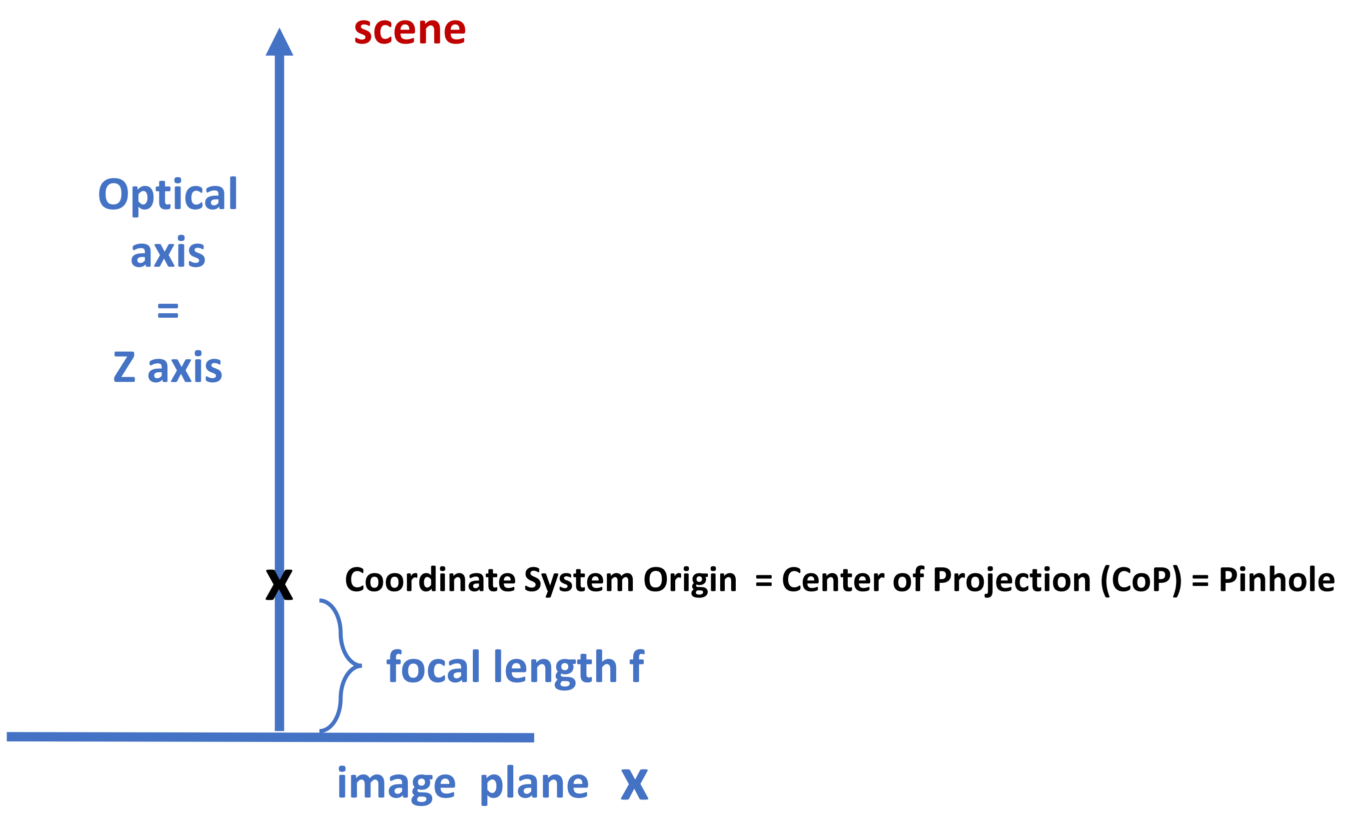
Image Formation: Pinhole Model, Binocular Stereo and Thin Lens Model

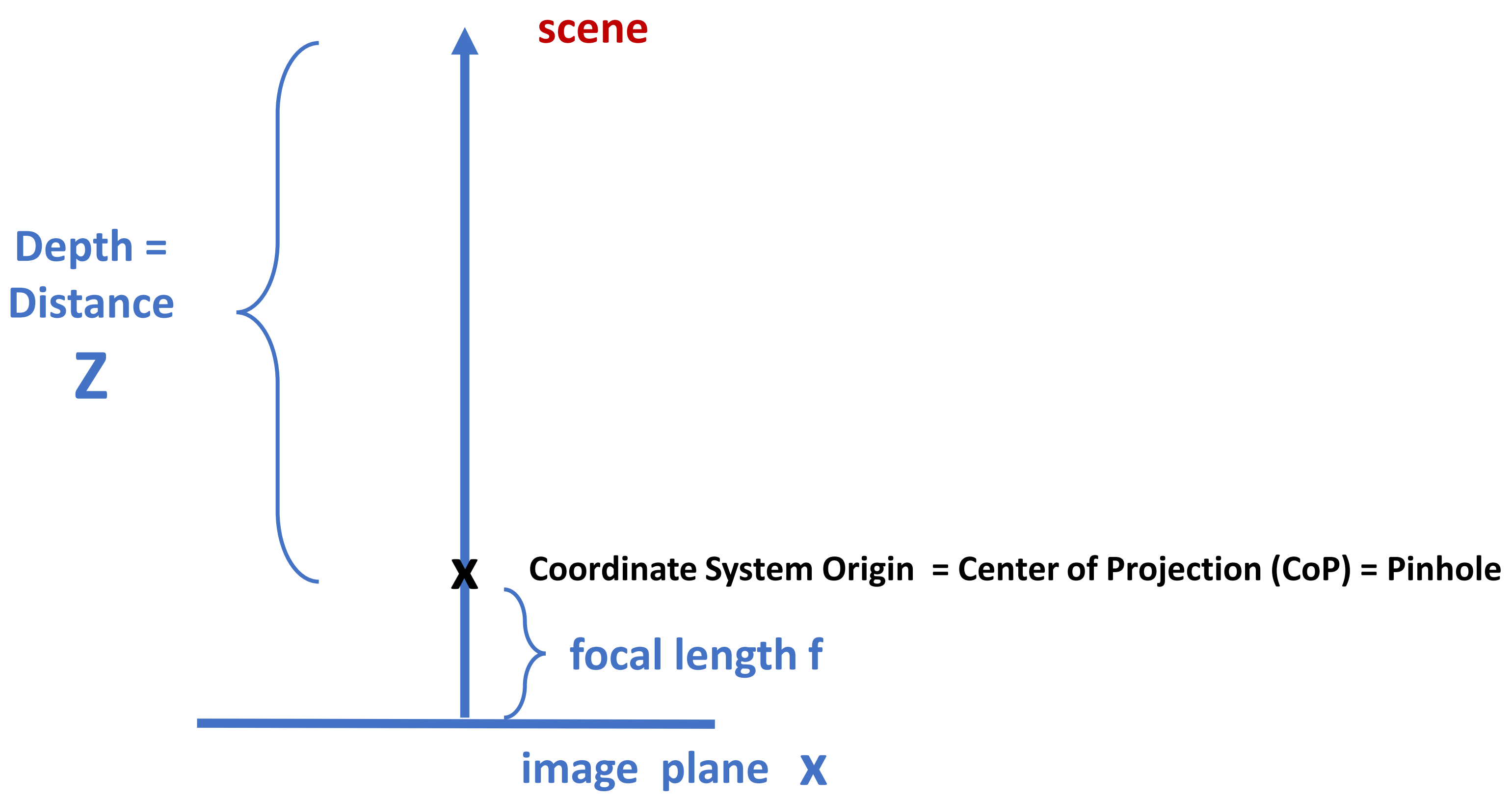
Lecture by Margrit Betke, CS 585, April 23, 2020

BOSTON UNIVERSITY

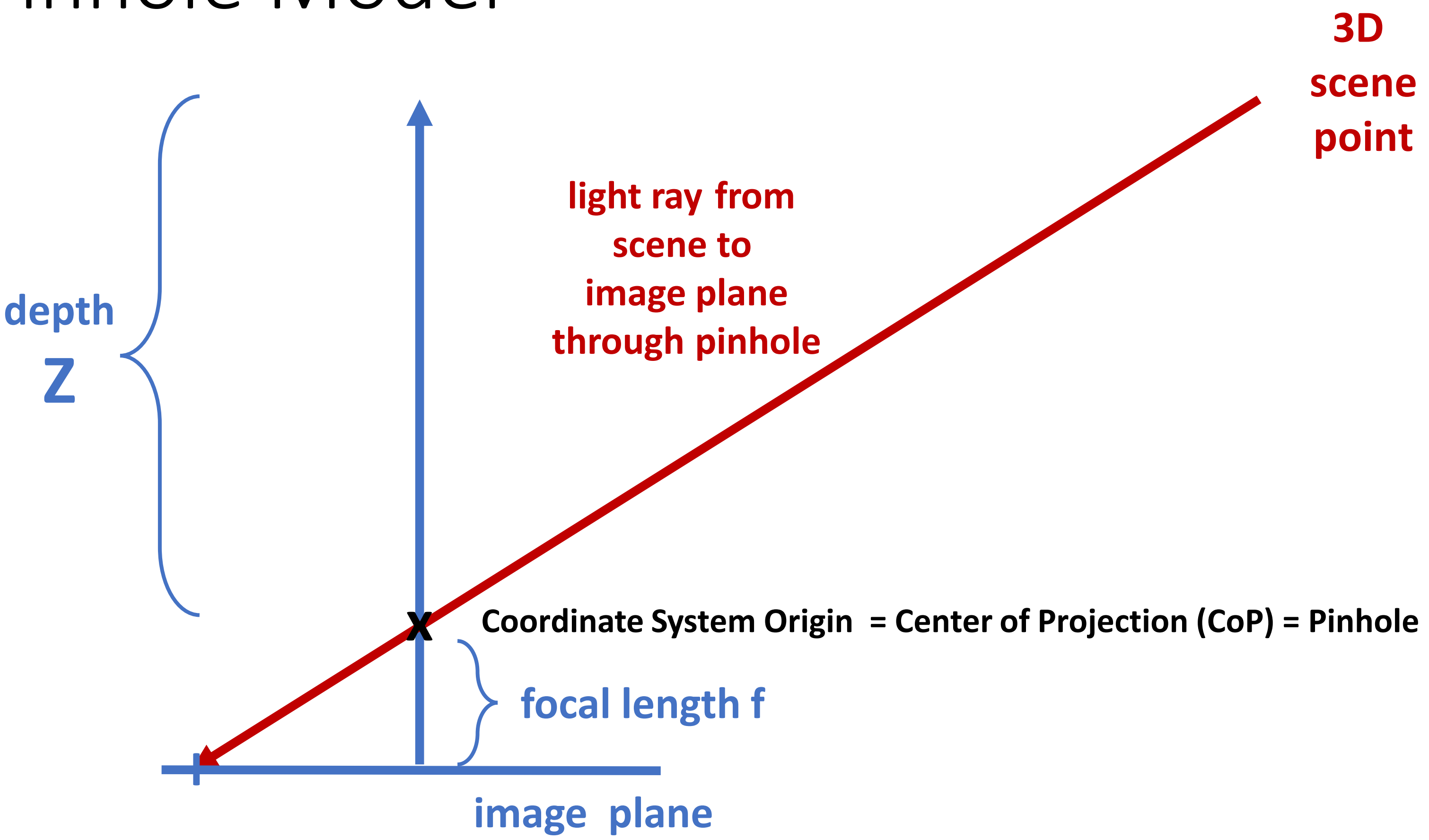
Pinhole Model: View from Top



Pinhole Model: View from Top



Pinhole Model



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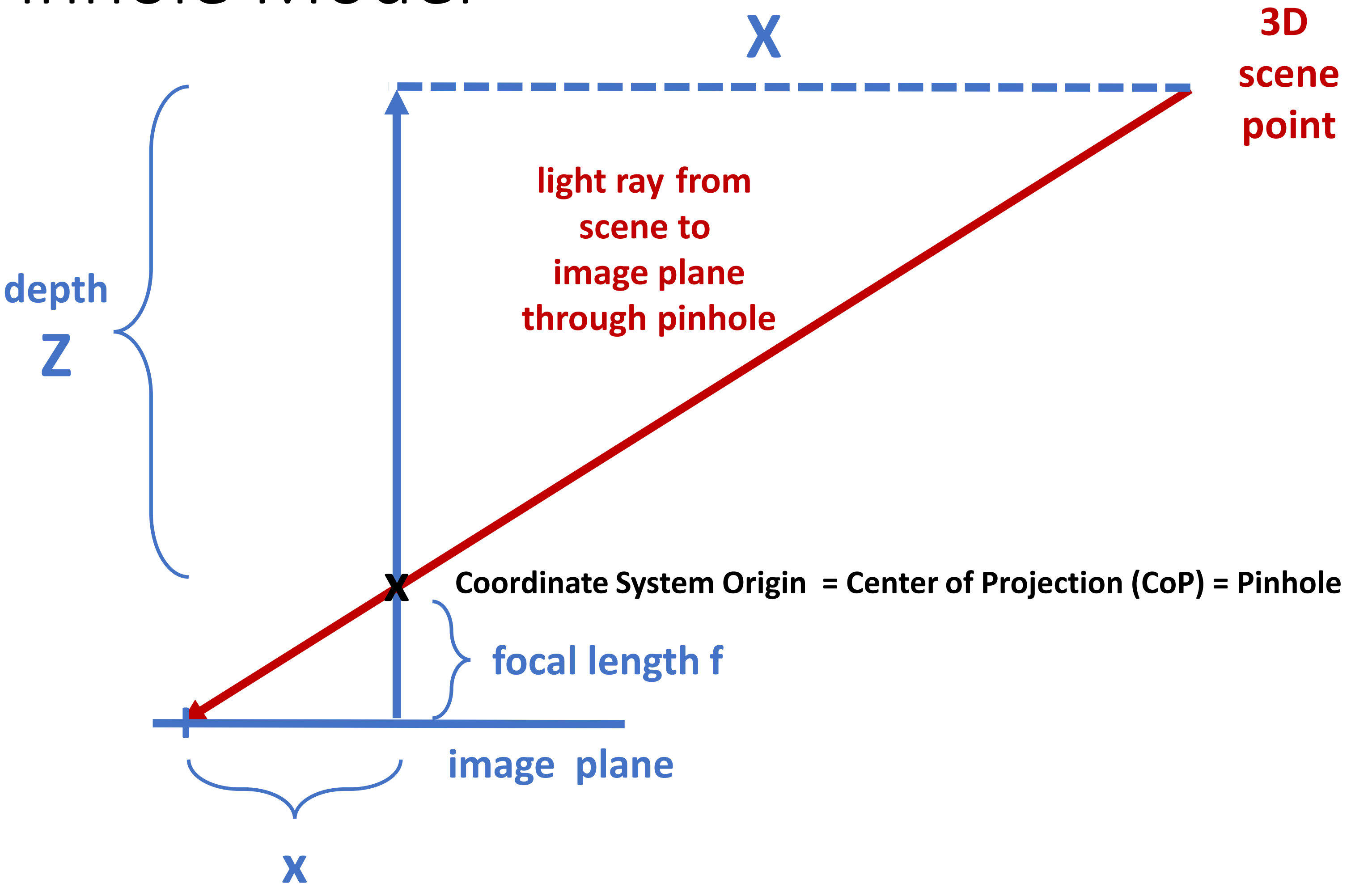
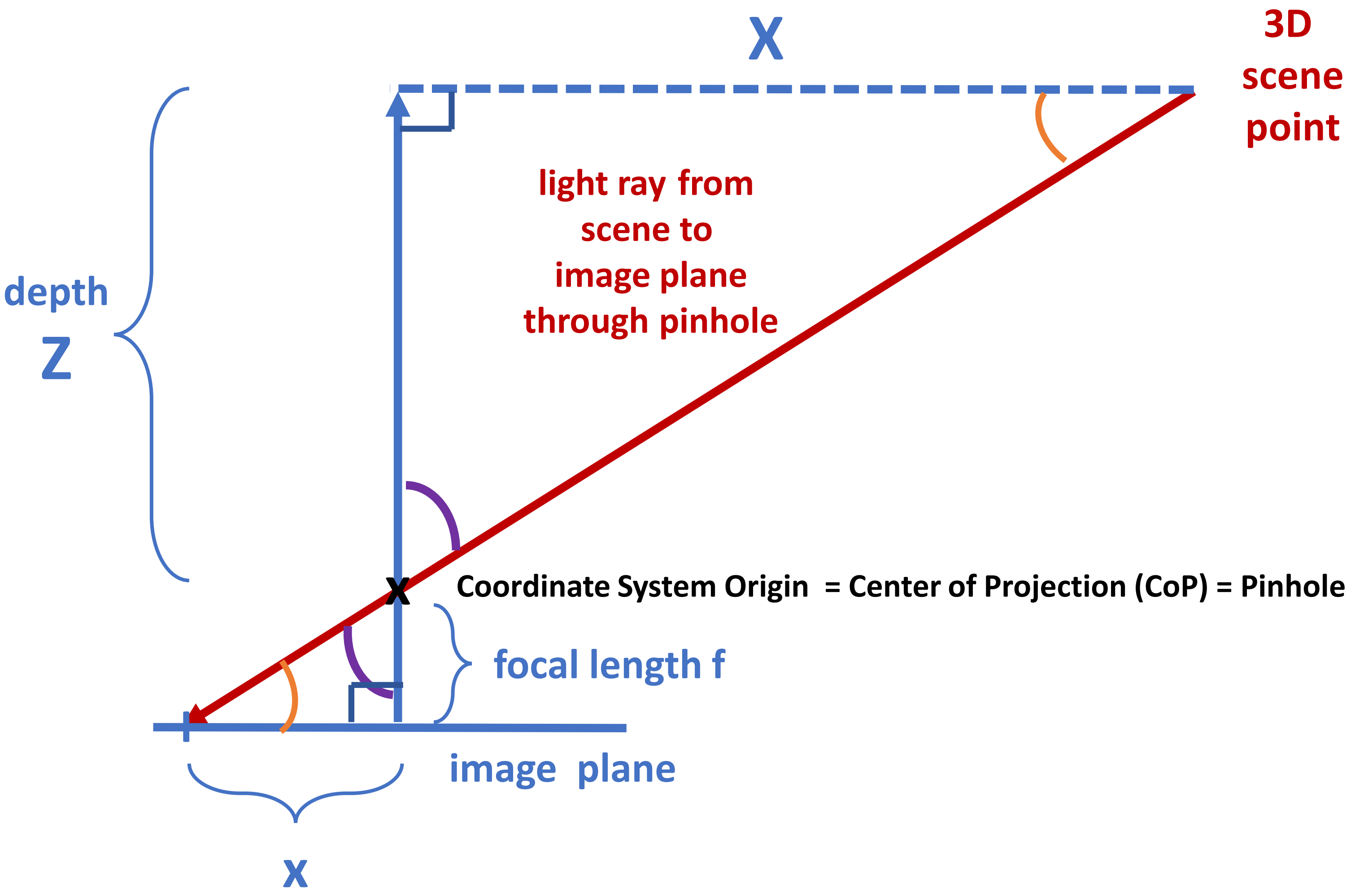


Similar Triangles



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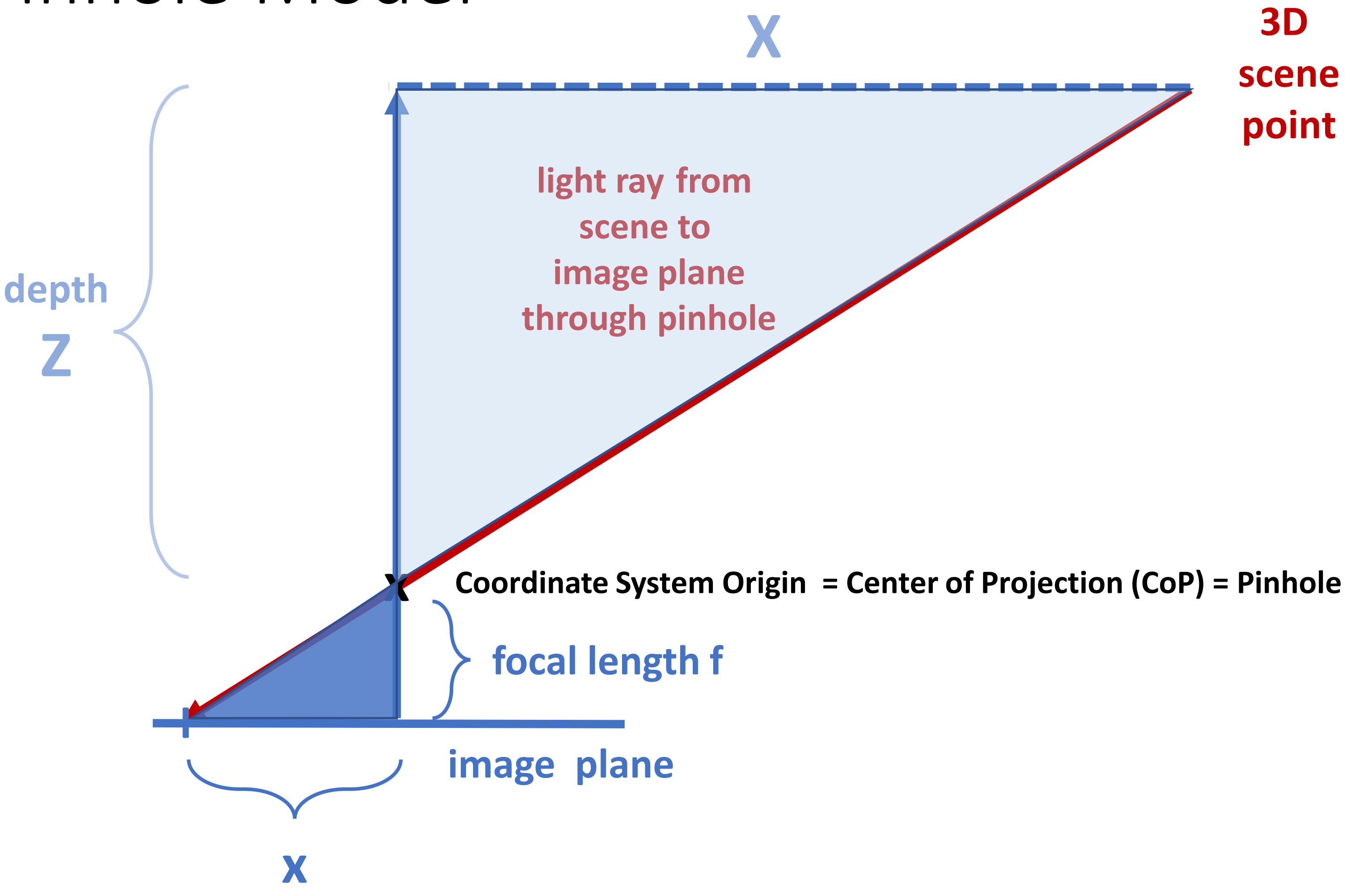

