Binocular Stereo, Part 2
Multiview Stereo
Epipolar Geometry
Active Stereo with Structured Light
Photometric Stereo

Lecture by Margrit Betke, CS 585, April 28, 2020

Camera Transformation Problems

1. Interior Orientation = Camera Calibration = Intrinsic Calibration:

   Simple version: Find focal length f and principal point p (= point where optical axis intersects image plane)

   Better: Correct for lens distortion, check if angle between x & y axes is 90^0

2. Exterior Orientation = Hand-Eye Calibration in Robotics:

   Find Center of Projection (CoP) of camera in world coordinate system

3. Absolute Orientation = Alignment of 2 Cameras

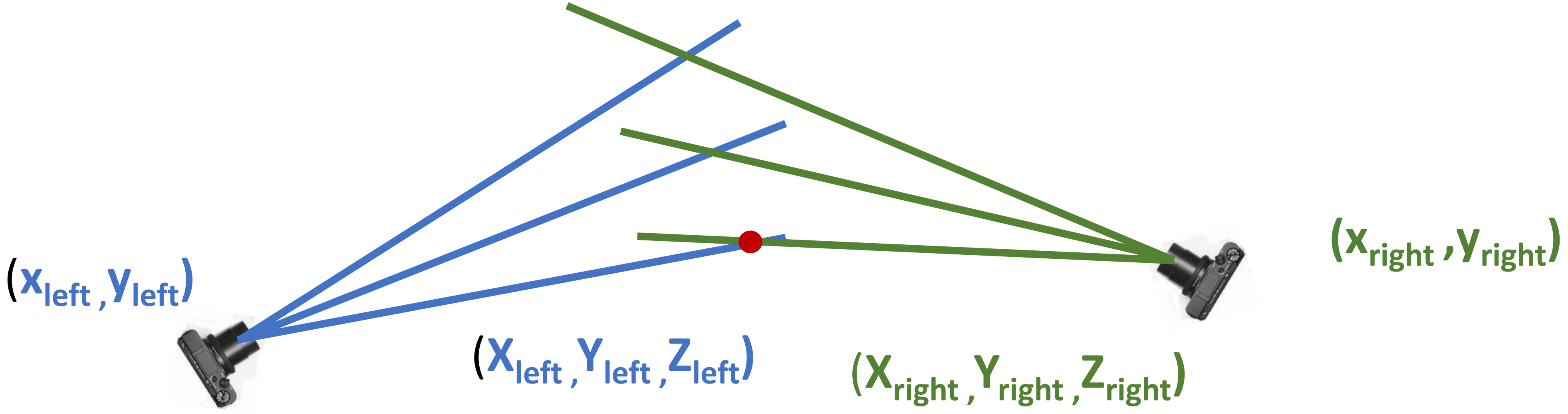
   Find relationship between cameras. 3D coordinates of points are known

4. Relative Orientation = Alignment of 2 Cameras

   Find relationship between cameras. 3D coordinates not known, only rays known

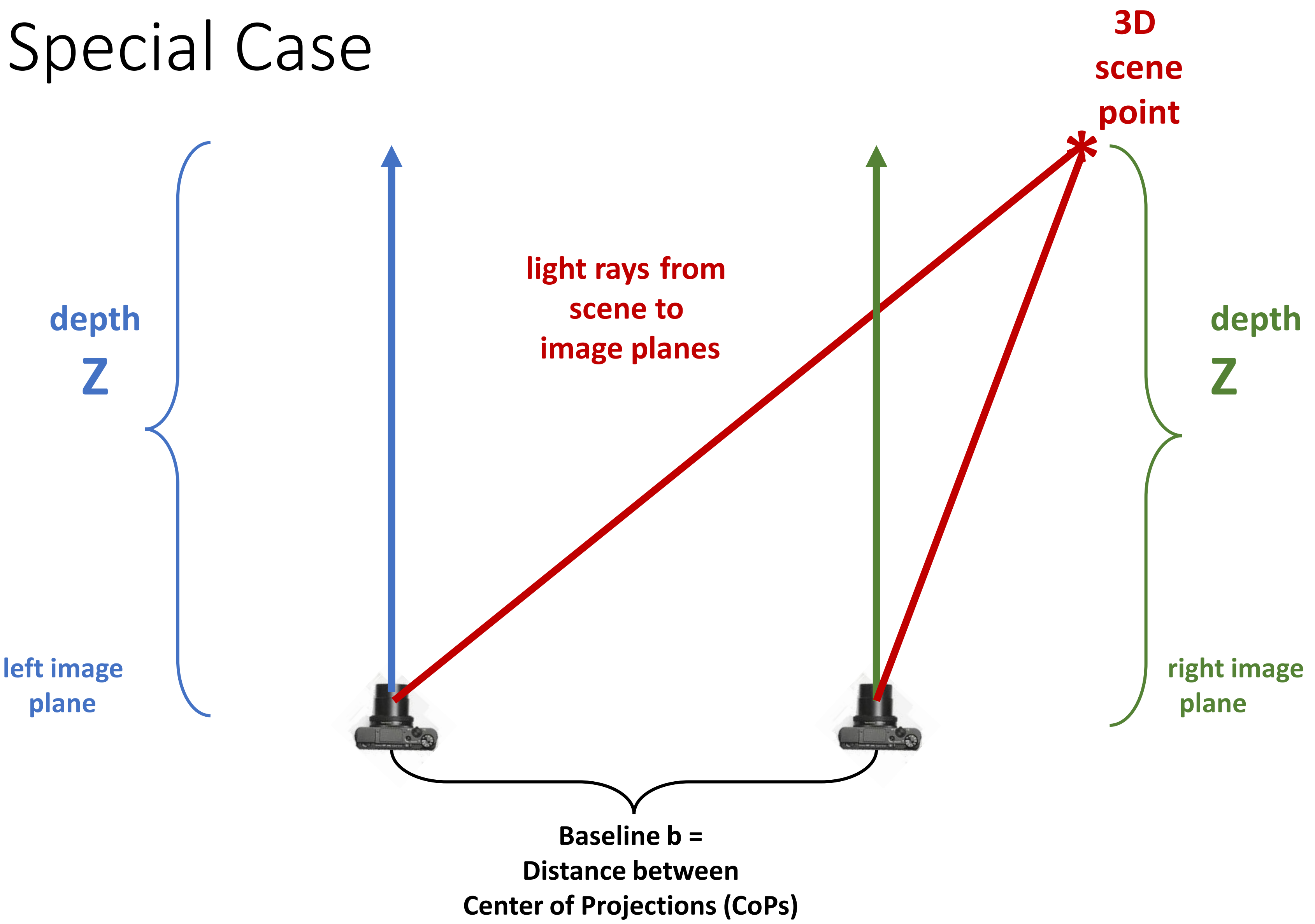
Relative Orientation – Binocular Stereo

Goal: Recovery of position and orientation of one imaging system relative to another from correspondences between rays



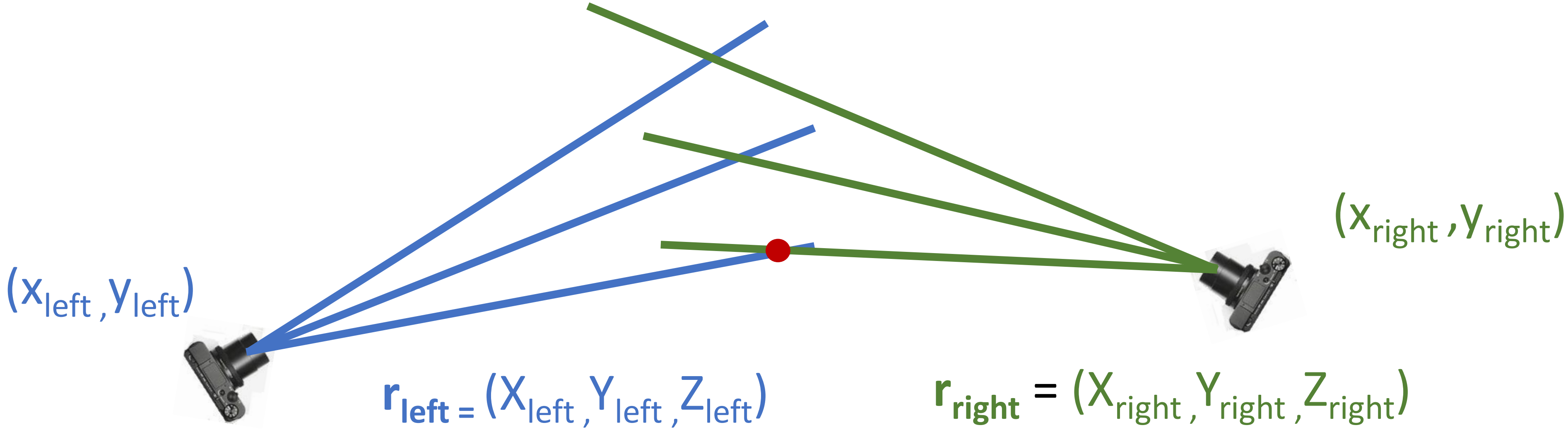
Given: 2D coordinates of image points of same world point

Special Case



$$Z = bf / \delta$$

Relative Orientation = Binocular Stereo



Use perspective projection equations:

$x_{left}/f_{left} = X_{left}/Z_{left}$ $\quad$ $y_{left}/f_{left} = Y_{left}/Z_{left}$

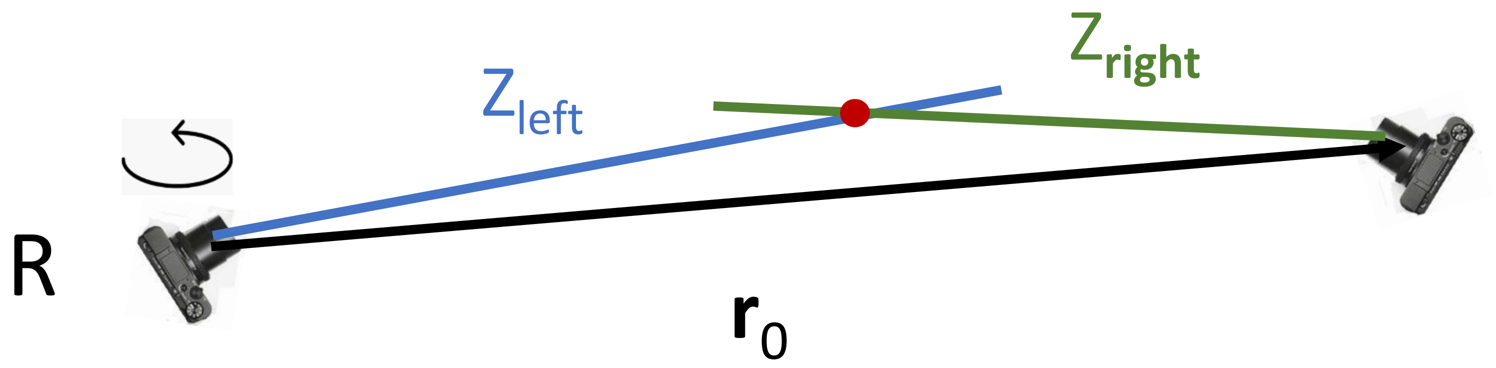
$x_{right}/f_{right} = X_{right}/Z_{right}$ $\quad$ $y_{right}/f_{right} = Y_{right}/Z_{right}$

Transformation equation: $R\,\mathbf{r}_{left} + \mathbf{r}_0 = \mathbf{r}_{right}$ $\quad$ R = rotation matrix, $\mathbf{r}_0$ = translation

Relative Orientation = Binocular Stereo

Transformation equation: $R\ \mathbf{r}_{left} + \mathbf{r}_0 = \mathbf{r}_{right}$

Unknown: Rotation matrix R, translation $\mathbf{r}_0$, Z coordinates of $\mathbf{r}_{left}$, $\mathbf{r}_{right}$

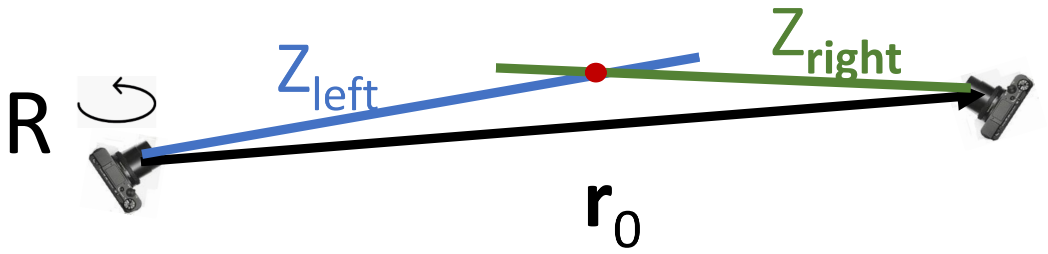


Relative Orientation = Binocular Stereo

$$R\,\mathbf{r}_{left} + \mathbf{r}_0 = \mathbf{r}_{right}$$

is equivalent to:

$$r_{11} X_{left} + r_{12}\ Y_{left} + r_{13} Z_{left} + r_{14} = X_{right}$$
$$r_{21} X_{left} + r_{22}\ Y_{left} + r_{23} Z_{left} + r_{24} = Y_{right}$$
$$r_{31} X_{left} + r_{32}\ Y_{left} + r_{33} Z_{left} + r_{34} = Z_{right}$$



Insert Perspective Projection Equations:

$x_{left}/f = X_{left}/Z_{left}$ $\quad y_{left}/f = Y_{left}/Z_{left}$

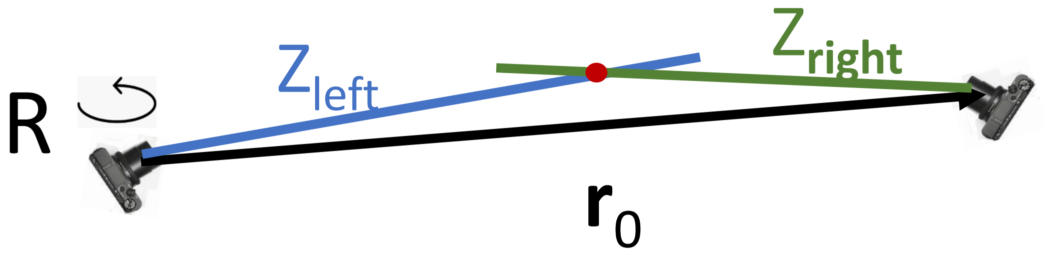
$x_{right}/f = X_{right}/Z_{right}$ $\quad y_{right}/f = Y_{right}/Z_{right}$

$$r_{11}\, x_{left}\, Z_{left}/f + r_{12}\ y_{left}\, Z_{left}/f + r_{13} Z_{left} + r_{14} = x_{right}\, Z_{right}/f$$
$$r_{21}\, x_{left}\, Z_{left}/f + r_{22}\ y_{left}\, Z_{left}/f + r_{23} Z_{left} + r_{24} = y_{right}\, Z_{right}/f$$
$$r_{31}\, x_{left}\, Z_{left}/f + r_{32}\ y_{left}\, Z_{left}/f + r_{33} Z_{left} + r_{34} = Z_{right}$$

Multiply by $f/\ Z_{left}$

Relative Orientation = Binocular Stereo

$R\ \mathbf{r}_{left} + \mathbf{r}_0 = \mathbf{r}_{right}$



is equivalent to:

$r_{11}\, x_{left} + r_{12}\, y_{left} + r_{13}\, f + r_{14}\, f/ Z_{left} = x_{right}\, Z_{right}/Z_{left}$

$r_{21}\, x_{left} + r_{22}\, y_{left} + r_{23}\, f + r_{24}\, f/Z_{left} = y_{right}\, Z_{right}/Z_{left}$

$r_{31}\, x_{left} + r_{32}\, y_{left} + r_{33}\, f + r_{34}\, f/Z_{left} = Z_{right}/Z_{left}$

One measurement pair (x_{left}, y_{left}) and (x_{right}, y_{right}) => 3 equations

with 14 unknowns $r_{11}, r_{12}, \ldots, r_{34}$, and Z_{right}, Z_{left}

Relative Orientation

$R\,\mathbf{r}_{left} + \mathbf{r}_0 = \mathbf{r}_{right}$

$$
\begin{aligned}
r_{11}\, x_{left} + r_{12}\, y_{left} + r_{13}\, f + r_{14}\, f/Z_{left} &= x_{right}\, Z_{right}/Z_{left} \\
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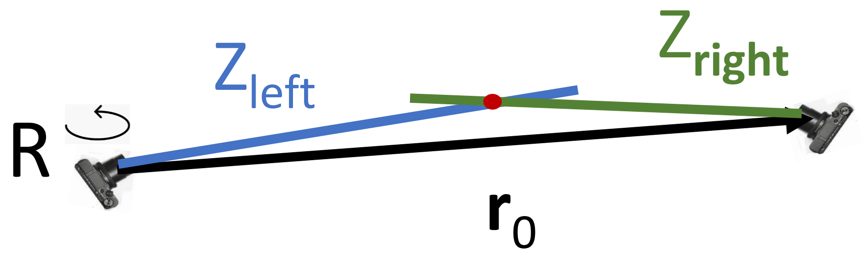


One measurement pair (x_{left}, y_{left}) and (x_{right}, y_{right}) => 3 equations

with 12 unknown $r_{11}, r_{12}, \ldots, r_{34}$ and 2 unknown Z_{right}, Z_{left}

Trick: To solve for 14 unknowns:

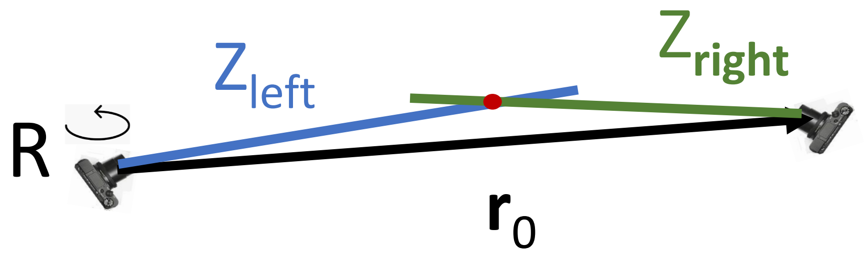
Use n measurements => 3n equations

Find additional equations

Relative Orientation

$R\, \mathbf{r}_{left} + \mathbf{r}_0 = \mathbf{r}_{right}$

$$\boxed{\begin{aligned} r_{11}\, x_{left} + r_{12}\, y_{left} + r_{13}\, f + r_{14}\, f/Z_{left} &= x_{right}\, Z_{right}/Z_{left} \\ r_{21}\, x_{left} + r_{22}\, y_{left} + r_{23}\, f + r_{24}\, f/Z_{left} &= y_{right}\, Z_{right}/Z_{left} \\ r_{31}\, x_{left} + r_{32}\, y_{left} + r_{33}\, f + r_{34}\, f/Z_{left} &= Z_{right}/Z_{left} \end{aligned}}$$



One measurement pair (x_{left}, y_{left}) and (x_{right}, y_{right}) => 3 equations

with 12 unknown $r_{11}, r_{12}, \ldots, r_{34}$, and 2 unknown Z_{right}, Z_{left}

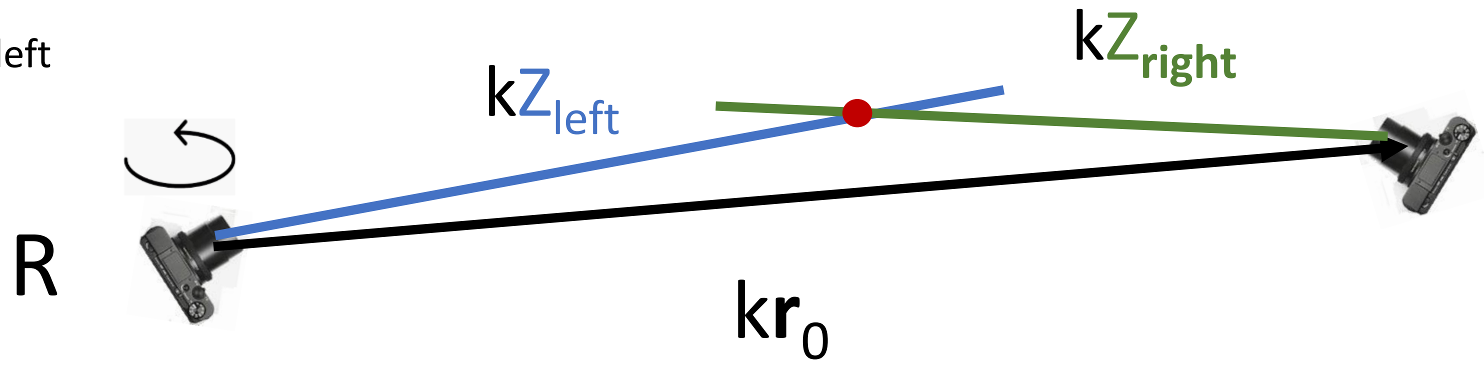
One extra equation:

Scale factor ambiguity $\mathbf{r}_0, Z_{right}, Z_{left}$

$\Leftrightarrow$ $k\mathbf{r}_0, kZ_{right}, kZ_{left}$

Force $\mathbf{r}_0$ to be unit vector

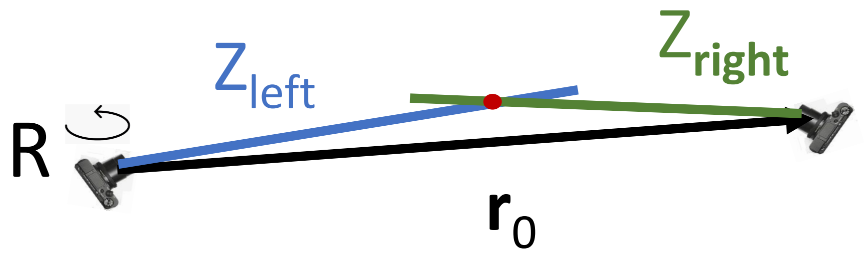
=> $|\mathbf{r}_0| = 1$



Relative Orientation

$$\begin{array}{l} r_{11}\, x_{left} + r_{12}\, y_{left} + r_{13}\, f + r_{14}\, f/Z_{left} = x_{right}\, Z_{right}/Z_{left} \\ r_{21}\, x_{left} + r_{22}\, y_{left} + r_{23}\, f + r_{24}\, f/Z_{left} = y_{right}\, Z_{right}/Z_{left} \\ r_{31}\, x_{left} + r_{32}\, y_{left} + r_{33}\, f + r_{34}\, f/Z_{left} = Z_{right}/Z_{left} \end{array}$$

