University of Colorado Boulder

ASEN 3200: Structures

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Lab 1: Attitude Sensors and Actuators

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Contents

This paper provides an analysis on how attitude sensors and actuators can be used to measure angular velocity, position, and momentum. Moreover this paper will provide a qualitative analysis on each type of attitude sensor and actuator which includes the the MEMS (Micro-Electromechanical System) Rate Gyro, Reaction Wheel actuator, CMG (Control Moment Gyro) and a Physical Rate Gyro. By plotting the data points collected, a mean K value of -1.1425 with a standard deviation of 0.0032 for the MEMS Rate Gyro was determined. The mean Moment of Inertia and standard deviation of the Reaction Wheel component with the Spin Module was computed to be 0.2266 and 0.043 kg$\cdot m^2$ respectively. For the Control Moment and Physical Rate Gyro, after qualitative analysis, it was determined that Conservation of Momentum holds true. Through this lab, the group members were able to gain a better understanding of how significant stabilization is in spacecrafts/satellites and how attitude sensors and actuators can assist with this process.

1 Preliminary Questions

1.1 Rate Gyro Calculations

To calculate the spacecraft's angular position from these measurements, first multiply the measurements by the sensitivity factor K. Then, multiplying the time change value with the rate measurement at one time point and then adding it to the position at the previous time step, one can find the position at the next time step. This can be repeated for the entire rate data set.

1.2 Angular Acceleration

To prevent the spacecraft from tumbling, the reaction wheel would need to produce an angular acceleration of:

$$M_d = I\dot{\omega} * \alpha \ \alpha = M_d/(I_\omega)$$

This will help the spacecraft remain stable and compensates for the drag disturbance. The required angular acceleration of the reaction wheel to compensate for the drag disturbance would be zero, as the spacecraft experiences constant disturbance torque, meaning that the force does not change. All the reaction wheel would have to do once it is spun up is counteract the forces of friction slowing it down. This of course is assuming that the reaction wheel is spun up to the proper speed to counteract the constant disturbance torque.

1.3 Physical Gyroscope

As the gyroscope is spun around on different axes, the holder will feel torques acting in different directions, depending on the way the gyroscope is oriented. If we imagine that we are holding a spinning gyroscope, we think we would feel it rotating against my hand as we rotate it. This is because gyroscopes force themselves to rotate the object to maintain the orientation of its rotation, or its angular velocity vector. Torque only occurs when rotating across the one perpendicular to the spin axis.

1.4 Wheel and Stool

The conservation of angular momentum dictates the movement of the wheel and stool. When starting out, the total angular momentum of the system is zero when the wheel is vertical, however when the wheel is turned it introduces momentum into the system. In order to compensate for this, the stool begins to spin in the opposing direction. As such, the wheel and its spin axis can be tilted 90 degrees to the right when holding it from the perspective of the person on the stool. The wheel will be spinning clockwise when looked at from above, so the stool will begin spinning left (counterclockwise) in order to compensate.