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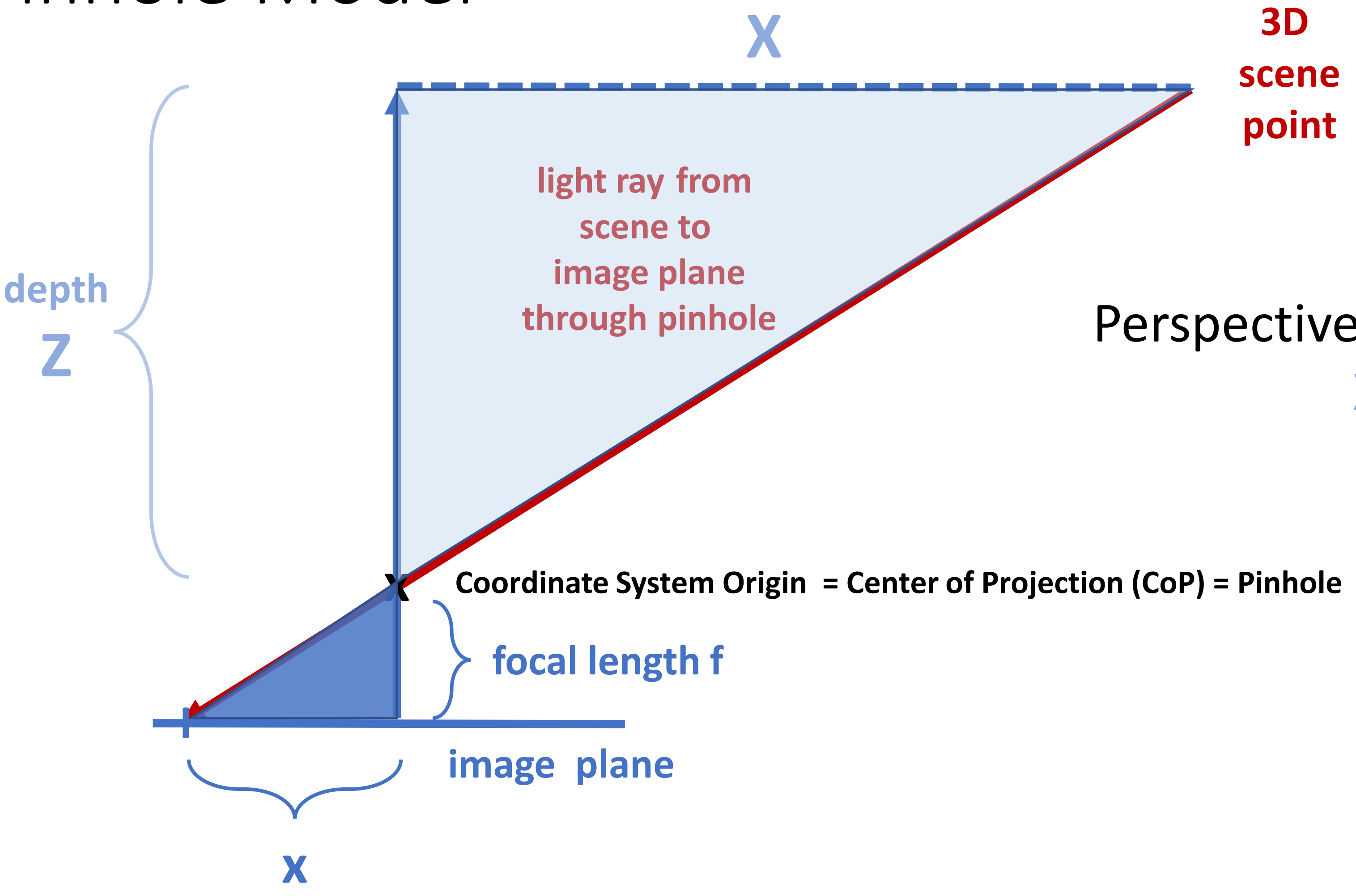
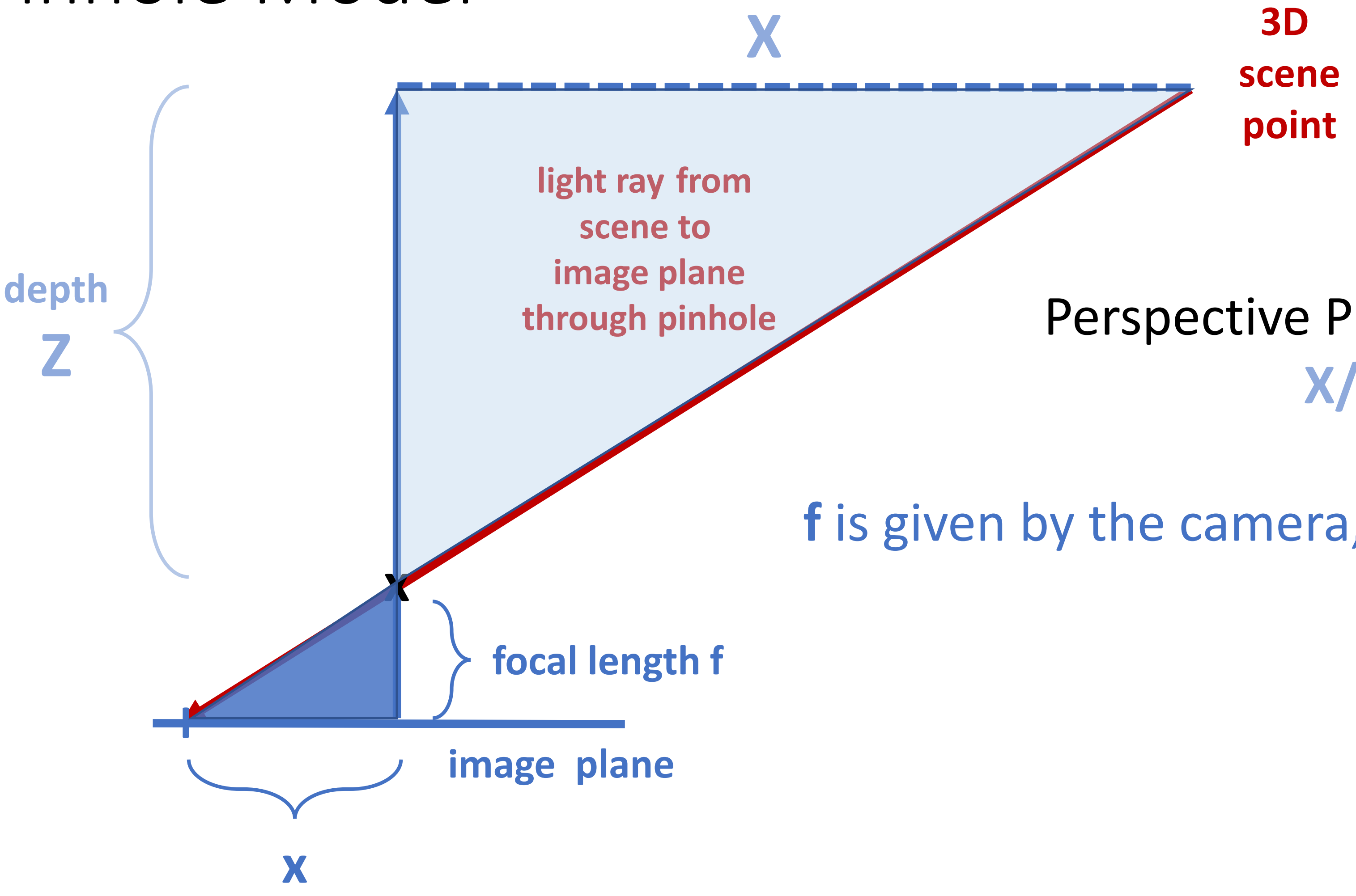


Perspective Projection Equation:

$$X/Z = x/f$$

Pinhole Model

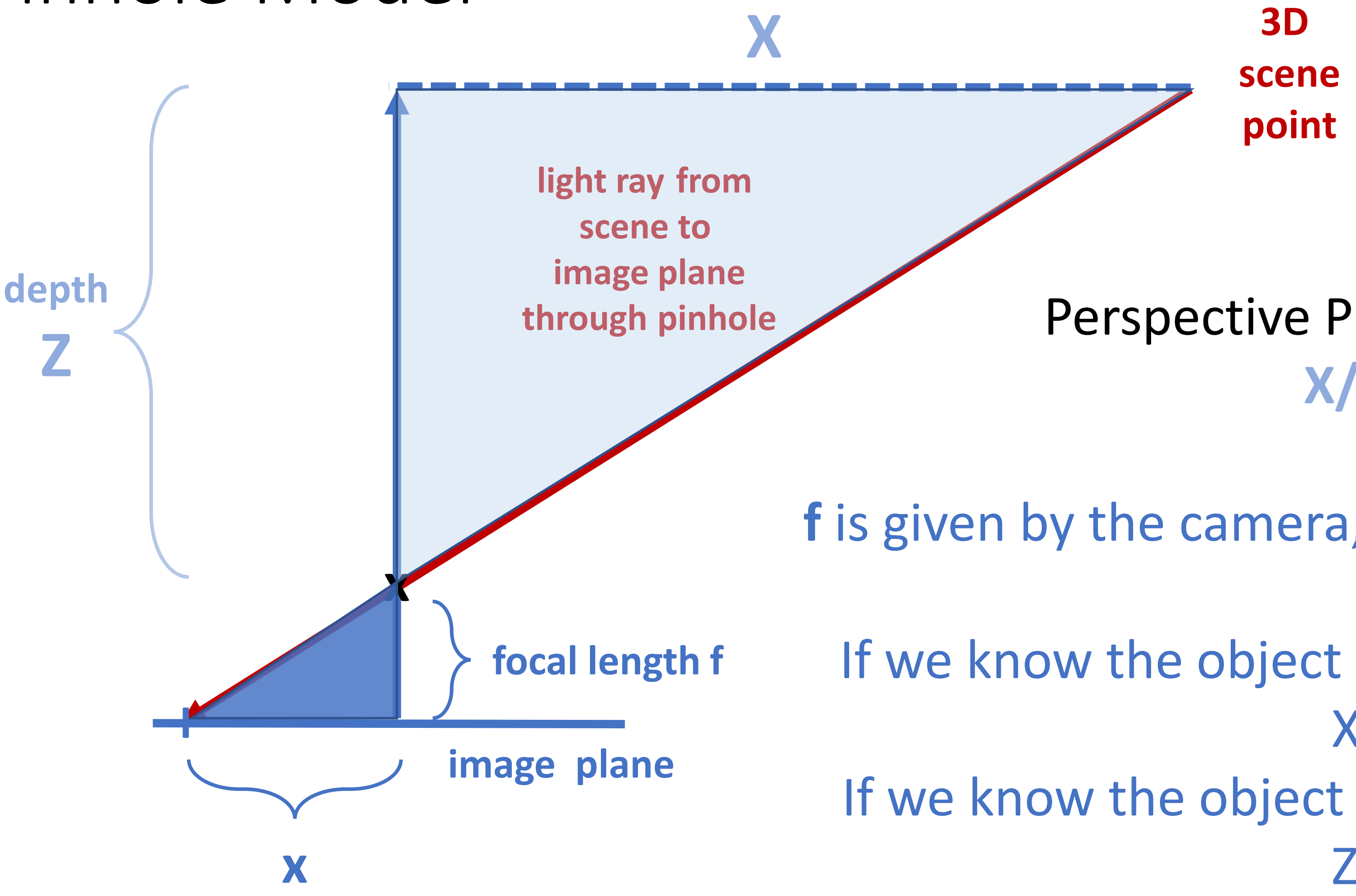


Perspective Projection Equation:

$$X/Z = x/f$$

f is given by the camera, **x** is our image measurement

Pinhole Model



Perspective Projection Equation:

$X/Z = x/f$

f is given by the camera, **x** is our image measurement

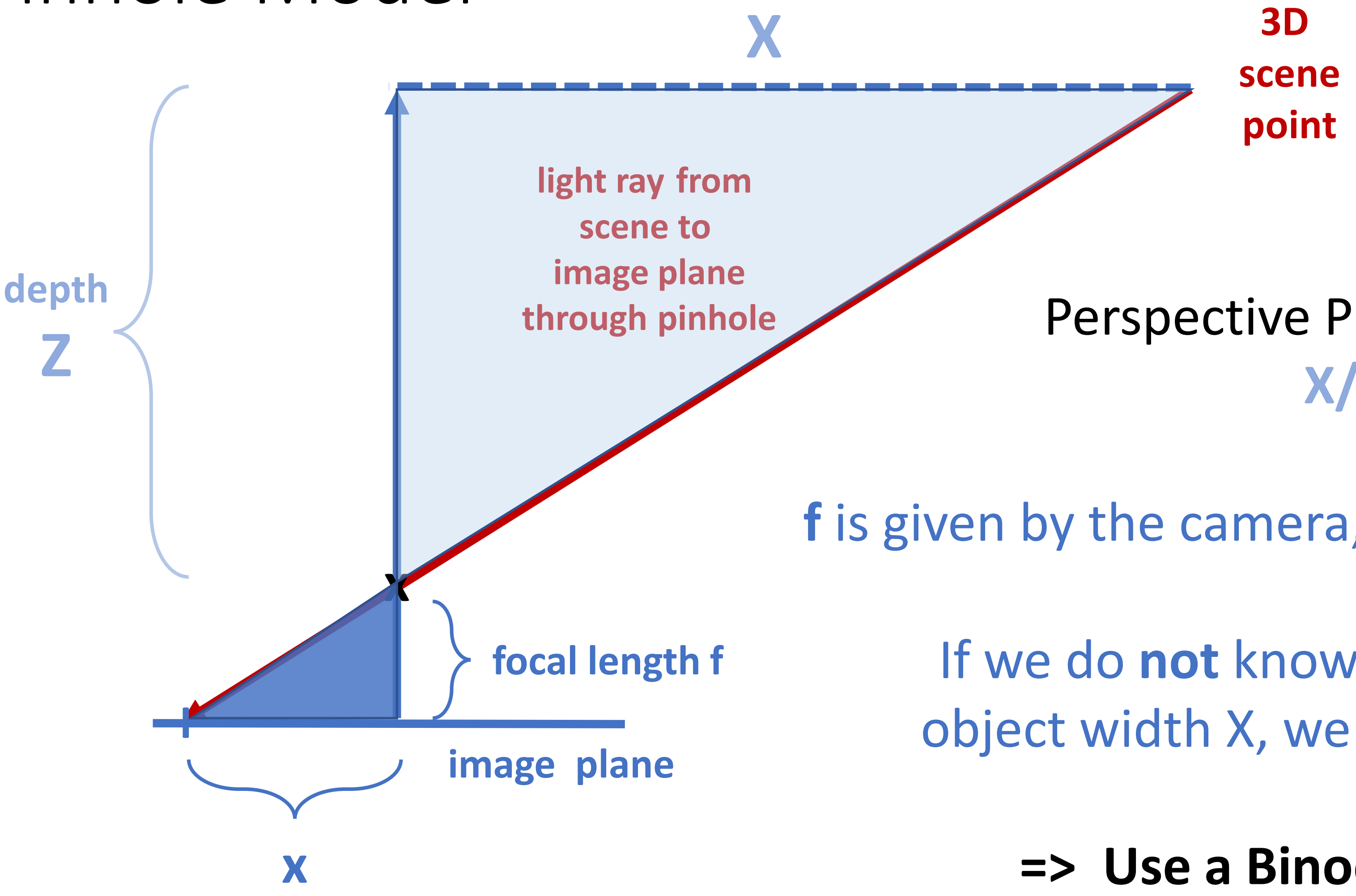
If we know the object depth Z, we can compute X:

$X = xZ/f$

If we know the object width X, we can compute Z:

$Z = Xf/x$

Pinhole Model



Perspective Projection Equation:

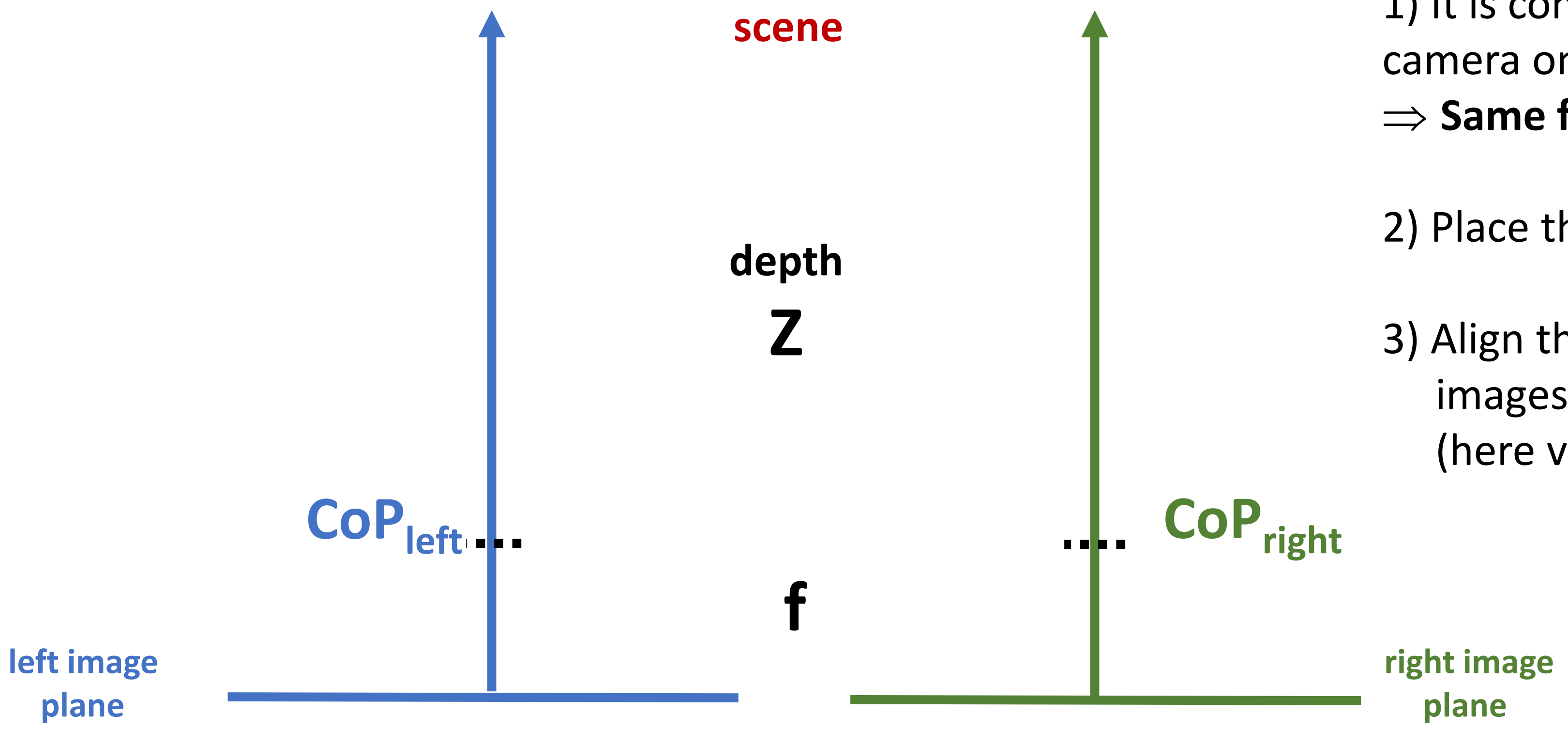
$X/Z = x/f$

f is given by the camera, **x** is our image measurement

If we do **not** know the object depth Z **and** object width X, we need another equation!

=> Use a Binocular Stereo System

Binocular Stereo



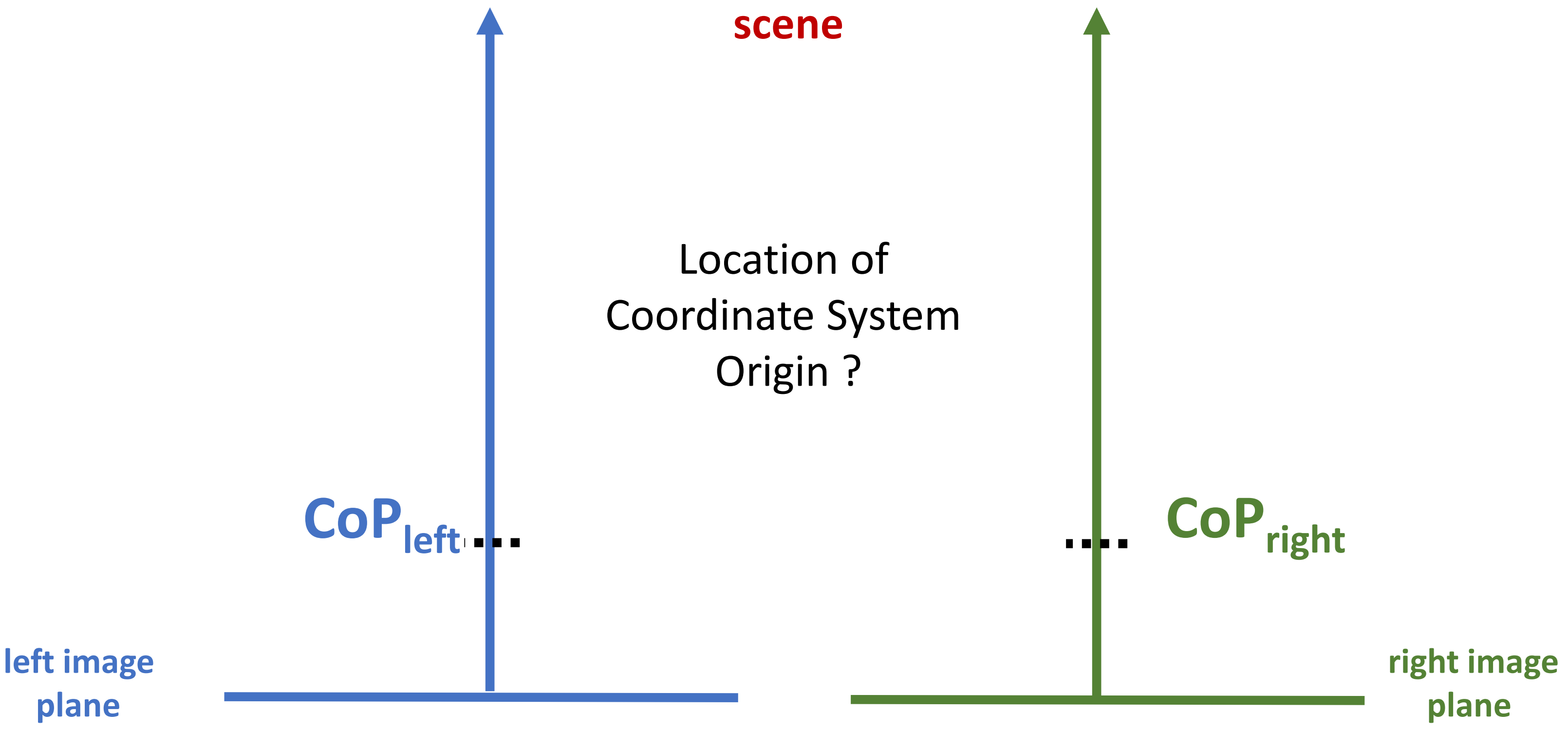
Special Considerations:

1) It is convenient to use the same type of camera on the left and the right
$\Rightarrow$ **Same focal length f**

2) Place the optical axes **parallel** to each other

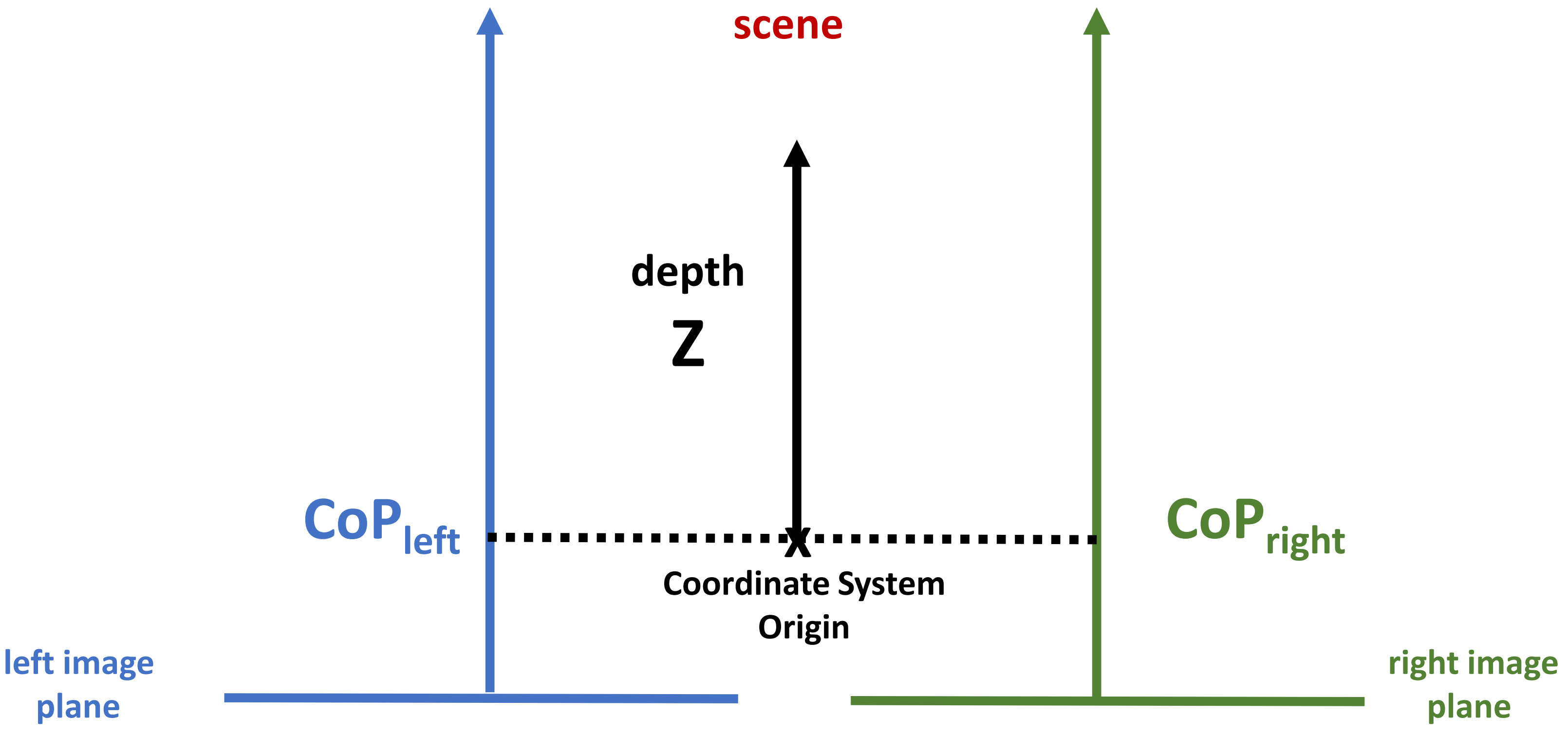
3) Align the cameras so that the left and right images are in the **same plane** (here viewed in cross section)