$R\, \mathbf{r}_{left} + \mathbf{r}_0 = \mathbf{r}_{right}$



One measurement pair (x_{left}, y_{left}) and (x_{right}, y_{right}) => 3 equations

with 14 unknowns $r_{11}, r_{12}, \ldots, r_{34}$, and Z_{right}, Z_{left}

# unknowns:	12	for R, $\mathbf{r}_0$
	2n	for Z_{right}, Z_{left} for each of n pairs of measurements

12 + 2n unknowns

Relative Orientation

$R\,\mathbf{r}_{left} + \mathbf{r}_0 = \mathbf{r}_{right}$

$$r_{11}\,x_{left} + r_{12}\,y_{left} + r_{13}\,f + r_{14}\,f/Z_{left} = x_{right}\,Z_{right}/Z_{left}$$
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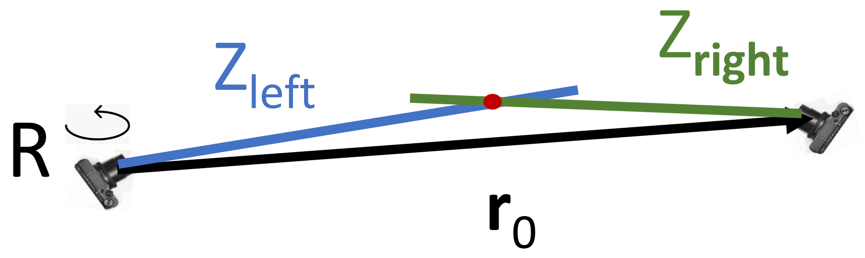

One measurement pair (x_{left},y_{left}) and (x_{right},y_{right}) => 3 equations

with 14 unknowns $r_{11}, r_{12}, \ldots, r_{34}$, and Z_{right}, Z_{left}

Number of equations: 6 for orthonormal R (columns sum to 1, dot products 0)

1 for unit length translation: $|\mathbf{r}_0|=1$

3n for 3 equations per measurement pair

7+ 3n equations

Relative Orientation

$R\,\mathbf{r}_{left} + \mathbf{r}_0 = \mathbf{r}_{right}$

$$
\begin{aligned}
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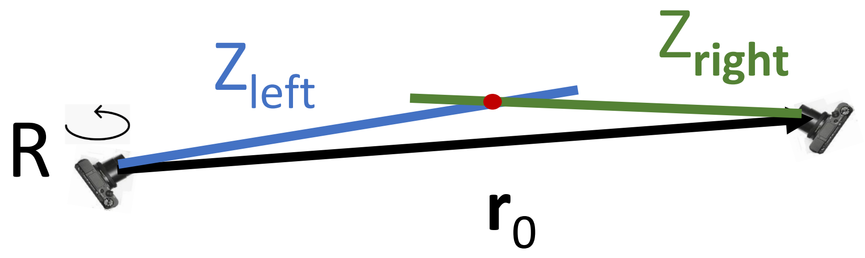

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with 14 unknowns r_{11}, r_{12}, …, r_{34}, and Z_{right}, Z_{left}

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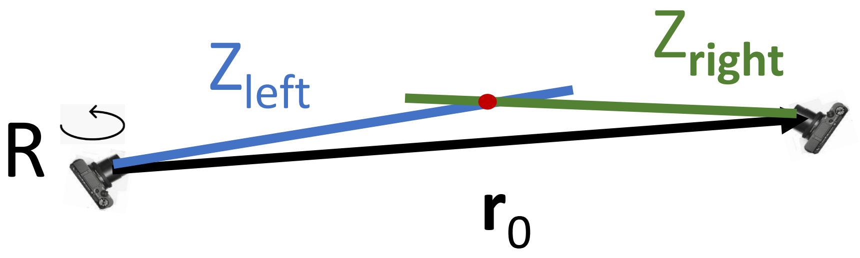
	2n	for Z_{right}, Z_{left} for each of n pairs of measurements
Number of equations:	6	for orthonormal R (columns sum to 1, dot products 0)
	1	for unit length translation $\mathbf{r}_0$ F
	3n	for 3 equations per measurement pair

Need at least n ? measurement pairs: $12 + 2 * n = 7 + 3*n$

Relative Orientation

$R\,\mathbf{r}_{left} + \mathbf{r}_0 = \mathbf{r}_{right}$

$$\boxed{\begin{aligned} r_{11}\,x_{left} + r_{12}\,y_{left} + r_{13}\,f + r_{14}\,f/Z_{left} &= x_{right}\,Z_{right}/Z_{left} \\ r_{21}\,x_{left} + r_{22}\,y_{left} + r_{23}\,f + r_{24}\,f/Z_{left} &= y_{right}\,Z_{right}/Z_{left} \\ r_{31}\,x_{left} + r_{32}\,y_{left} + r_{33}\,f + r_{34}\,f/Z_{left} &= Z_{right}/Z_{left} \end{aligned}}$$



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# unknowns:	12	for R, $\mathbf{r}_0$
	2n	for Z_{right}, Z_{left} for each of n pairs of measurements
Number of equations:	6	for orthonormal R (columns sum to 1, dot products 0)
	1	for unit length translation $\mathbf{r}_0$ F
	3n	for 3 equations per measurement pair

Need at least 5 measurement pairs: $12 + 2 * 5 = 22 = 7 + 3*5$

Algorithms to Solve the Problem of Binocular Stereo = Relative Orientation

- Longuet-Higgins' 8-point Algorithm (1981):

$$(x_{left}, y_{left}, 1_{left})^T F \;\; (x_{right}, y_{right}, 1_{right}) = 0$$

F is called the 3x3 "fundamental matrix"

Algorithm is sensitive to how accurate point pairs were located (= numerically unstable)

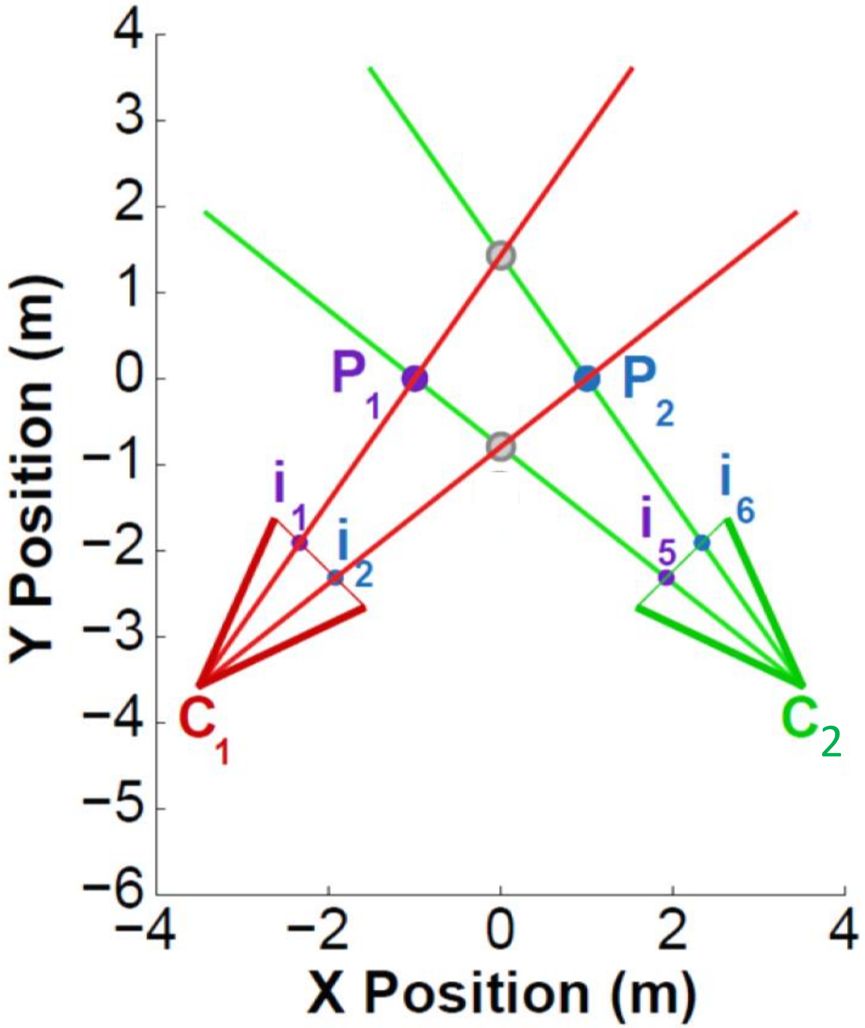
- Variations of the 8-point Algorithm

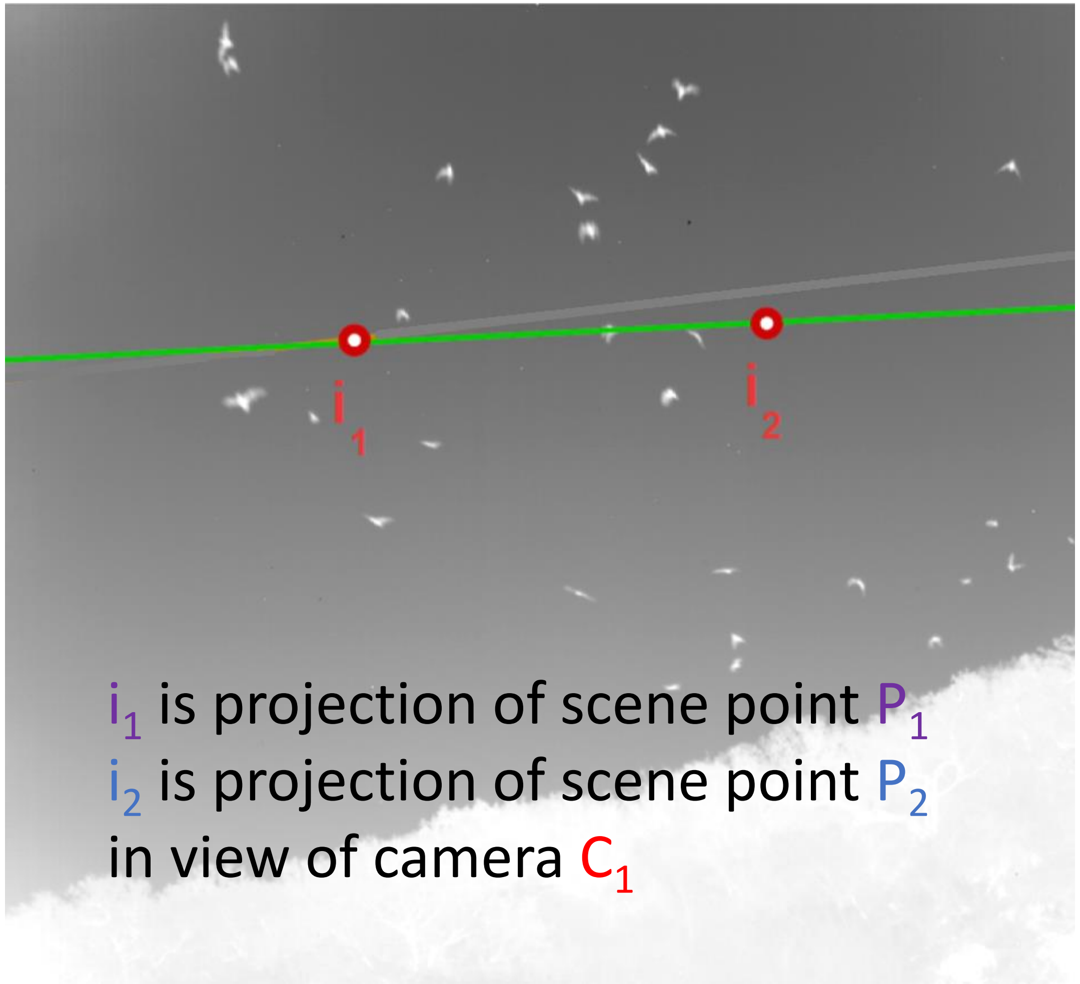
  e.g. Hartley's Normalized 8-point algorithm (1997)

- Horn's Iterative Method (1991)
- Deep Fundamental Matrix Estimation without Correspondences (e.g., Poursaeed et al. 2018)

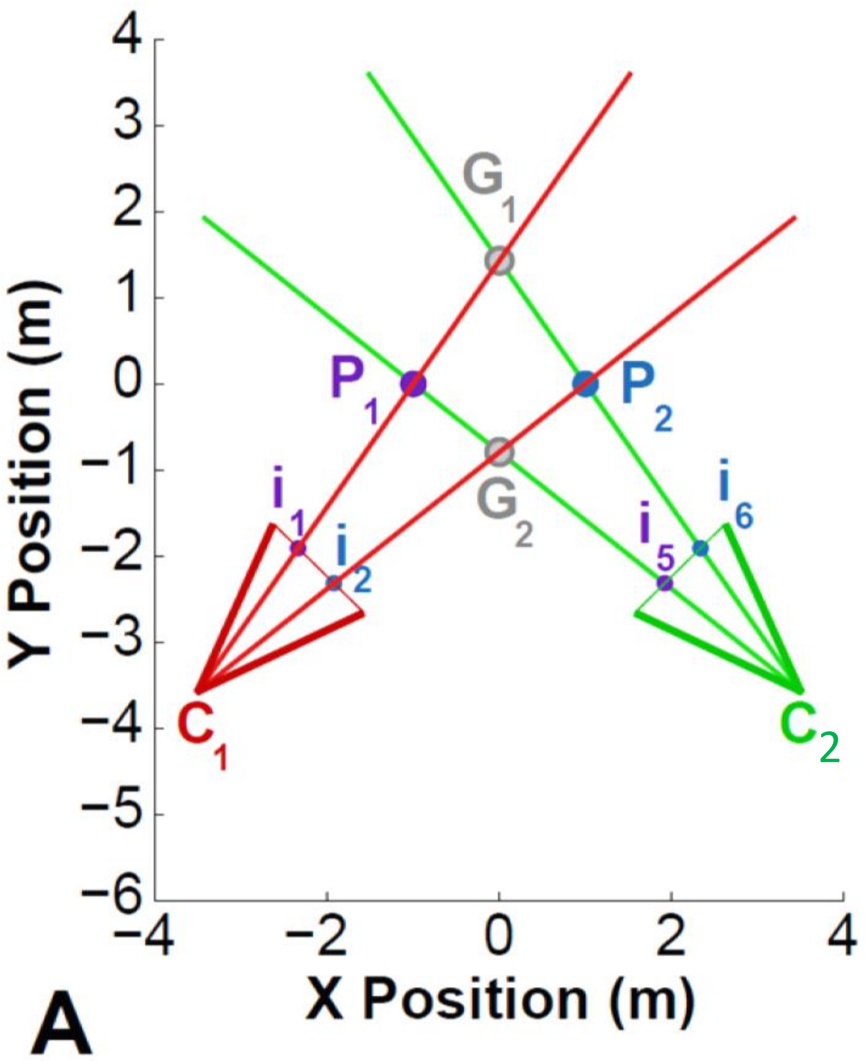
Multi-Camera Stereo

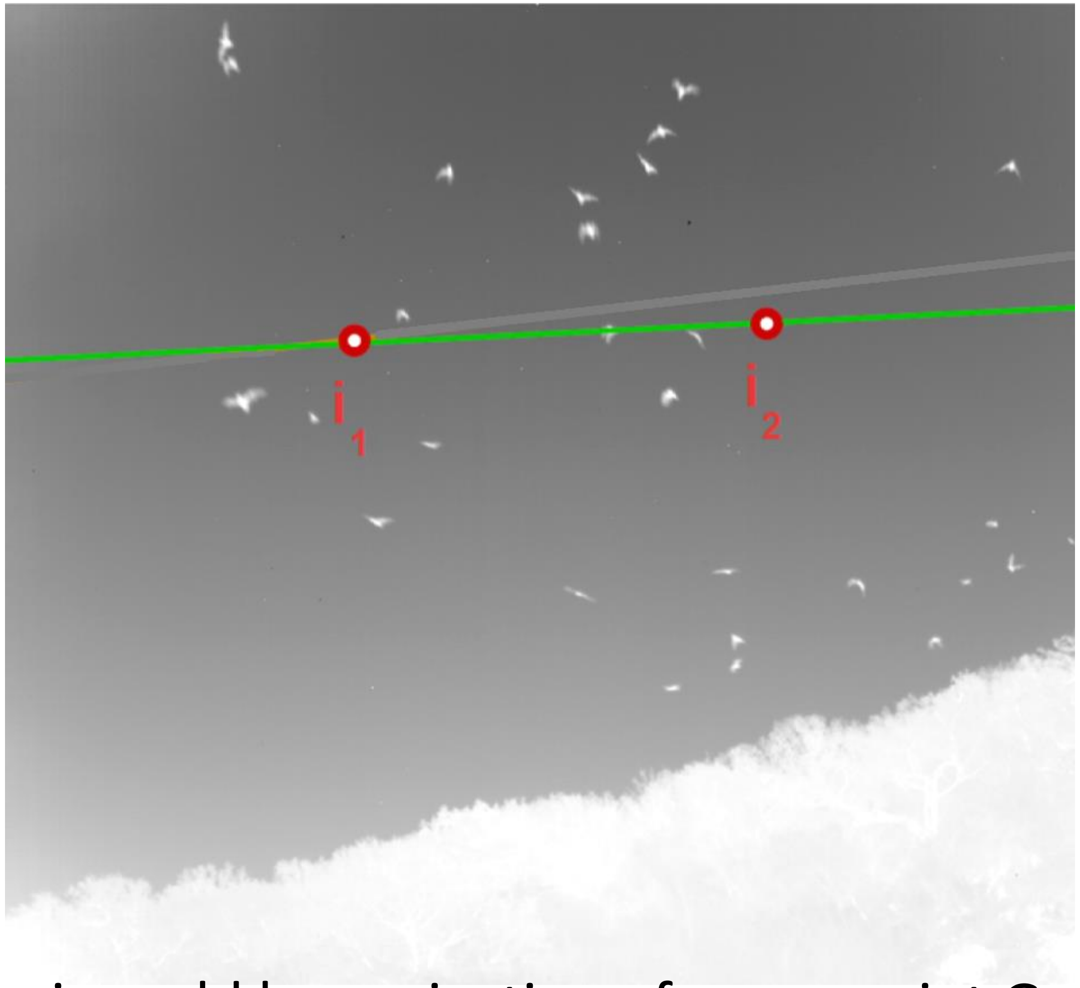
View from above: Z axis = direction of gravity