2 MEMS Rate Gyro Characterization

2.1 Block Diagram

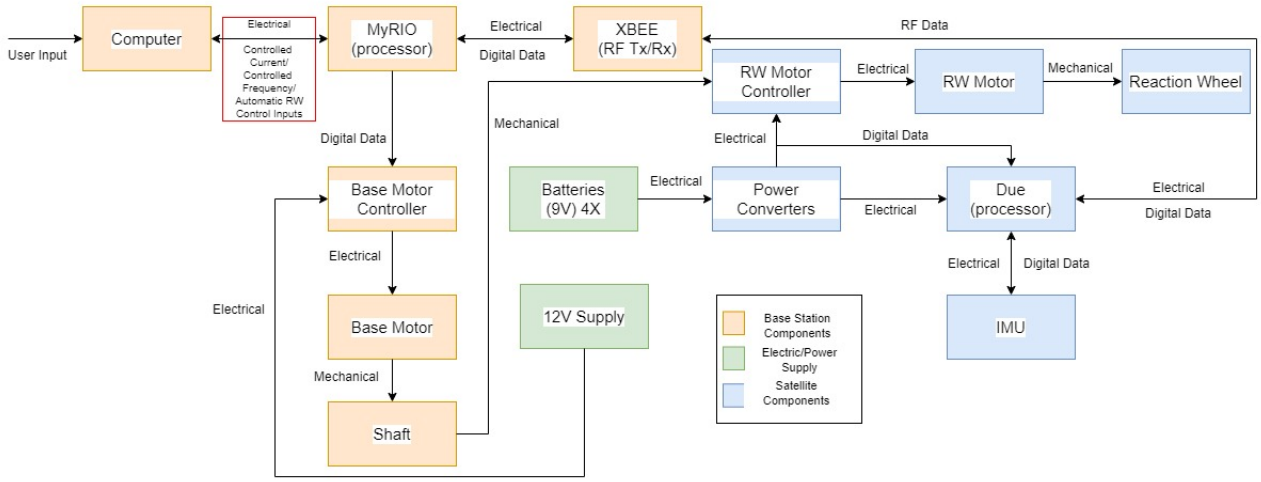

Figure 1: Functional Block Diagram Reaction Wheel

2.2 Theory

By graphing the input and output data sets against each other, a line of best fit can be determined from them with the form:

$$y = mx + b \tag{1}$$

Here, the slope coefficient m represents the sensitivity factor K, while the y-intercept b is the bias number. The output data can then be multiplied by K and the bias number added to it to find the calibrated values. This data can then be applied to the equation:

$$\theta_2 = \theta_1 + \omega * \Delta t \tag{2}$$

By taking a previous position point (starting from zero) and then multiplying the rate point by the timestep, we can find the next position point. This process repeats over the entire timespan.