Binocular Stereo



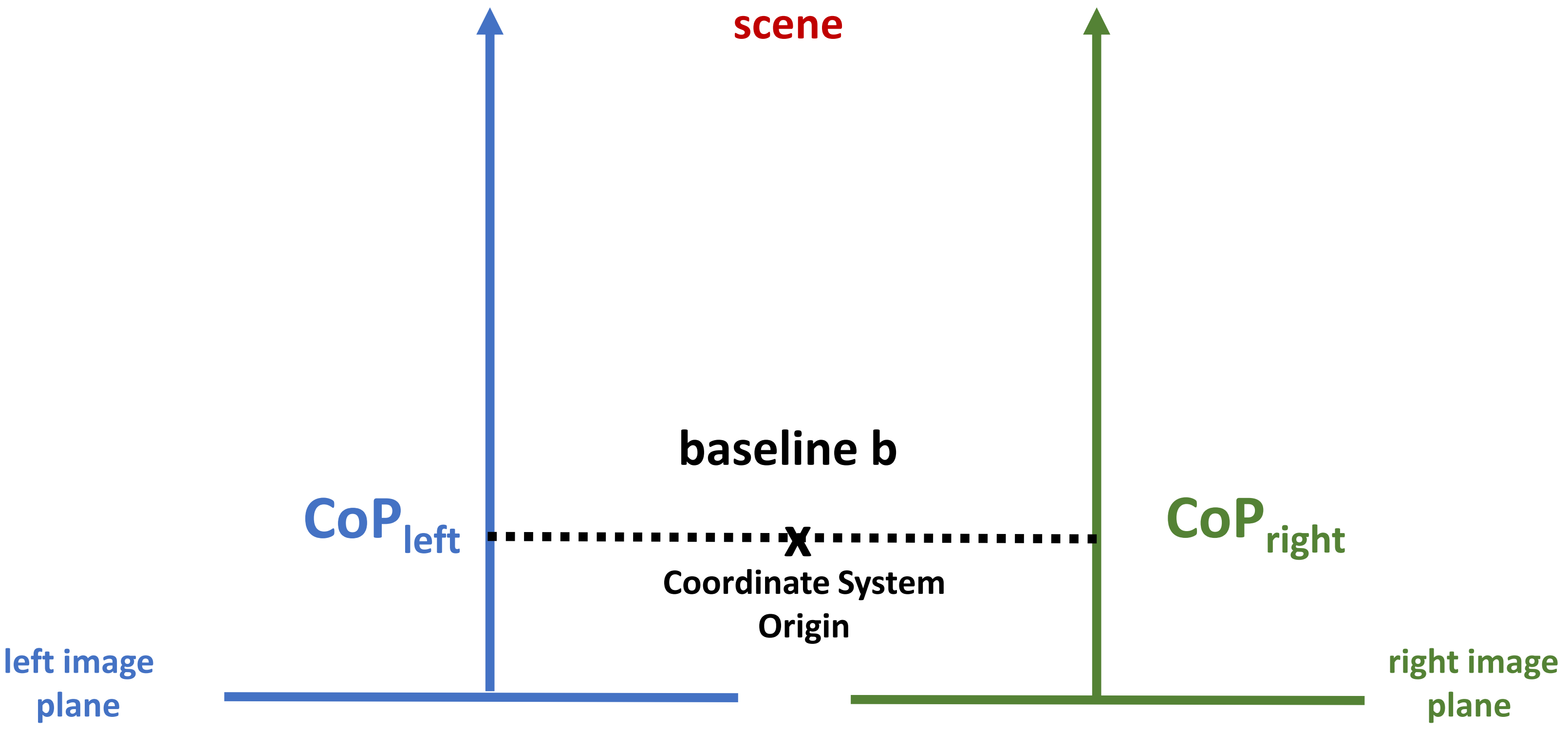
In a Monocular System: Coordinate System Origin = Center of Projection (CoP) = Pinhole

Binocular Stereo



In a Monocular System: Coordinate System Origin = Center of Projection (CoP) = Pinhole
In a Binocular System: Coordinate System Origin in the middle between CoPs

Binocular Stereo



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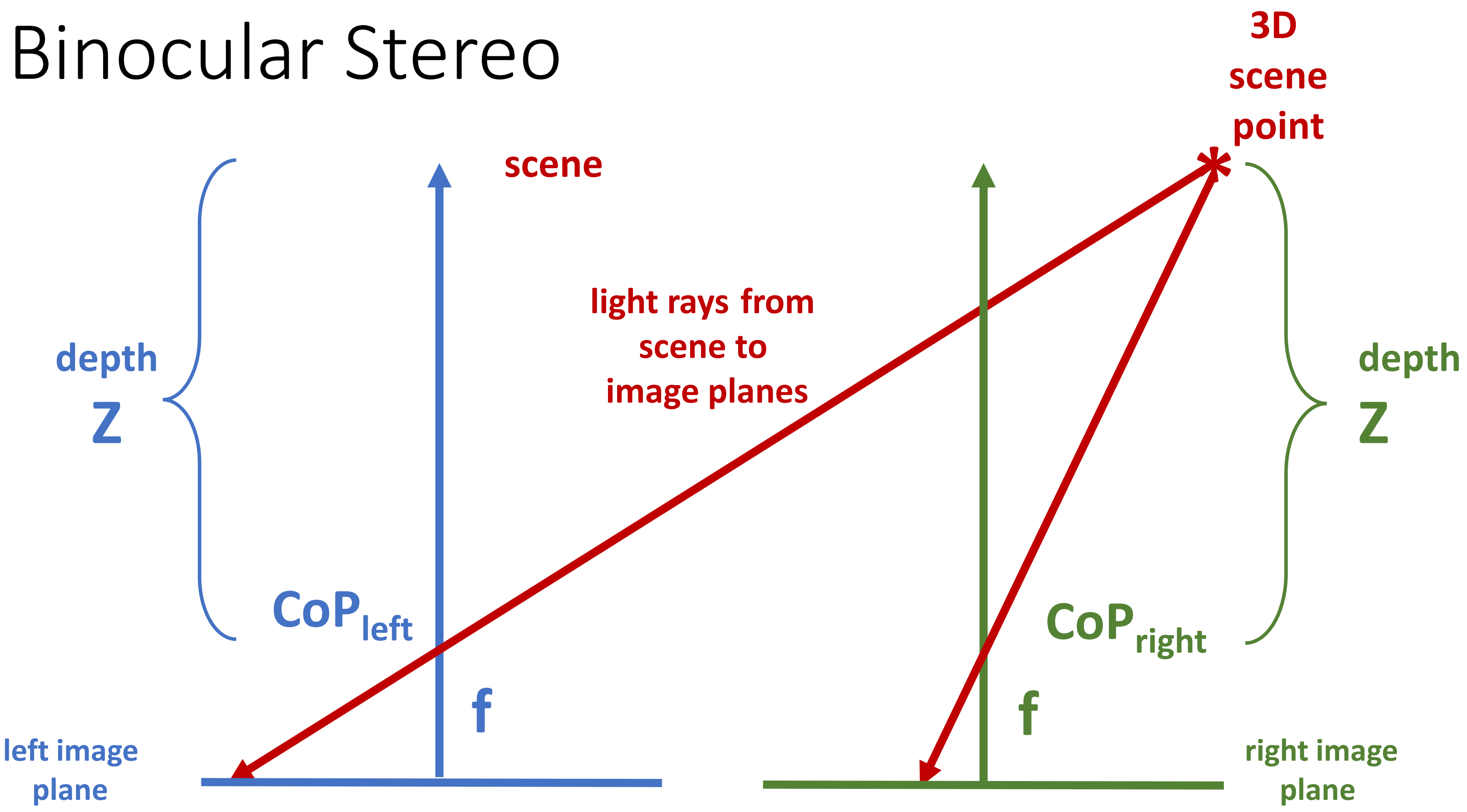
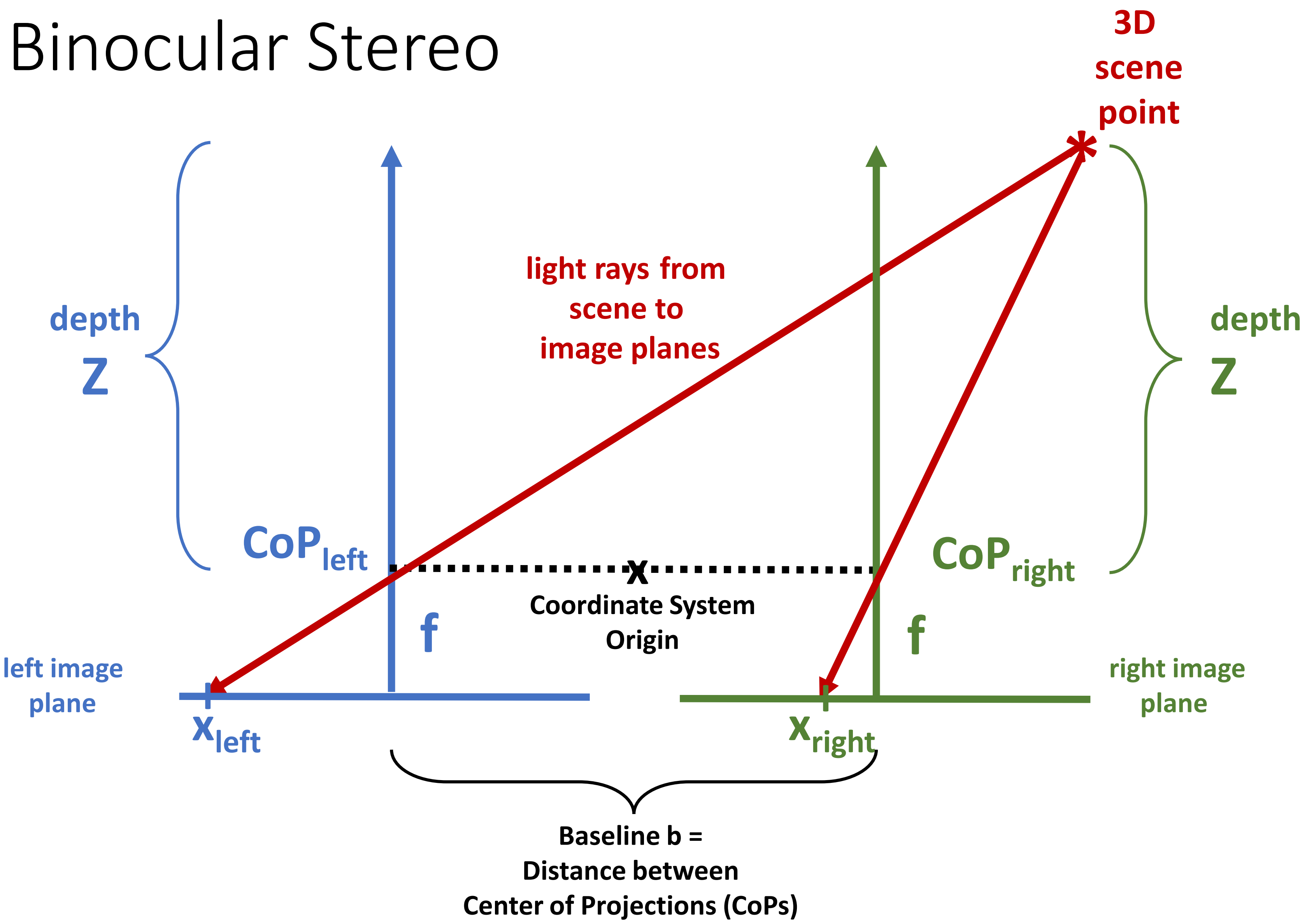
Binocular Stereo



Binocular Stereo



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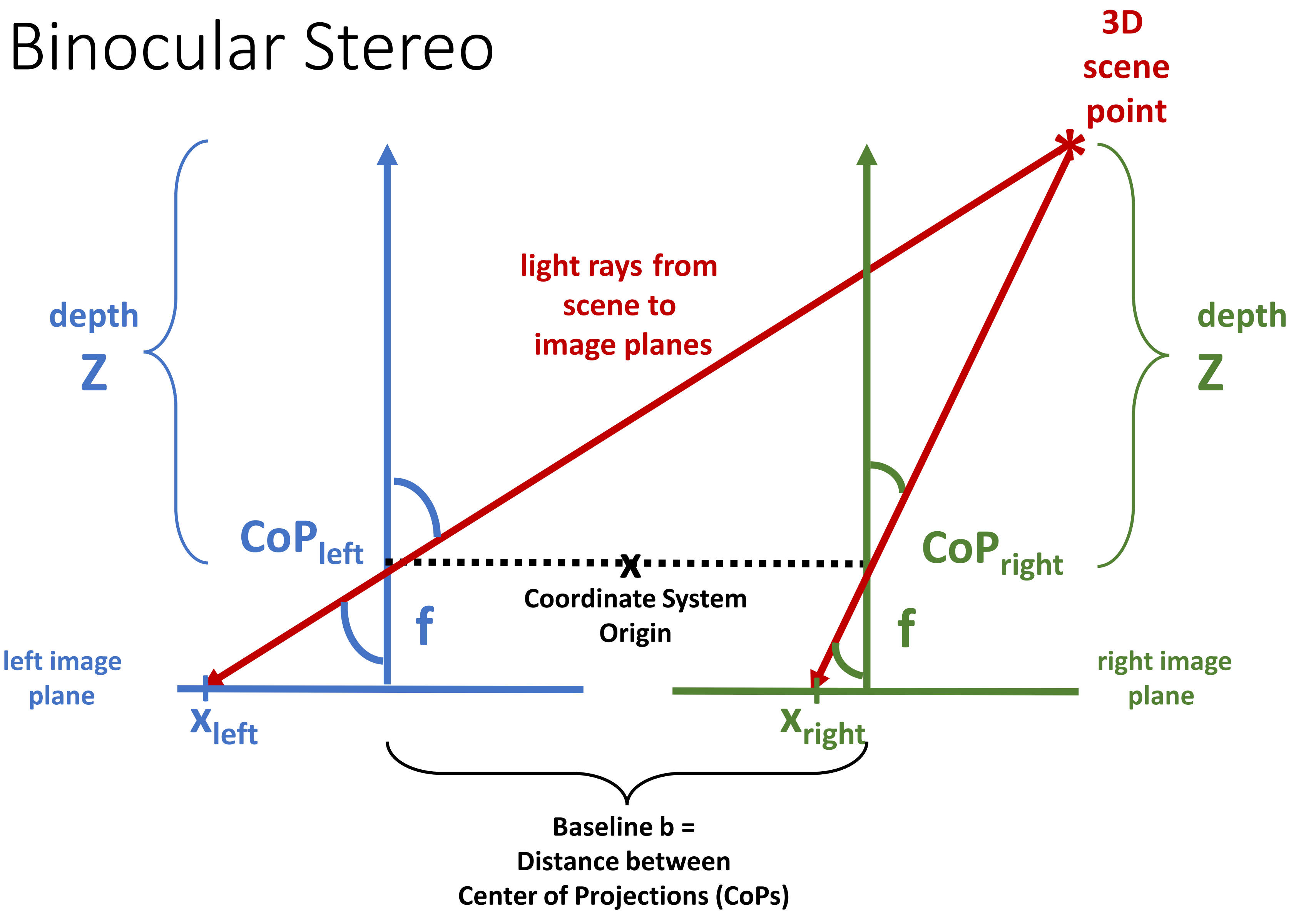

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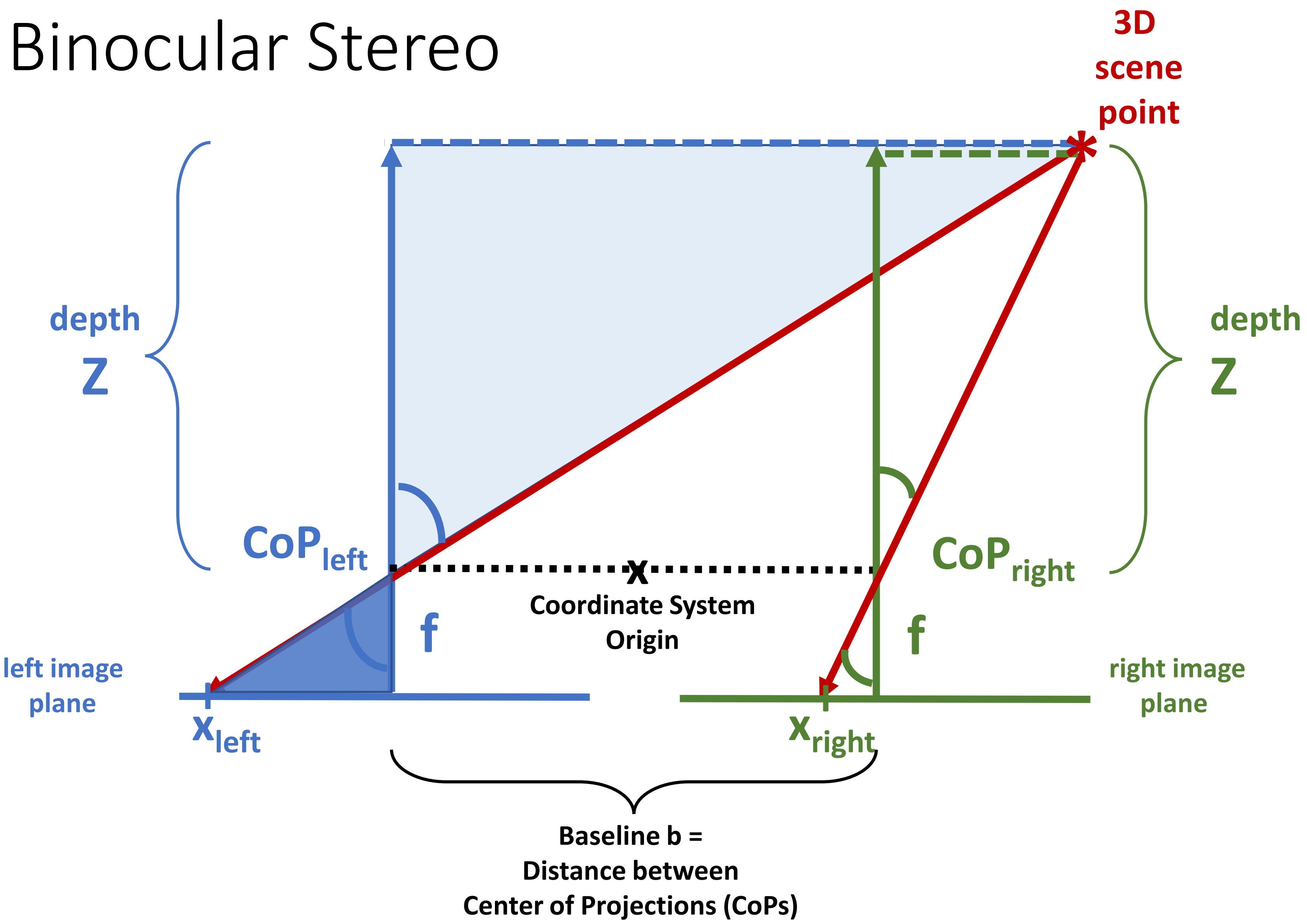
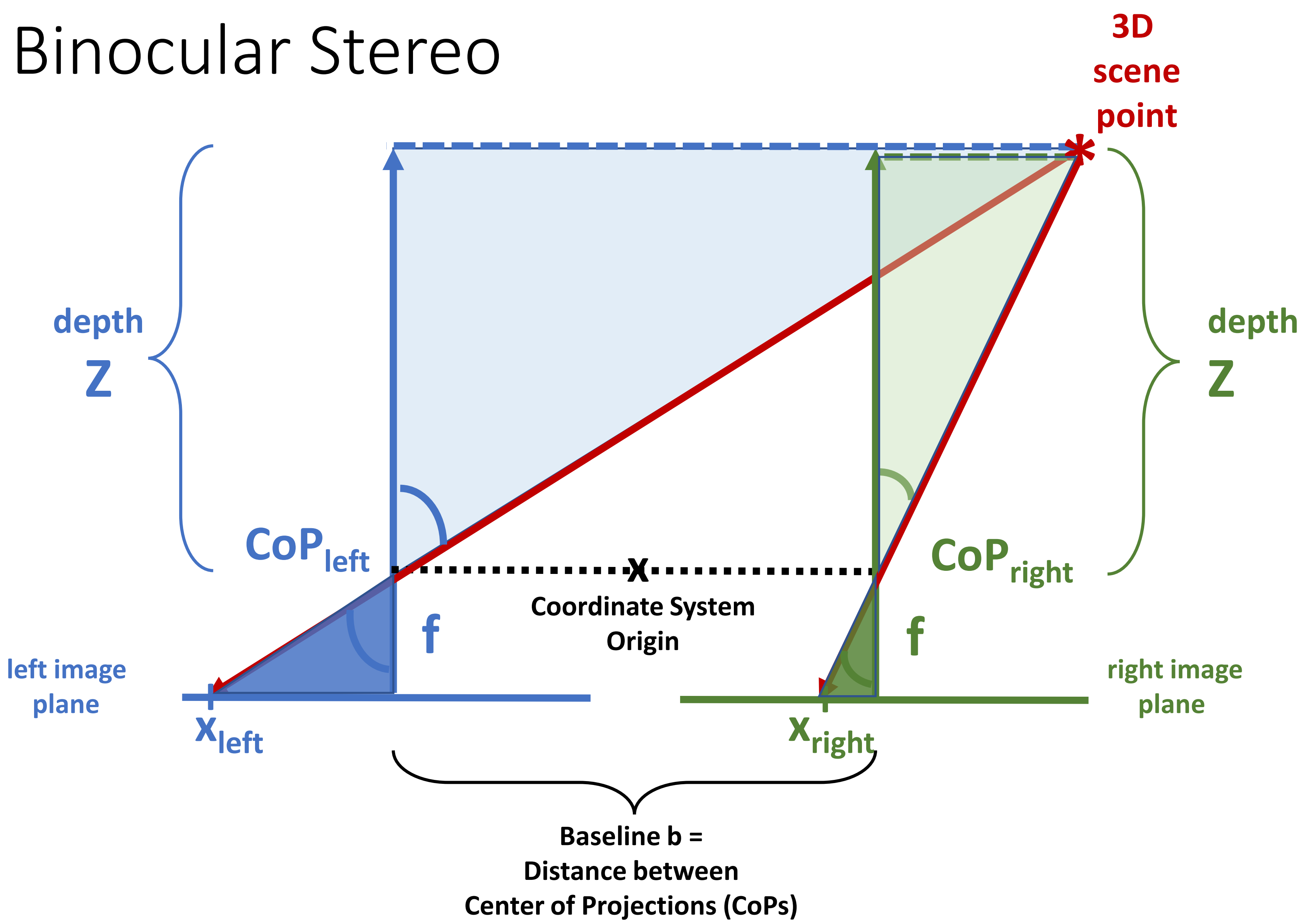


Binocular Stereo



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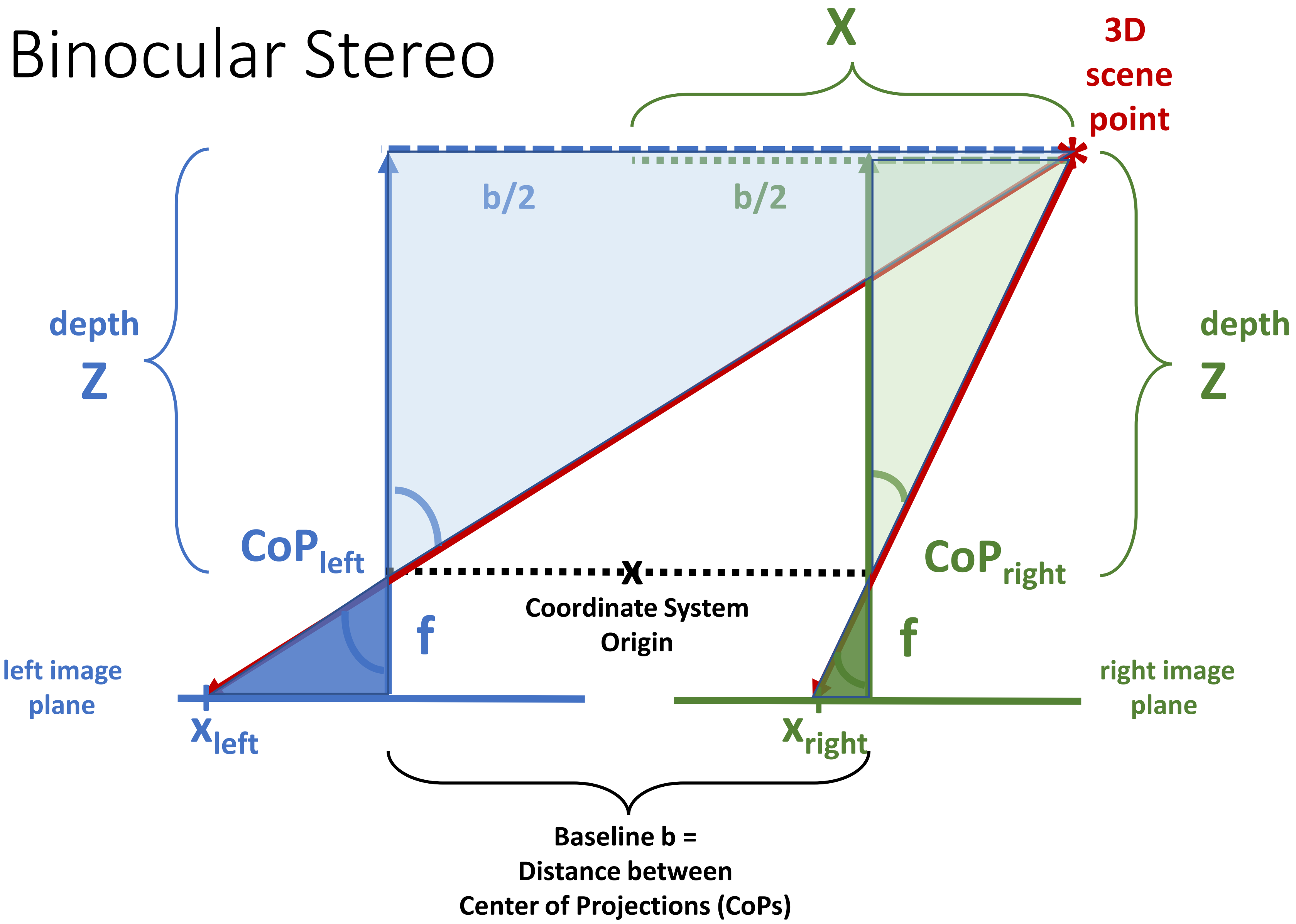
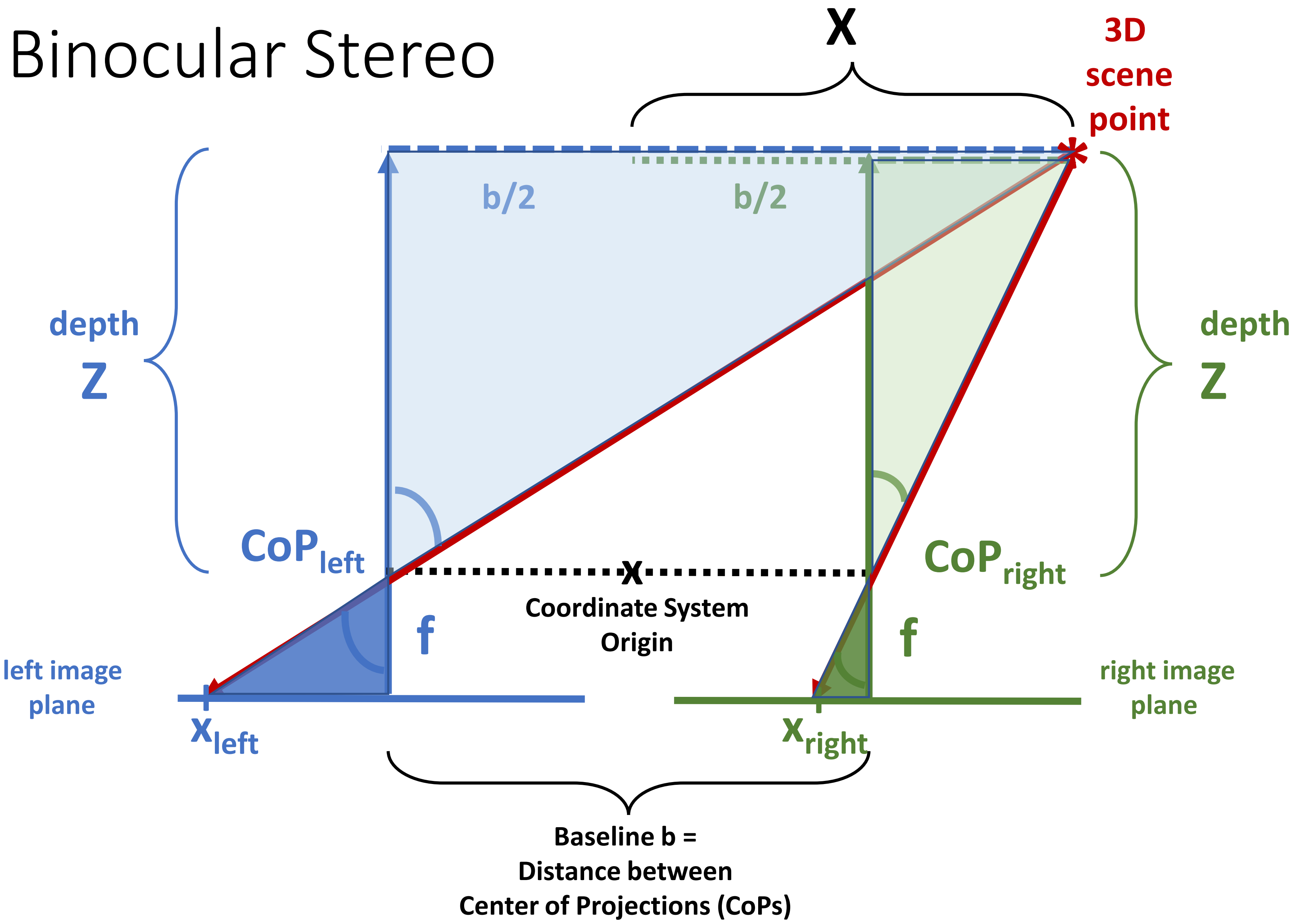