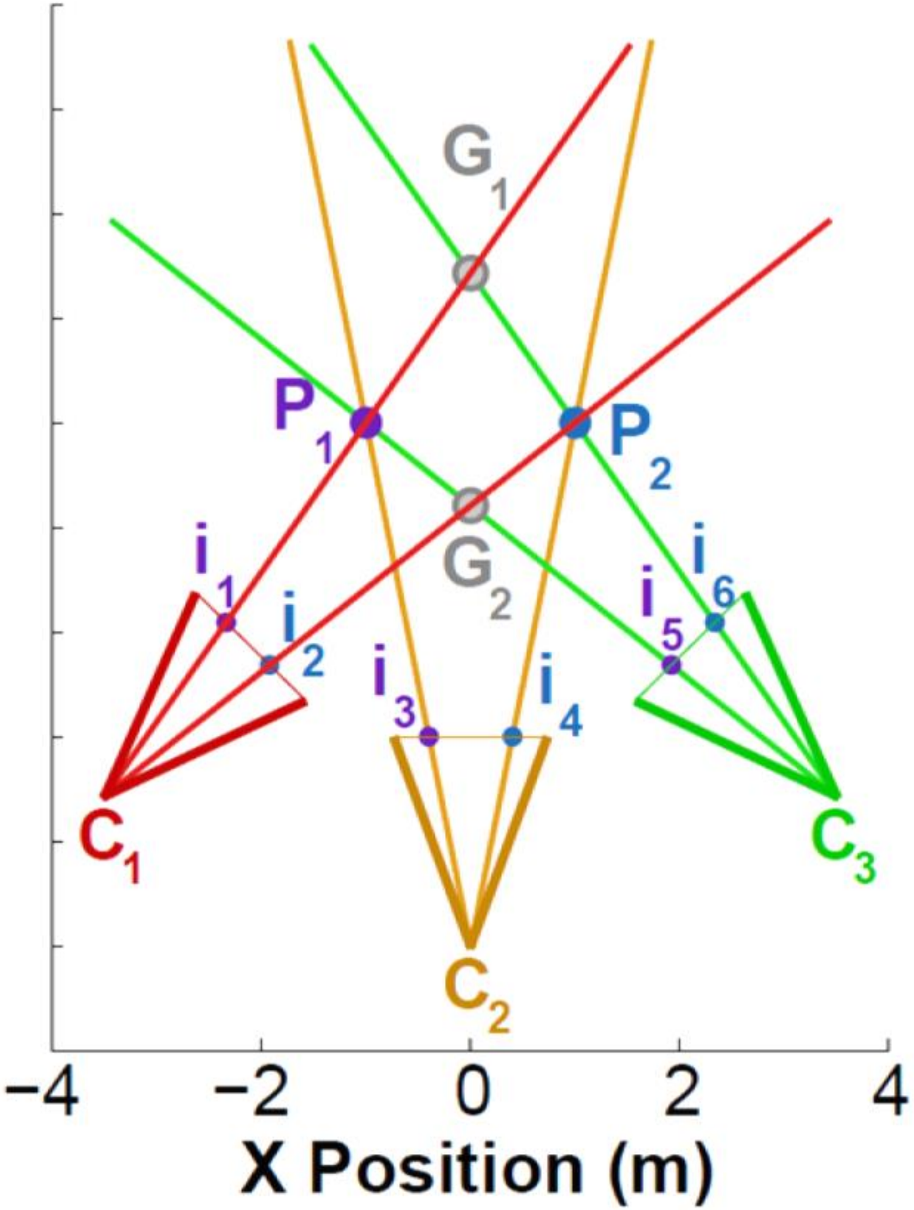
Multi-Camera Stereo

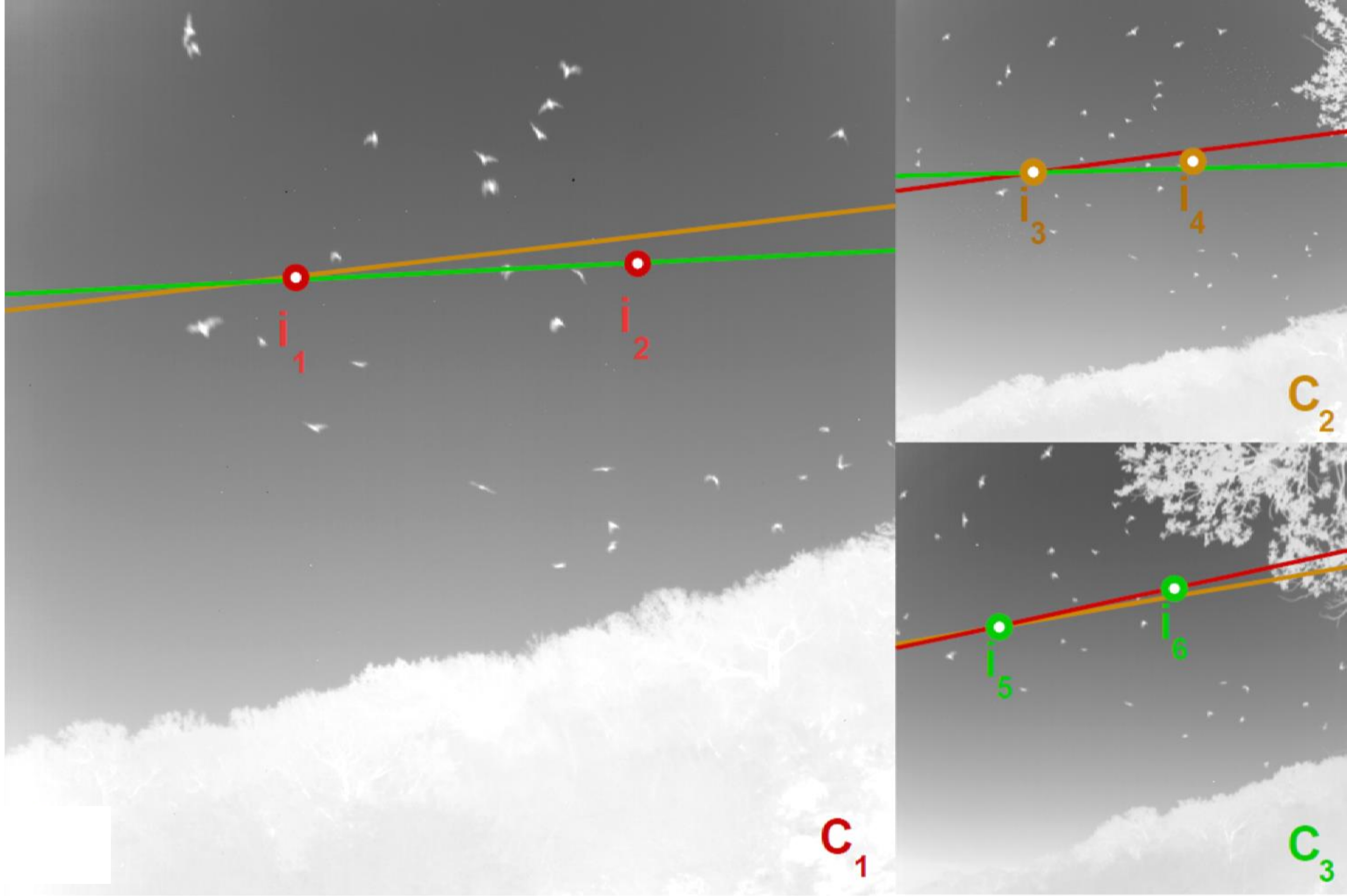




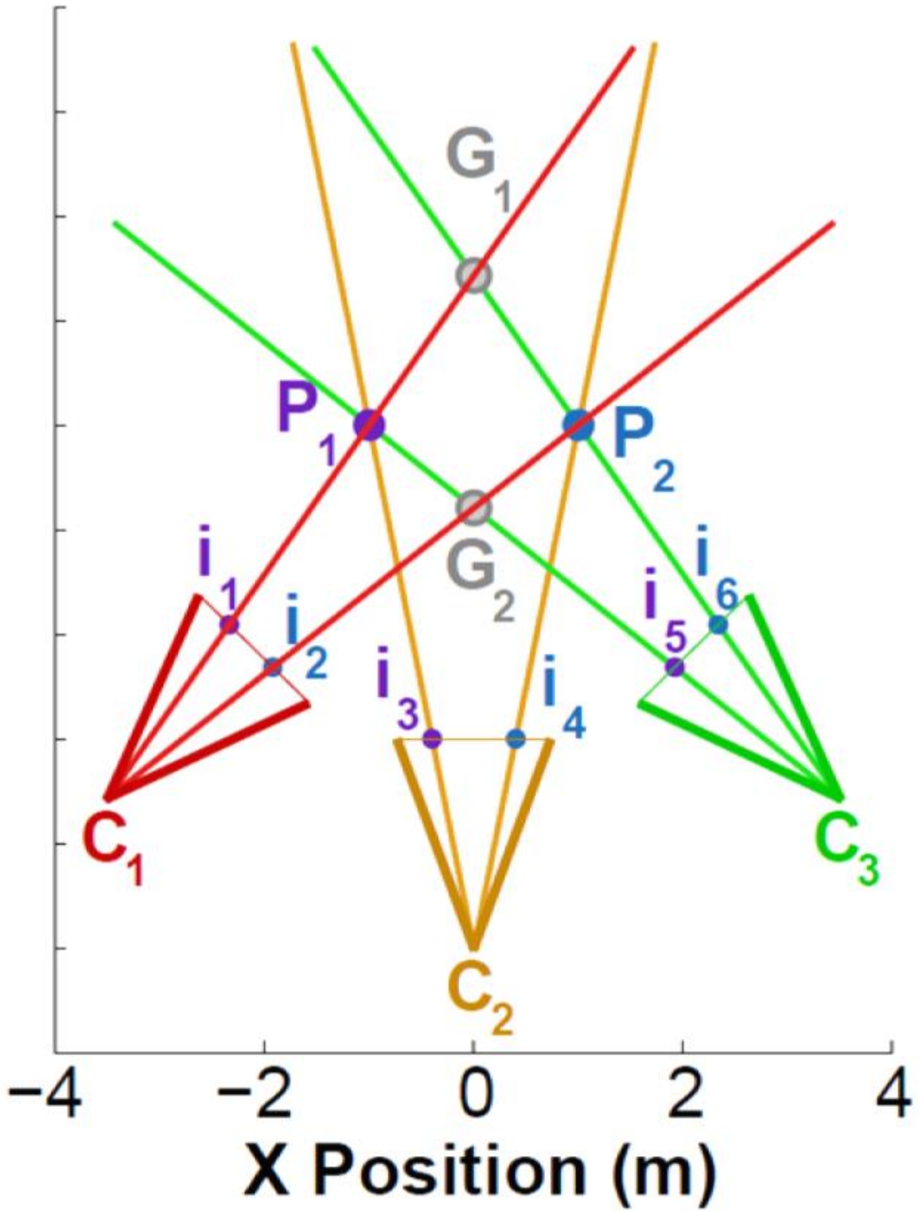
i_1 could be projection of scene point G_1

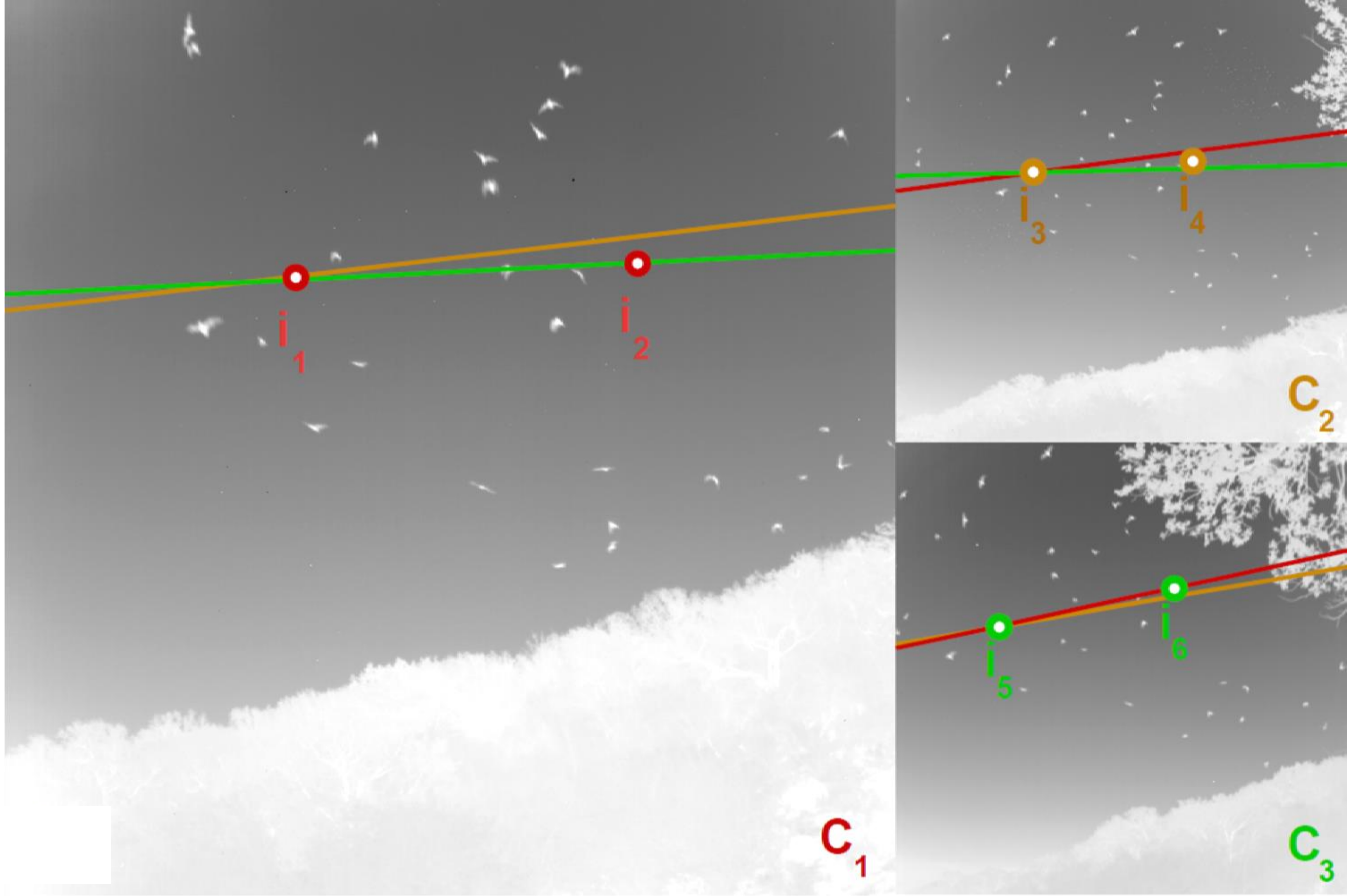
i_2 could projection of scene point G_2



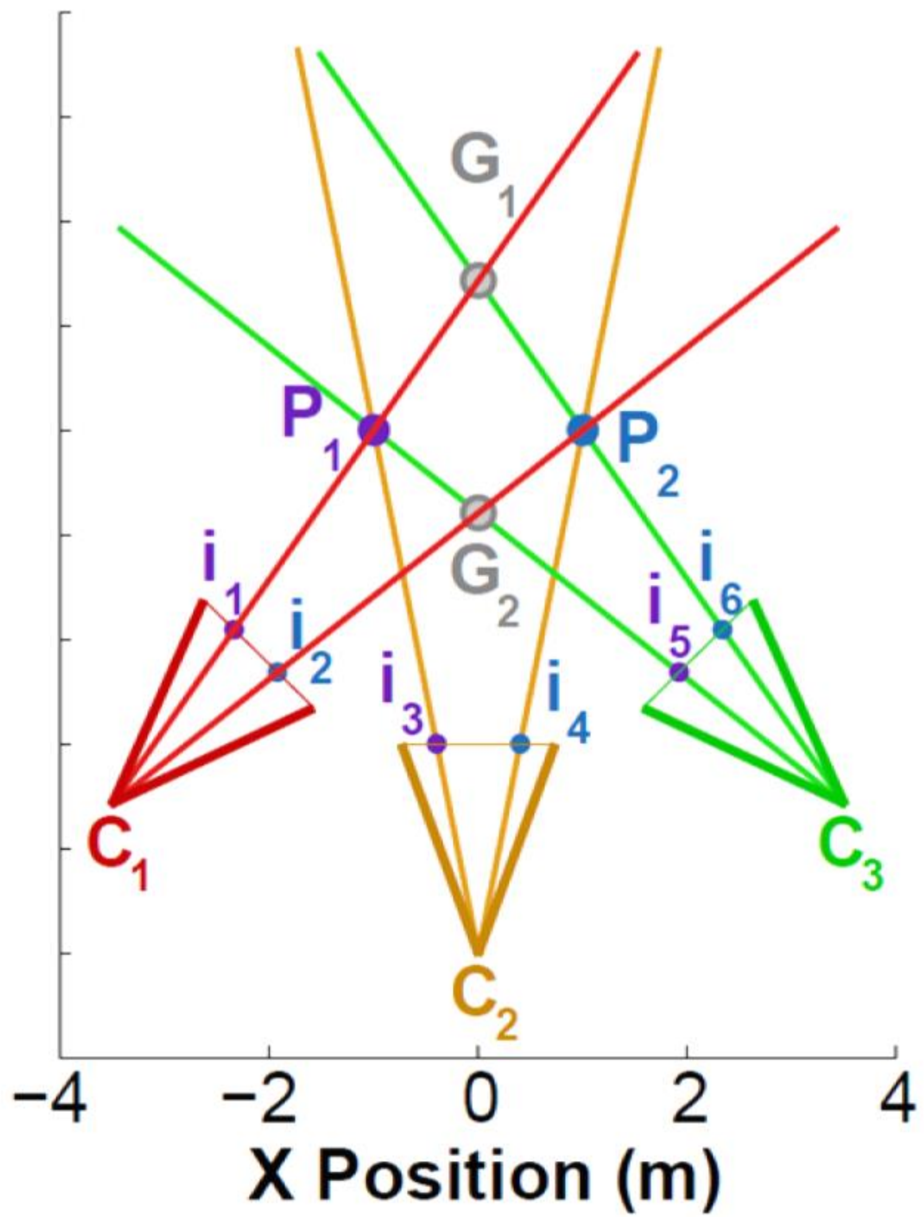


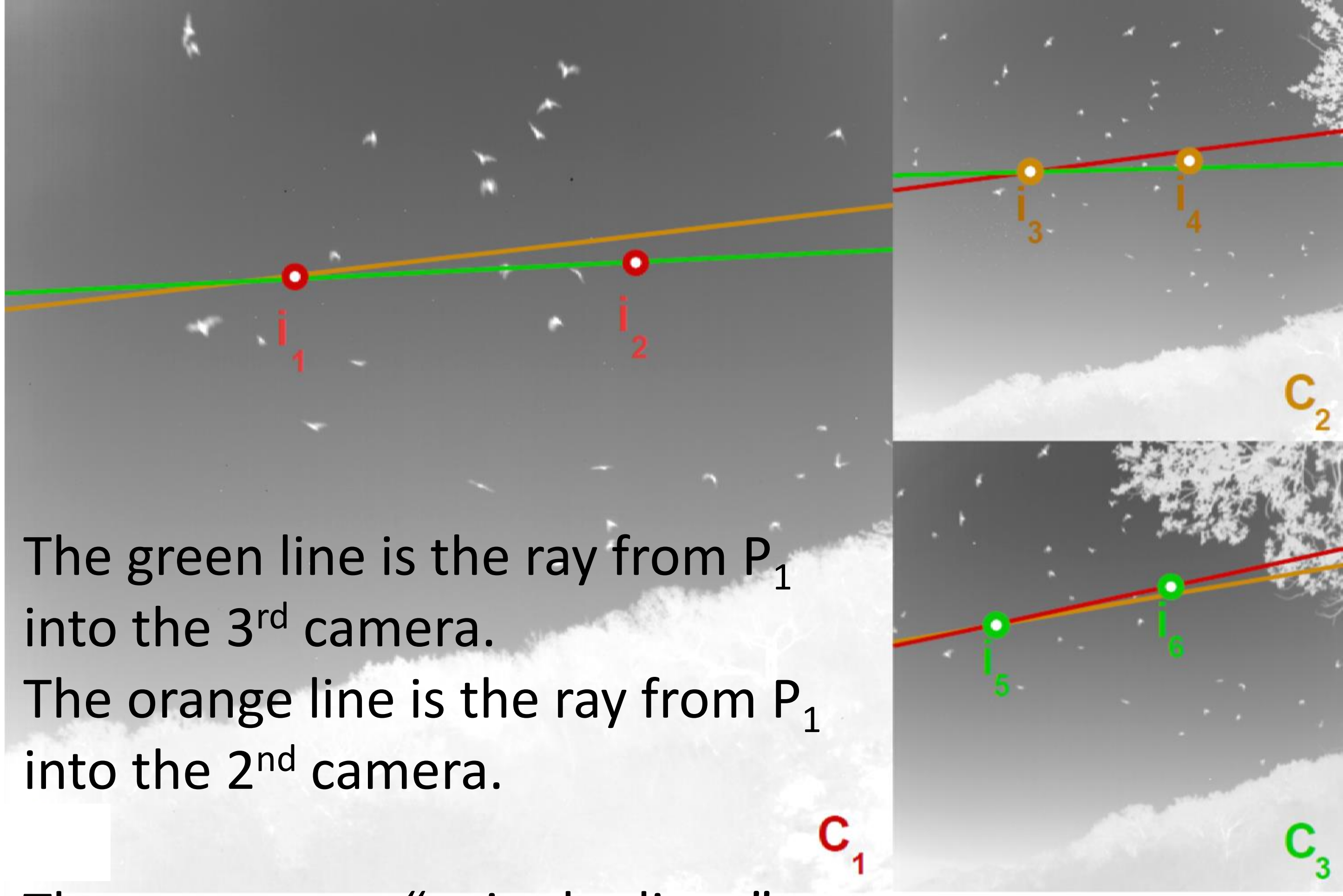
3rd Camera resolves the ambiguity:
G_1 and G_2 are "ghosts" (non-exisiting points)
P_1 and P_2 are the true scene points





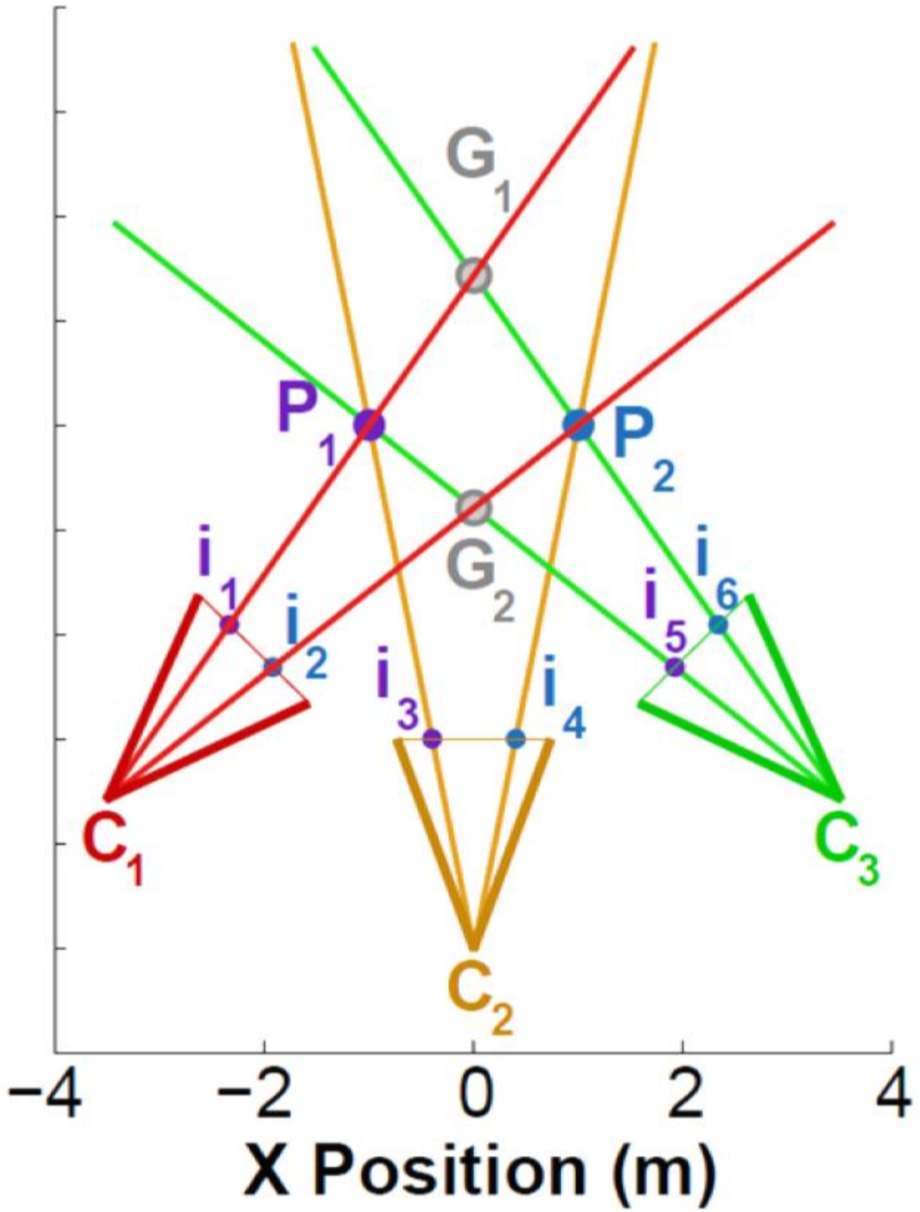
Green line is ray from P_1 into camera C_3.
It appears as an “epipolar line” in the image of camera C_1