2.3 Results

2.3.1 Data for Frequency = 0.2, Current = 0.5

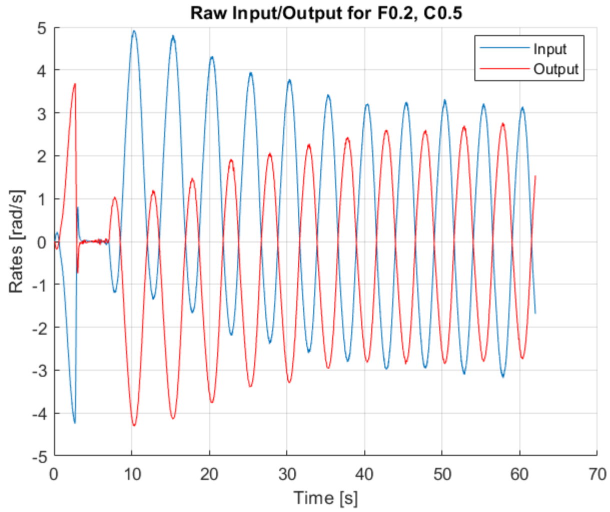

Figure 2: Raw Input/Output Data - F0.2, C0.5

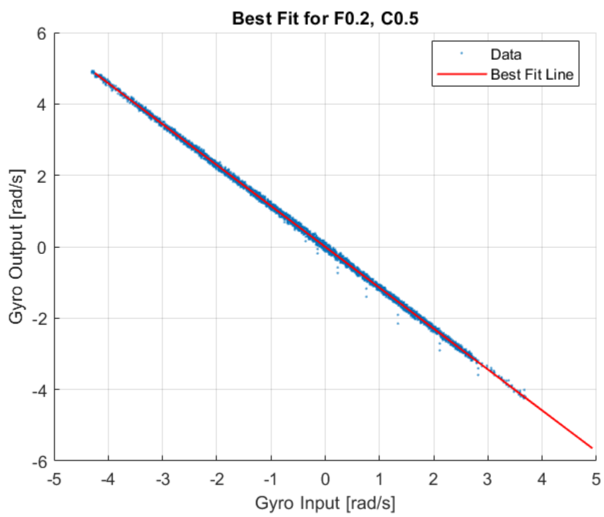

Figure 3: Best Fit Data - F0.2, C0.5

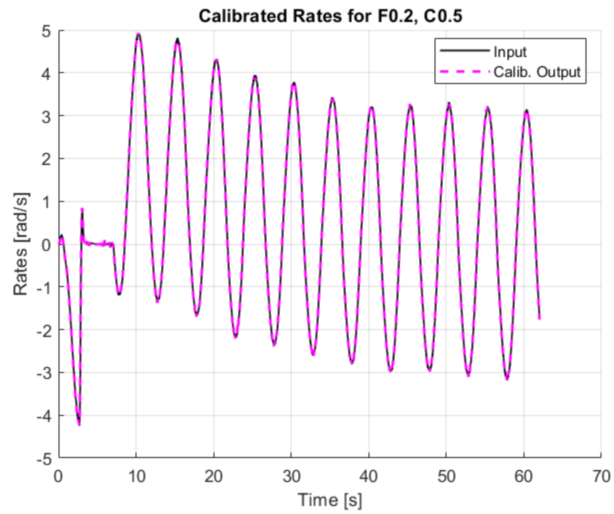

Figure 4: Calibrated Output Data - F0.2, C0.5

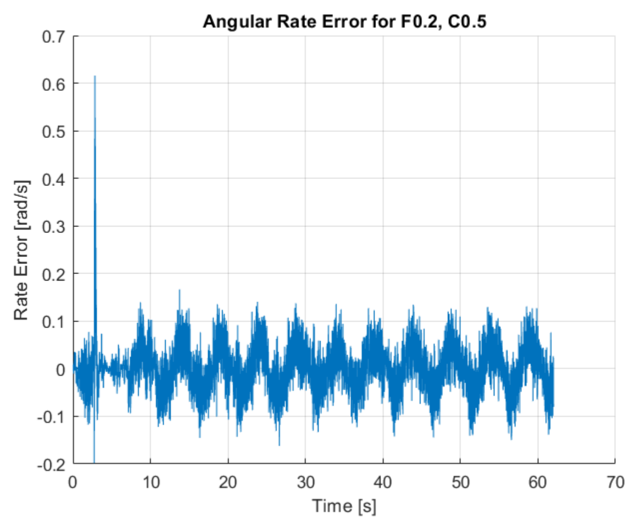

Figure 5: Angular Rate Measurement Error - F0.2, C0.5

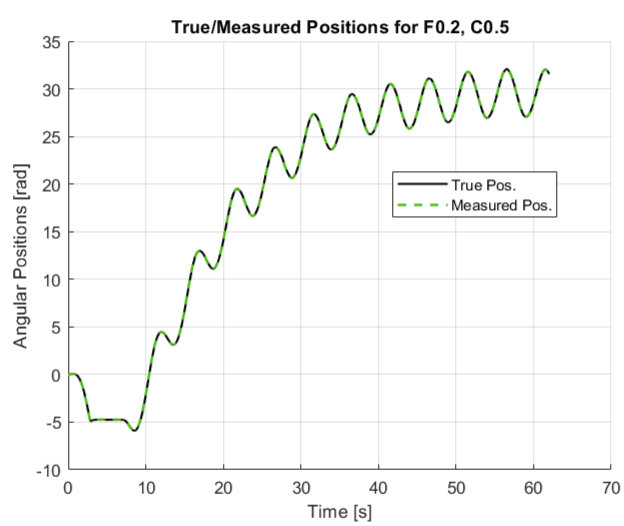

Figure 6: True and Measured Position Data - F0.2, C0.5

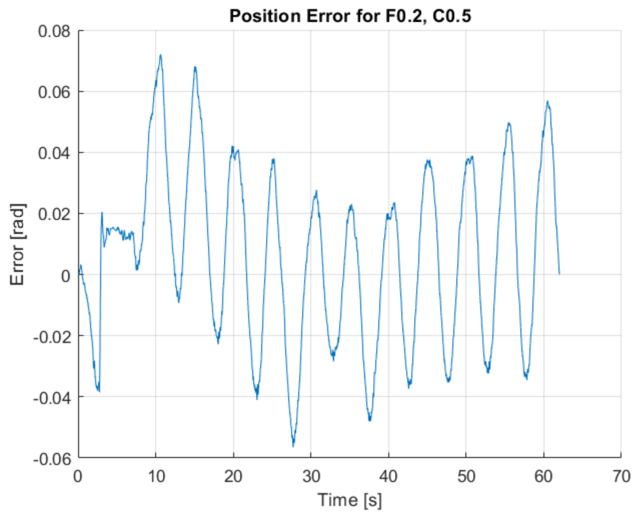

Figure 7: Angular Position Error - F0.2, C0.5

2.3.2 Data for Frequency = 0.4, Current = 0.75

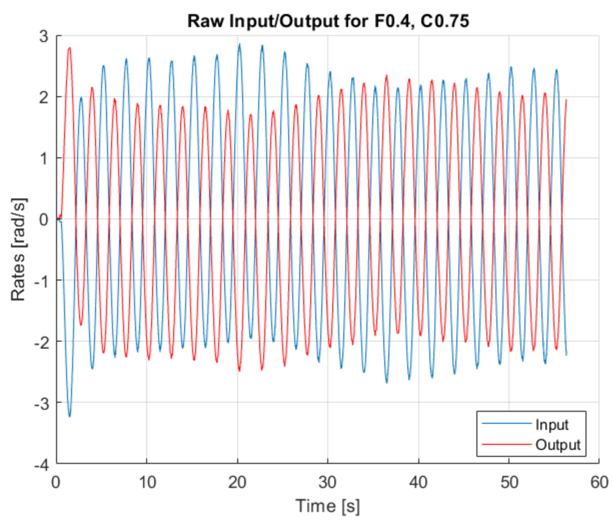

Figure 8: Raw Input/Output Data - F0.4, C0.75

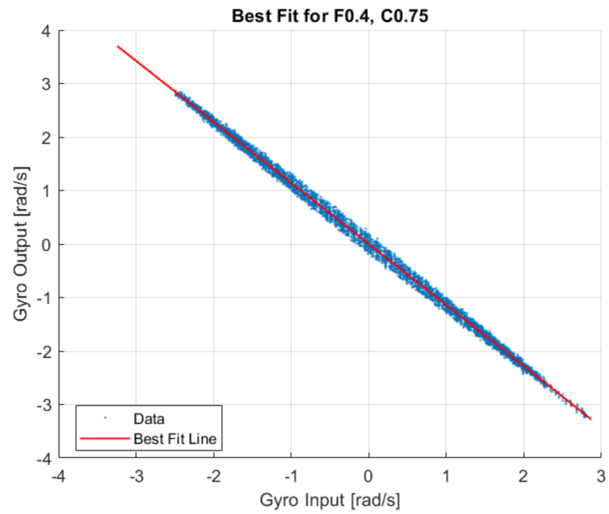

Figure 9: Best Fit Data - F0.4, C0.75