Binocular Stereo

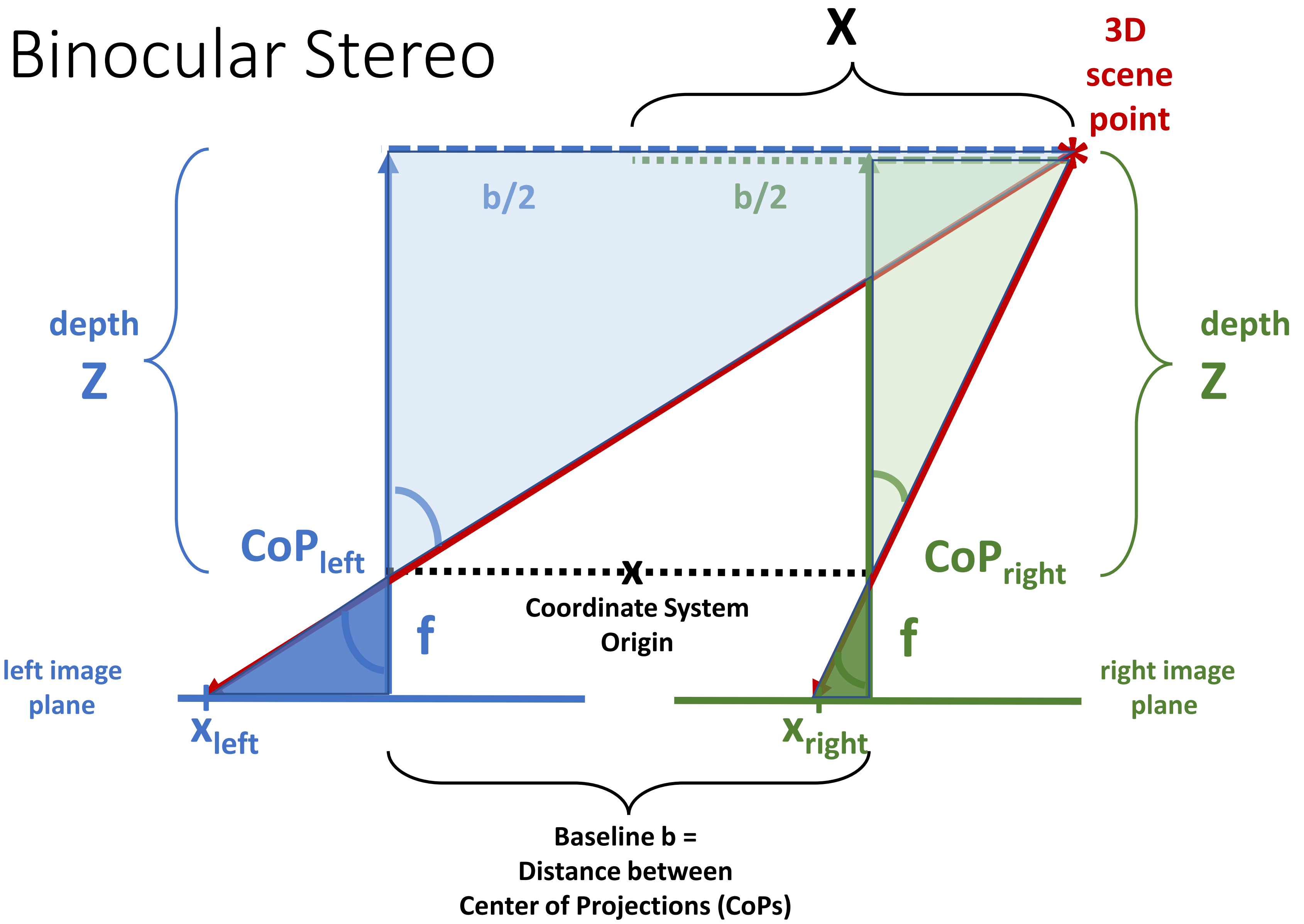


Projection Equations:

$x_{left}/f = (X+b/2)/Z$

$x_{right}/f = (X-b/2)/Z$

Binocular Stereo



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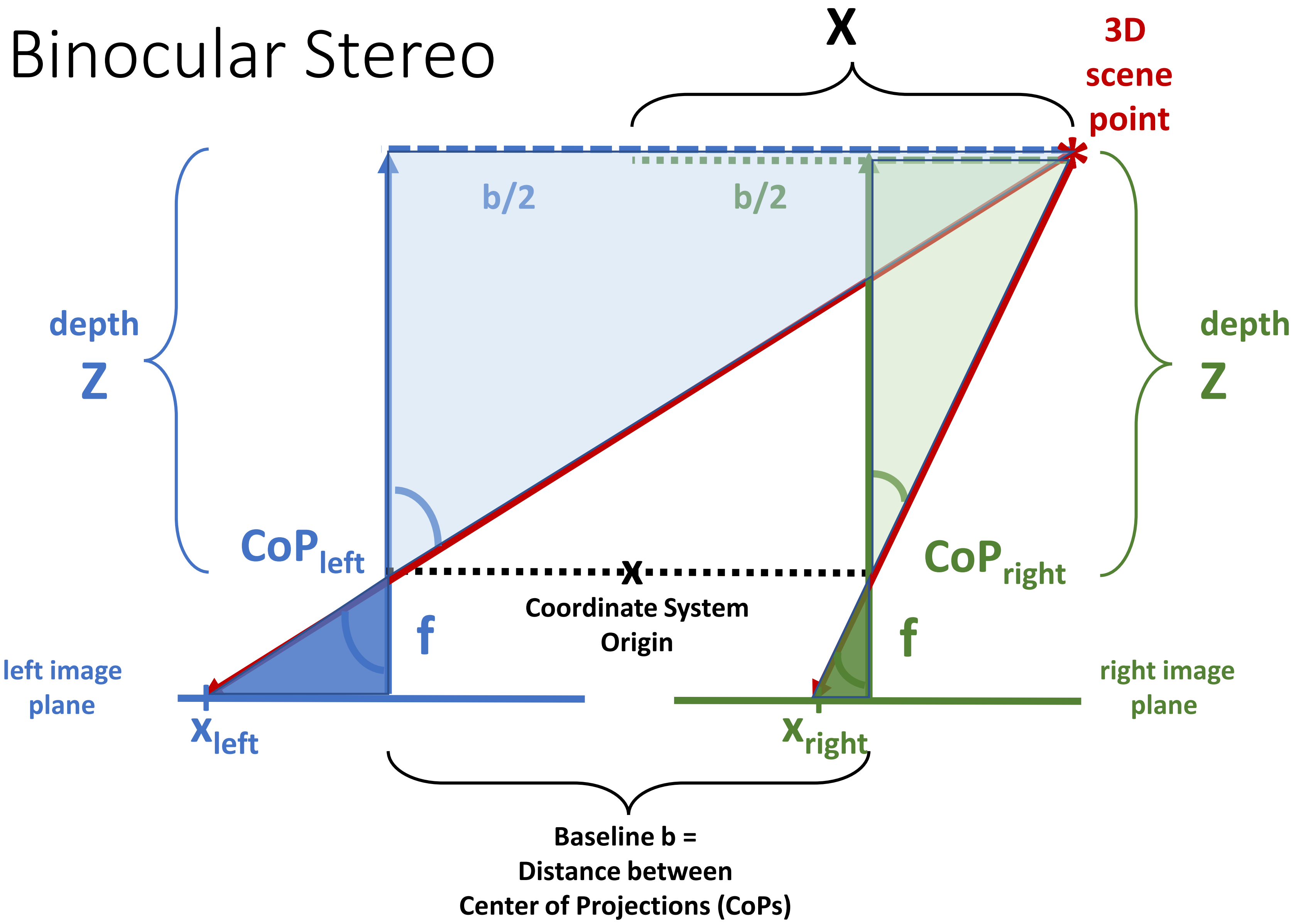


Projection Equations:

$x_{left}/f = (X+b/2)/Z$

$x_{right}/f = (X-b/2)/Z$

Subtract the 2[nd] equation from the 1[st] equation:

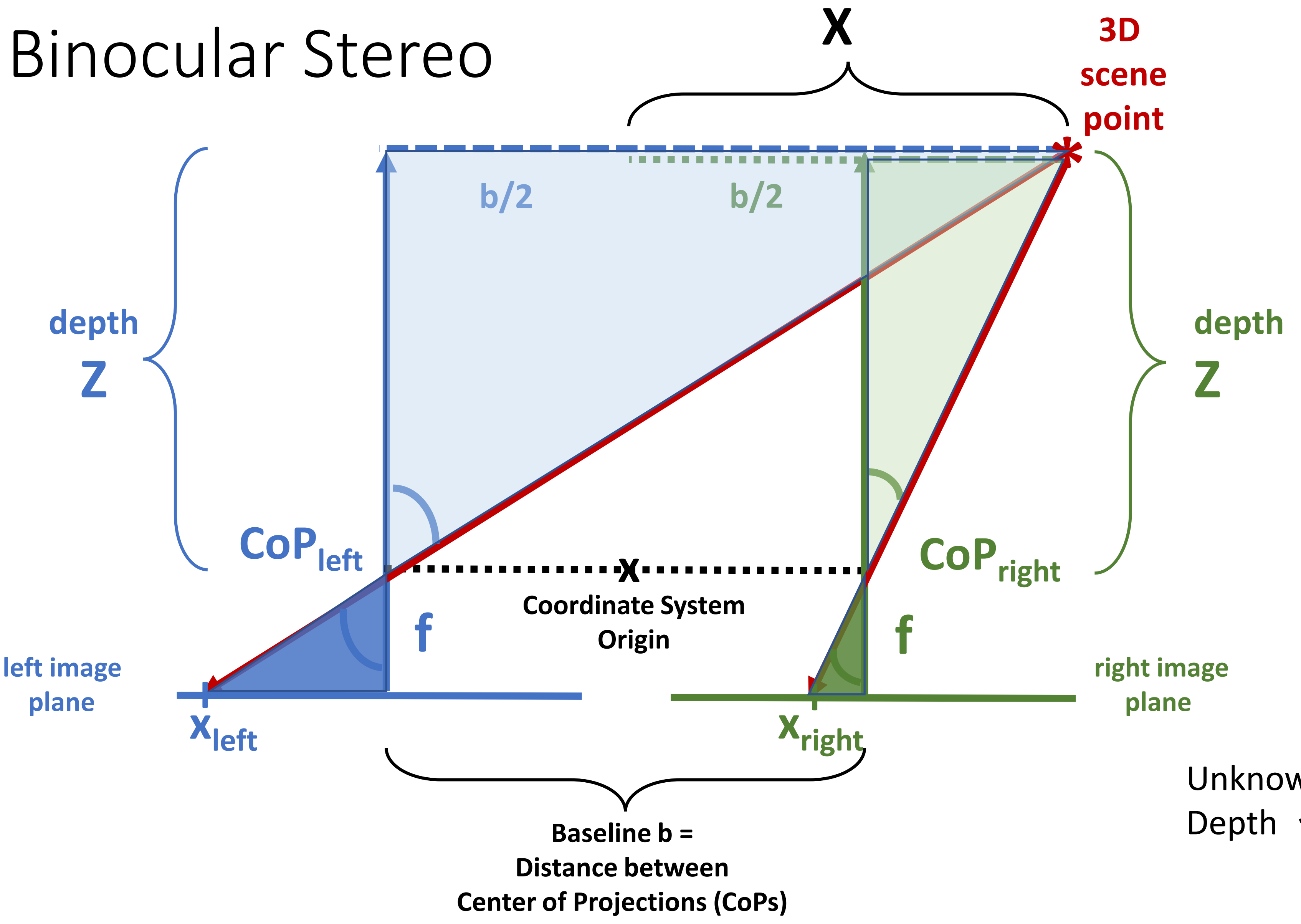
$(x_{left} - x_{right})/f = (X+b/2-X+b/2))/Z$

which results in:

$(x_{left} - x_{right})/f = b/Z$ or:

$Z = bf/(x_{left} - x_{right})$

Binocular Stereo



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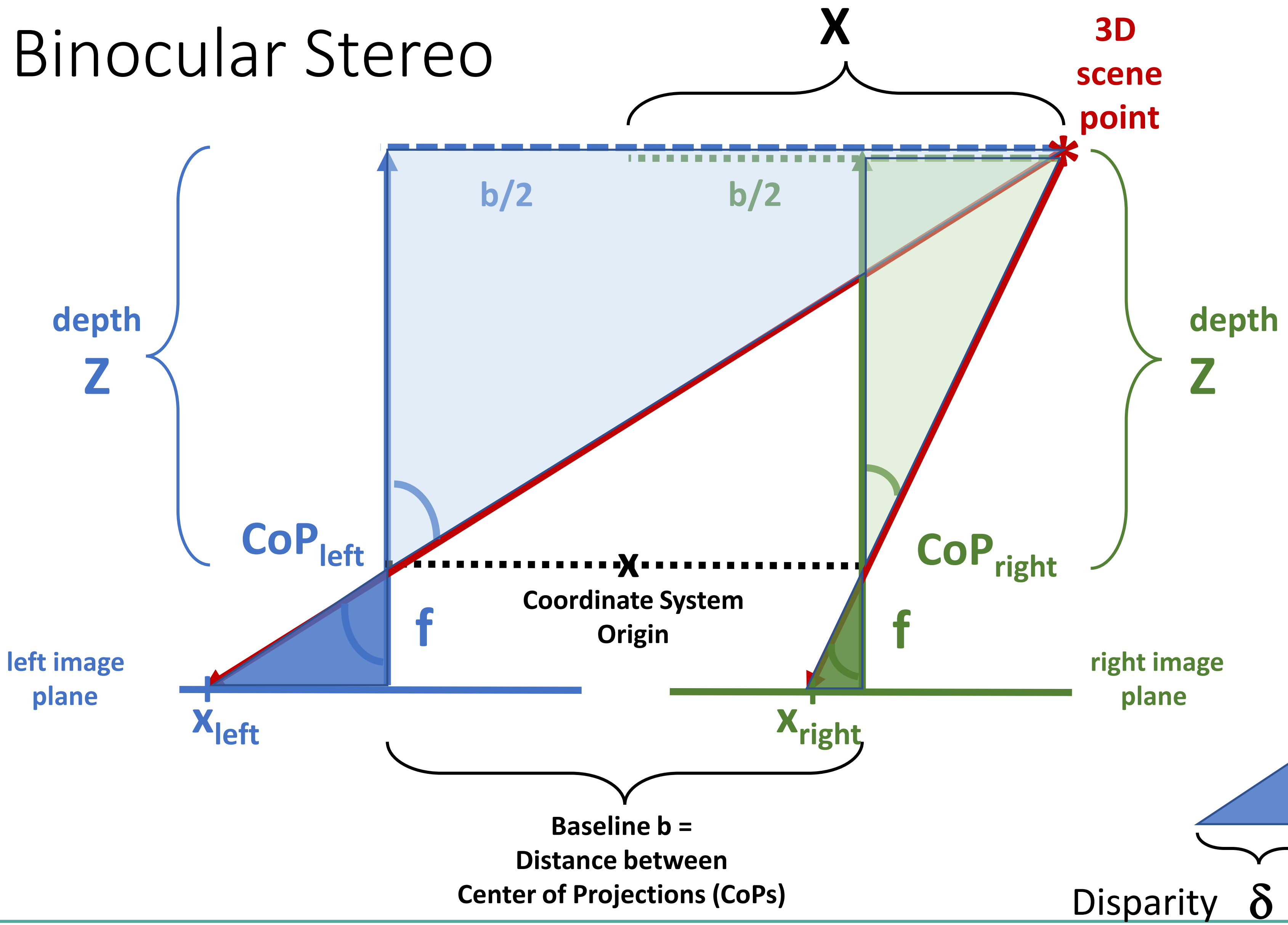
Unknown Depth → $\underbrace{Z}_{} = bf/\underbrace{(x_{left} - x_{right})}_{\text{measured}}$

known

measured

Binocular Stereo



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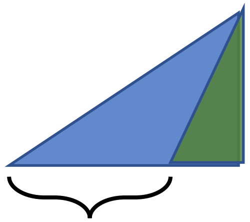
which results in:

$(x_{left} - x_{right})/f = b/Z$ or:

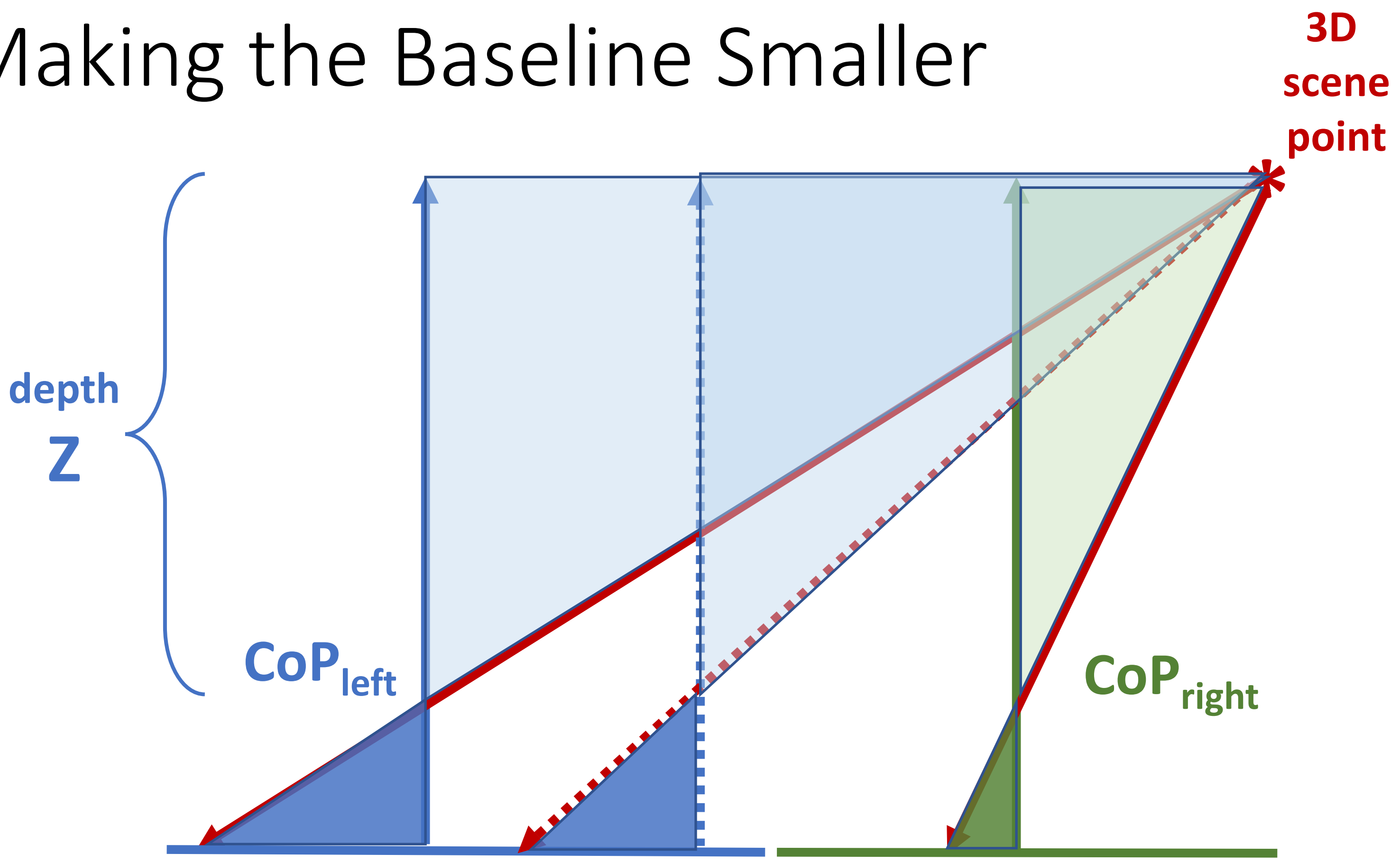
$Z = bf/\underbrace{(x_{left} - x_{right})}_{\delta}$ or:



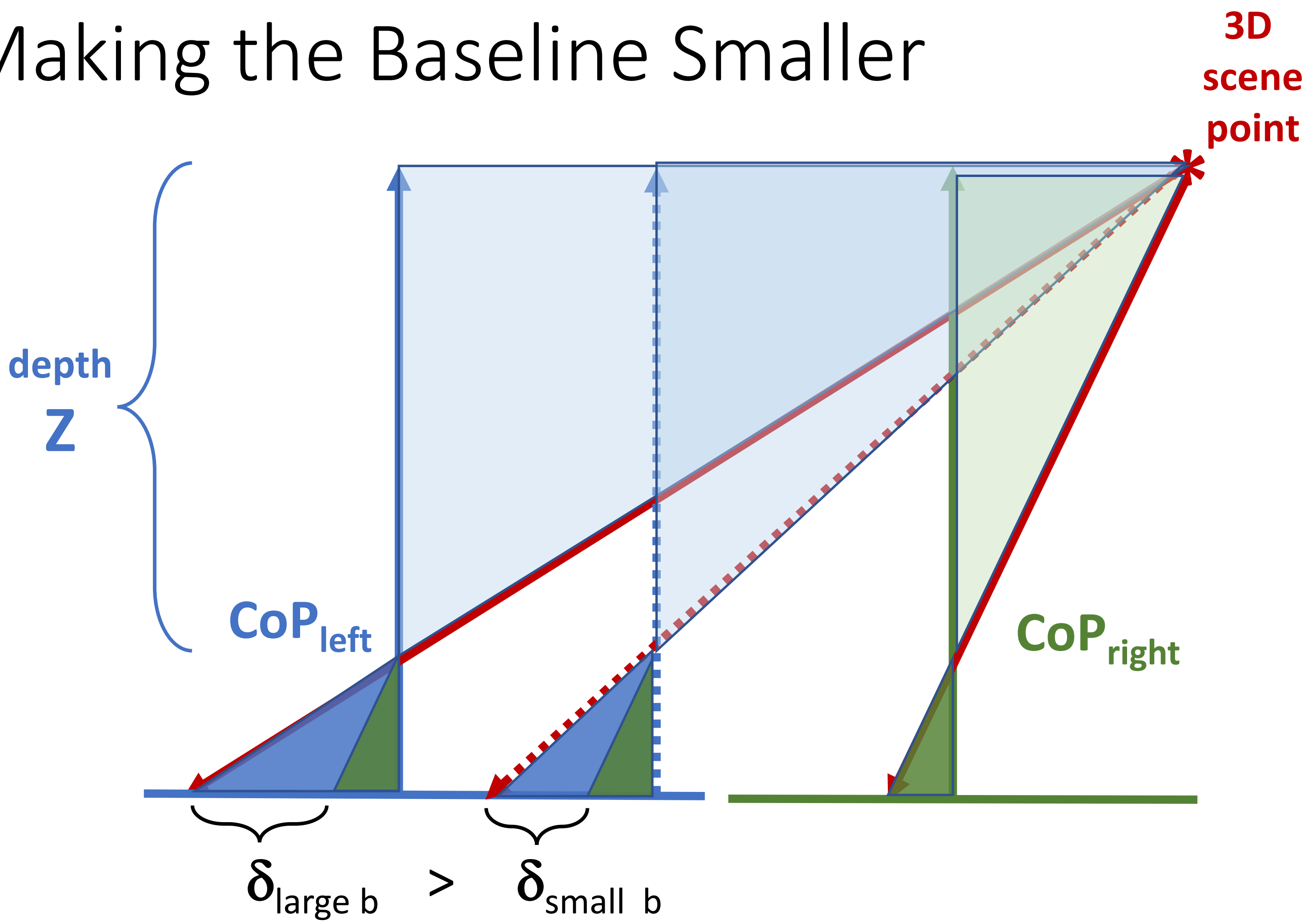
Disparity δ

$\boxed{Z = bf/\delta}$

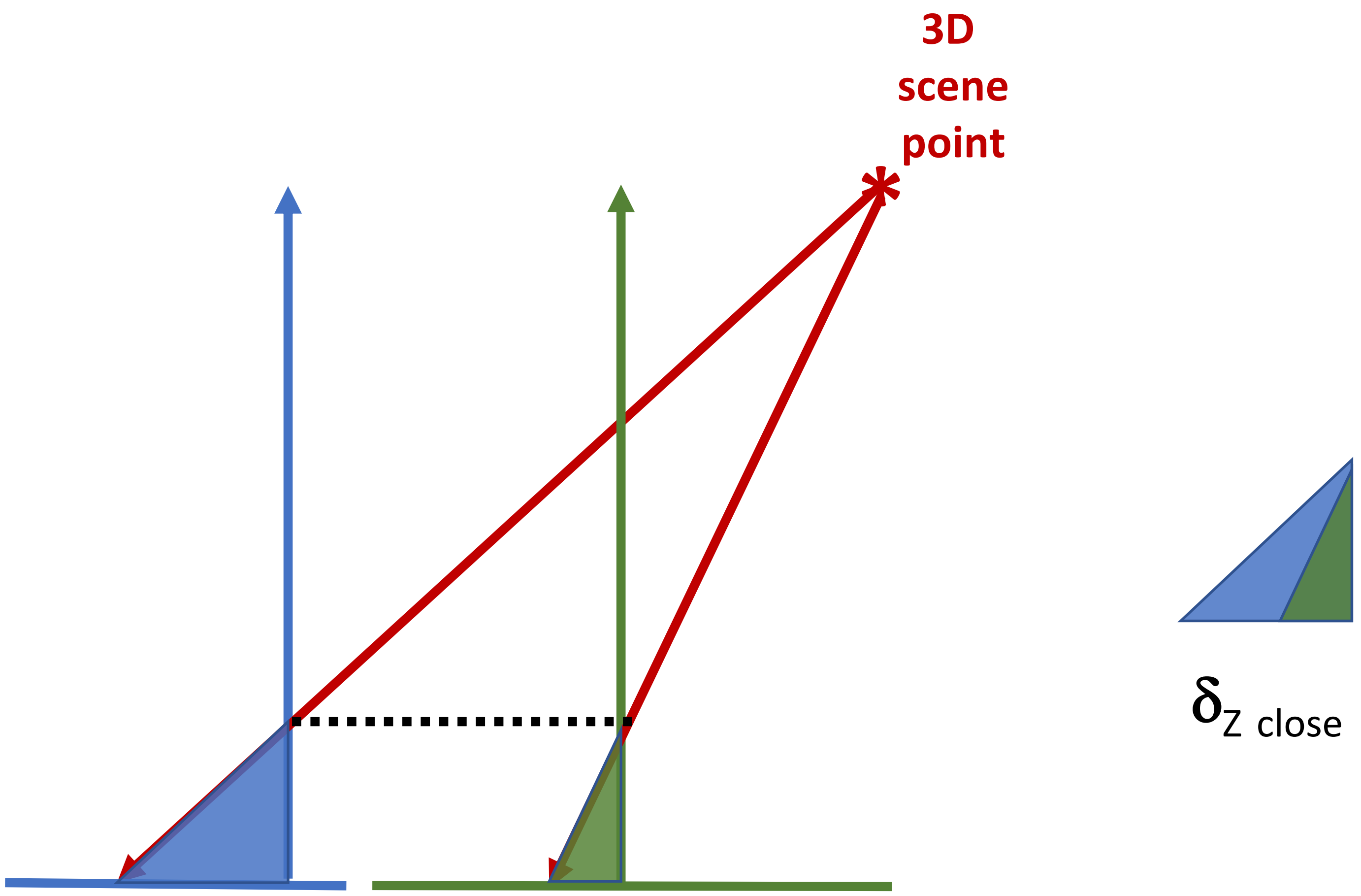
Making the Baseline Smaller



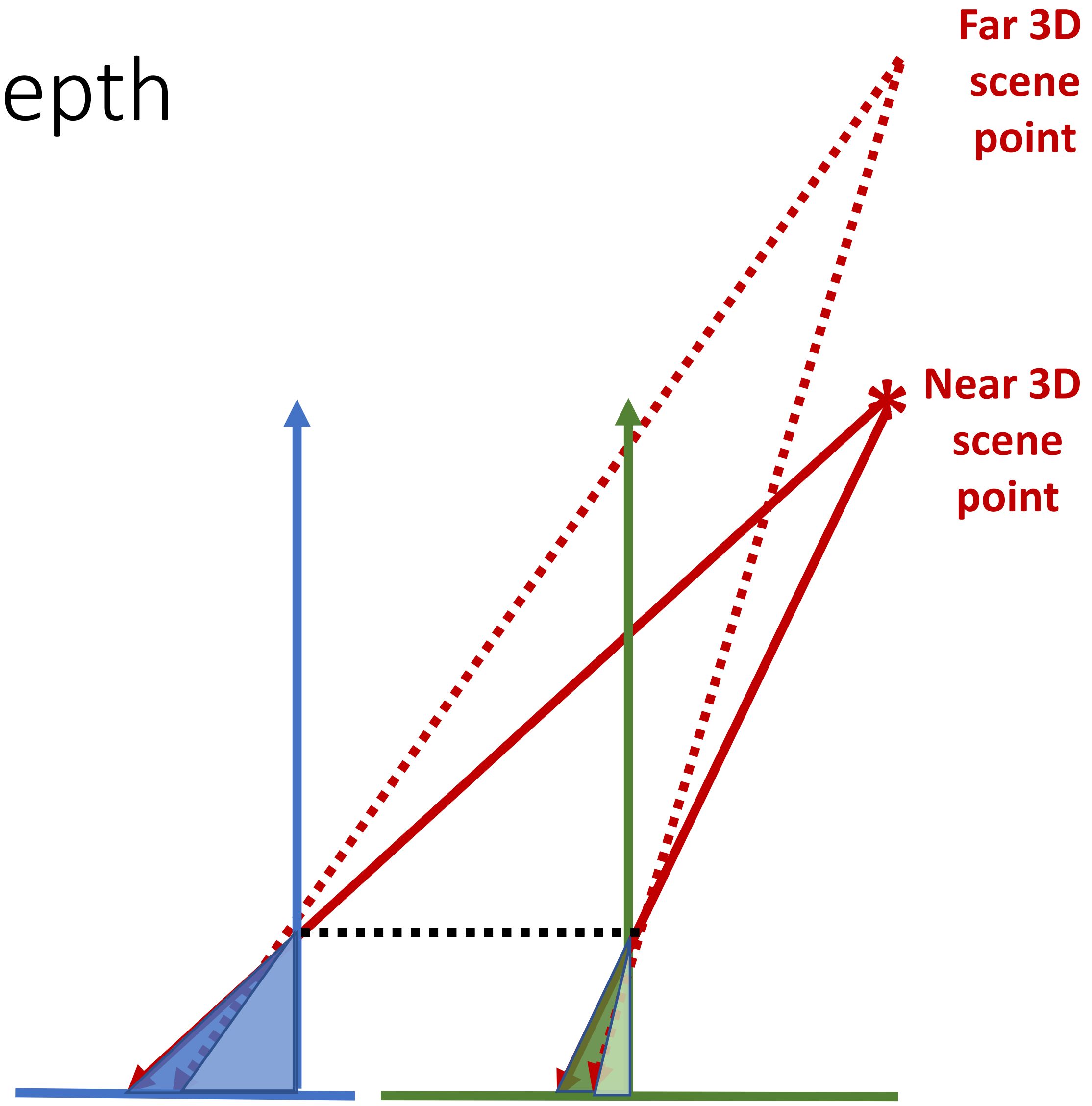
Making the Baseline Smaller



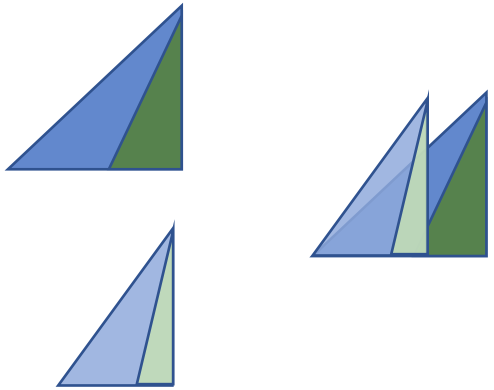
Increasing Depth



Increasing Depth



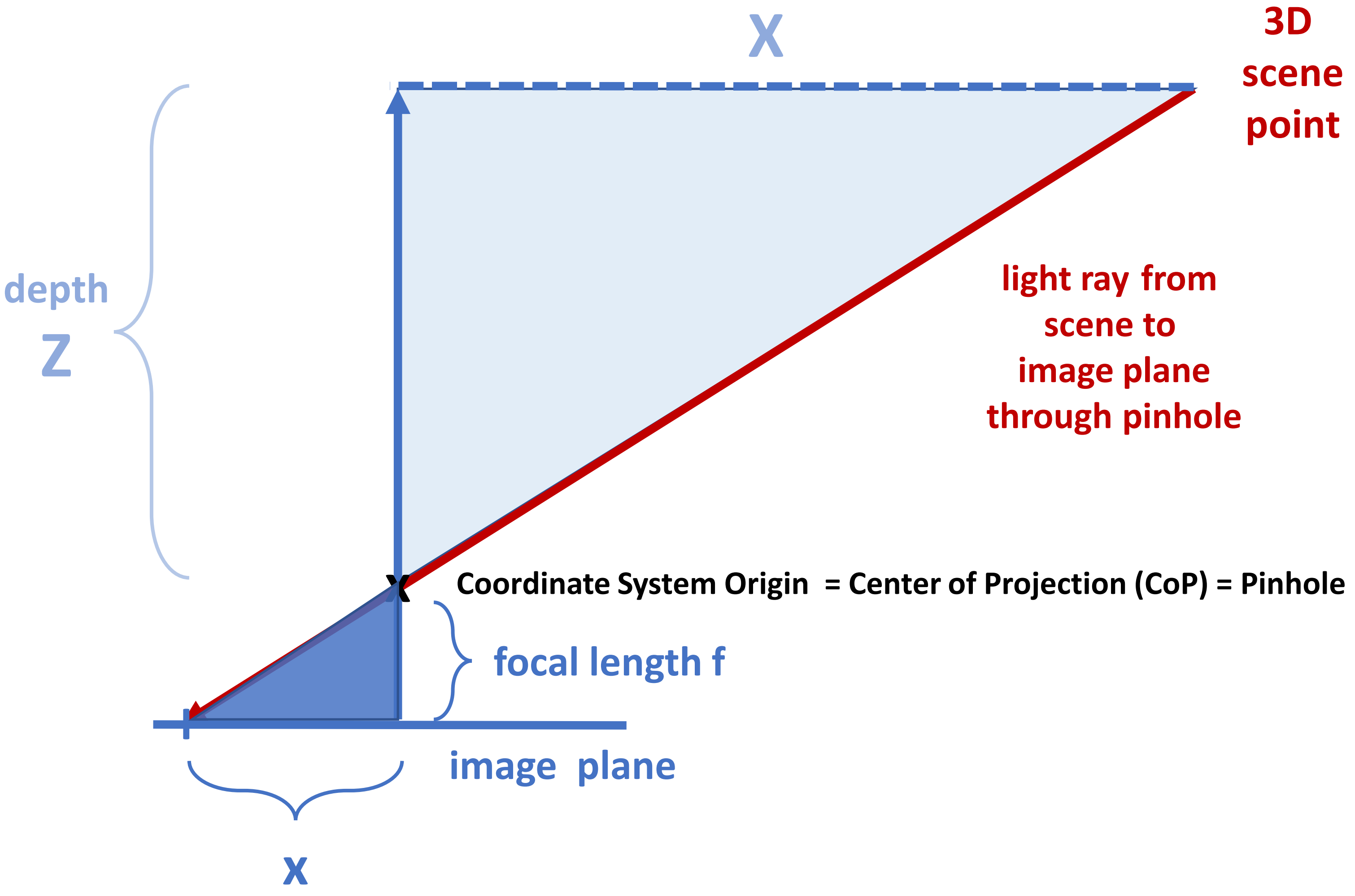
$\delta_{\text{near Z}} > \delta_{\text{far Z}}$



Summary of Concepts: Binocular Stereo

- Considered only special case: Parallel optical axes, image planes aligned, same focal length
- Combining perspective projection equations for both cameras yields formula $Z = bf/\delta$
- Disparity changes with changes in b or Z

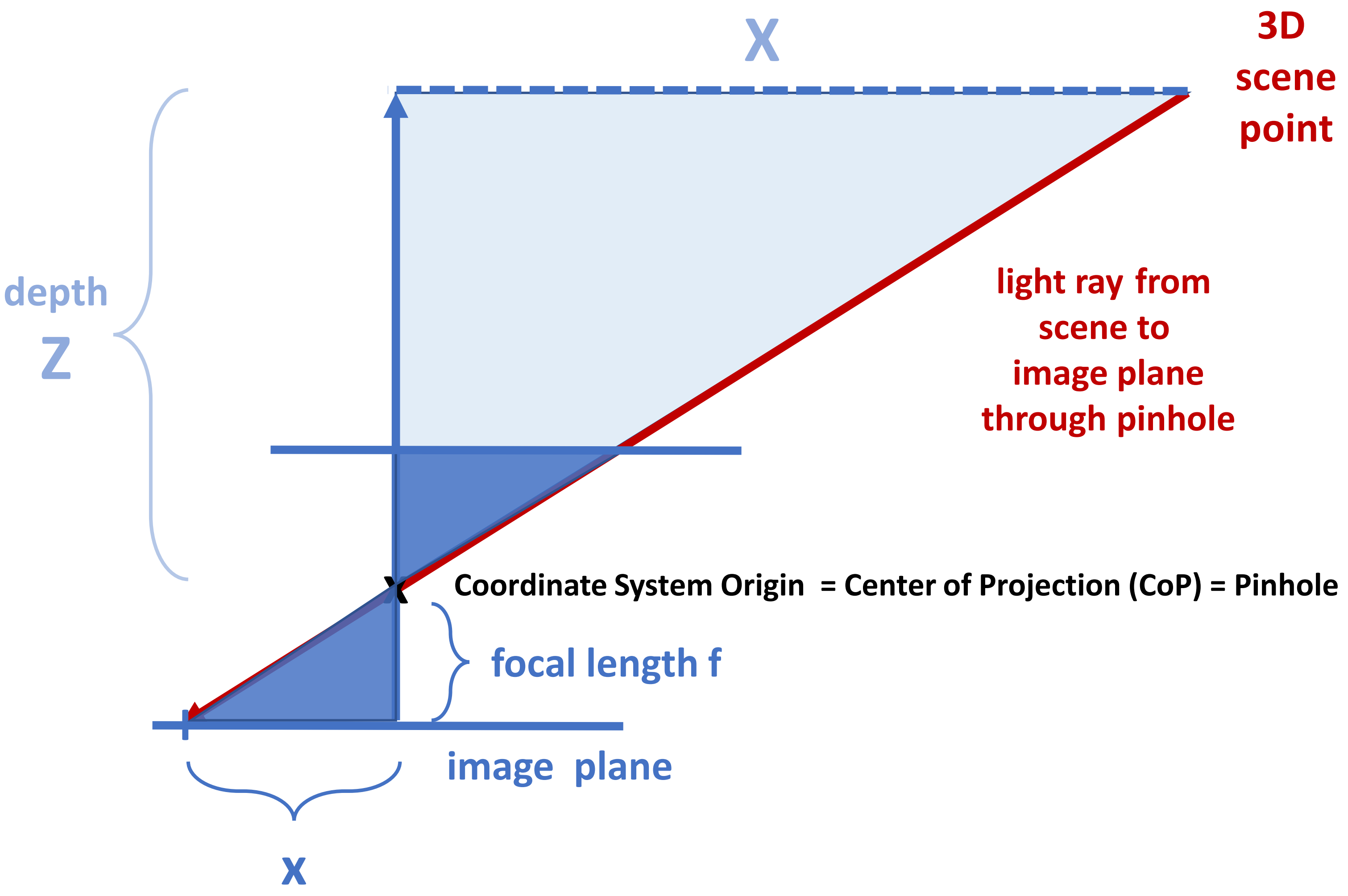
Back to the Single Camera Pinhole Model



Projection Equation:

$X/Z = x/f$

Placing Image Plane in Front of Pinhole



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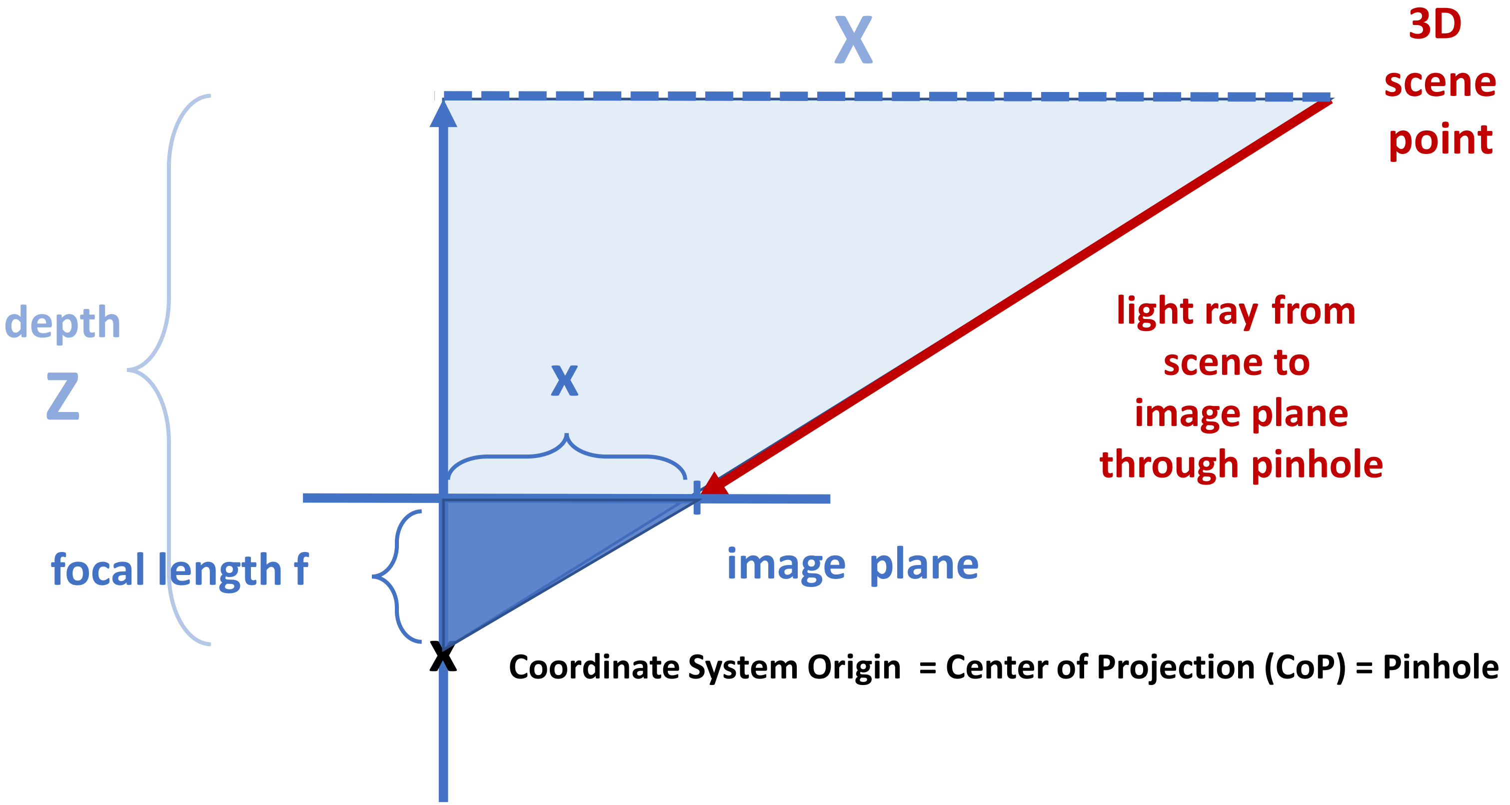
Placing Image Plane in Front of Pinhole

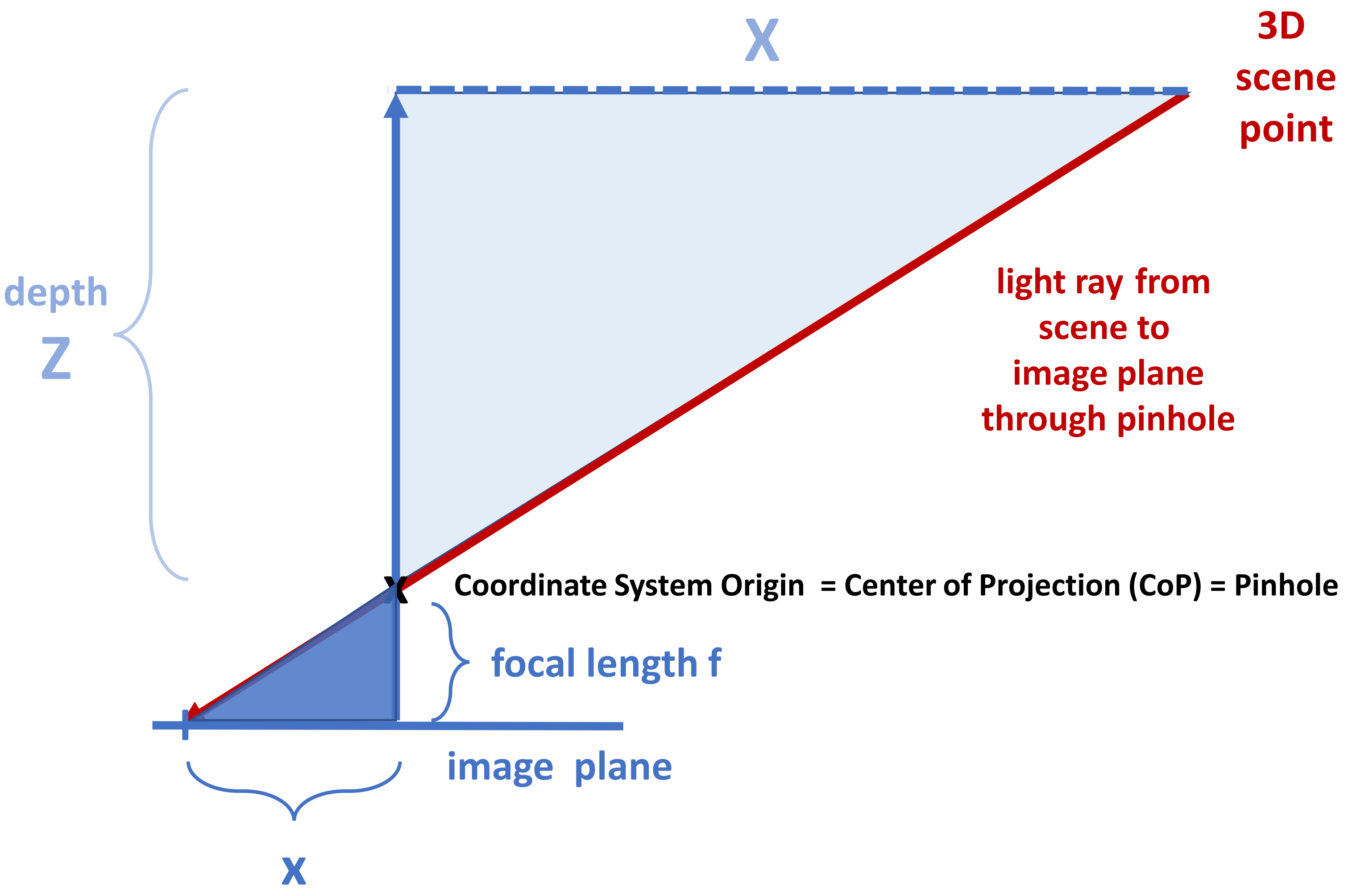


Projection Equation:

$$X/Z = x/f$$

This is done for mathematical convenience: Image x and scene X are measured in the same direction on the x-axis (a positive x means a positive X). The focal length is often set to f=1.

