

A Project Report
in
**CSE 486: INTRODUCTION TO NEURAL AND COGNITIVE
MODELLING**

Topic: SINGLE-NEURON & POPULATION MODELS:
Variations on LIF models

Submitted to
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(March 2020)

INTRODUCTION AND MOTIVATION

Computational neuroscience is the field of study in which mathematical tools and theories are used to investigate brain function. It also incorporates diverse approaches from electrical engineering, computer science and physics in order to understand how the nervous system processes information.

The integrate-and-fire neuron model is one of the most widely used models for analyzing the behavior of neural systems. It describes the membrane potential of a neuron in terms of the synaptic inputs and the injected current that it receives. An action potential (spike) is generated when the membrane potential reaches a threshold, but the actual changes associated with the membrane voltage and conductances driving the action potential do not form part of the model.

In this project we look at the various Leaky Integrate and Fire Models such as: 1) Quadratic LIF 2) Exponential LIF and 3) Adaptive Exponential LIF

What we studied in class was a simple Spiking LIF which fires when the voltage crosses the threshold voltage it doesn't have any special complexity which can distinguish its firing pattern. This mini project grows beyond that. We explored other LIF models stated above to get a deeper understanding of various types of behaviour shown by different single neuron models.

AIM

To see the various spiking rates and patterns for each type of Integrate and Fire model, for varying parameters and plotting them.

Functions:

This section describes/states the mathematical differential equations on which the studied models are based upon.

Quadratic Integrate and Fire Model:

The equation for Quadratic Integrate and Fire model is given by:

$$\tau \frac{d}{dt} u = a_0 (u - u_{\text{rest}}) (u - u_c) + RI,$$

with parameters $a_0 > 0$ and $u_c > u_{\text{rest}}$. For $I=0$ and initial condition $u < u_c$, the voltage decays to the resting potential u_{rest} . For $u > u_c$ it increases so that an action potential is triggered. The parameter u_c can therefore be interpreted as the critical voltage for spike initiation by a short current pulse.

Exponential Integrate and Fire Model:

The equation for Exponential Integrate and Fire model is given by:

$$\tau \frac{d}{dt} u = - (u - u_{\text{rest}}) + \Delta_T \exp\left(\frac{u - \vartheta_{rh}}{\Delta_T}\right) + RI;$$

The first term on the right-hand-side describes the leak of a passive membrane. The second term is an exponential nonlinearity with 'sharpness' parameter Δ_T and 'threshold' ϑ_{rh} .

Adaptive Exponential Integrate and Fire Model:

The equations for Adaptive Exponential Integrate and Fire model is given by:

$$\begin{aligned} \tau_m \frac{du}{dt} &= f(u) - R \sum_k w_k + RI(t) \\ \tau_k \frac{dw_k}{dt} &= a_k (u - u_{\text{rest}}) - w_k + b_k \tau_k \sum_{t^{(f)}} \delta(t - t^{(f)}). \end{aligned}$$

A single equation is, however, not sufficient to describe the variety of firing patterns that neurons exhibit in response to a step current. We therefore couple the voltage equation to abstract current variables w_k , each described by a linear differential equation.

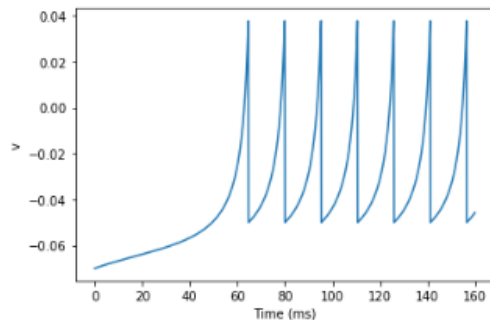
Results

Quadratic LIF

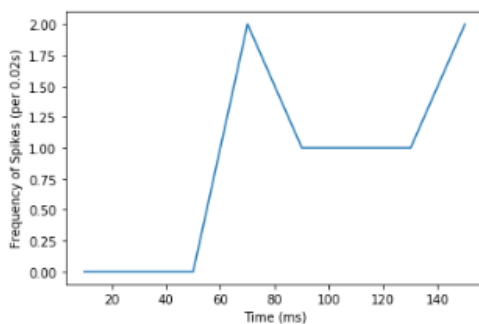
For quadratic LIF we plotted the figures with the parameters, $C=1500$ pF, $V_{\text{thresh}}=-60$ mV, $V_{\text{rest}}=-50$ mV and for input current of 550 pico Amperes.