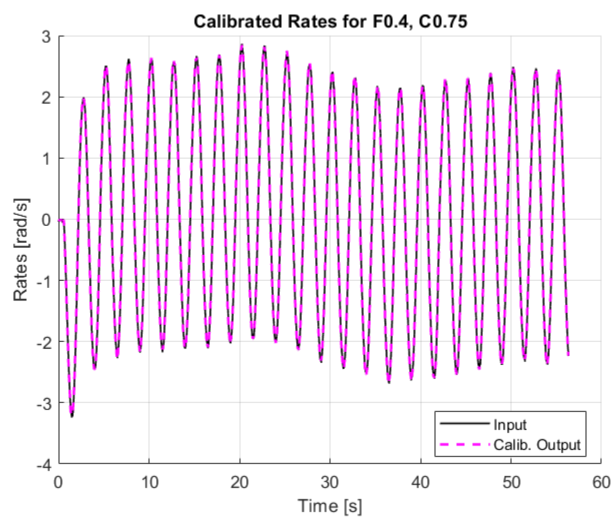

Figure 10: Calibrated Output Data - F0.4, C0.75

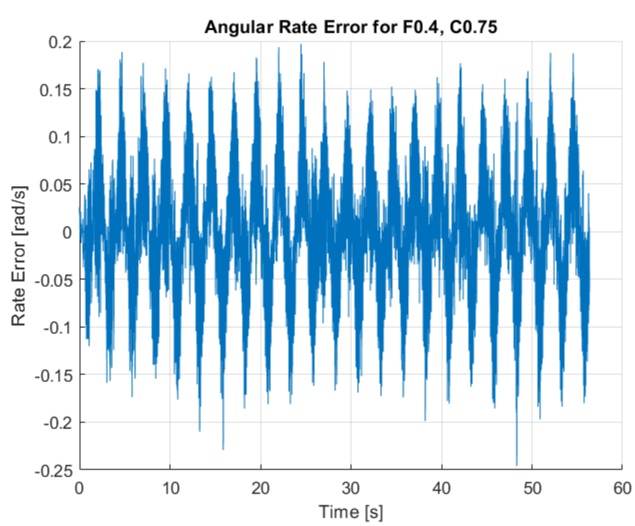

Figure 11: Angular Rate Measurement Error - F0.4, C0.75

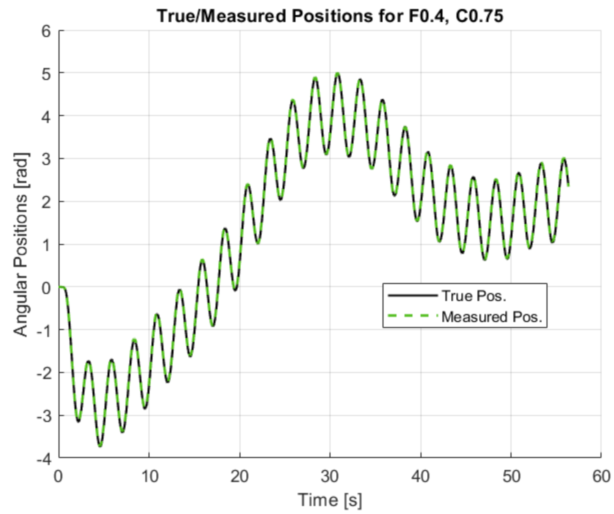

Figure 12: True and Measured Position Data - F0.4, C0.75

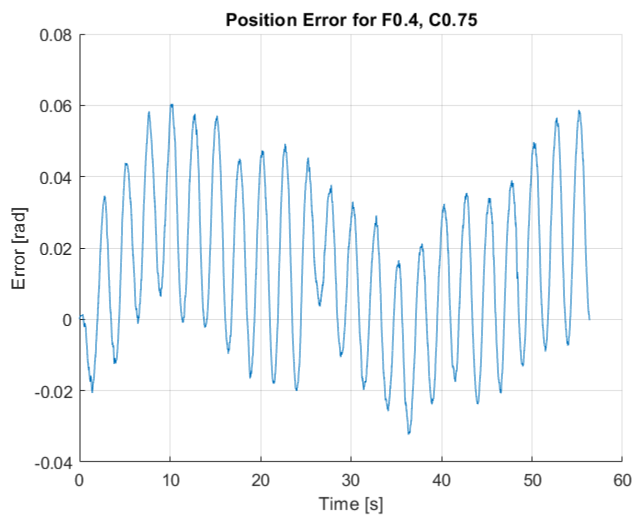

Figure 13: Angular Position Error - F0.4, C0.75

2.3.3 Data for Frequency = 0.5, Current = 0.3

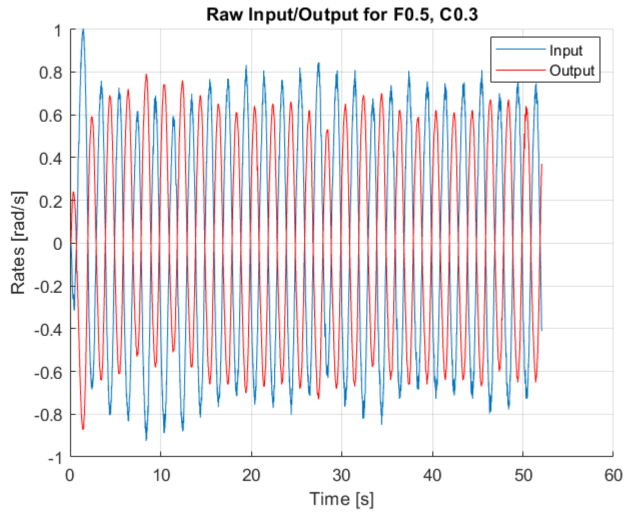

Figure 14: Raw Input/Output Data - F0.5, C0.3

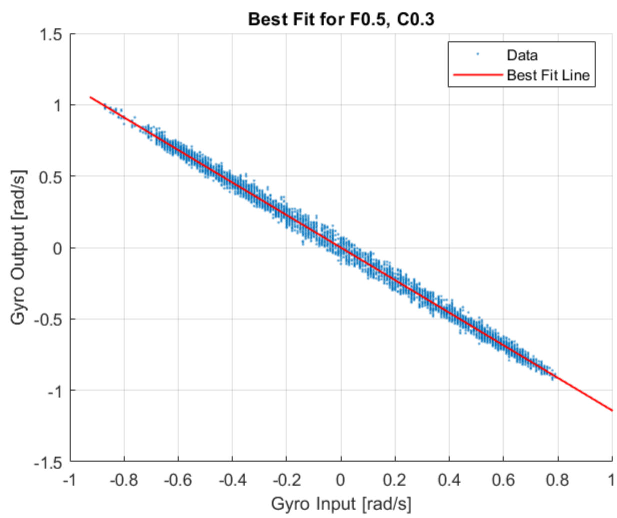

Figure 15: Best Fit Data - F0.5, C0.3

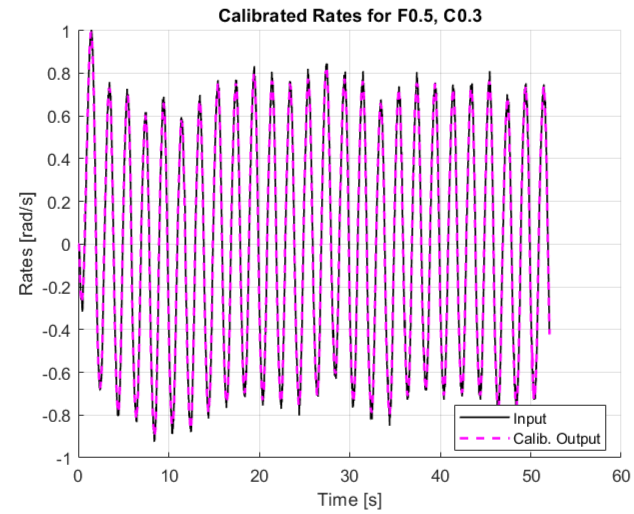

Figure 16: Calibrated Output Data - F0.5, C0.3

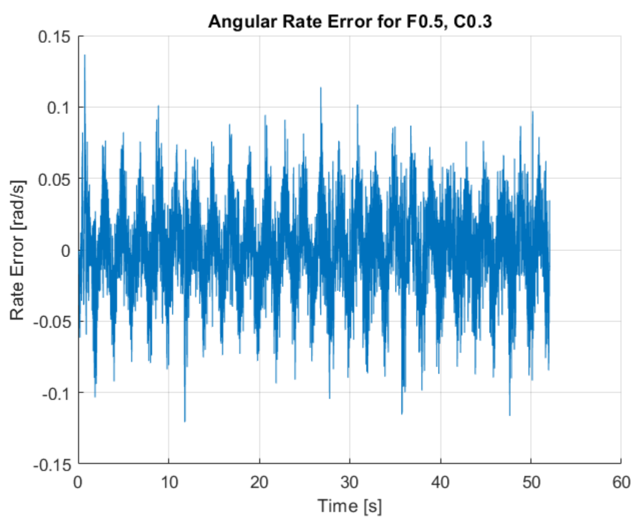

Figure 17: Angular Rate Measurement Error - F0.5, C0.3