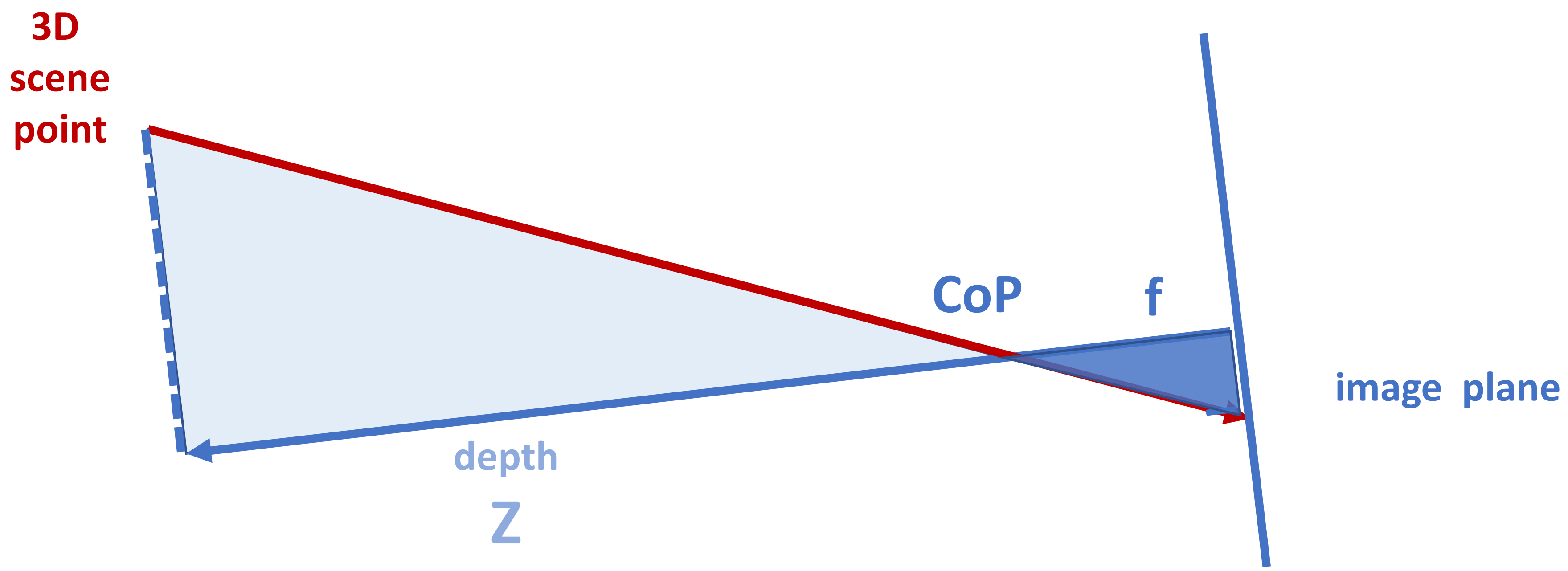
Back to the Original Single Camera Pinhole Model



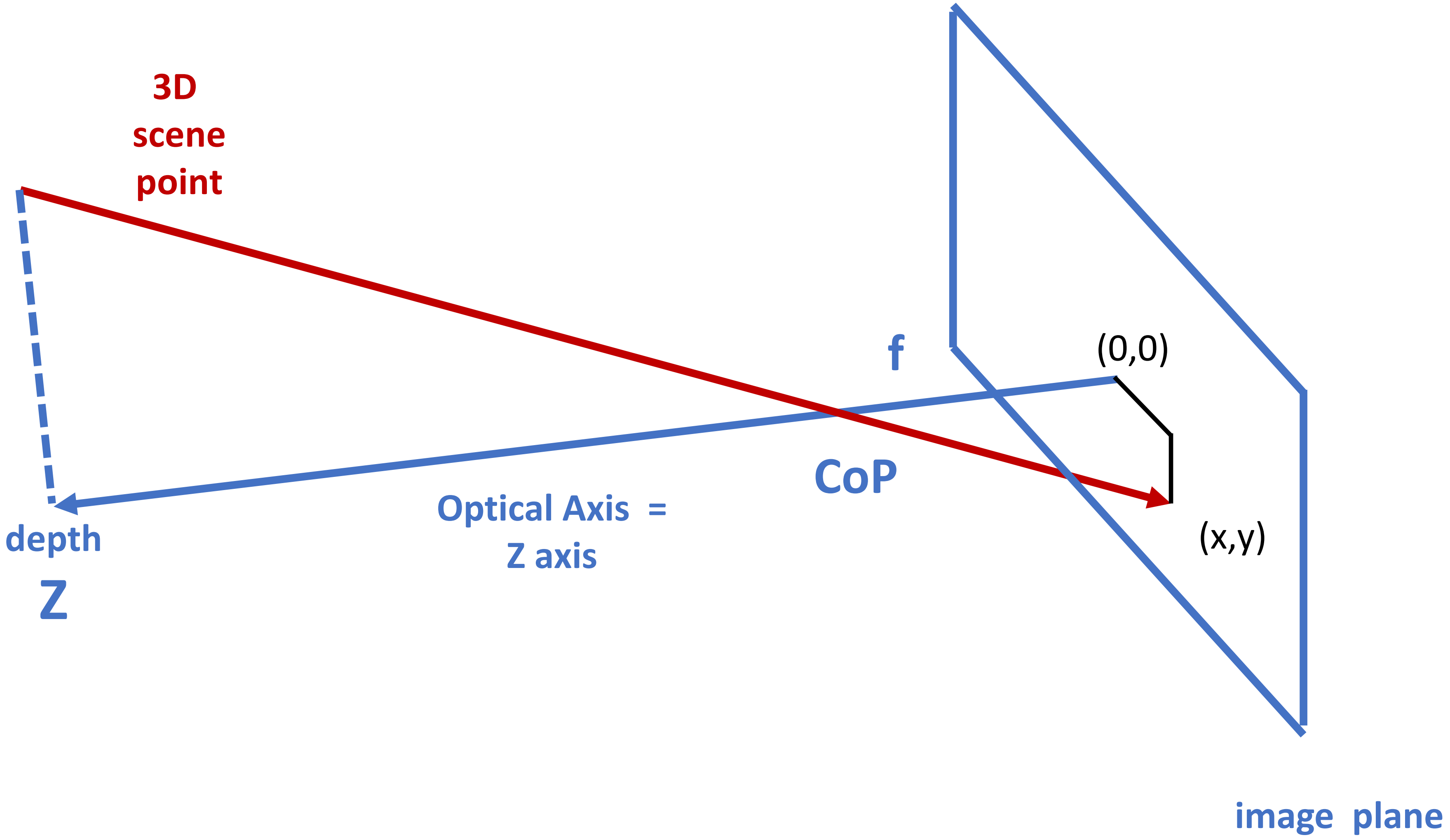
Projection Equation:

$X/Z = x/f$

Let's rotate the camera counterclockwise:



Vertical Dimension Included:



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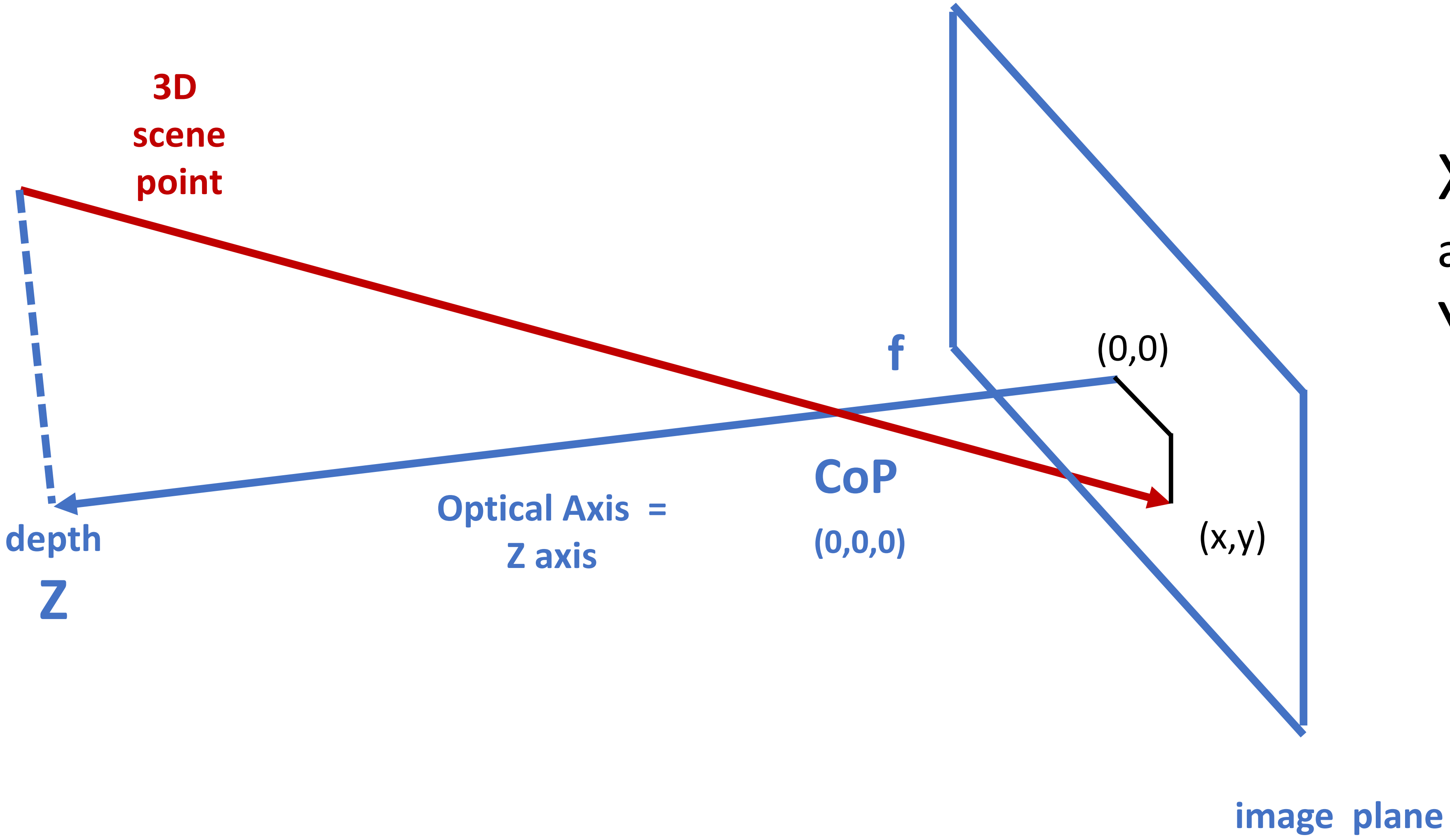
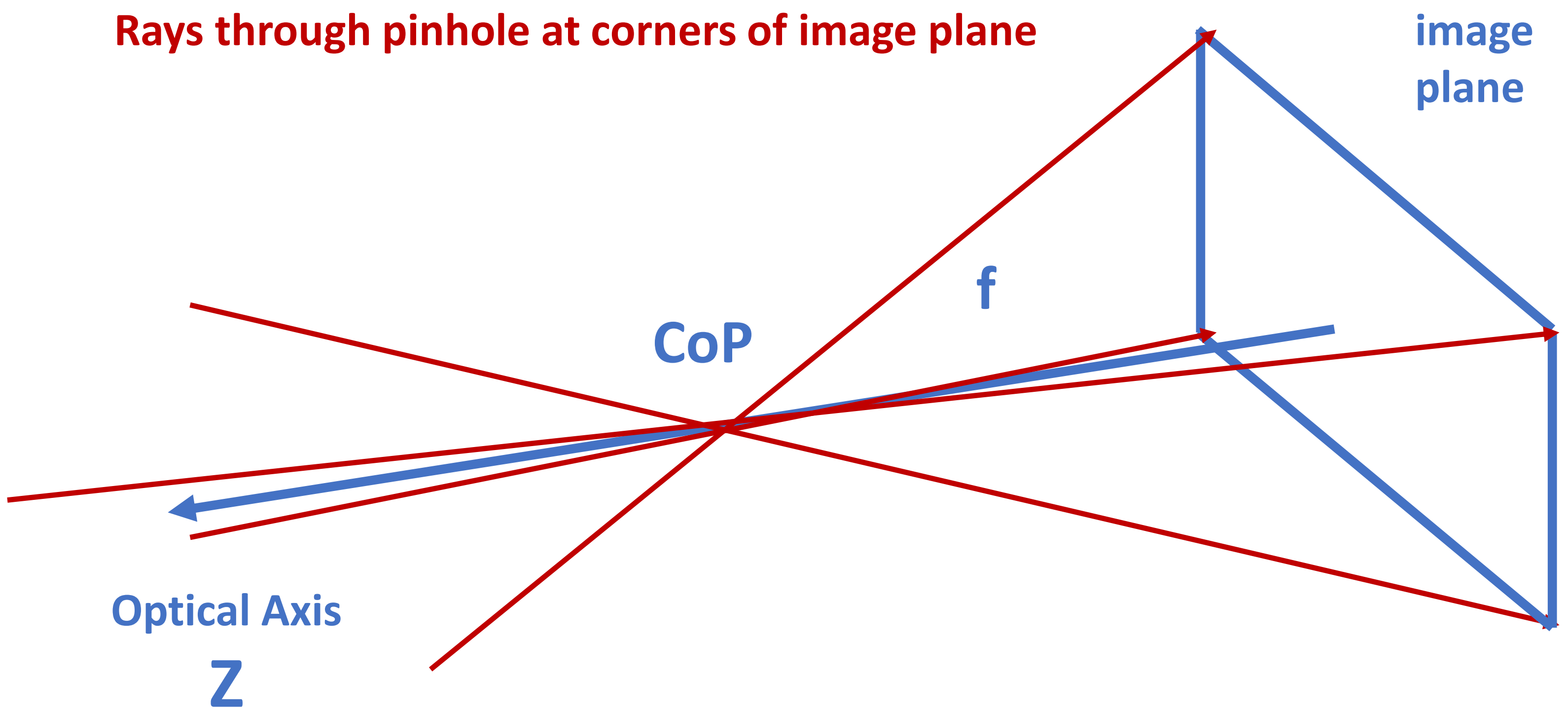


$$X/Z = x/f$$

and

$$Y/Z = y/f$$

Field of View



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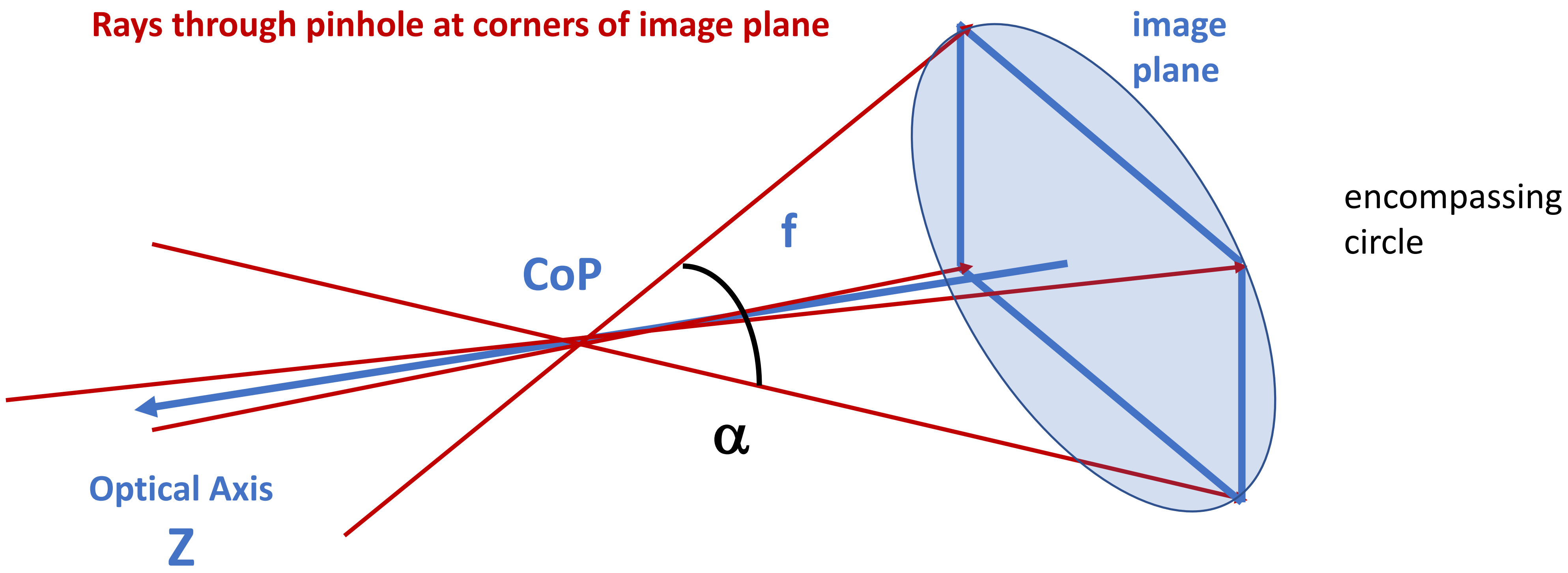
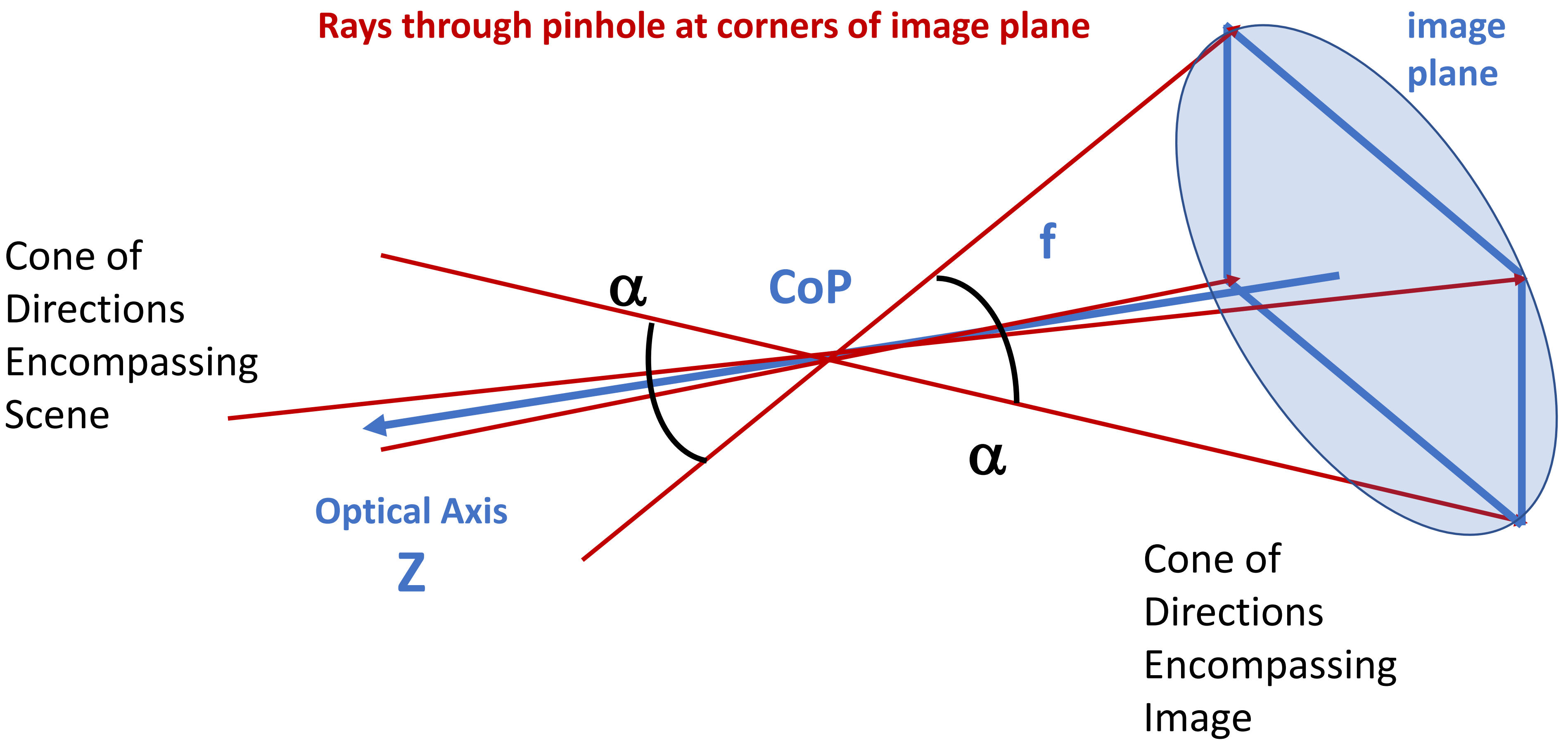
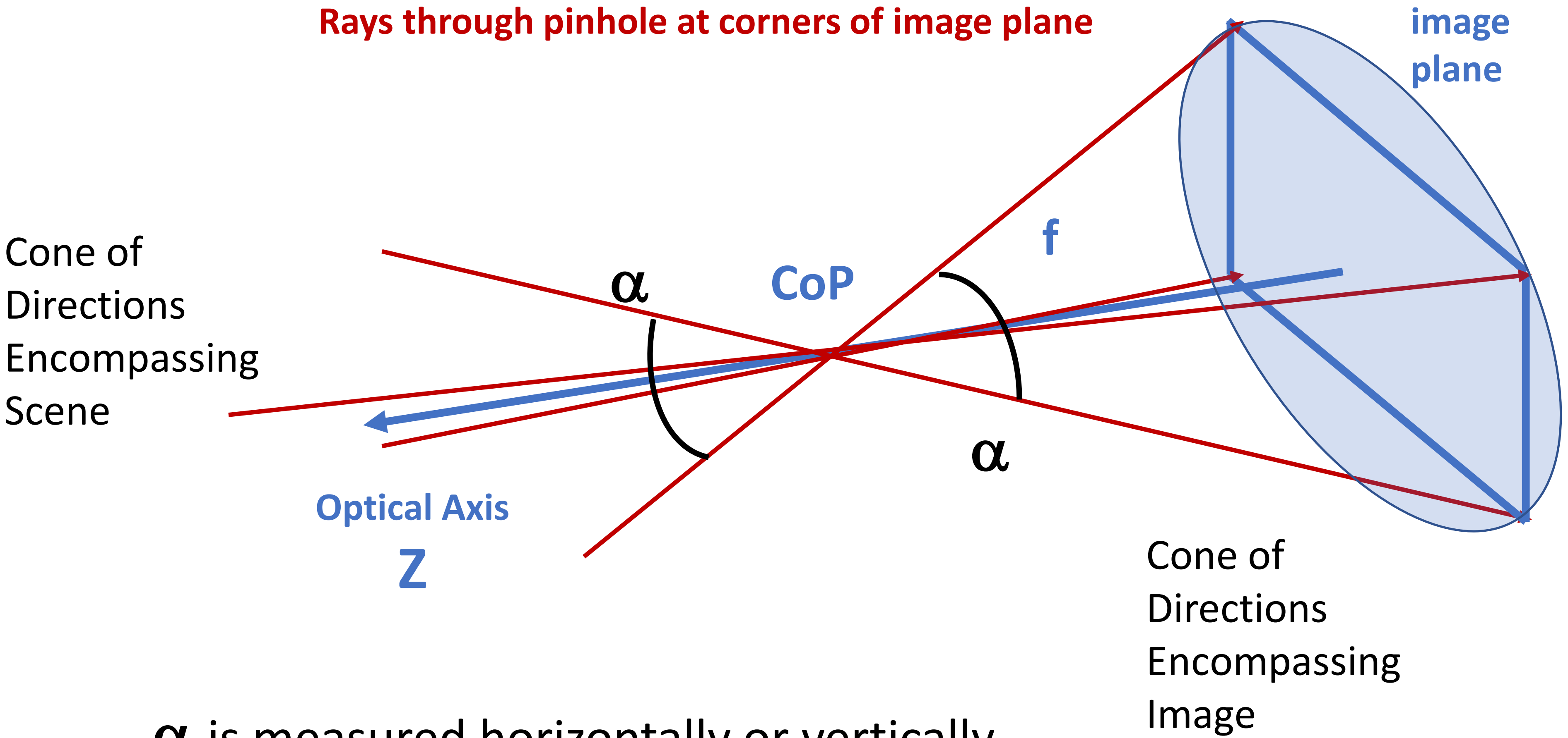