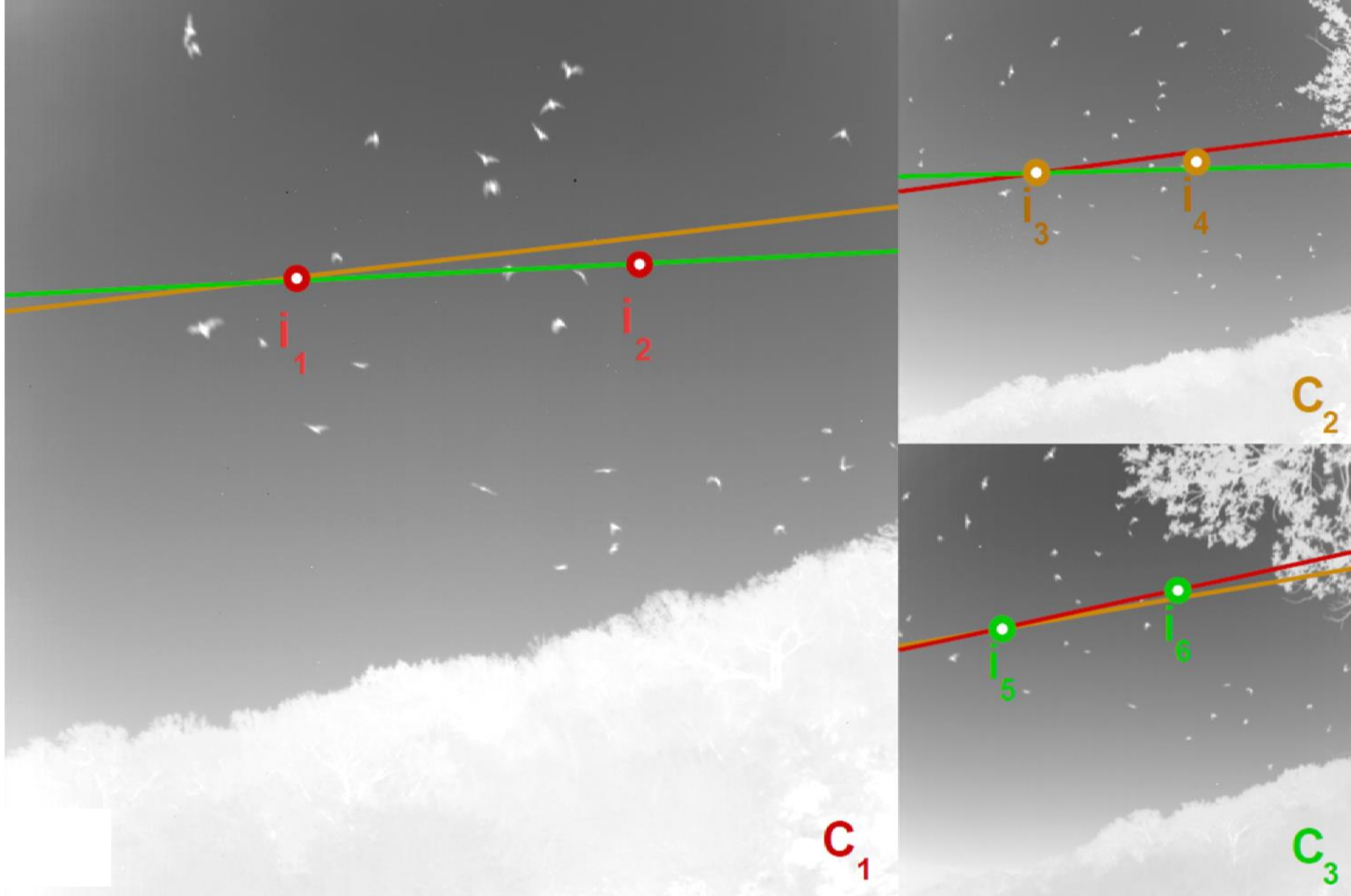




The green line is the ray from P_1 into the 3rd camera.
The orange line is the ray from P_1 into the 2nd camera.

They appear as "epipolar lines" in the image of camera C_1 and must intersect at the same image point i_1.



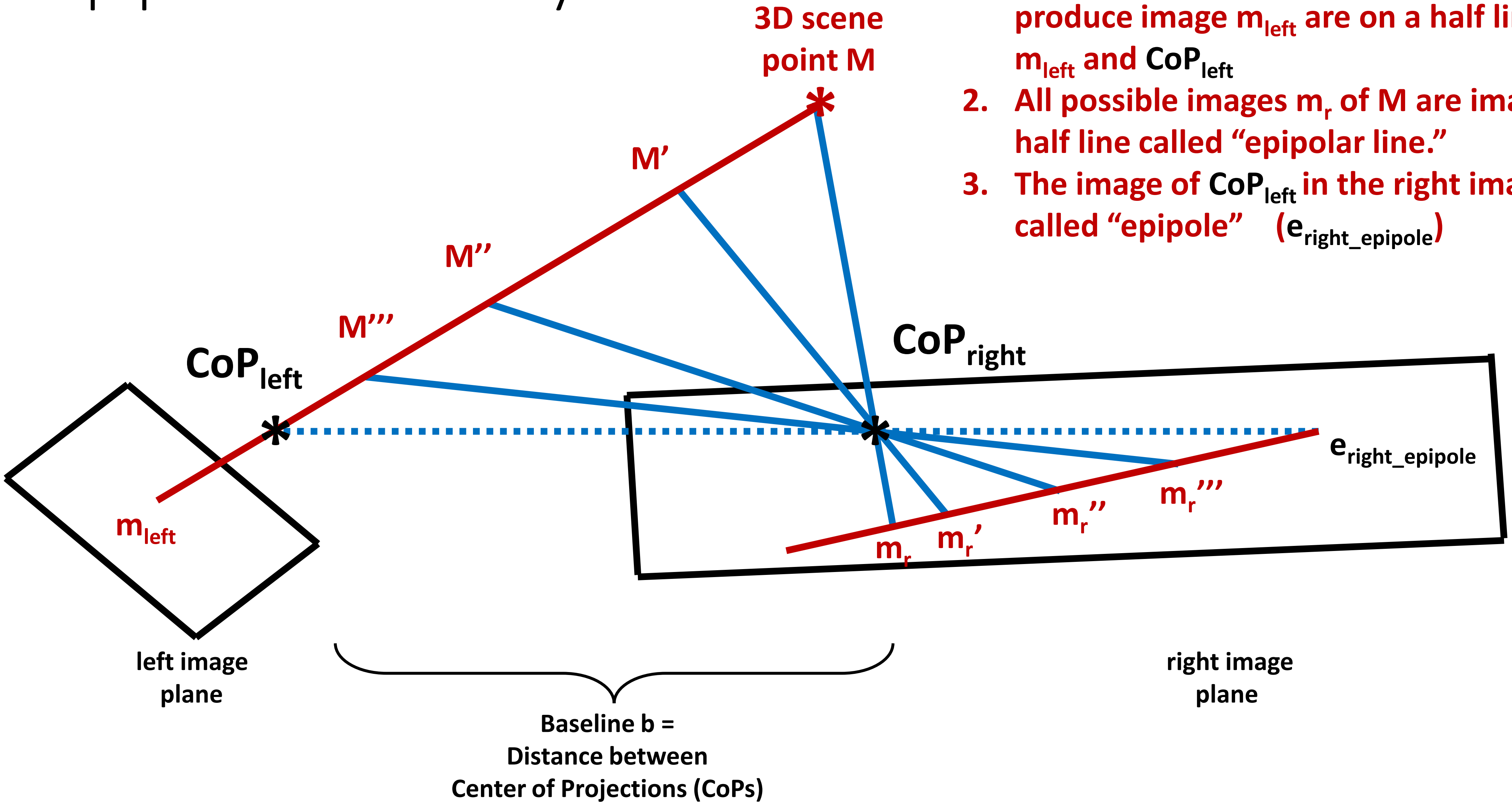


The green and red epipolar lines in the camera C_2 intersect at image point i_3.

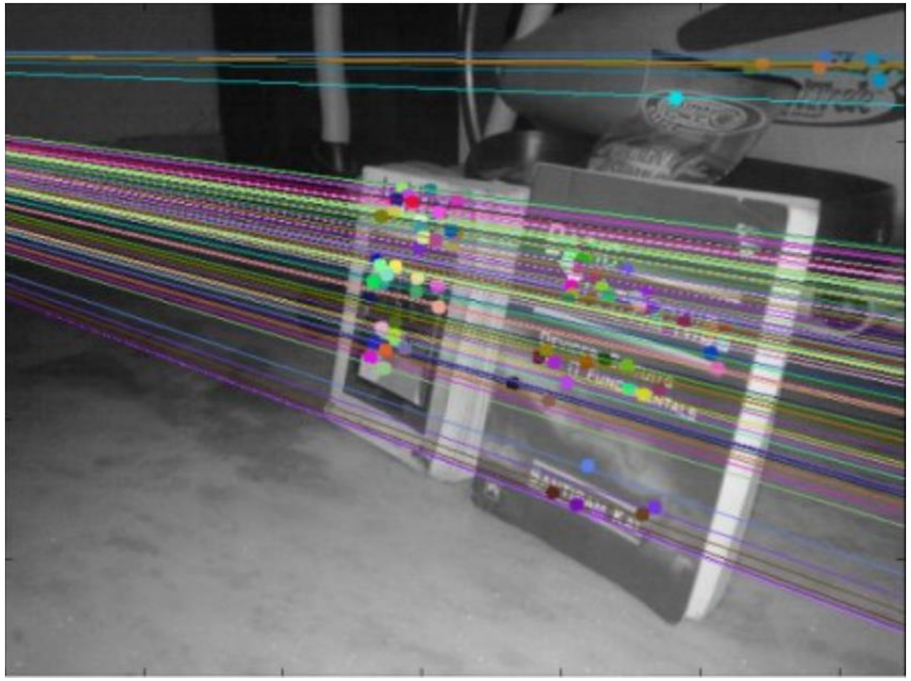
The orange and red epipolar lines in the camera C_3 intersect at image point i_3

Epipolar Geometry

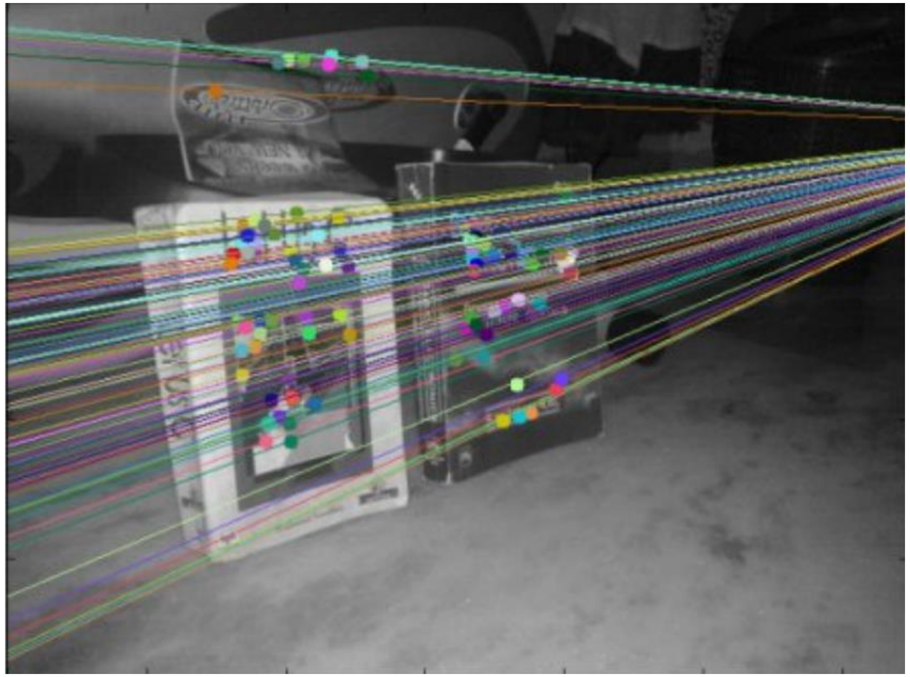
1. All possible scene points M (M', M'', ...) that produce image m_{left} are on a half line through m_{left} and CoP_{left}
2. All possible images m_r of M are images of this half line called "epipolar line."
3. The image of CoP_{left} in the right image plane is called "epipole" ($e_{right\_epipole}$)



Epipolar Geometry



left image



right image

Image Credit: OpenCV.org

Epipolar Geometry: Special Case Parallel Optical Axes

left image

right image

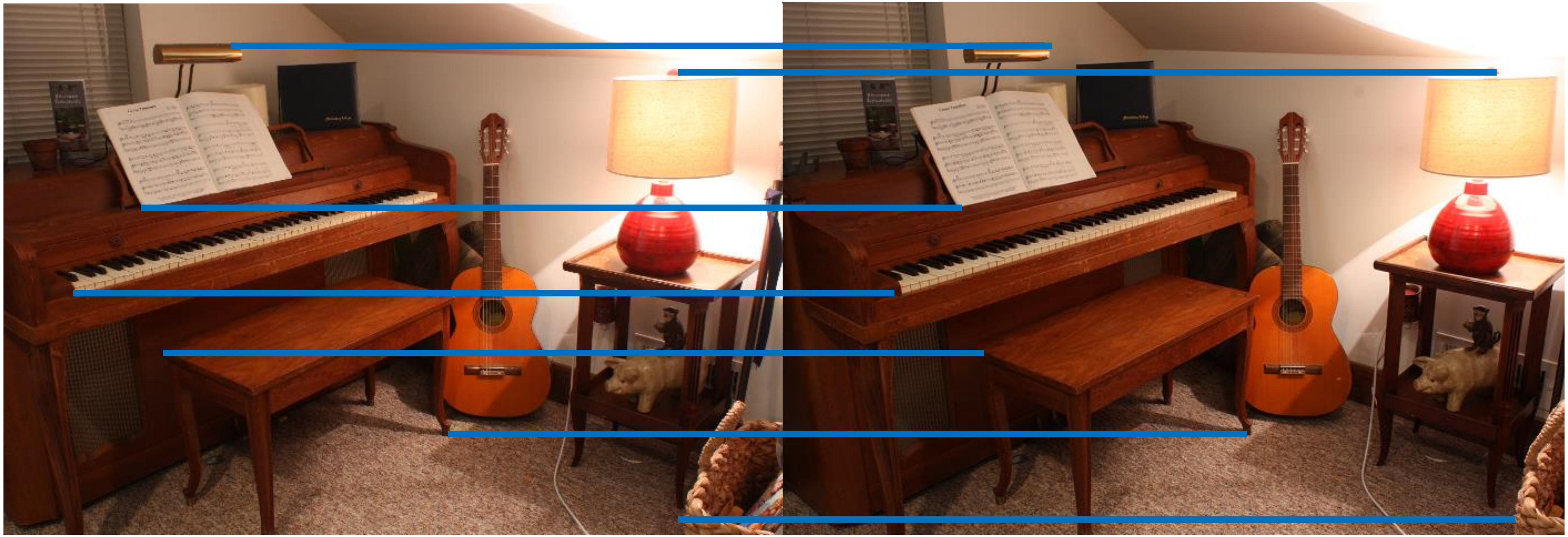
Image Credit: Scharstein, 2014

left image

right image

Image Credit: OpenCV.org

Epipolar Geometry



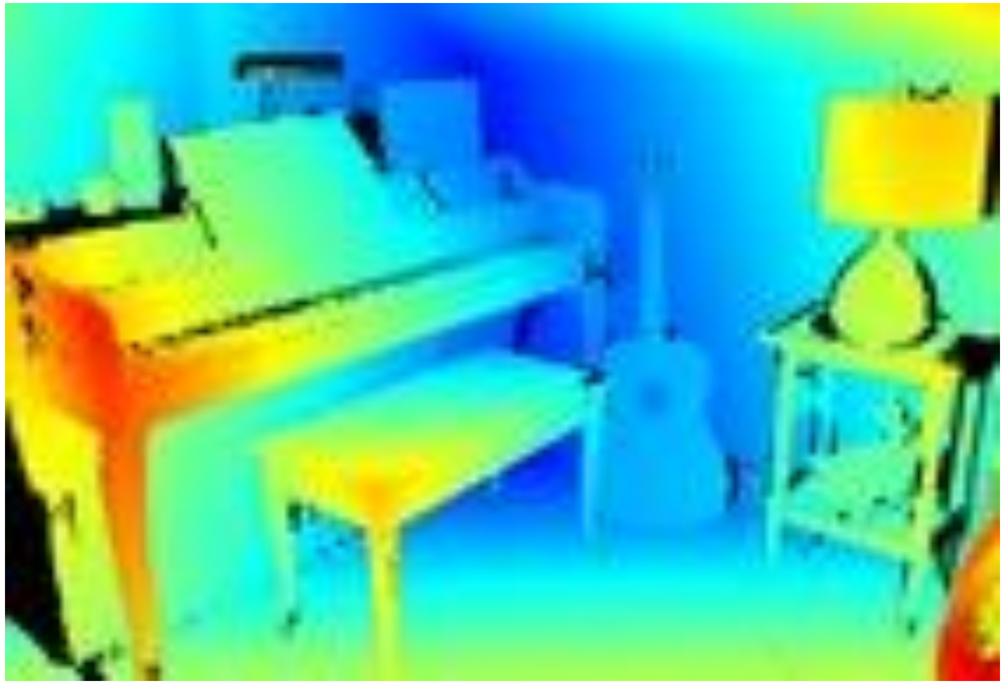
left image

right image

Epipolar lines are parallel = along image rows (epipoles are at infinity)

Algorithm: Find corresponding points in same image rows, e.g., via template matching

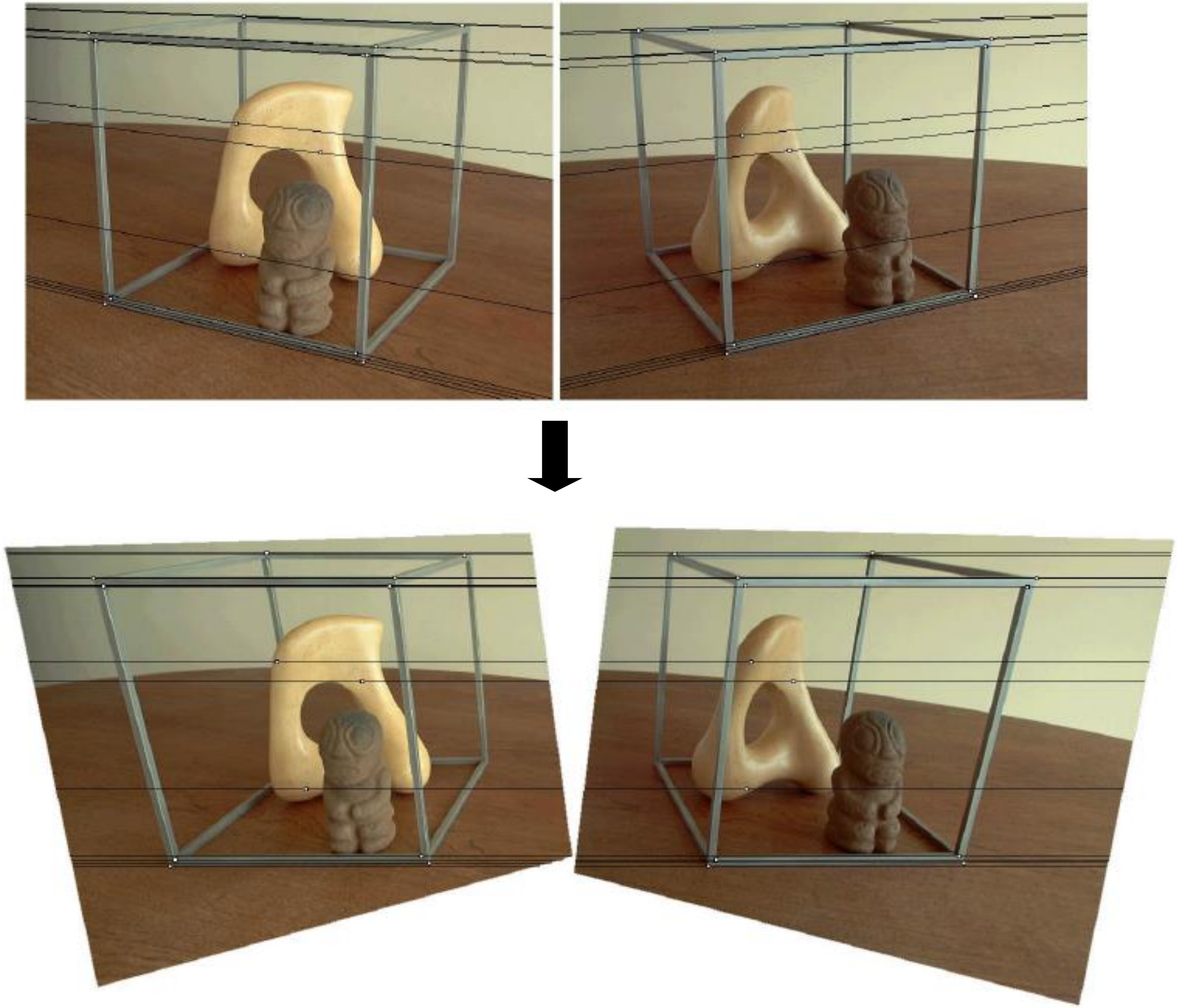
Result of Binocular Stereo Matching: Depth Map



$Z = bf/\delta$

http://vision.middlebury.edu/stereo/data/scenes2014/

Rectification of Binocular Stereo Images: Undo Foreshortening



Why?