We checked C for values : 500, 600, 700, 800, 900, 1200, 2000 the outcome we saw was that the spikes get less frequent.

```
After v = -45.62584136 mV
{'t': {0: array([ 64.6,  79.9,  95.2, 110.5, 125.8, 141.1, 156.4]) * msecond}, 'u': {0: array([41.36560989, 41.4274494, 41.4274494, 41.4274494, 41.4274494, 41.4274494, 41.4274494]) * mvolt}}
```



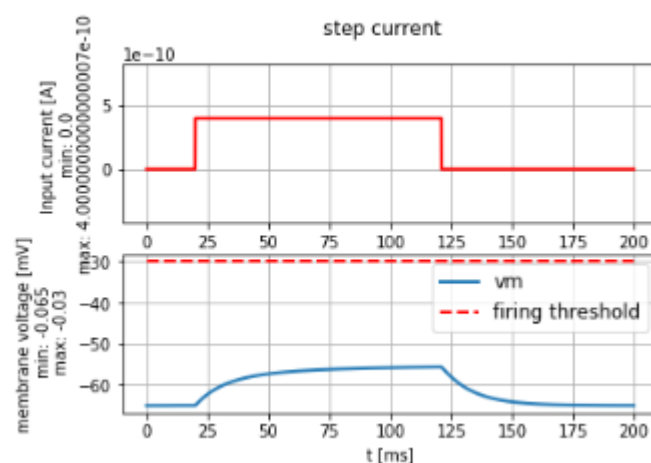
```
[0, 0, 0, 2, 1, 1, 1, 2]
[10. * second, 30. * second, 50. * second, 70. * second, 90. * second, 110. * second, 130. * second, 150. * second]
```



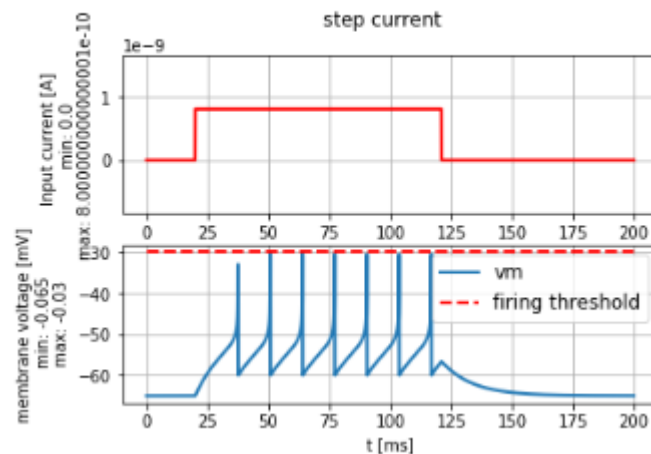
Exponential LIF

For exponential LIF we plotted the figures with the parameters resting potential -65 mV, reset potential -60 mV, threshold -30 mV and current amplitude 0.4 and 0.8 nanoAmperes. We got spikes and for 0.8 nA but no spikes for 0.4 nA. We can see the plots for both the cases below.

```
No spikes : current = 0.4 nA
nr of spikes: 0
```



7 repetitive spikes : current = 0.8 nA
nr of spikes: 7



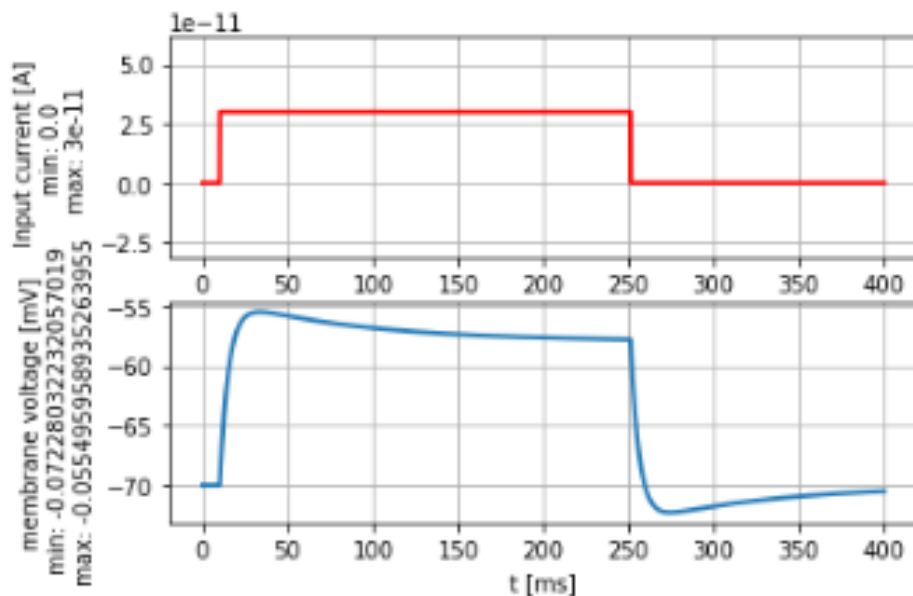
Adaptive Exponential LIF

For adaptive exponential LIF we plotted the figures with the parameters resting potential