Field of View



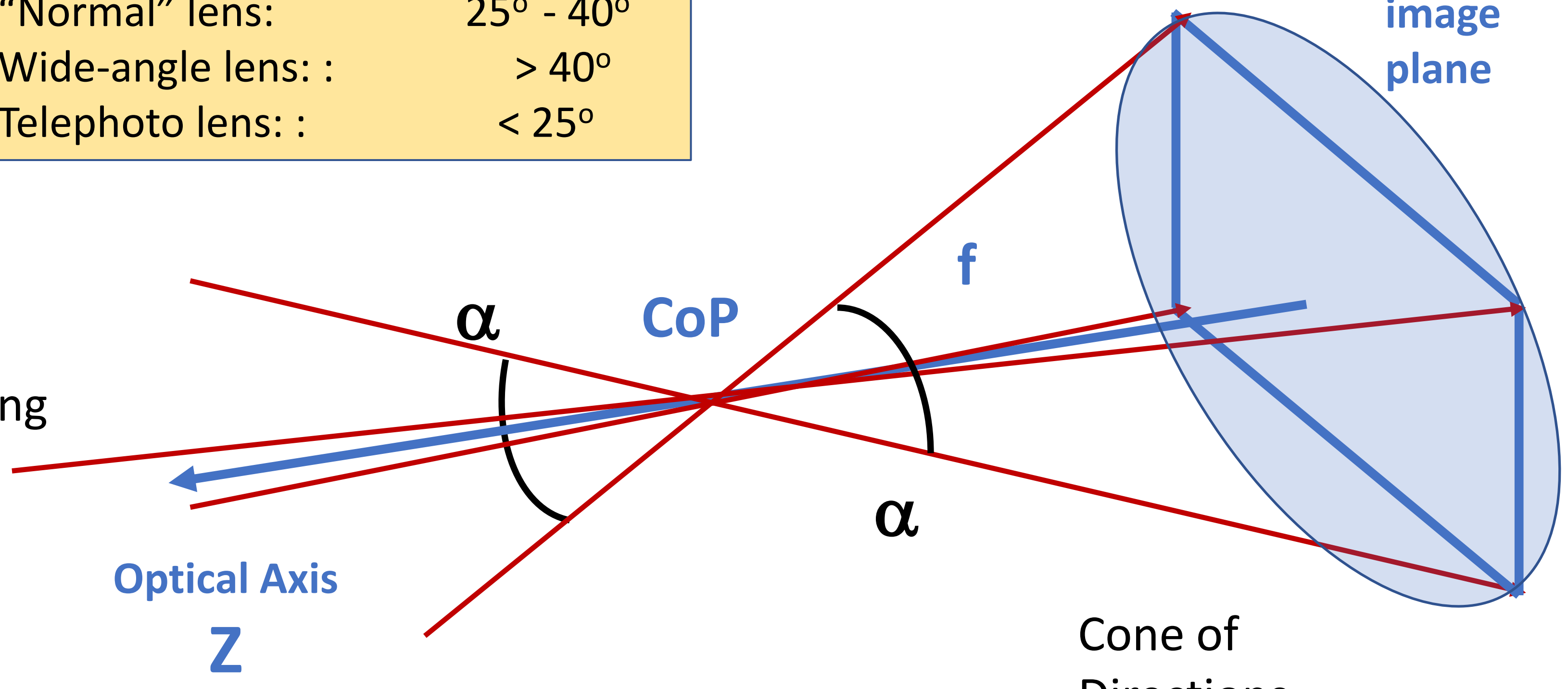
Field of View α



α is measured horizontally or vertically

Field of View of Lenses

"Normal" lens:	25º - 40º
Wide-angle lens: :	> 40º
Telephoto lens: :	< 25º

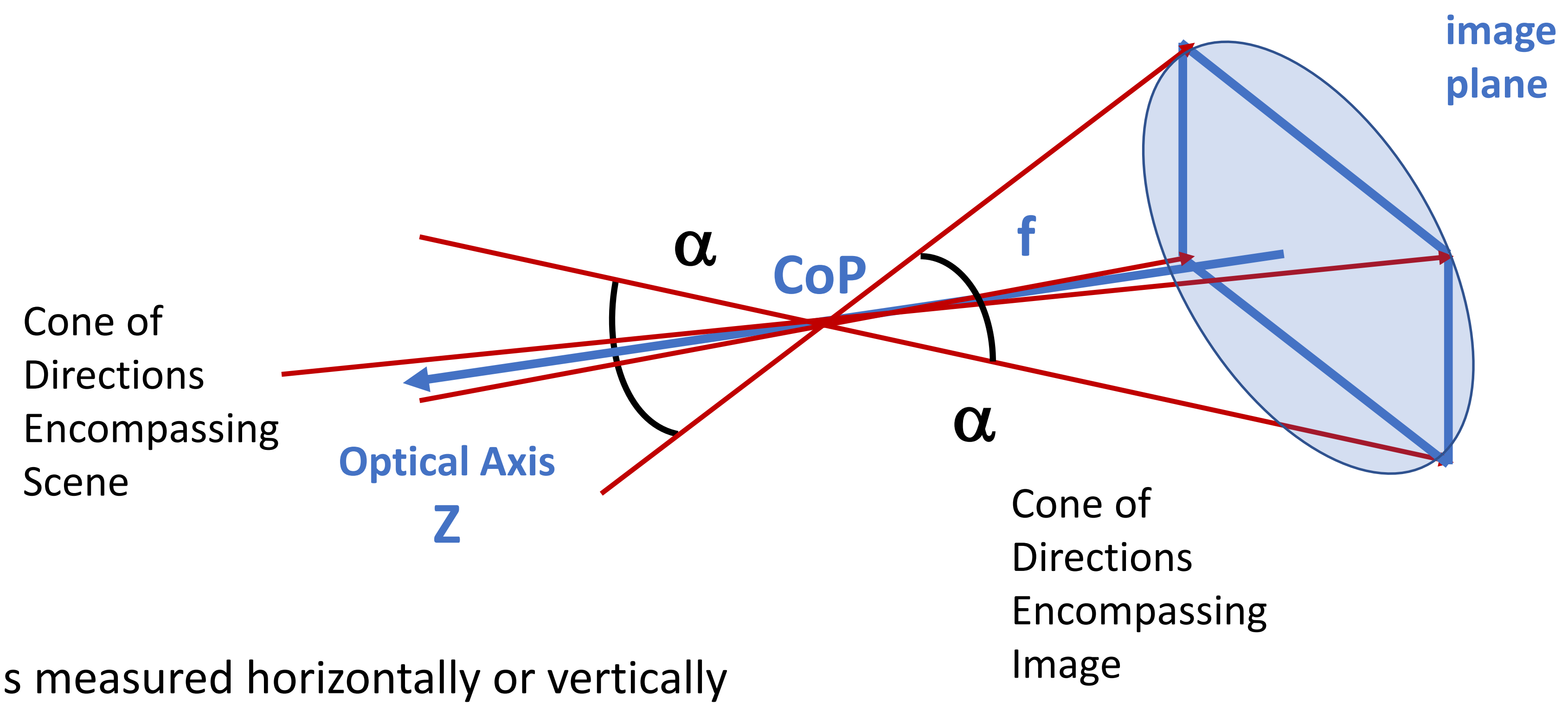


α is measured horizontally or vertically

Field of View of Lenses

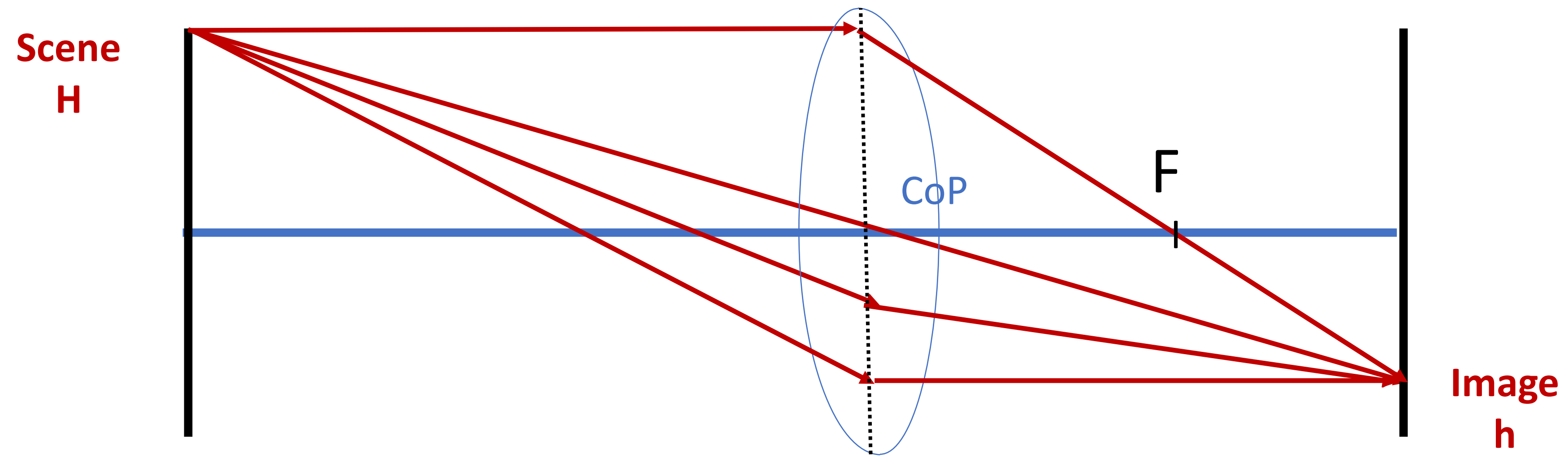
"Rule of thumb:"

"Normal" lens:	25^o - 40^o	
Wide-angle lens: :	> 40^o	Use perspective projection (f << image size)
Telephoto lens: :	< 25^o	Use orthographic projection (i.e., rays parallel to optical axis)



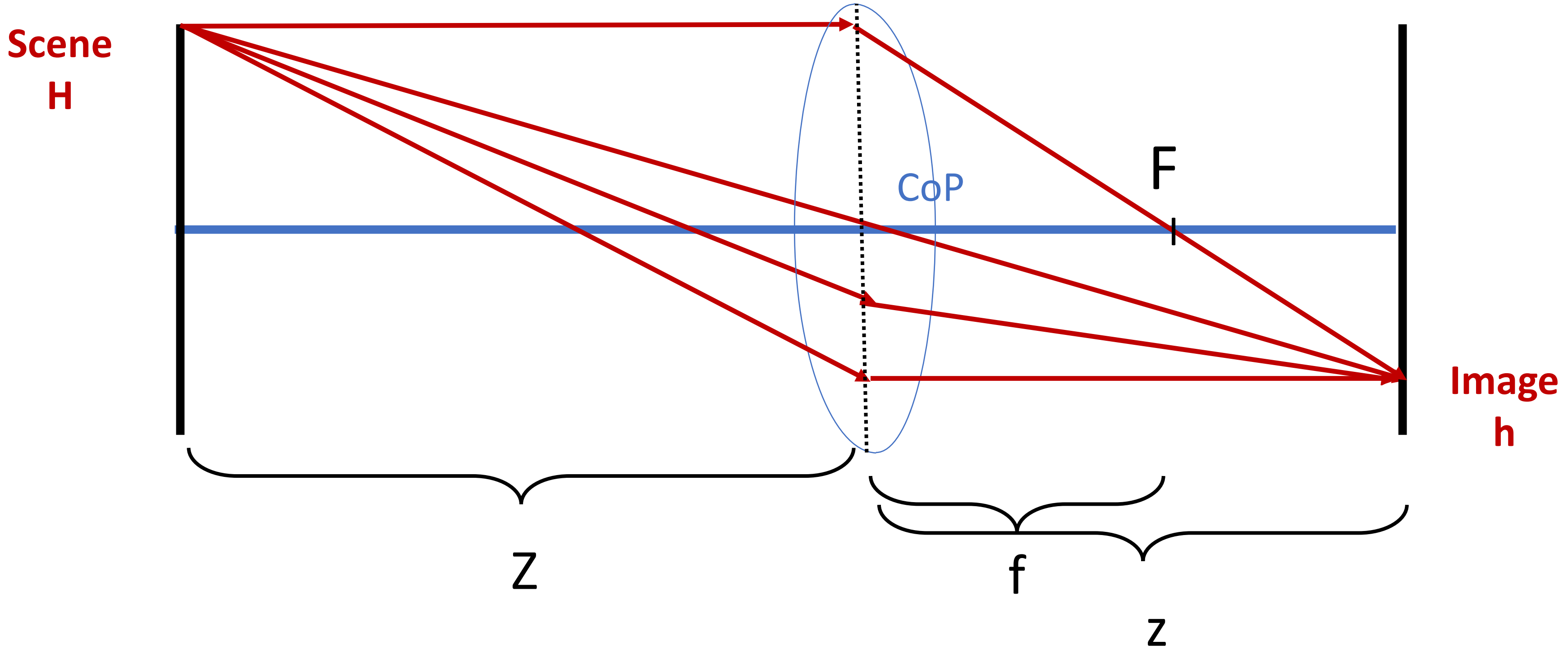
α is measured horizontally or vertically

Ideal Lens: Same projection as pinhole camera but gathers more light



1) Ray through center of lens is not reflected
2) Parallel ray intersects optical axis at F from CoP
3) Spherical shape of lens -> well-focused images only at particular distance
4) Well-focused system = all rays from scene point H reach same image point h as central ray

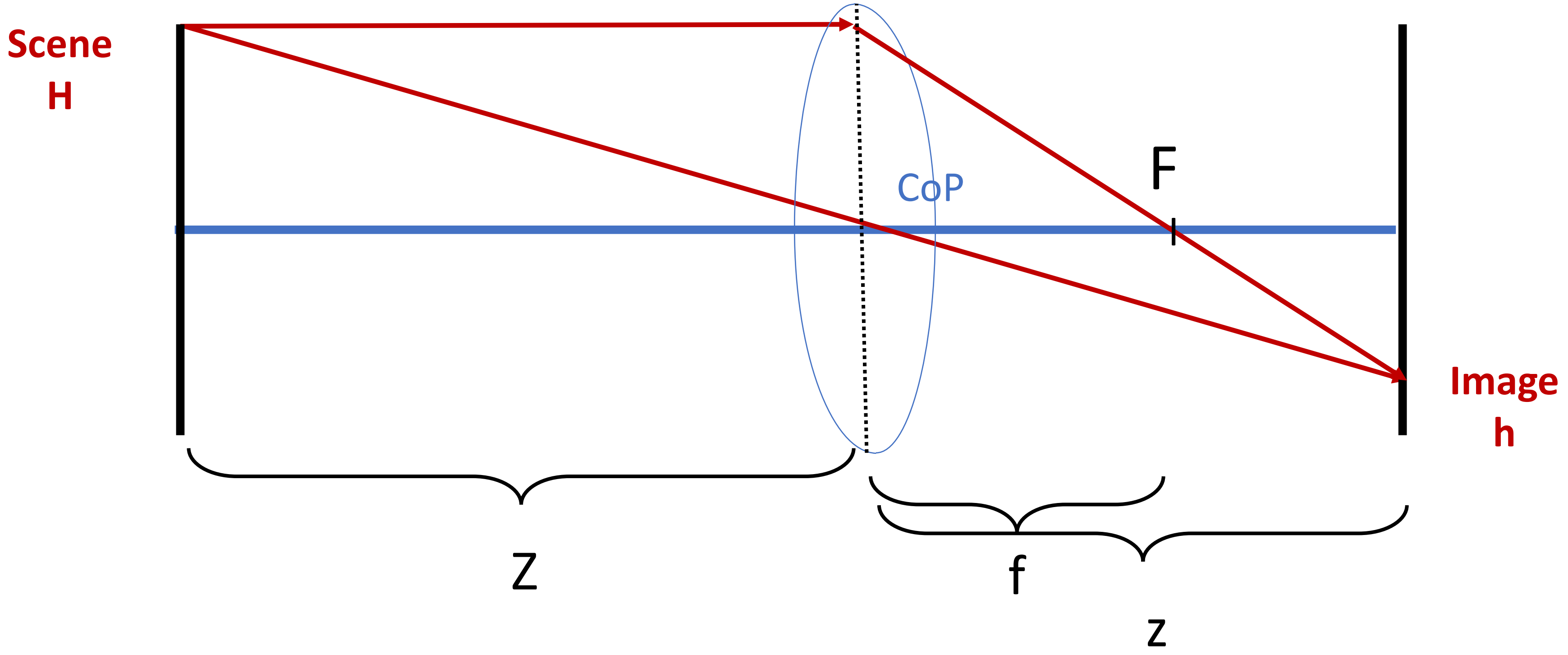
Ideal Lens: Same projection as pinhole camera but gathers more light



z = principal distance
Z = depth

F = Focal point
f = focal length

Derivation of the Lens Equation



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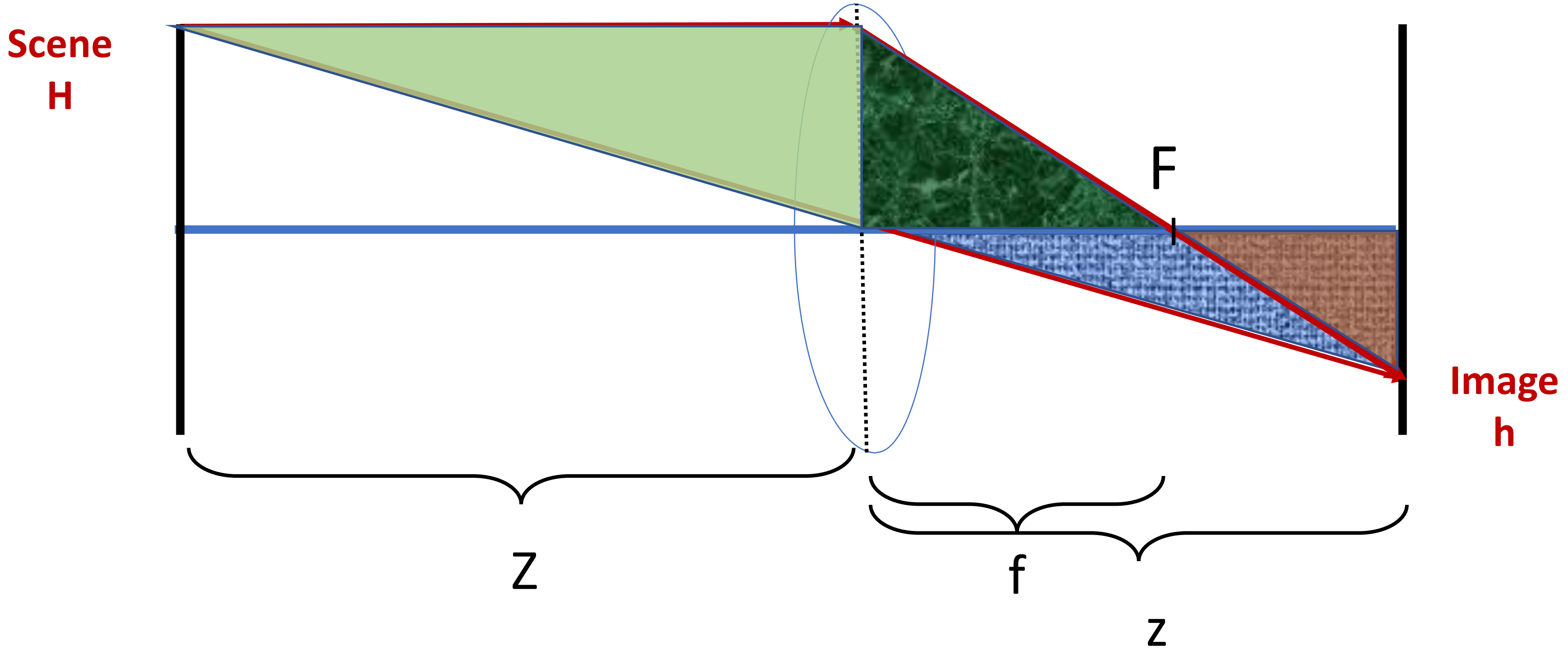

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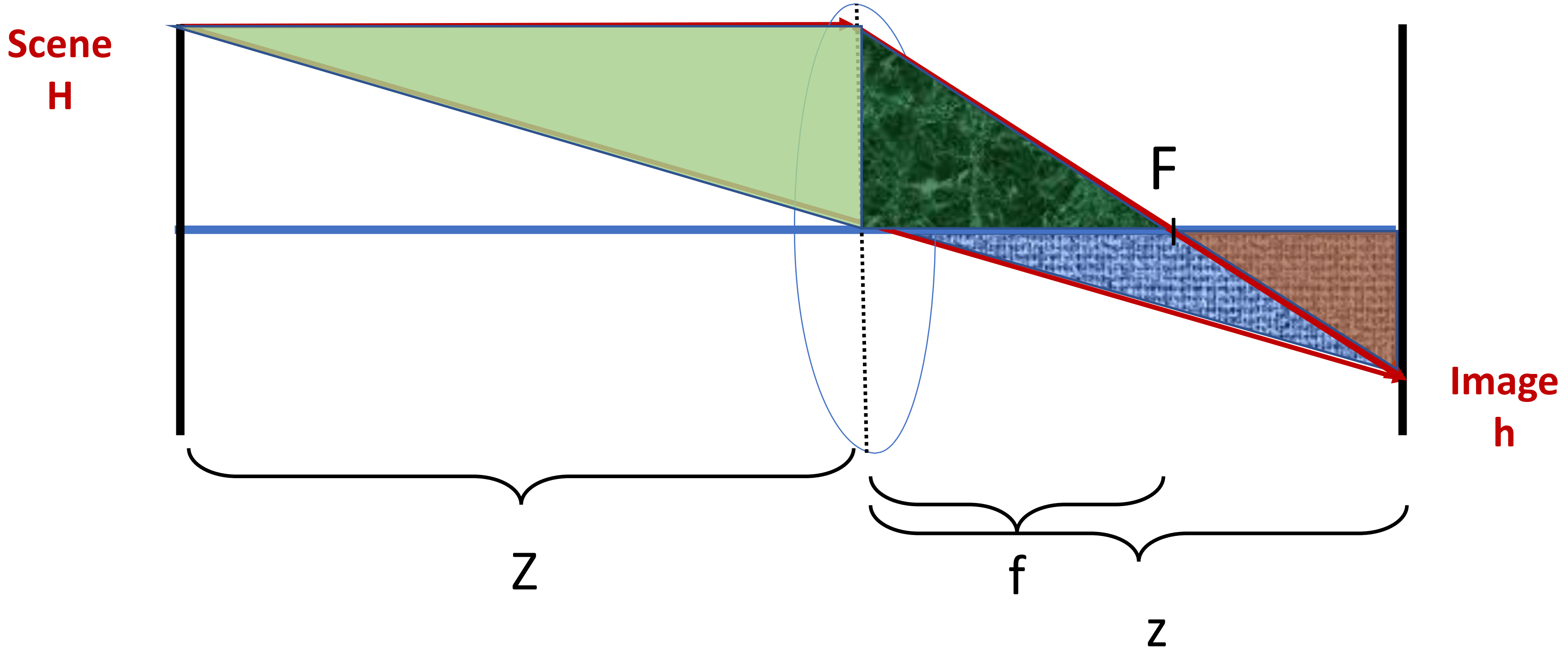
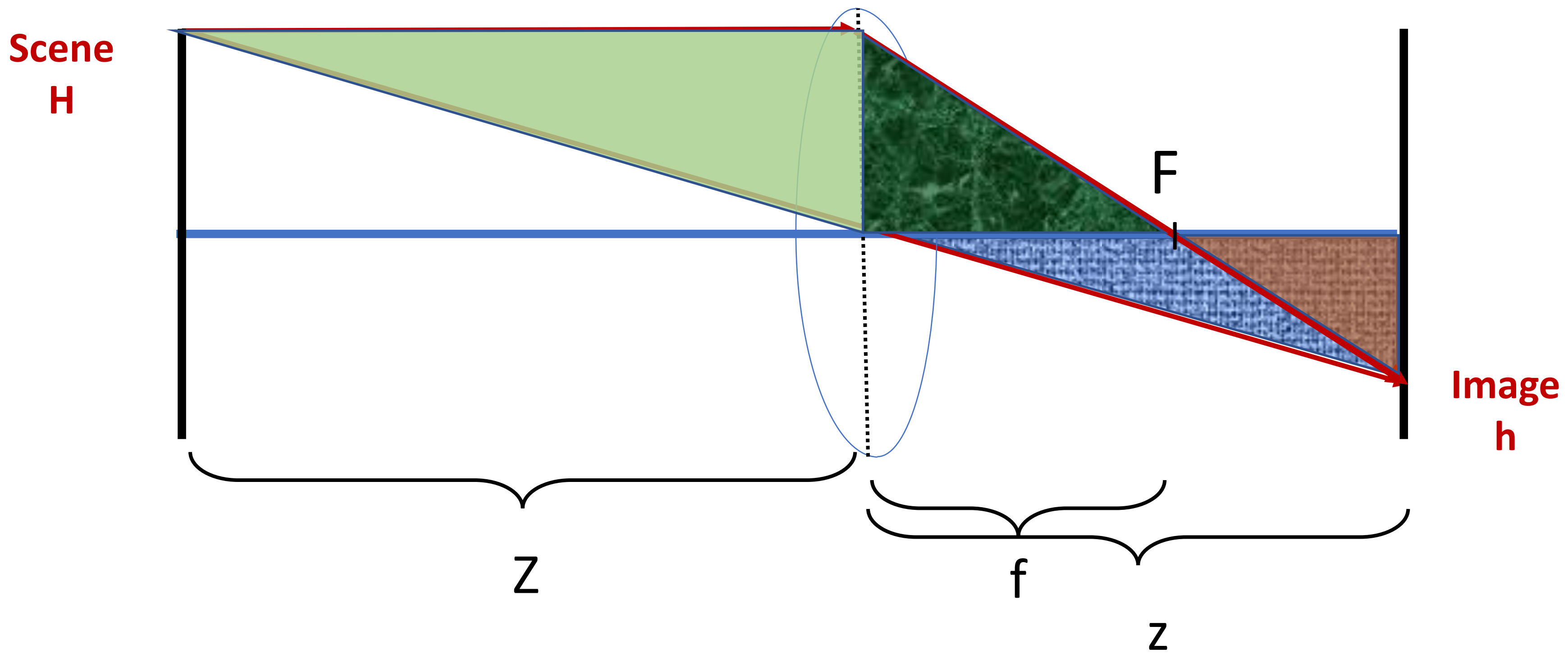


$H/Z \quad = \quad h/z \qquad \text{and} \qquad H/f \quad = \quad h/(z-f)$

Derivation of the Lens Equation



H/Z = h/z and H/f = h/(z-f)