Epipolar lines are
now parallel,
enabling a simple
search for corresponding
points along image
rows

Image Source: Alyosha Efros

Rectification of Binocular Stereo Images: Undo Foreshortening

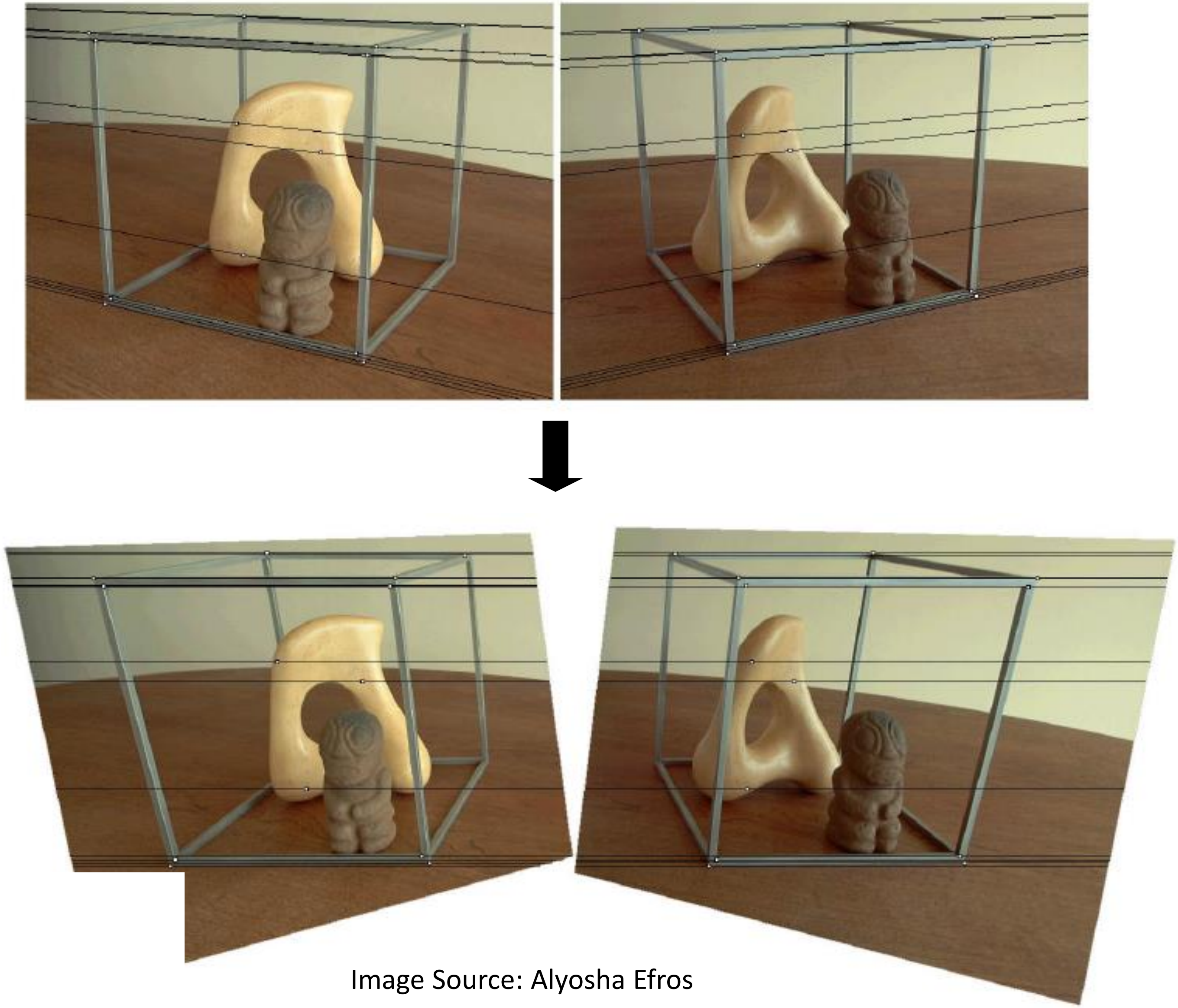


Image Source: Alyosha Efros

How?
Iterative Scheme

We want

$$I_{left}(x + \delta/2, y) = I_{right}(x - \delta/2, y)$$

Least Squares Method:

$$\min_{\delta} \sum_{\mathbf{p}} [I_{left}(x + \delta/2, y) - I_{right}(x - \delta/2, y)]^2$$

p = patch
size of patch **p**: tradeoff
too small instability
too large smearing

Use current estimate of disparity δ to warp

Then solve LSM & update disparity

Binocular Stereo Solution Paths: 2 Alternatives

1. "Weak Calibration"
   - If needed: Use rectification to ensure epipolar lines are along image rows
   - Find corresponding points in both views and calculate disparity δ
   - Compute depth: $Z = bf/\delta$

2. "Strong Calibration"
   - Calibrate each camera (= interior orientation): f, pp
   - Find geometric transformation of cameras (= relative orientation): R, r_0
   - Find 3D coordinates

Binocular Stereo Solution Paths: 2 Alternatives

1. “Weak Calibration”
   - If needed: Use rectification to ensure epipolar lines are along image rows
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2. “Strong Calibration”
   - Calibrate each camera (= interior orientation): f, pp
   - Find geometric transformation of cameras (= relative orientation): R, r_0
   - Find 3D coordinates

In our animal tracking research, “strong calibration” was the better solution

Binocular Stereo Solution Path: “Strong Calibration”

Throw wand in the air several times (mark out bird flying space)

Identify wand position in all views

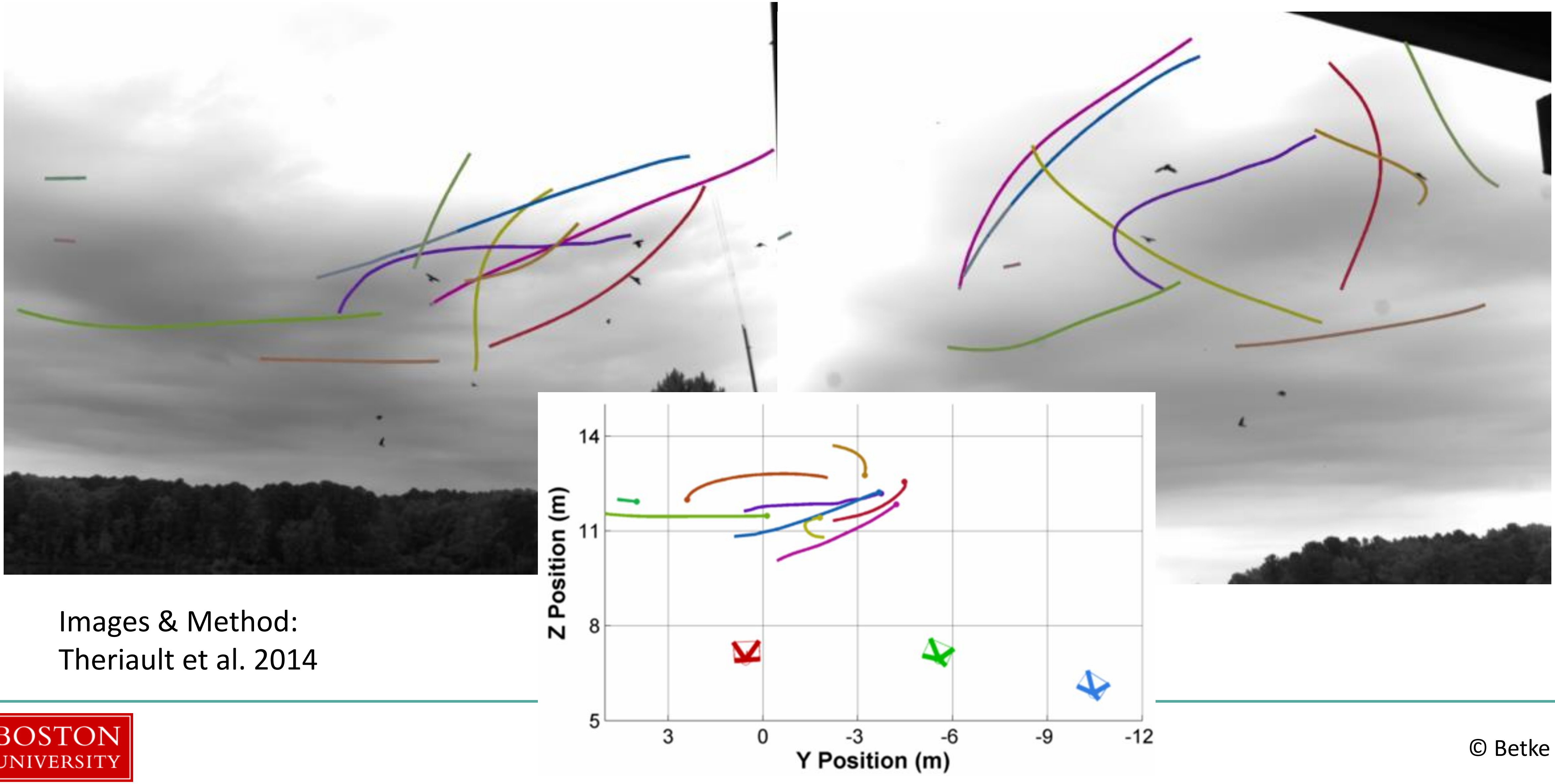
Take advantage of knowing the dimensions of the wand

Estimate R and r_0

Images & Method:
Theriault et al. 2014

Binocular Stereo Solution Path: "Strong Calibration"

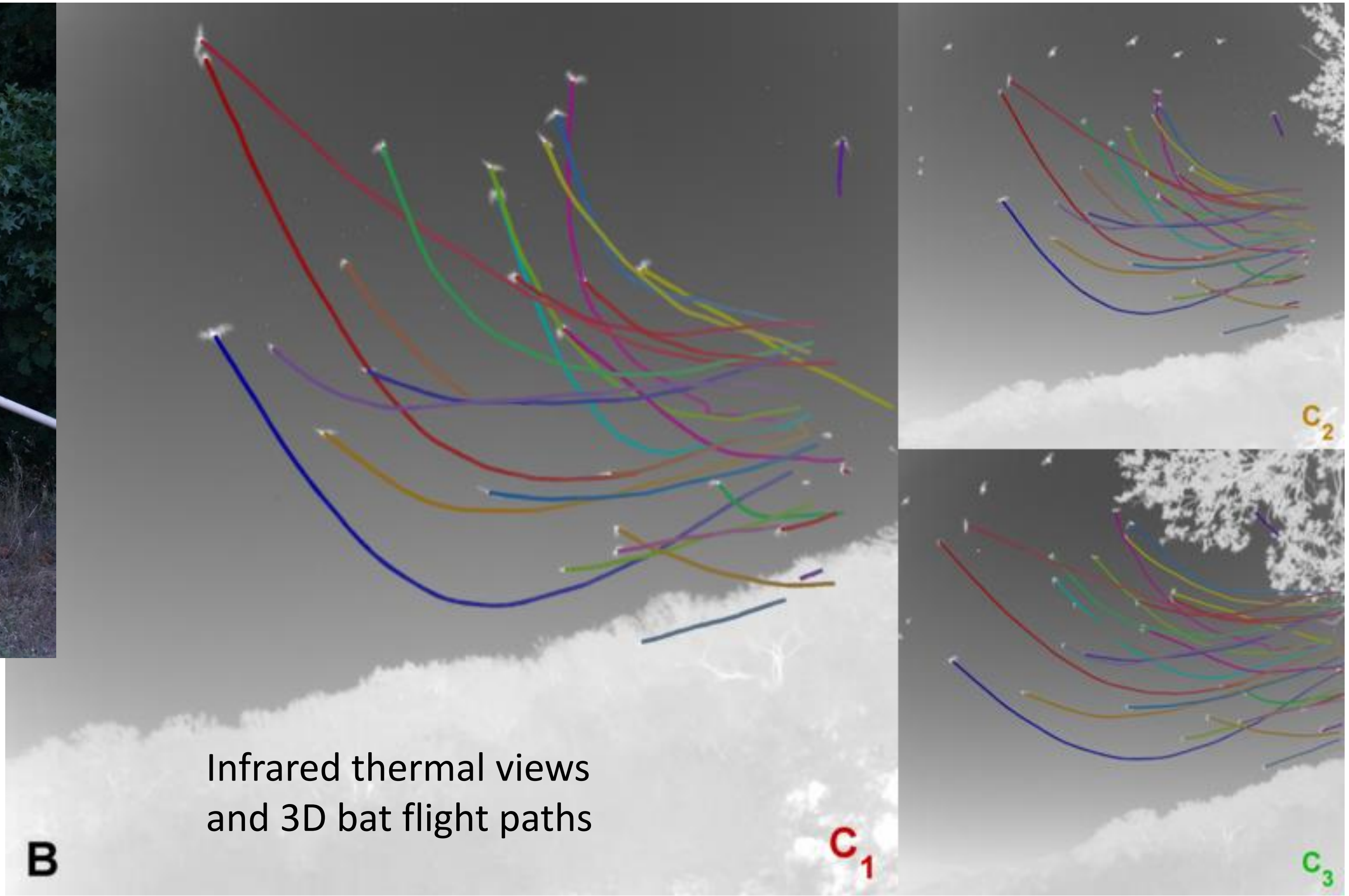
Binocular Stereo for 3D Bird Flight Analysis



Images & Method:
Theriault et al. 2014

Binocular Stereo for 3D Bat Flight Analysis

Calibration tool with ice packs



Infrared thermal views
and 3D bat flight paths

Binocular Stereo Solution Path: "Strong Calibration"

Indoor scenario is much easier:

Instead of wand, use "checker board" as calibration device

Take many images at different positions & orientations

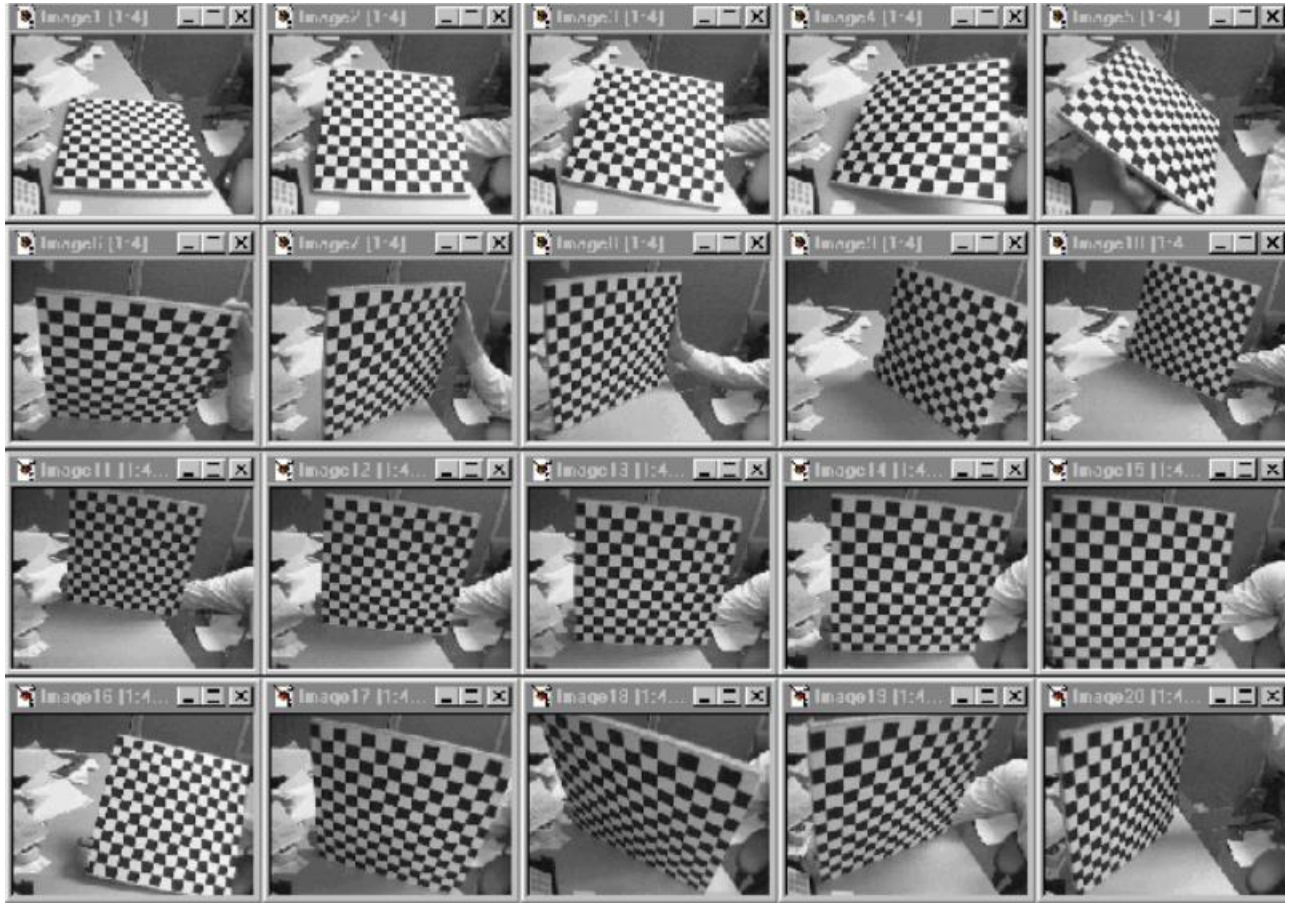


Image Source: Jean-Yves Bouguet

Binocular Stereo Solution Path: "Strong Calibration"

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Instead of wand, use "checker board" as calibration device

Take many images at different positions & orientations

Use http://www.vision.caltech.edu/bouguetj/calib_doc/index.html

Or OpenCV

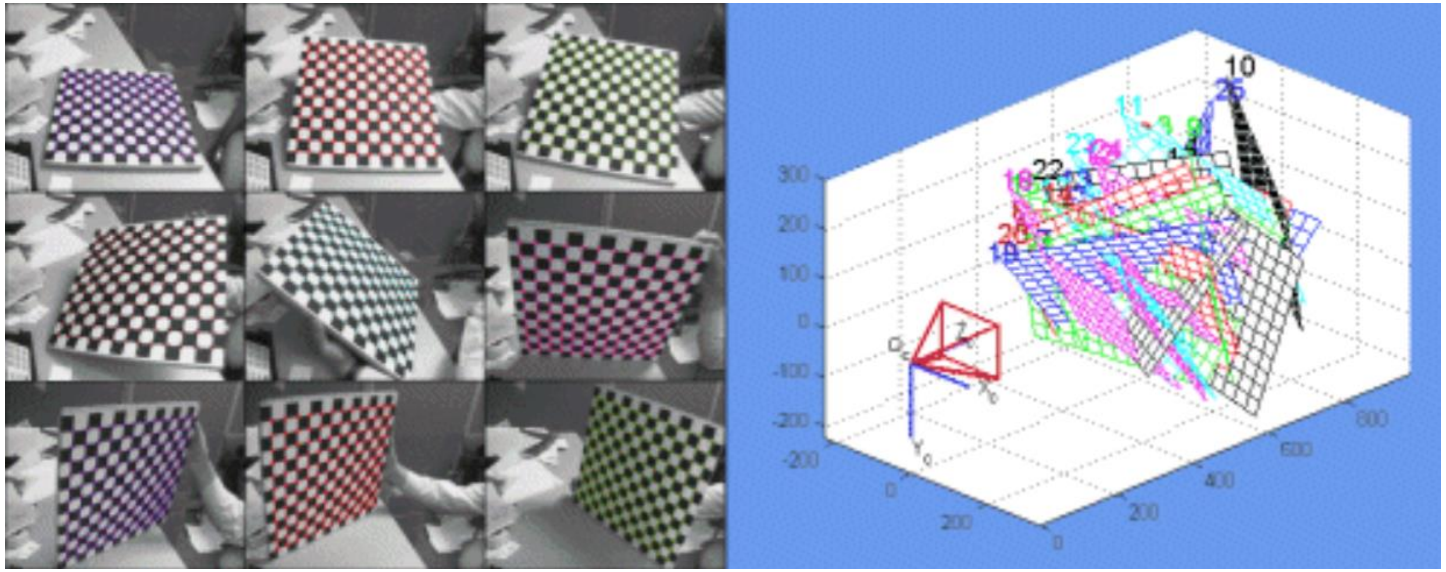


Image Source: Jean-Yves Bouguet

Active stereo with structured light

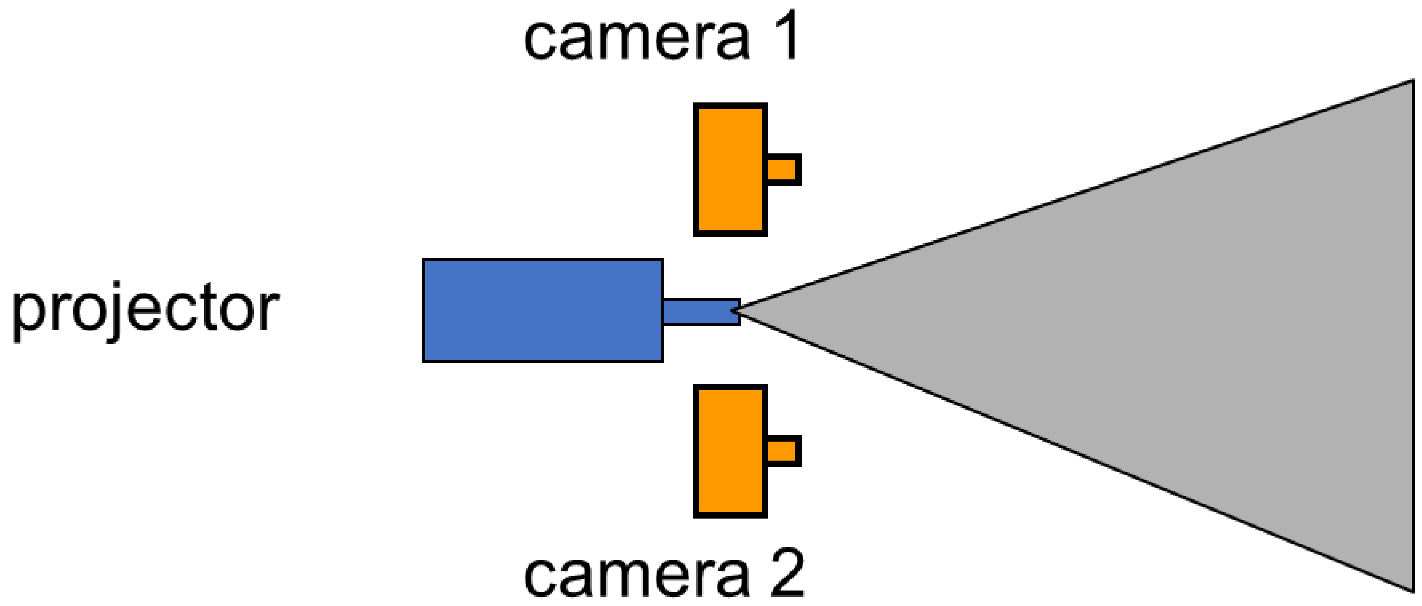
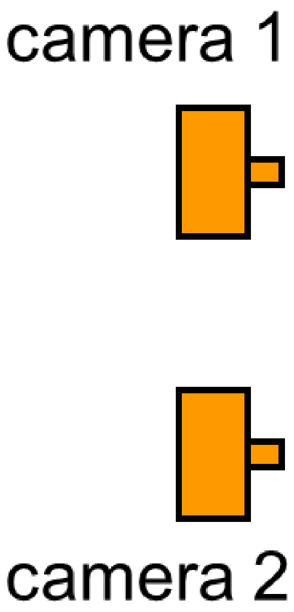