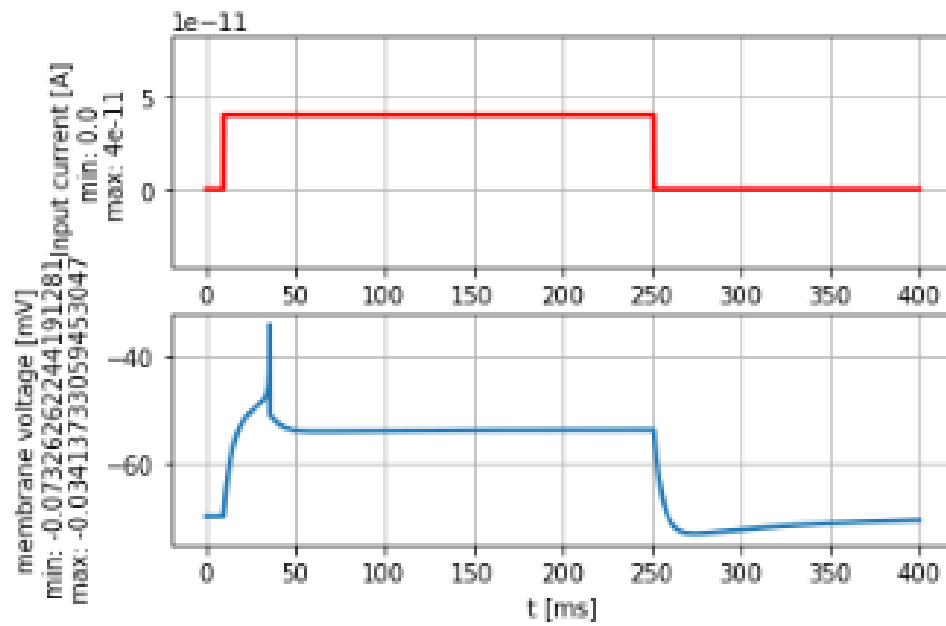
-70mV, reset potential -51mV, threshold -30mV, rheo threshold = -50mV and current amplitude 30, 40 and 65 picoAmperes. We got 10 spikes and for 65 pA but

no spikes for 30 pA and one single spike for 40pA. We can see the plots for both the cases below.

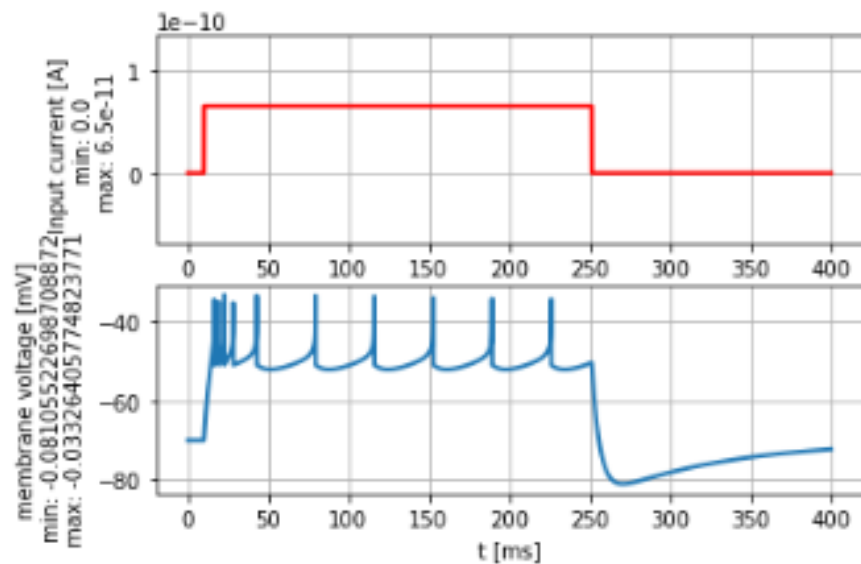
No spike : current=30pA
nr of spikes: 0



1 single spike : current=40pA
nr of spikes: 1



10 spikes : current=65pA
nr of spikes: 10



Discussion and Conclusion

Leaky integrate and fire (LIF) model represents neuron as a parallel combination of a “leaky” resistor (conductance, g_L) and a capacitor (C). A current source $I(t)$ is used as synaptic current input to charge up the capacitor to produce a potential $V(t)$.

We saw various leaky integrate and fire models and their different behavior in various conditions. It helped in capturing the neuron behavior and its spiking frequency dependence on input. As we know brain is a very complex organ and it can be understood by multiple simple models working together. This project helped us in achieving that. It helped understanding the brain functionalities. This kind of study can help in mapping the above simulations into circuits and hence make one small step closer to mimicking the brain artificially.

We also saw that even little changes in current input i.e a change as small as 10^{-12} can change the firing pattern of the neuron.

References:

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