H=h Z/z substitute into:
f/H = (z-f)/h

f/(h Z/z) = (z-f)/h

Multiply by h:
f z/Z = z-f

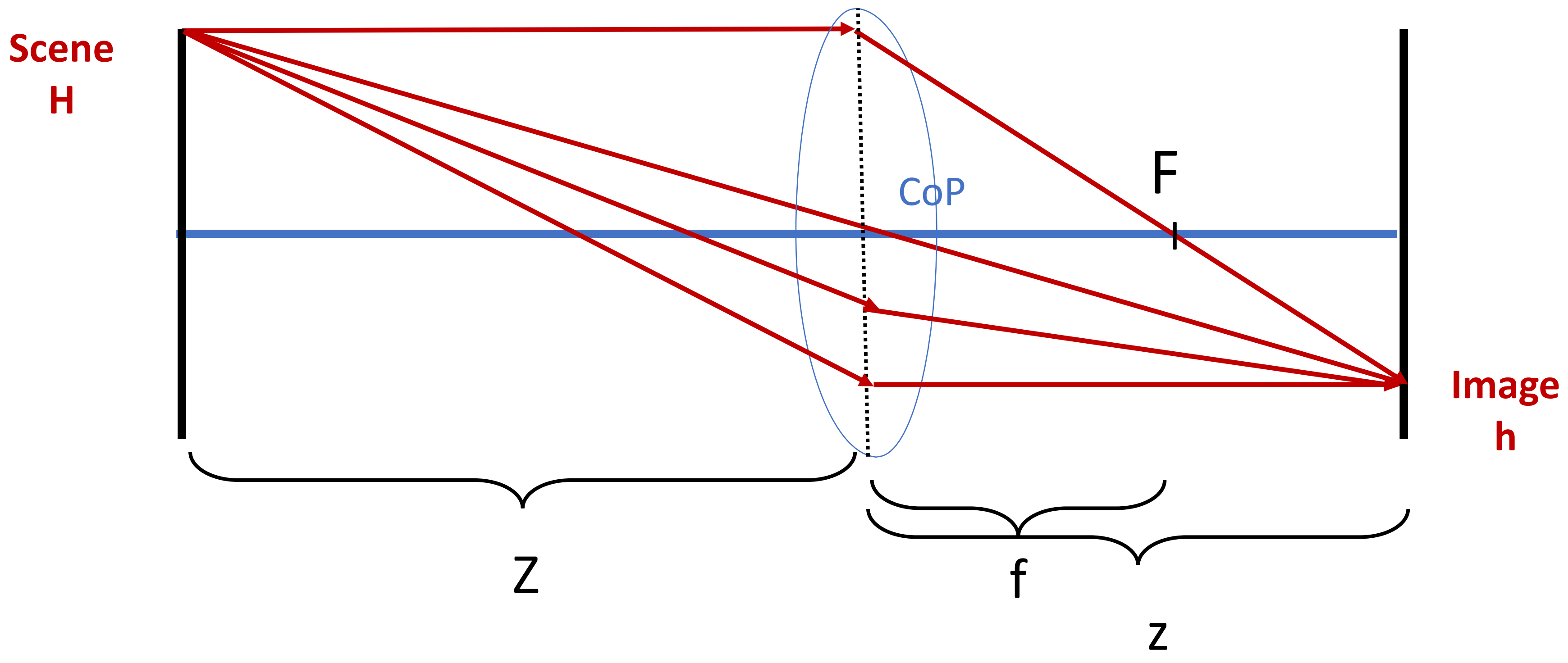
Add f:
f z/Z + f = z

Divide by f and z:

$$\boxed{1/Z + 1/z = 1/f}$$

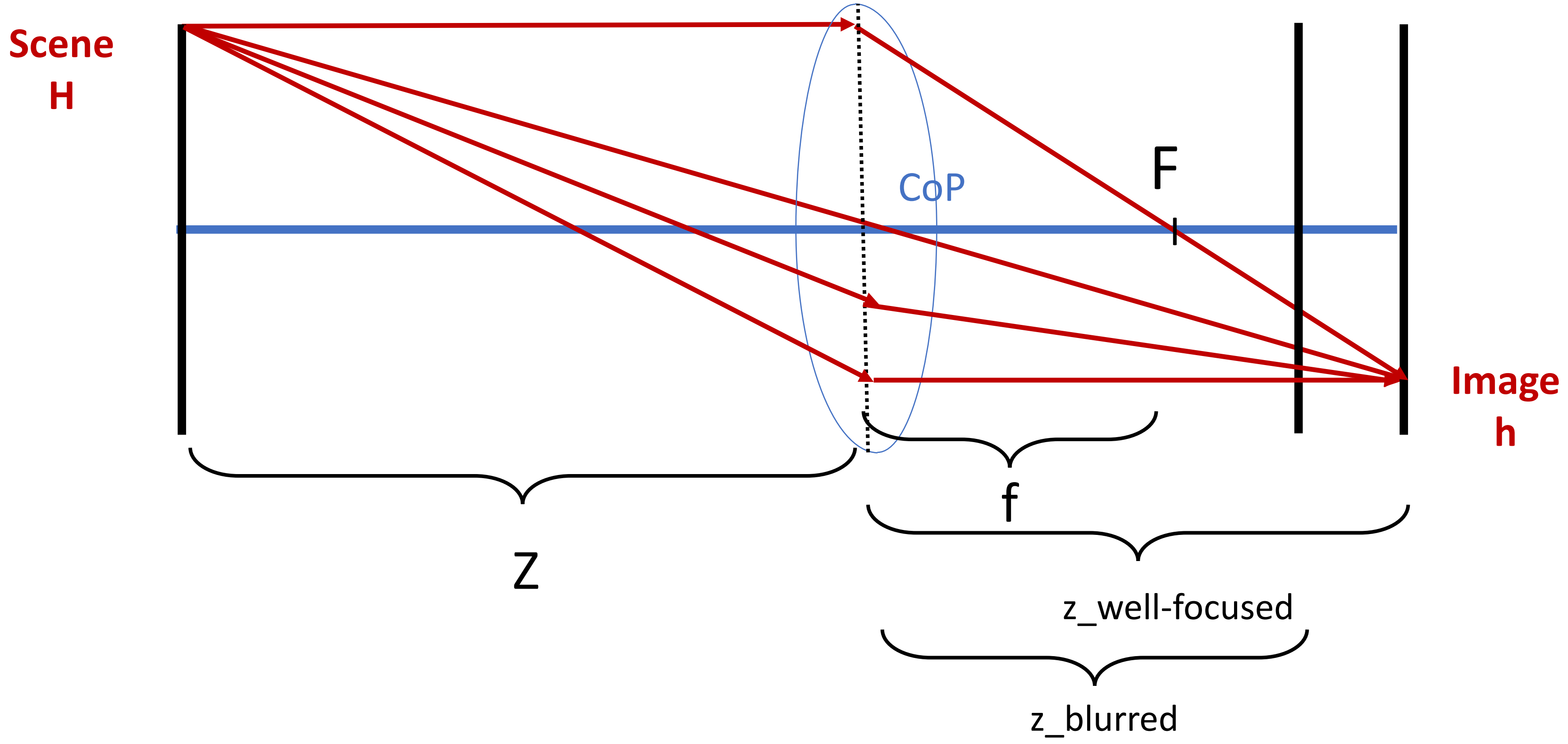
Interpretation of the Lens Equation

$1/Z + 1/z = 1/f$



Lens equation determines how far image plane can be placed to have image in focus

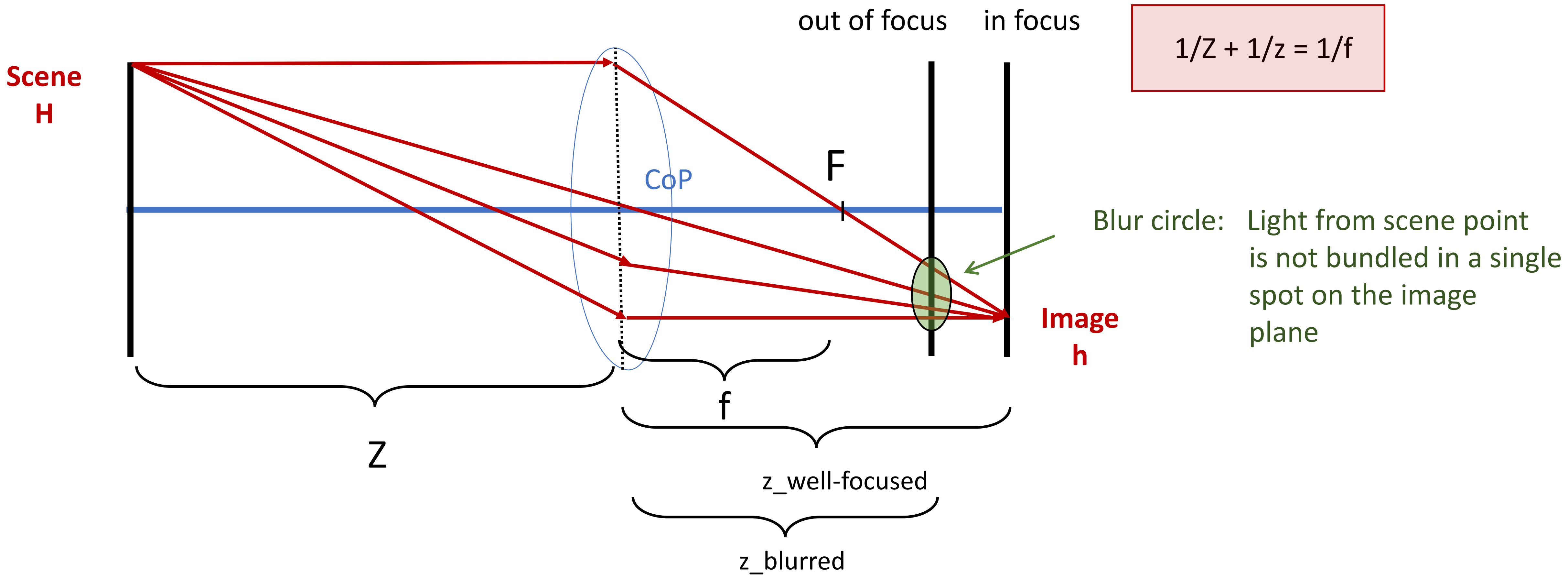
Interpretation of the Lens Equation



$1/Z + 1/z = 1/f$

Lens equation determines how far image plane can be placed to have image in focus

Interpretation of the Lens Equation



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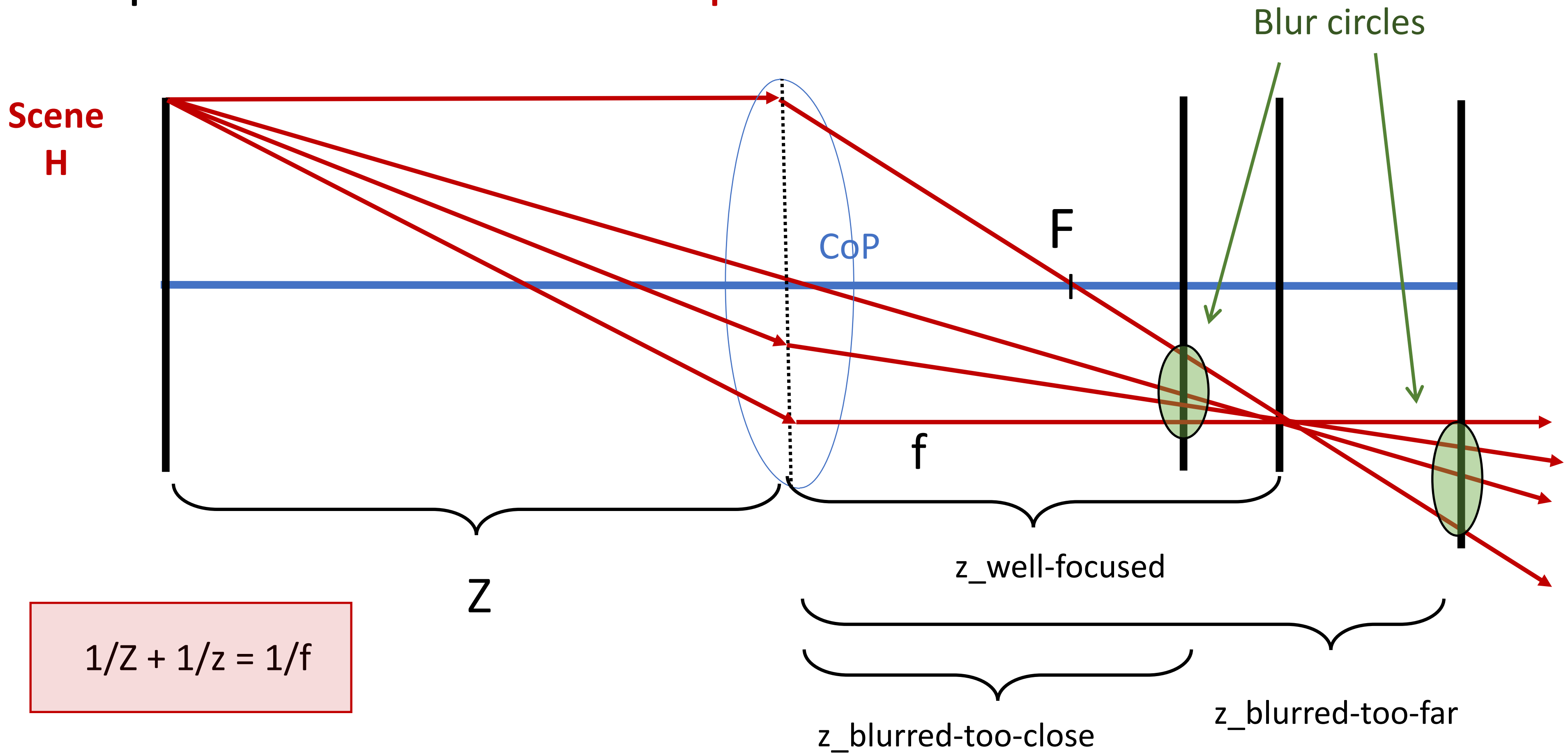
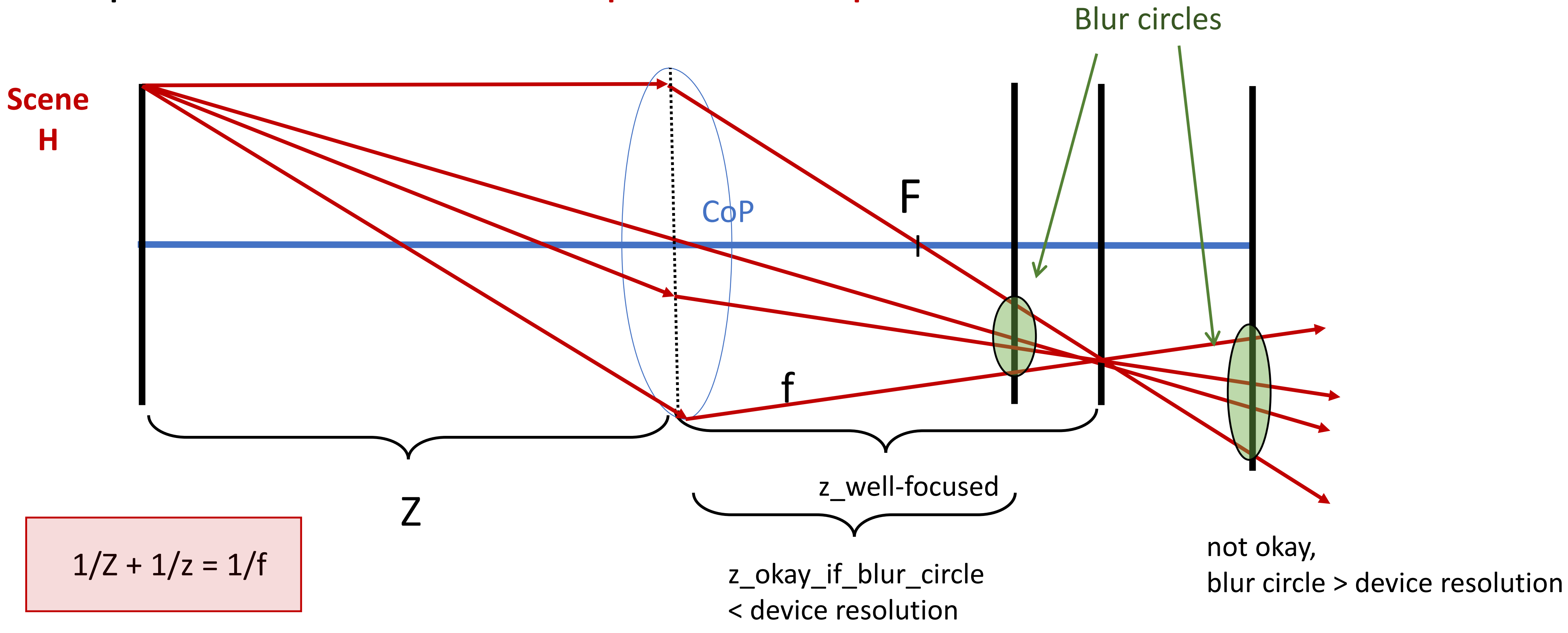
Interpretation of the Lens Equation



$1/Z + 1/z = 1/f$

Lens equation determines how far image plane can be placed to have image in focus

Interpretation of the Lens Equation: Depth of Focus

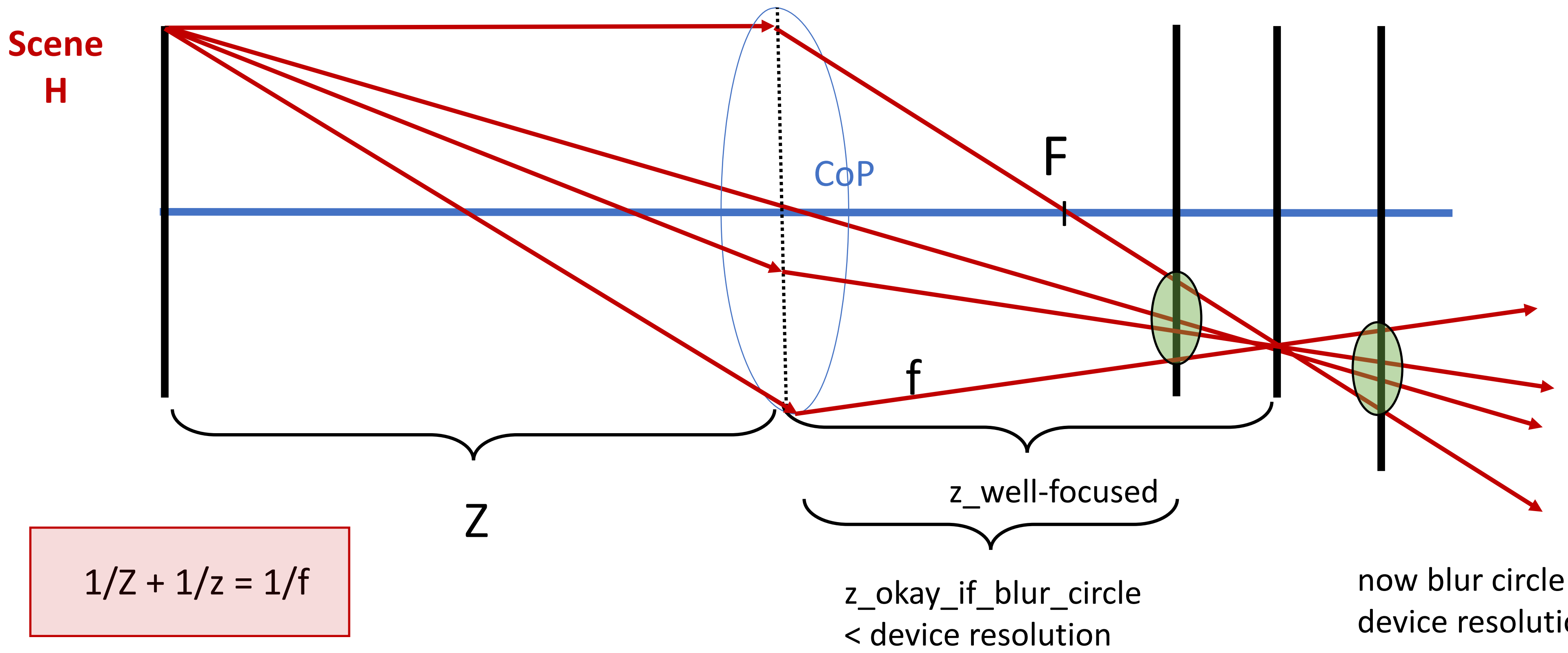


$$1/Z + 1/z = 1/f$$

Depth of focus = range of image plane placement so that objects are focused sufficiently well

Blur circle must be < resolution of the image device

Interpretation of the Lens Equation: Depth of Focus

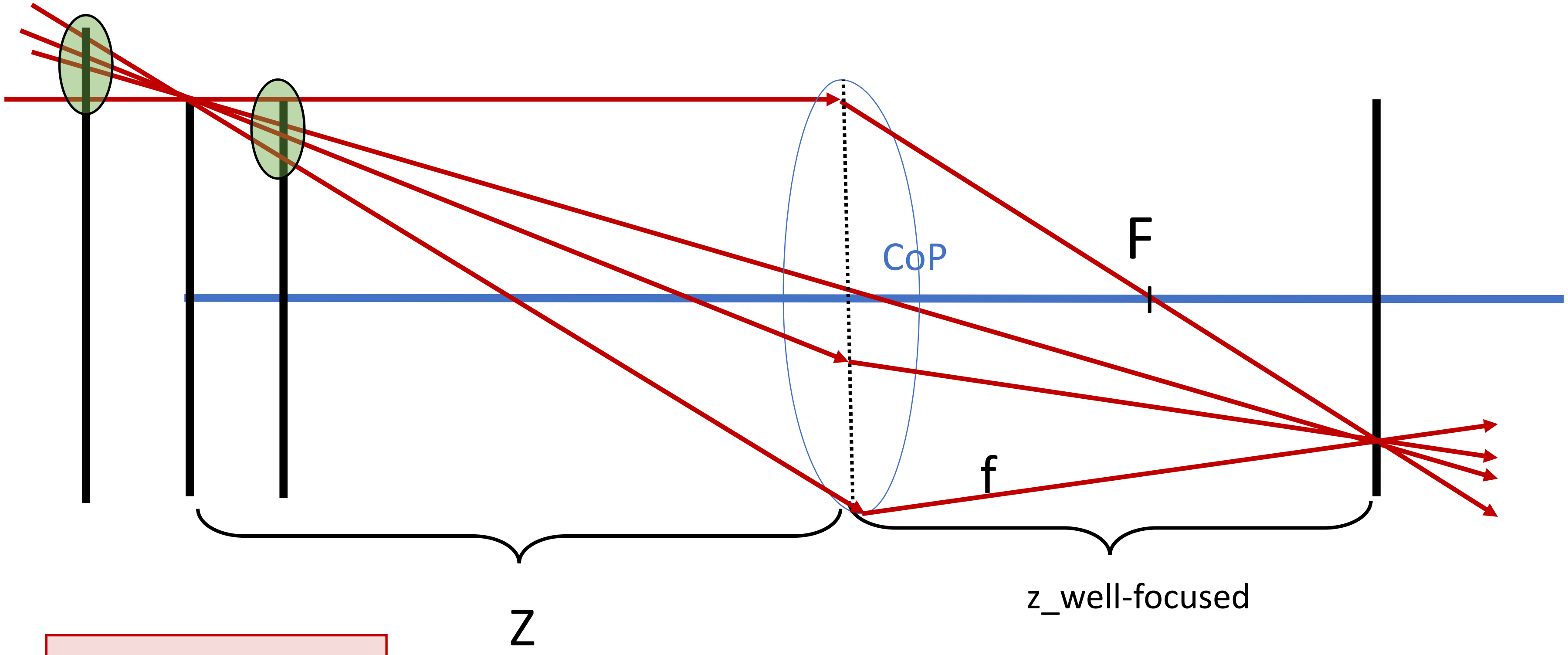


$1/Z + 1/z = 1/f$

Depth of focus = range of image plane placement so that objects are focused sufficiently well

Blur circle must be < resolution of the image device

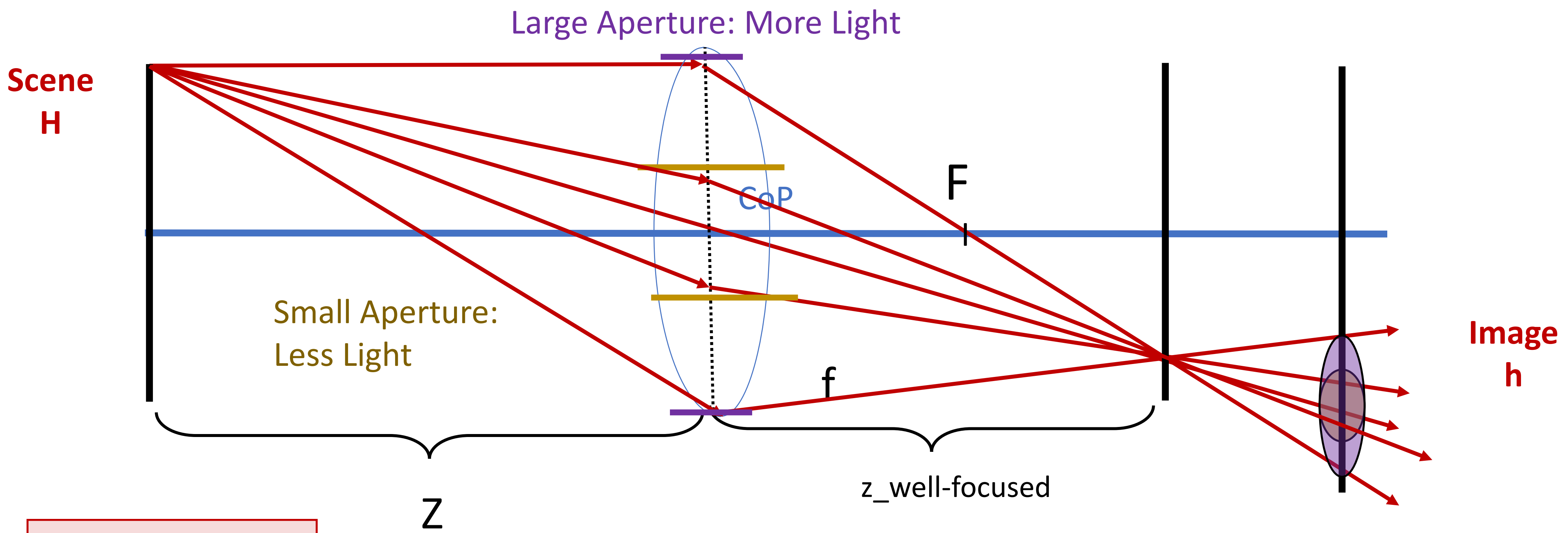
Interpretation of the Lens Equation: Depth of Field



$$1/Z + 1/z = 1/f$$

Depth of field = range of distances over which objects are focused sufficiently well

Interpretation of the Lens Equation: Aperture of Lens



$1/Z + 1/z = 1/f$

Large Aperture:
⇒ Large blur circle
⇒ Small depth of field

Small Aperture:
⇒ Small blur circle
⇒ Large depth of field

Summary of Concepts: Ideal Thin Lens

- Field of View
- Imaging rules for lenses
- Focal Point
- Lens Equation
- Depth of Focus
- Depth of Field
- Aperture