Image credit: Li Zhang

Active stereo with structured light

View without structured light



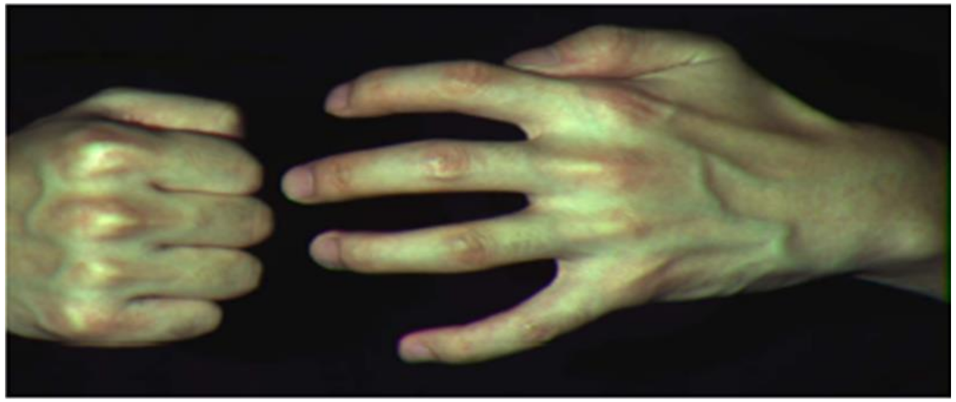
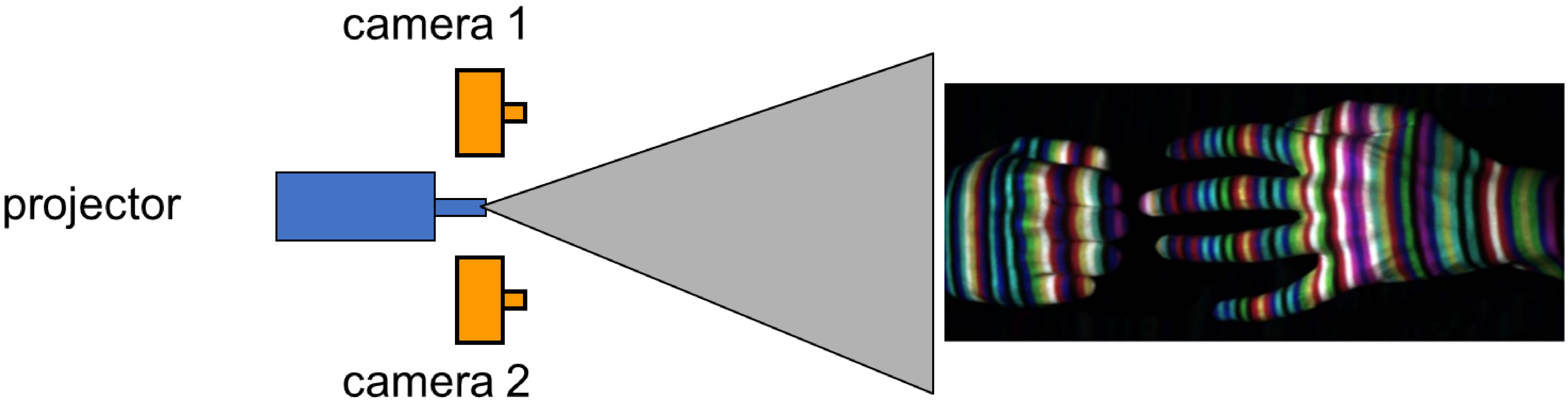


Image credit: Li Zhang

Active stereo with structured light

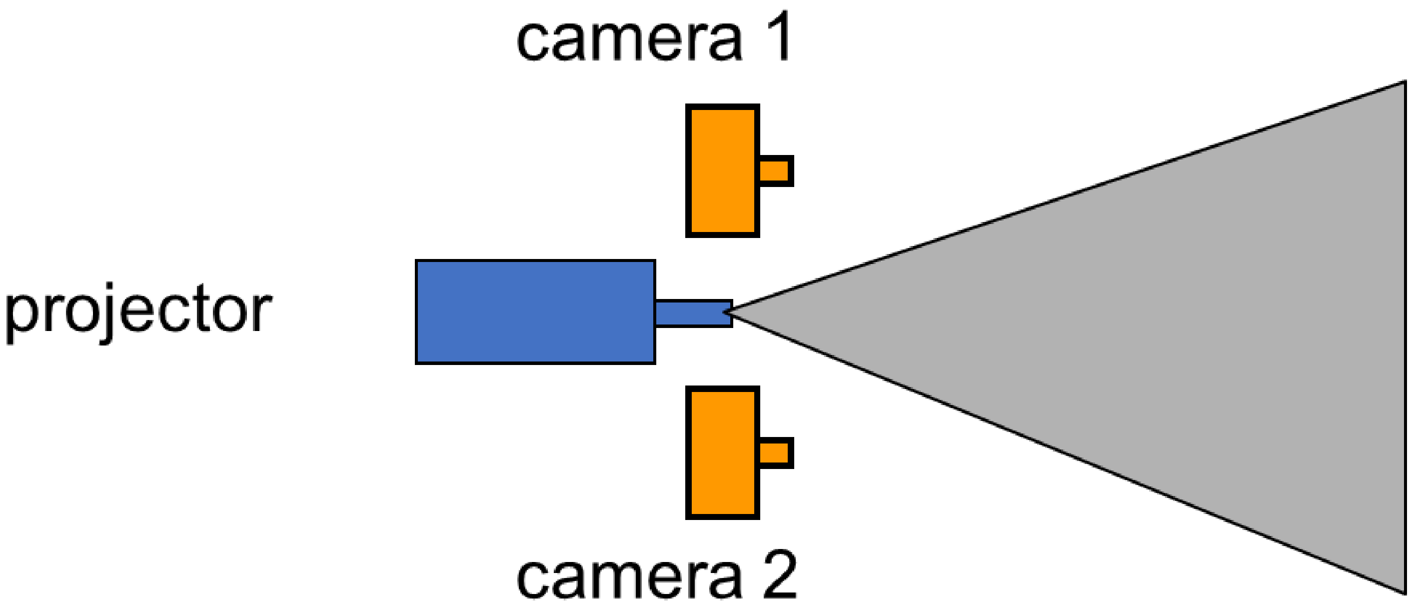


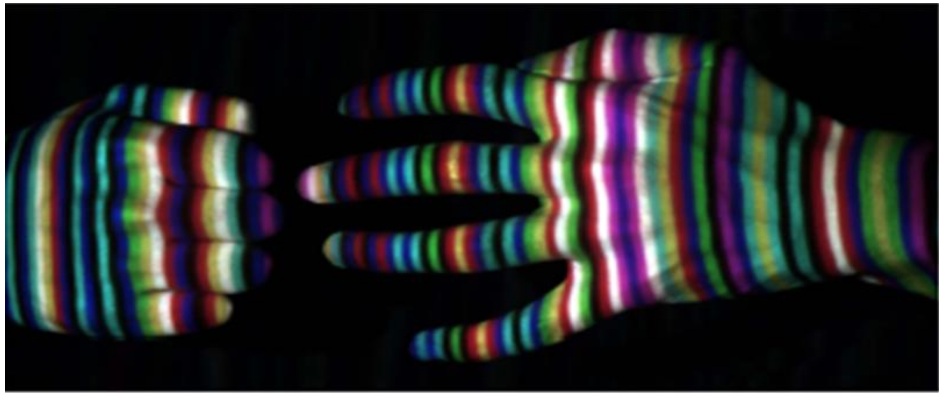
Project "structured" light patterns onto the object

simplifies the correspondence problem

Image credit: Li Zhang

Active stereo with structured light





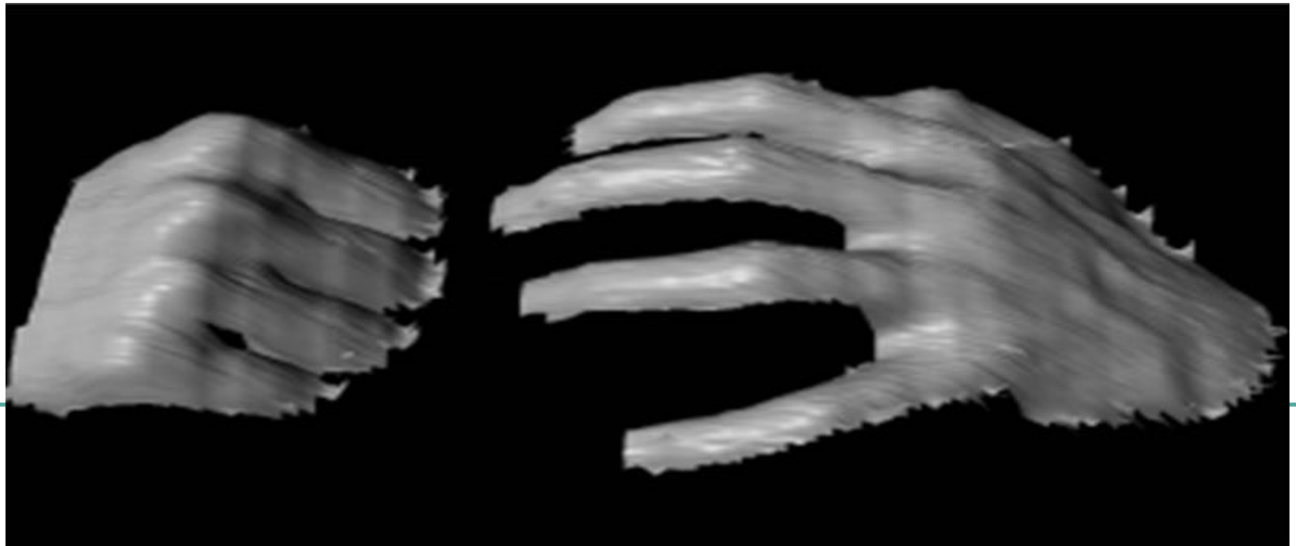


Image credit: Li Zhang

Active stereo with structured light

Image credit: Li Zhang et al.

What if we do not have 2 cameras?

Can we still do 3D reconstruction of a scene?

YES !!!

2 Algorithms

Problem Definitions

- Shape from Shading (SfS)

  Find 3D shape in scene from a single 2D image

- Photometric Stereo $\neq$ binocular stereo

  Find 3D shape in scene from a set of 2D images that are taken under different lighting conditions

“stereo” = “solid” in Greek, used to refer to solidity, three-dimensionality

Photometric Stereo

Example:

Find 3D shape in scene from these images of faces

Photometric Stereo

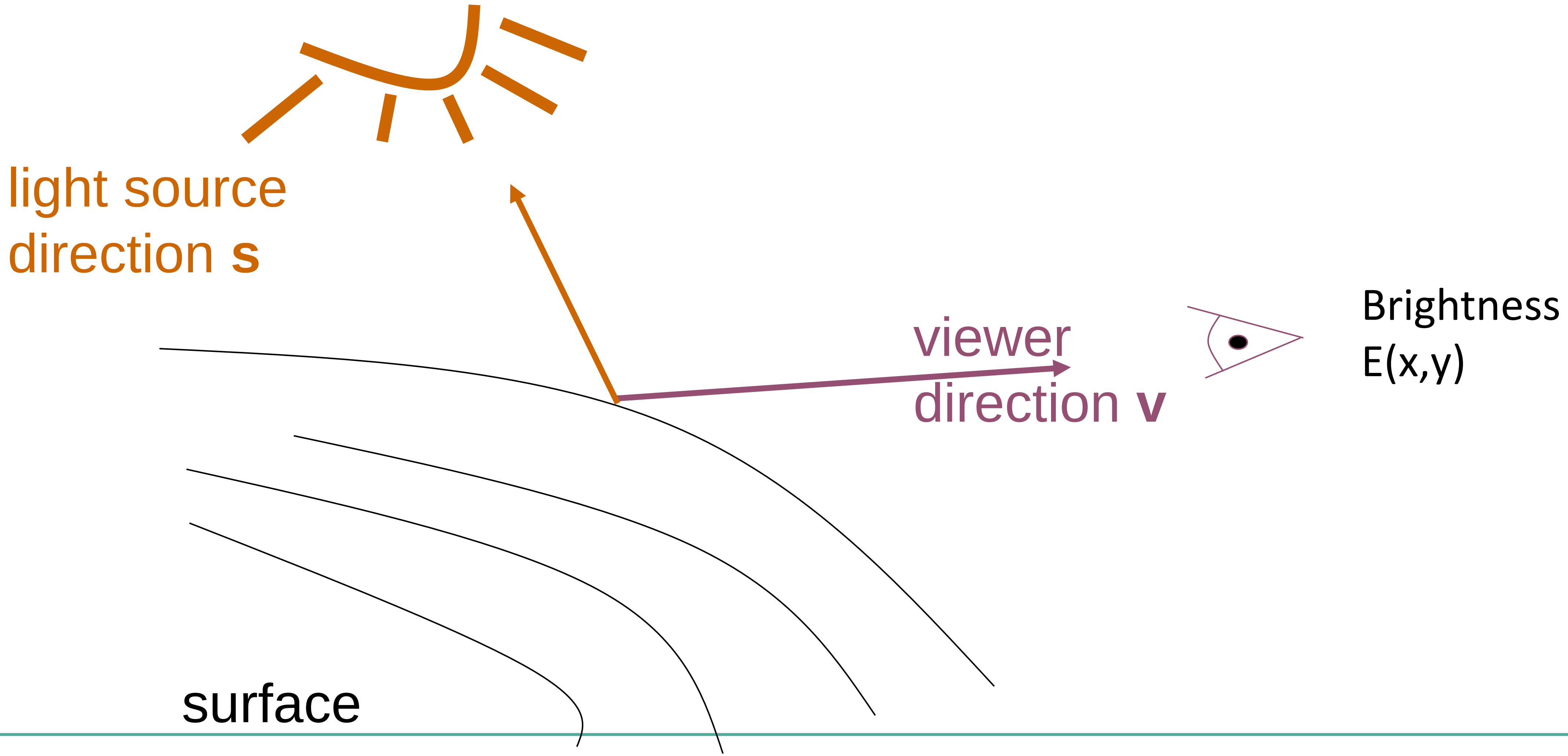
3D shape visualized with texture from 1st image

Photometric Stereo & SfS

Light reflected at surface patch depends on

- surface orientation
- reflectance properties of surface
- distribution of light sources illuminating surface

Light source **s**, Viewer Direction **v**, Image E



Photometric stereo & SfS

Light reflected at surface patch depends on

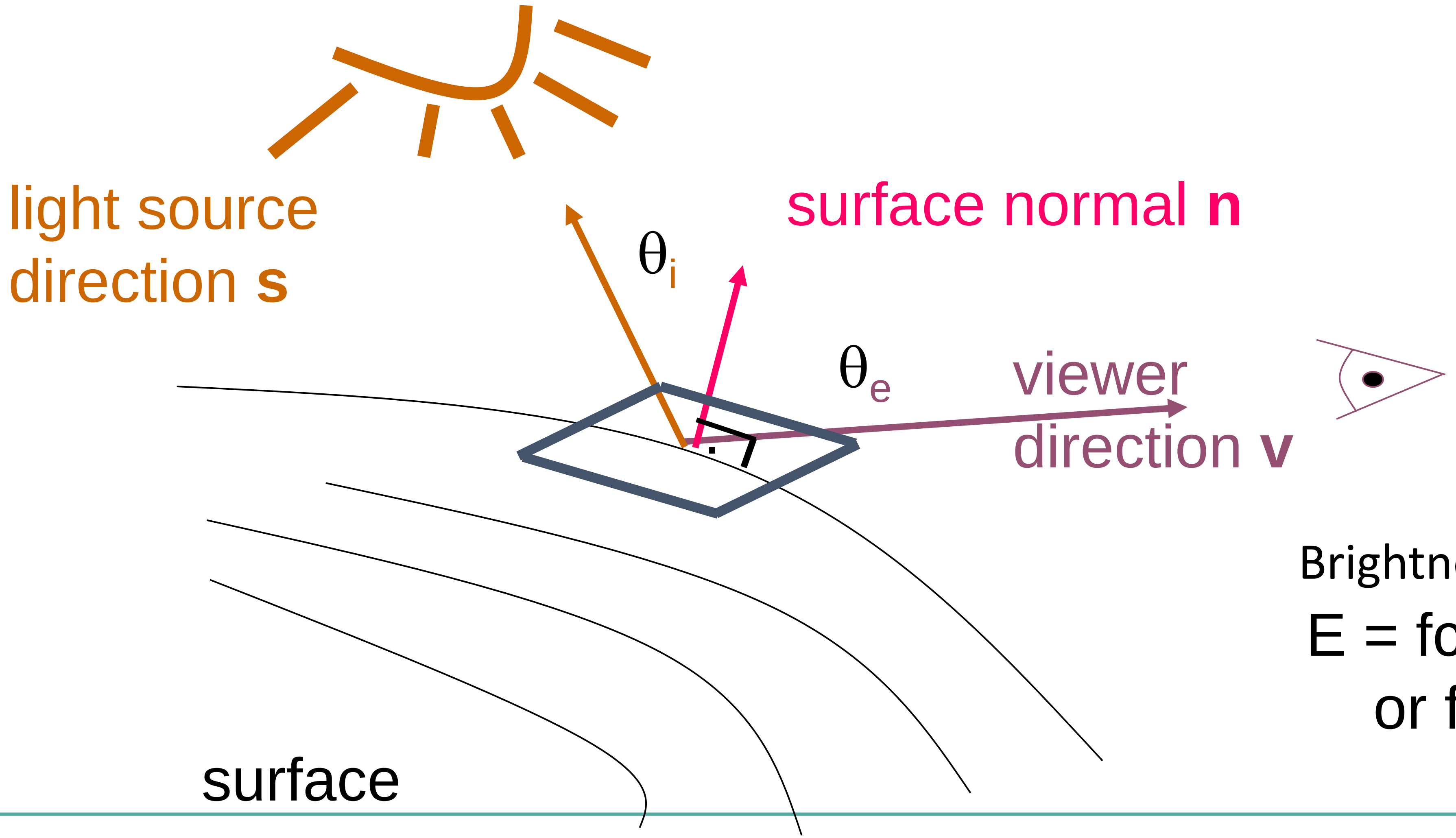
- surface orientation
- reflectance properties of surface
- distribution of light sources illuminating surface

Reconstruction Method:

Determine surface reflectance properties and direction of light source(s)

Compute surface orientation

Light source **s**, Viewer Direction **v**, Surface Normal **n**, Image E



Brightness

$E = fct(\theta_i, \theta_e)$

or $fct(n, s, v)$

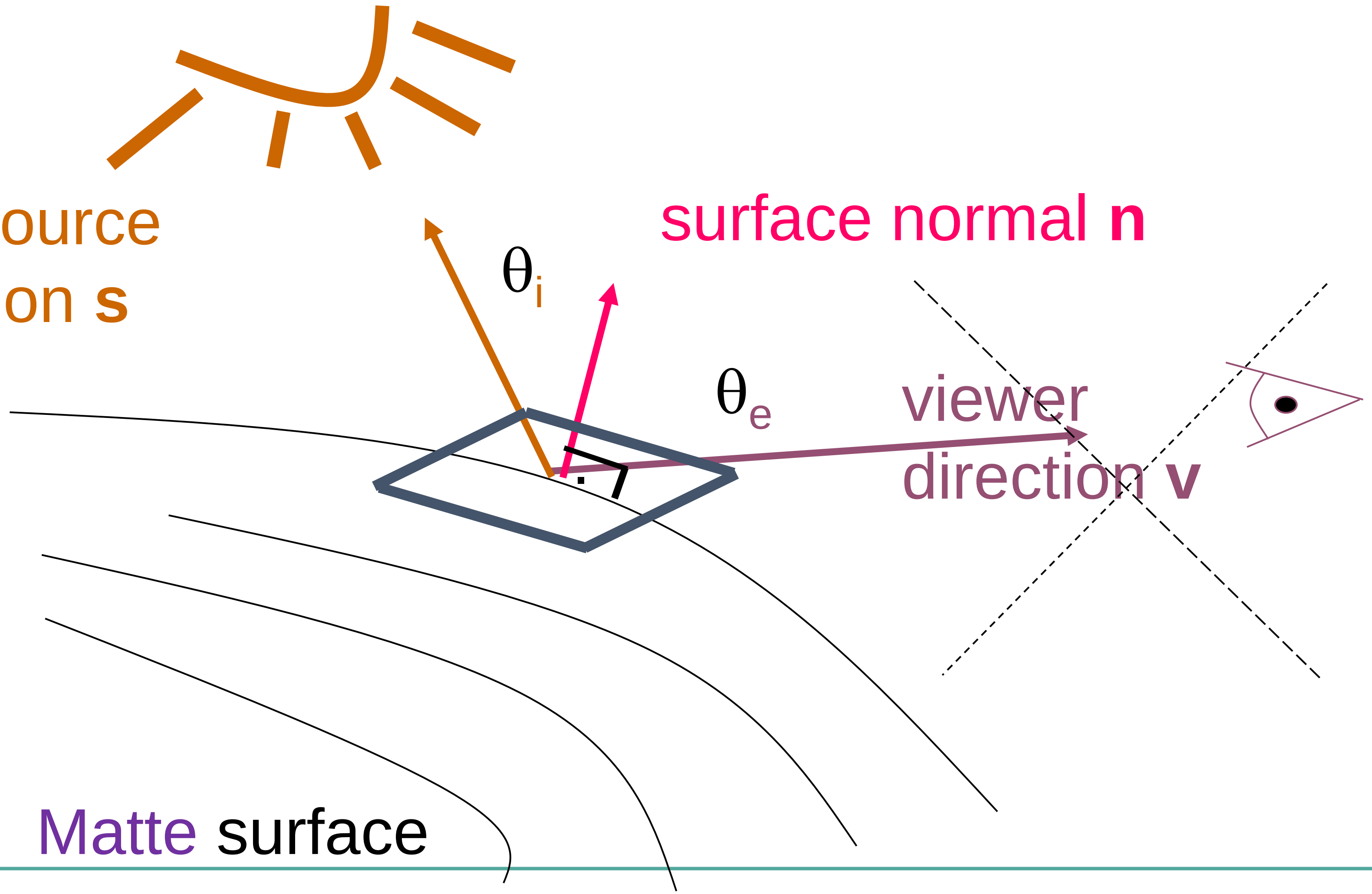
Ideal Lambertian Surface looks equally bright from all directions

