear
```

```
## [1] 322.5598
```

```r
error_quad
```

```
## [1] 375.0977
```

```r
error_nat_cubic_spline
```

```
## [1] 310.8196
```

```r
error_smooth_spline
```

```
## [1] 184.6008
```

- Repeat the entire process 200 times. Record and report the mean, median, and standard deviation of the errors for each method.

```r
#Making error vectors for 200 iterations
error_linear <- rep(0, 200)
error_quad <- rep(0, 200)
error_nat_cubic_spline <- rep(0, 200)
error_smooth_spline <- rep(0, 200)

#generate test data
x_test <- seq(from = -1,
              to = 1,
              length.out = 1000)
y_test <- sin(pi * x_test)
data_test <- data.frame(cbind("x" = x_test, "y" = y_test))

#Running 200 options
for (i in 1:200) {
  #data generation
  n <- 30
  x <- runif(n,-1, 1)
  y <- sin(pi * x) + rnorm(n, 0, 1)
  data_train <- data.frame(cbind(x, y))

  #linear spline
  knots <- c(-0.5, 0, 0.5)
  linear_spline_data <- my_linear_spline(x, knots)
```

```r
  linear_spline <- lm(y ~ . , data = linear_spline_data)

  #quad spline
  knots <- c(-0.5, 0.5)
  quad_spline <-
    lm(y ~ bs(x, knots = knots, degree = 2), data = data_train)

  #natural cubic spline
  knots <- c(-0.5, 0, 0.5)
  nat_cubic_spline <-
    lm(y ~ ns(x, knots = knots), data = data_train)

  #Smooth spline
  smooth_spline <- smooth.spline(x, y, cv = TRUE)

  #predictions
  linear_spline_data <- my_linear_spline(x_test, knots)
  y_hat_linear <-
    predict(linear_spline, newdata = linear_spline_data)
  y_hat_quad <- predict(quad_spline, newdata = data_test)
  y_hat_nat_cubic_spline <-
    predict(nat_cubic_spline, newdata = data_test)
  y_hat_smooth_spline <- predict(smooth_spline, x_test)$y

  #Calculating error
  error_linear[i] <- sum((y_test - y_hat_linear) ^ 2)
  error_quad[i] <- sum((y_test - y_hat_quad) ^ 2)
  error_nat_cubic_spline[i] <-
    sum((y_test - y_hat_nat_cubic_spline) ^ 2)
  error_smooth_spline[i] <- sum((y_test - y_hat_smooth_spline) ^ 2)

}

# Mean error
mean_error <-
  c(
    mean(error_linear),
    mean(error_quad),
    mean(error_nat_cubic_spline),
    mean(error_smooth_spline)
  )
mean_error
```

```
## [1] 207.8037 403.6787 206.1959 266.6894
```

```r
#Median error
median_error <-
  c(
    median(error_linear),
    median(error_quad),
    median(error_nat_cubic_spline),
    median(error_smooth_spline)
  )
```

```r
median_error
```

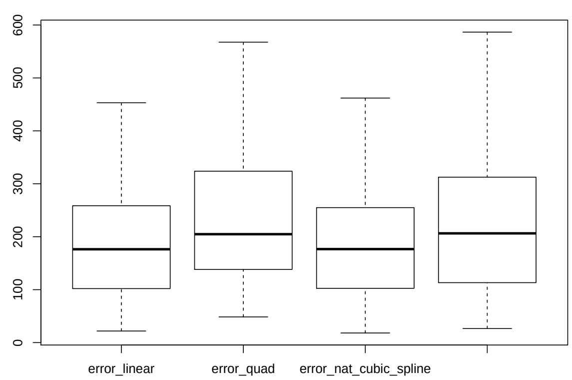
```
## [1] 176.3182 204.7921 176.6087 206.3612
```

```r
#Standard deviation of error
sd_error <-
  c(
    sd(error_linear),
    sd(error_quad),
    sd(error_nat_cubic_spline),
    sd(error_smooth_spline)
  )
sd_error
```

```
## [1]  141.3511 1499.1924  150.0872  280.6365
```

```r
#combining error data for boxplot
error_data <-
  as.matrix(cbind(
    error_linear,
    error_quad,
    error_nat_cubic_spline,
    error_smooth_spline
  ))

#boxplot with outliers removed for better view
boxplot(error_data, outline = FALSE)
```



The mean error is minimum for natural cubic spline. The standard deviation of error is also minimum for natural cubic spline. Thus NCS is more stable as compared to other models. Hence, I would prefer natural cubic spline for this data.