Brightness

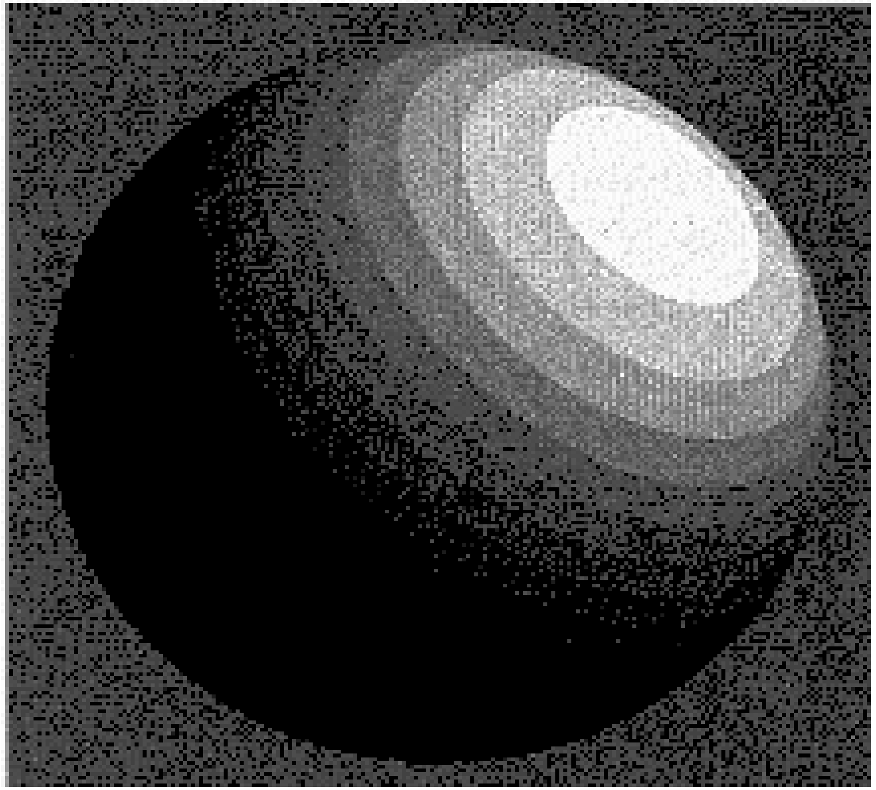
$E = \text{fct}(\theta_i, \theta_e)$

or $\text{fct}(n, s, v)$

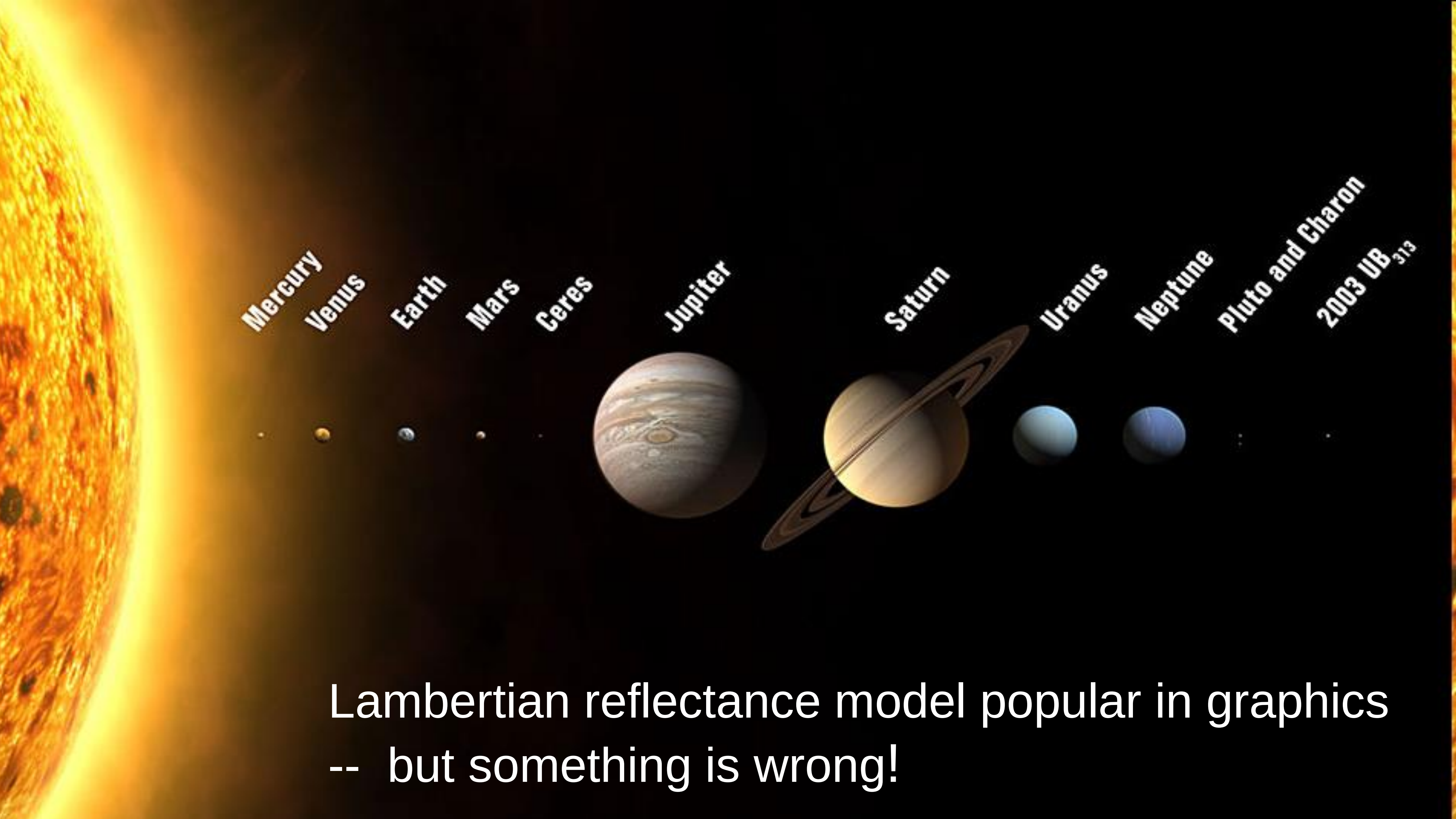
$E = \cos\theta_i = \mathbf{n.s}$



Example of Lambertian Surface: Matte Sphere



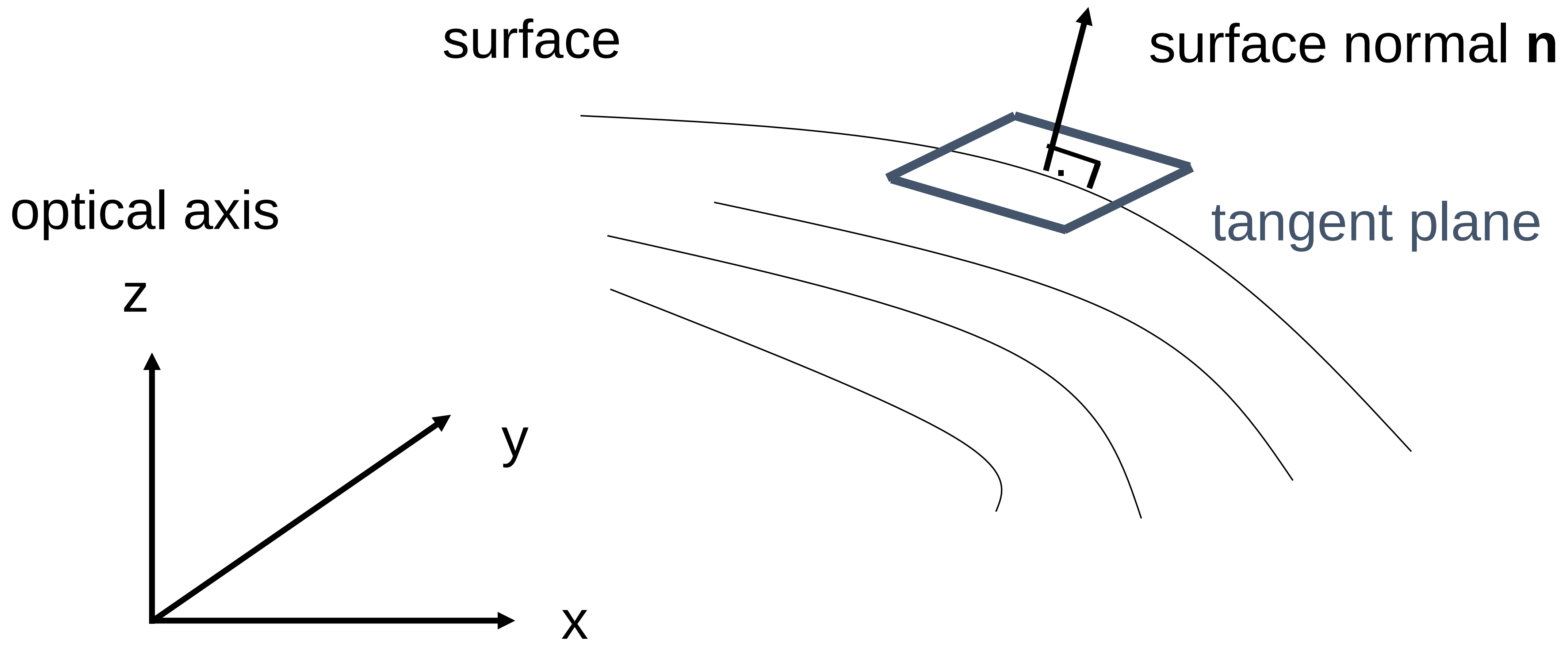
Synthetic image with deliberately few gray values
Brightness (gray levels) does not change linearly from bright to dark
Equation? cos (n, s)



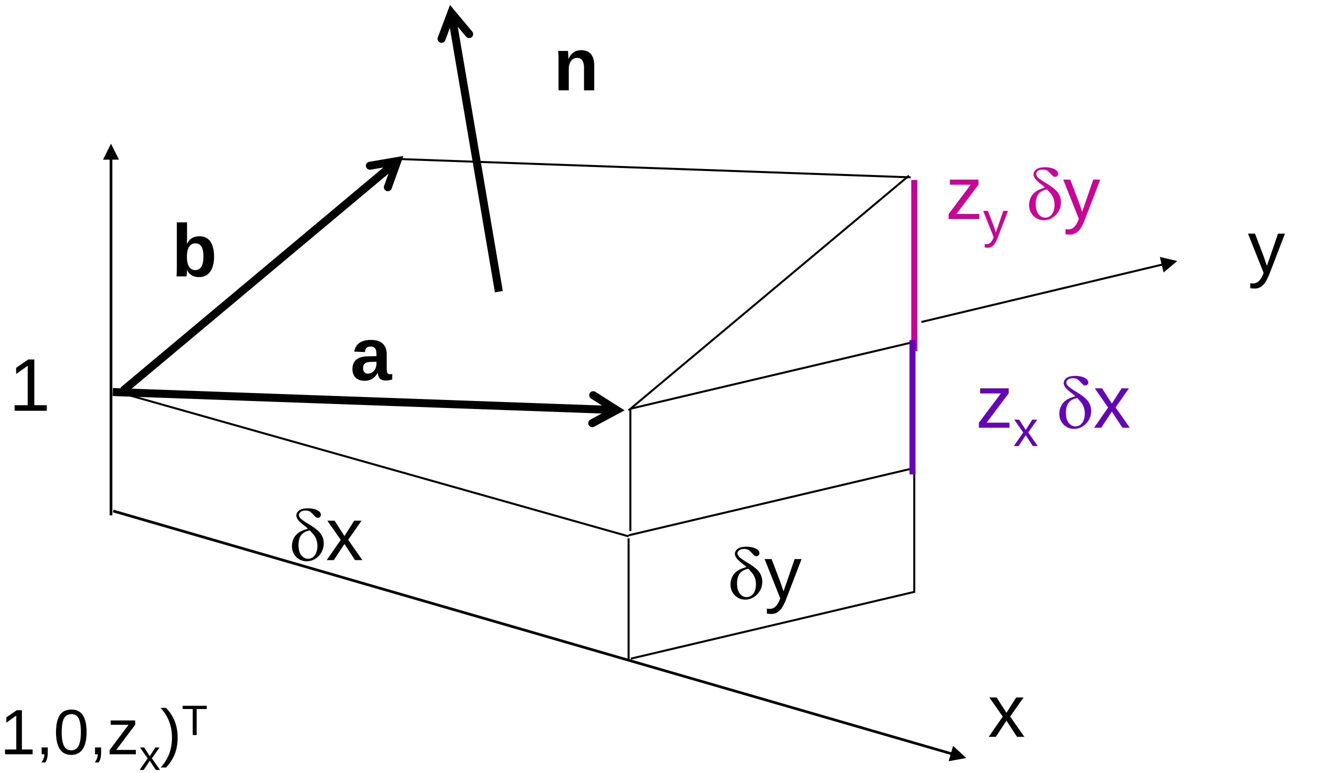
Lambertian reflectance model popular in graphics

-- but something is wrong!

Surface Orientation



Surface Orientation



Surface Gradient $(z_x, z_y)^T$

$\mathbf{a} = (\delta x,\ 0,\ z_x\delta x)^T = \delta x\ (1,0,z_x)^T$

$\mathbf{b} = (0,\ \delta y,\ z_y\delta y)^T = \delta y\ (0,1,z_y)^T$

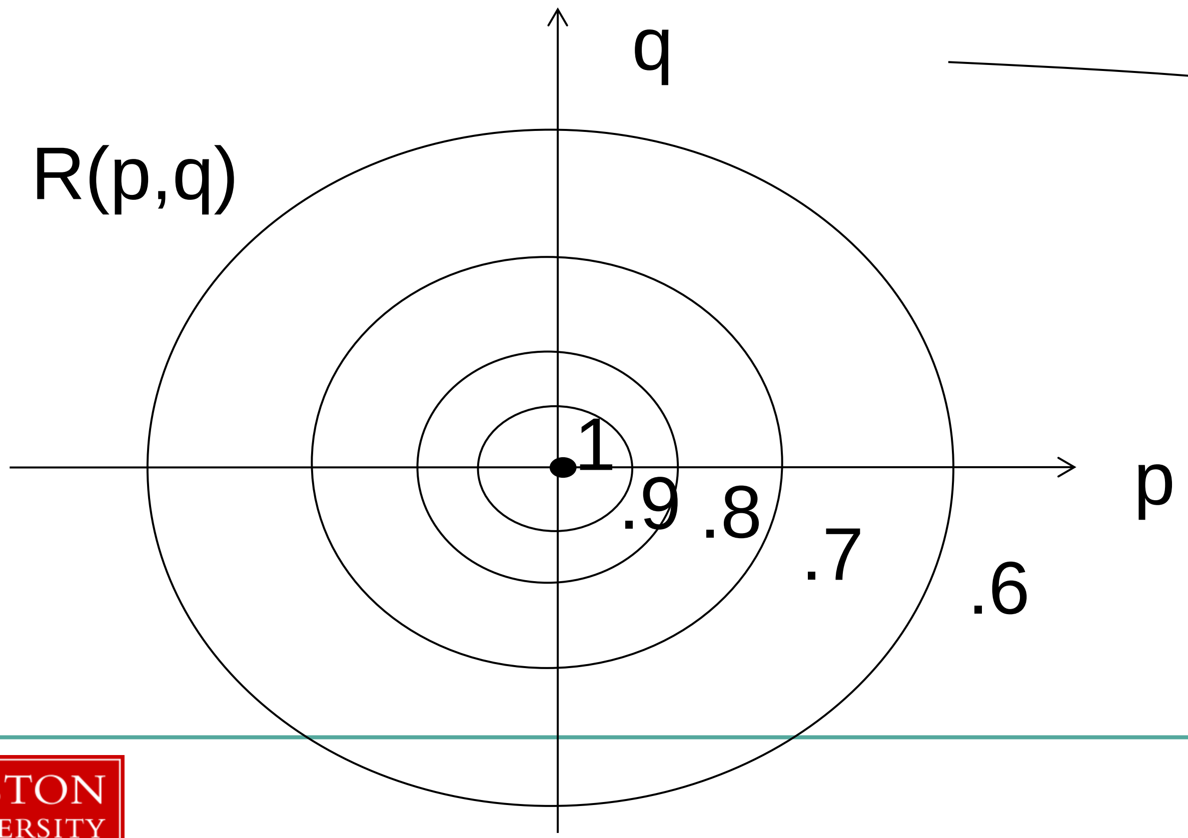
$\mathbf{n} \parallel (\mathbf{a} \times \mathbf{b})$

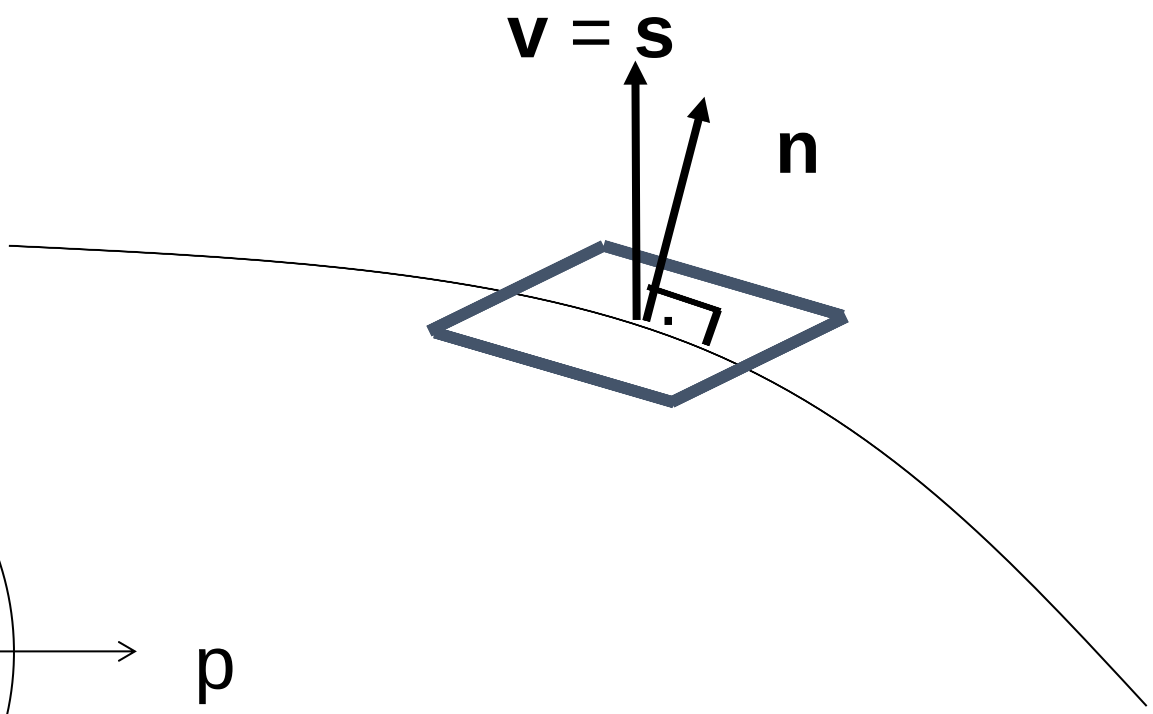
$\mathbf{n} = (-z_x,\ -z_y, 1)^T = (-p,-q,1)^T$

Reflectance Map of Matte Surface

Ideal Lambertian surface looks equally bright from all directions

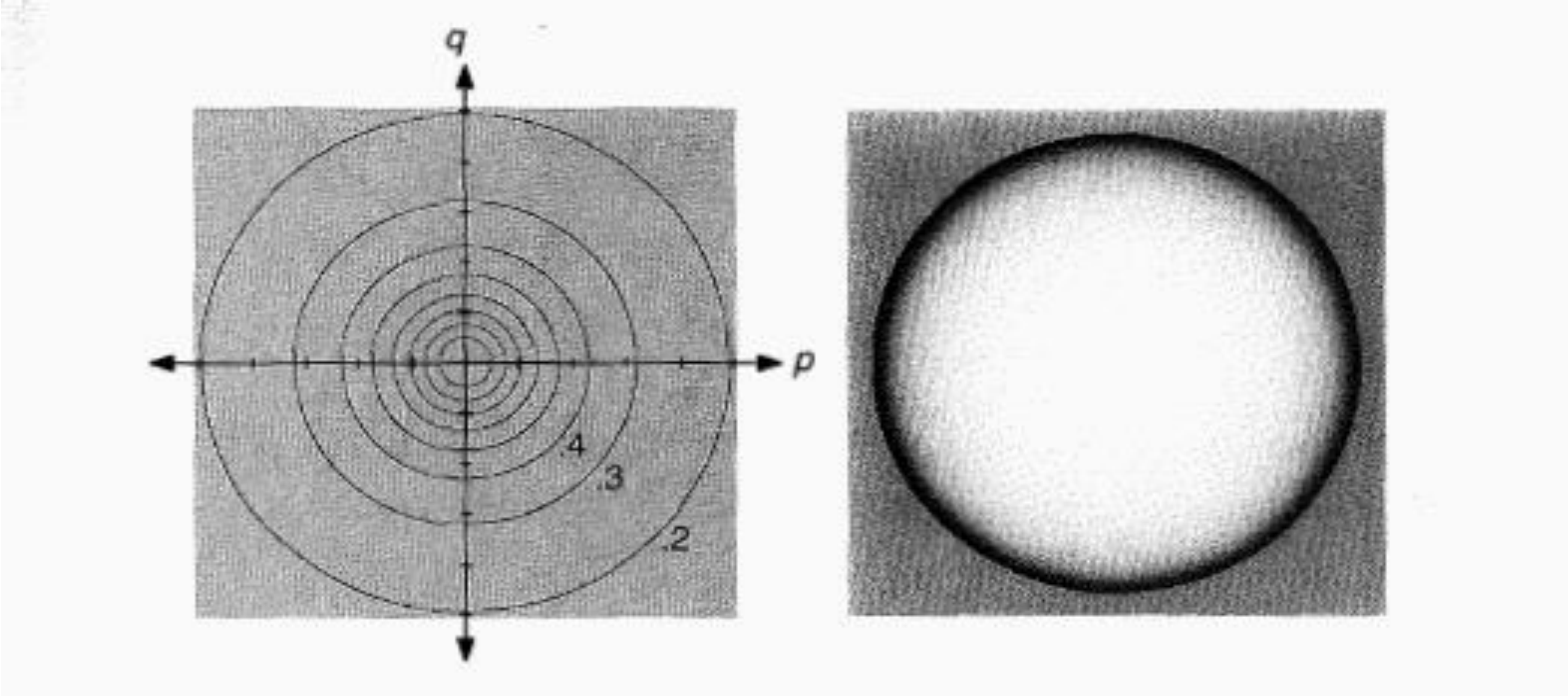
$\cos\theta$ = **n.s**





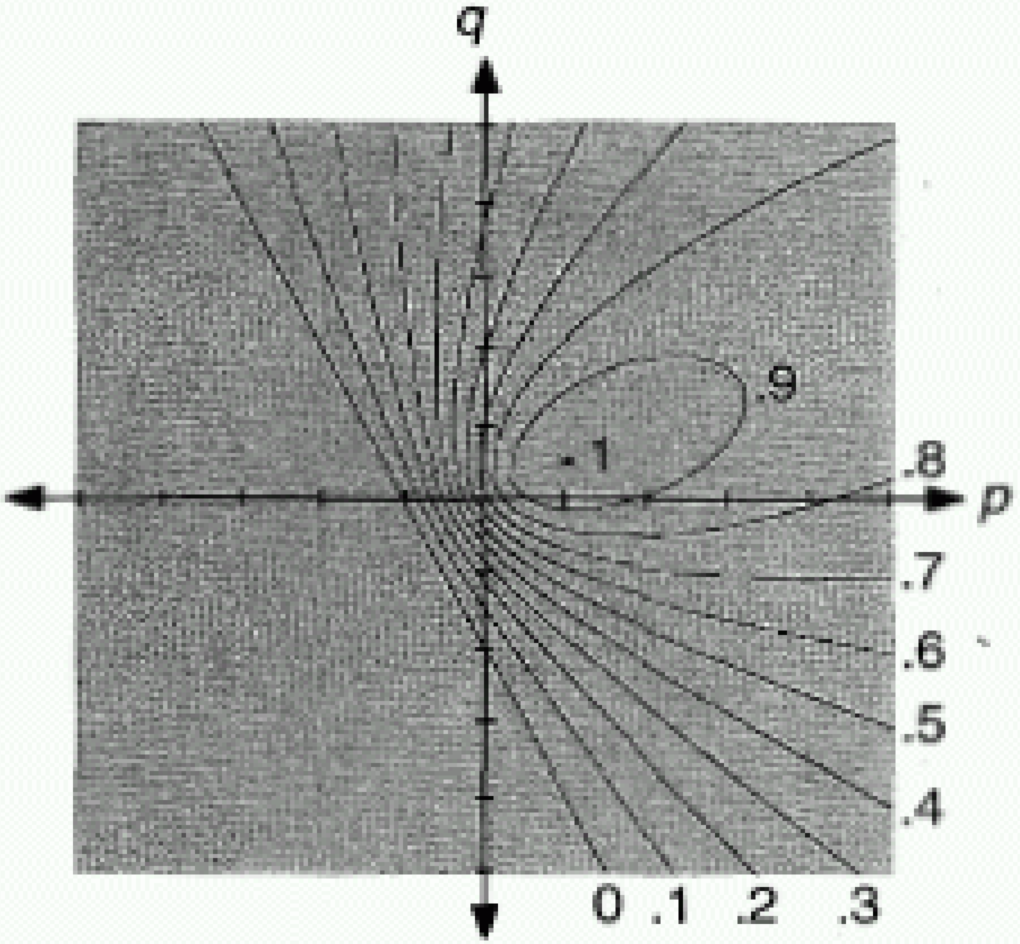
Example:
light source near viewer

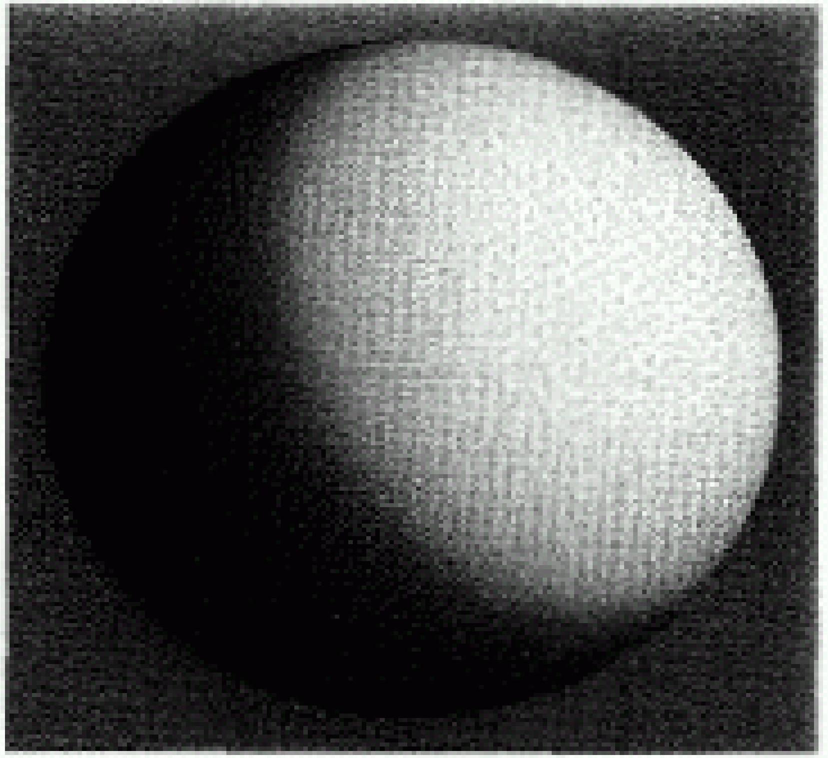
R(p,q) of Lambertian Surface



Nalwa '93

R(p,q) of Lambertian Surface





Nalwa '93

Reflectance Map

Two different projections can create maps of the surface gradients on "Gaussian" (or unit) sphere:

Stereographic plane:

- Whole sphere is projected
- Includes occluding boundary of sphere

Reflectance map:

- Upper hemisphere of sphere is projected
- Isobrightness lines extend to infinity

Photometric Stereo

Goal: Given images E_1 and E_2 under 2 lighting conditions (p_1,q_1) and (p_2,q_2), find surface orientation $\mathbf{n} = (-p,-q,1)^T$, i.e., find p & q.

2 nonlinear equations:

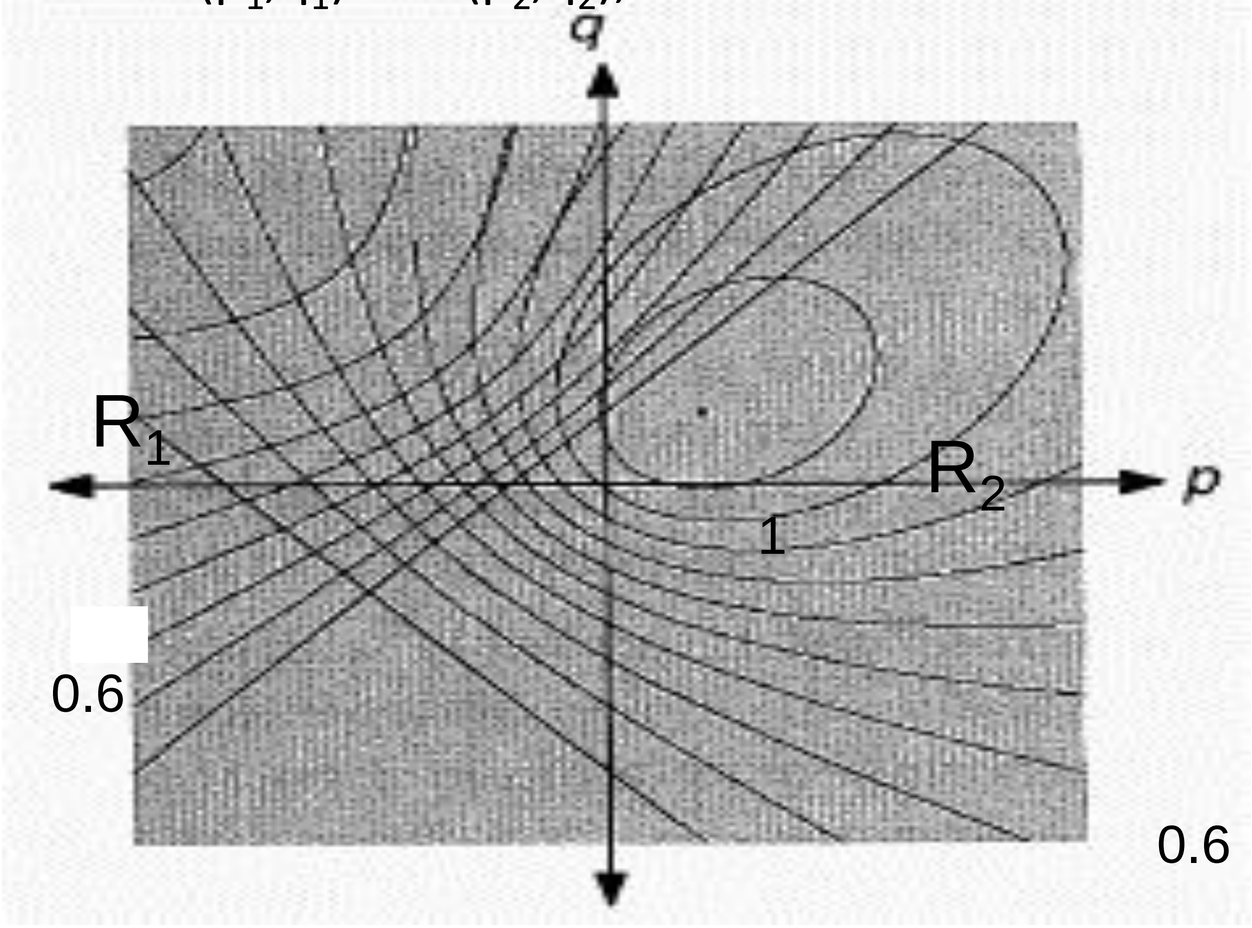
$E_1 = R_1(p,q)$

$E_2 = R_2(p,q)$

If $(p_1,q_1) = (p_2,q_2)$
  infinite number of solutions

else 0, 1, or 2 solution(s)

Better, use *N* images &
  least-squares method



Mars

Viking
Lander I
1977

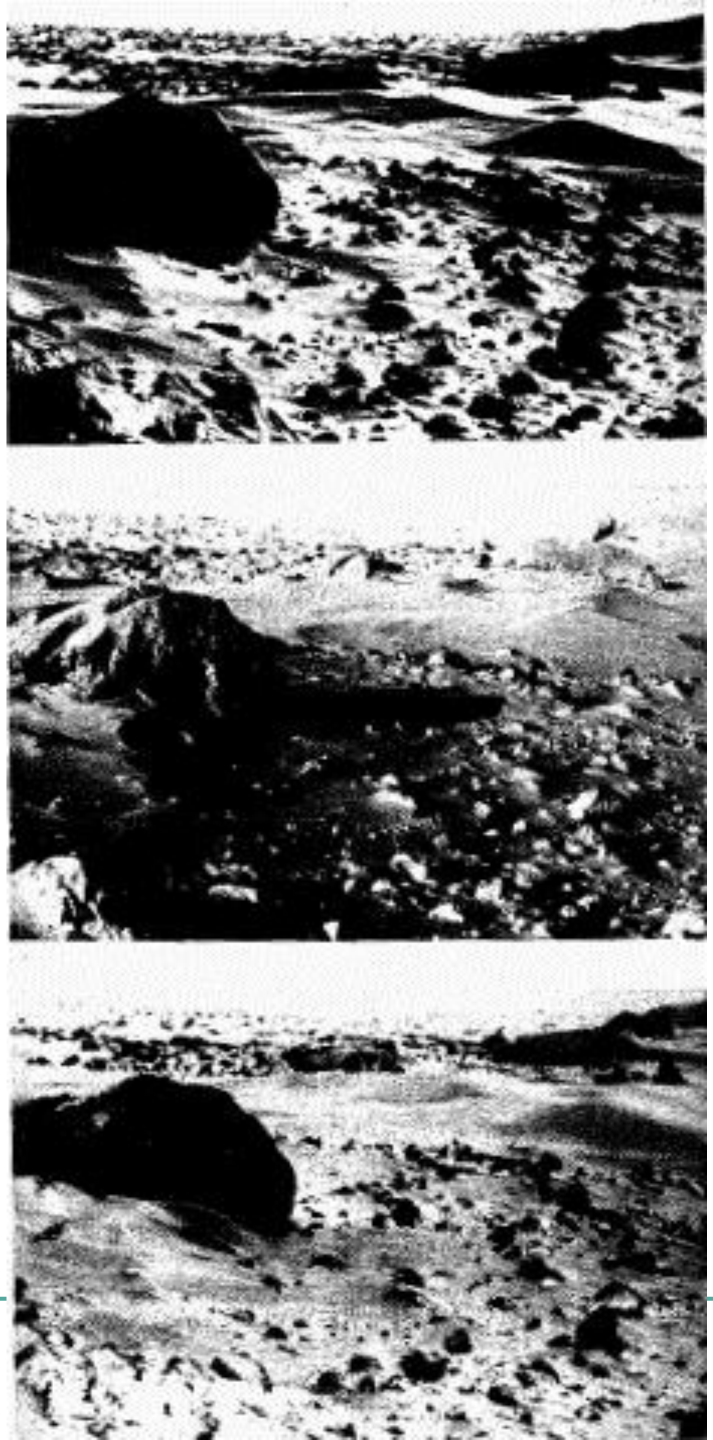
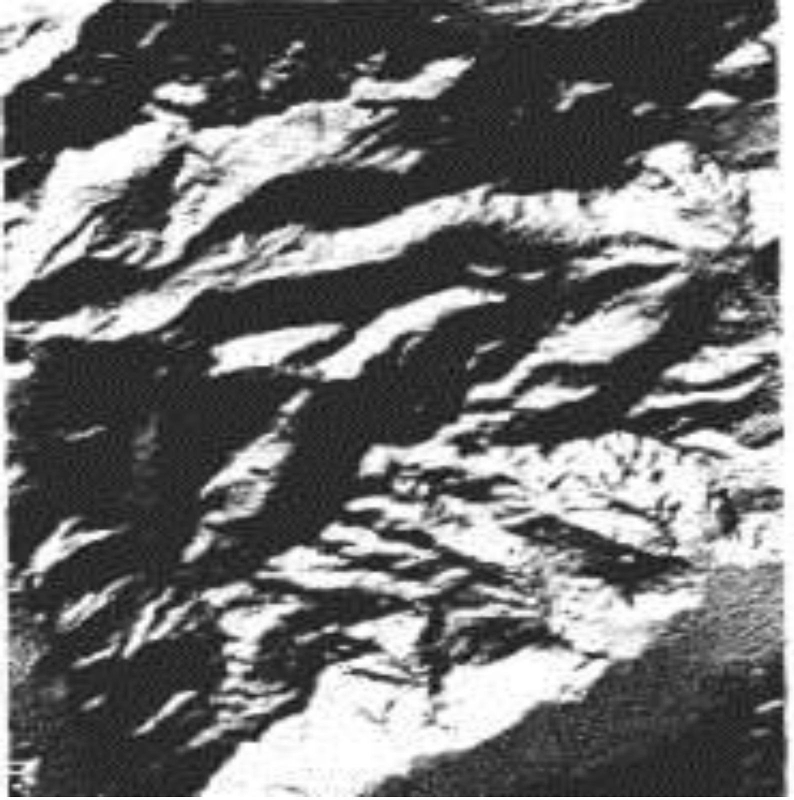
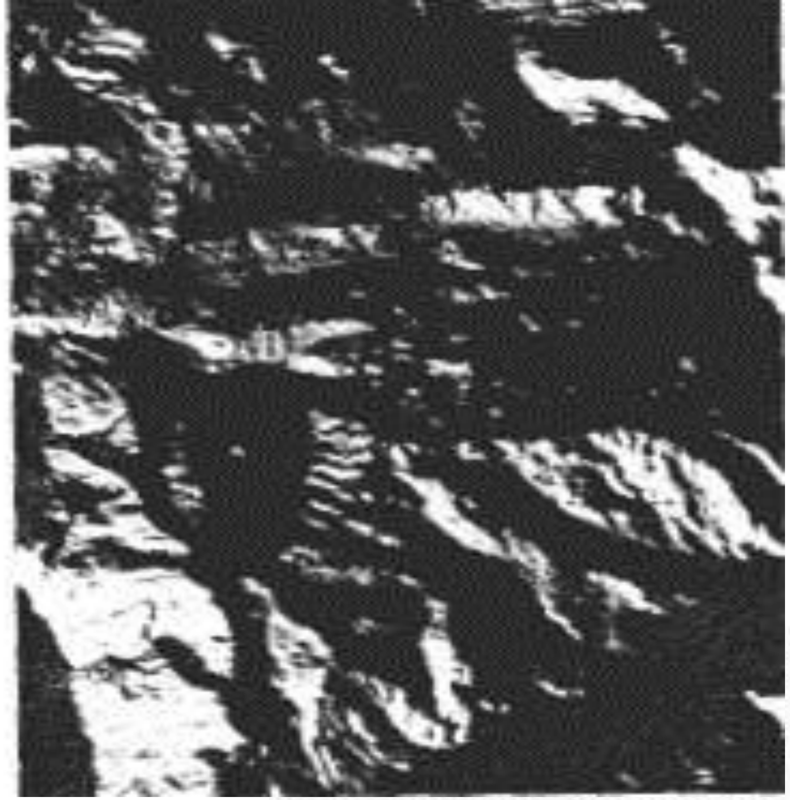


Image Credit: Horn





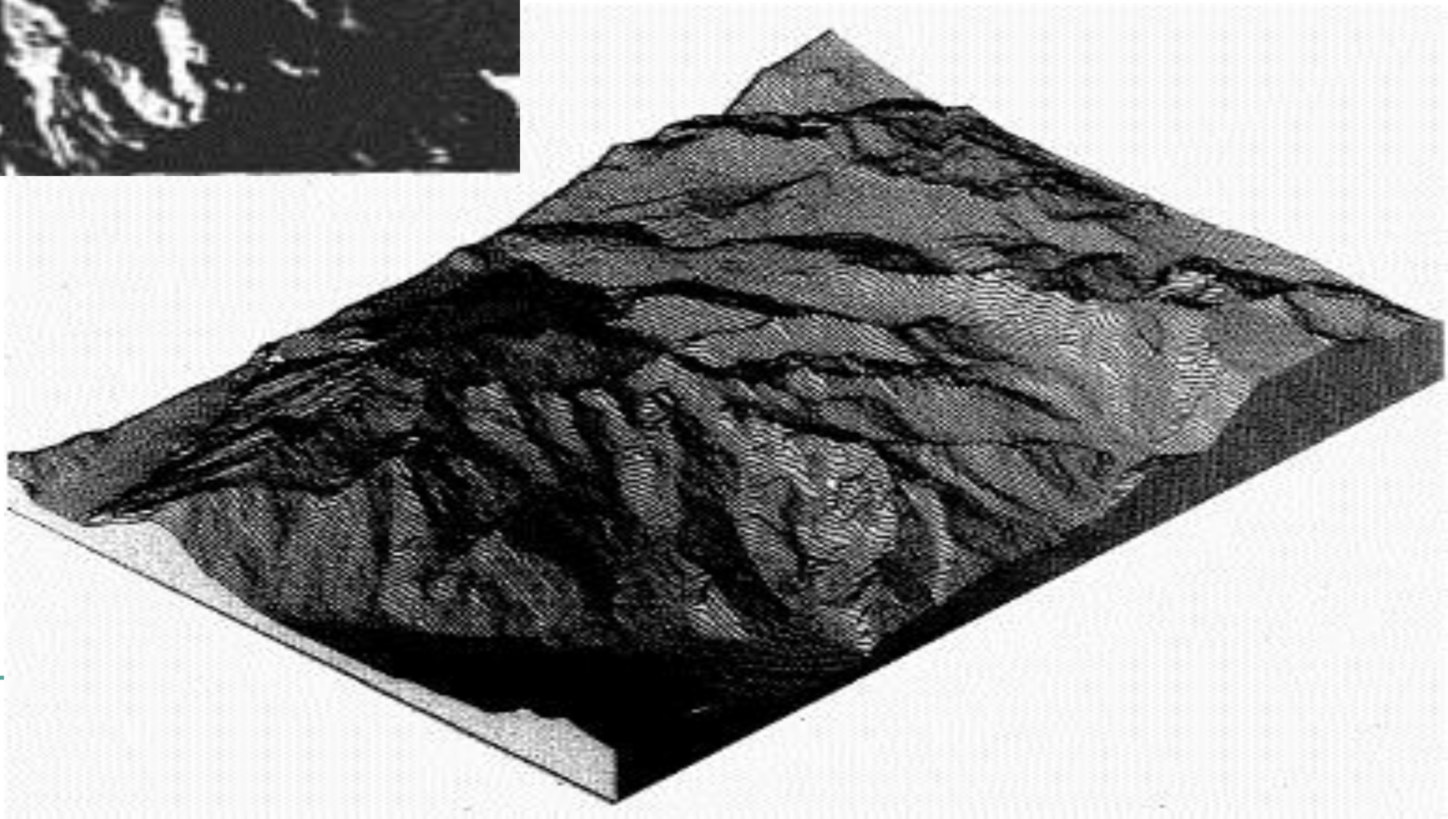


Image Credit: Horn

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