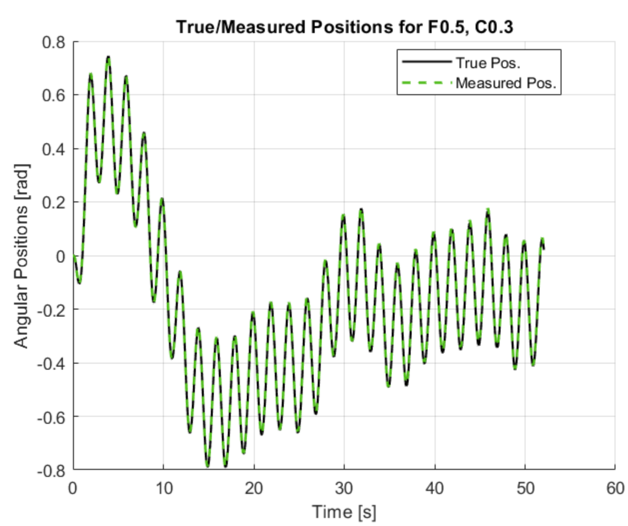

Figure 18: True and Measured Position Data - F0.5, C0.3

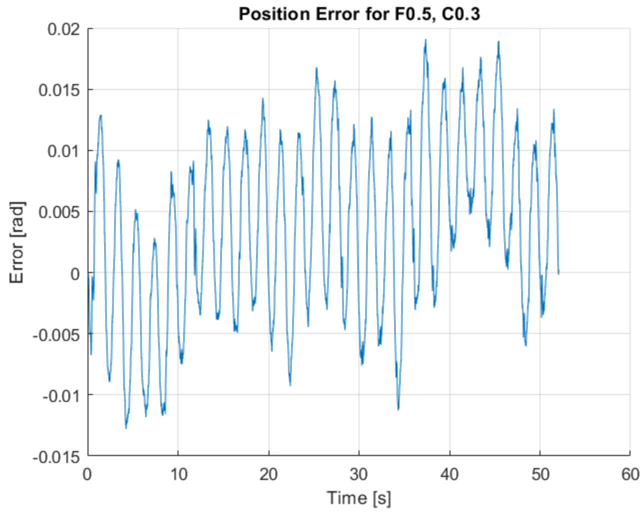

Figure 19: Angular Position Error - F0.5, C0.3

2.3.4 Key Results - Mean and Standard Deviation

	Bias	Sensitivity (K)
Trial 1	-0.0027	-1.1460
Trial 2	-0.0030	-1.1417
Trial 3	-0.0016	-1.1397

Figure 20: Table of Bias and Sensitivity Values

	Mean	Standard Deviation
Bias	-0.0023	9.245e-4
Sensitivity (K)	-1.1425	0.0032

Figure 21: Table of Mean and Standard Deviation Values

All three trials of data were used when determining the mean and standard deviation in order to maximize the accuracy of these values. Some of the data was cut at the beginning of each set, as it was recorded when the gyroscope was stationary and thus not yielding anything useful.

The group was able to determine the sensitivity quite accurately, only being off by a factor of about -1.15. Based on the error data, this value of K could be useful for values less than 0.2 radians per second in either direction. The gyro bias is easily repeatable, as it is highly consistent (a fact reinforced by a tiny standard deviation) and therefore important. This means that the effects of the bias are easily predictable and can be taken into account for future experiments.

The position estimates lined up well with the known position data, as the error between the two stays under 0.08 radians for each trial. This means that the sensors measure the data accurately, but not perfectly. The differences between the two data sets is likely due a number a factors, such as the shift of wheel changing direction throwing the data collection off as the sensors try to keep up. This cause is supported by the manual data tests, as observations of this data collection demonstrated that gyro became less accurate at tracking when undergoing higher frequencies and more forceful shifts in direction. Although this trend was not consistent within the automatic data (the error for frequency of 0.5 was less than those of the two previous sets with smaller values), it's likely due to the decreased current number.

There are many possible sources of error within this experiment. Human involvement could be a significant factor, including reasons such as improper operation of the assembly, interference with data collection (like touching the machine while running), and others. There could also be defective or faulty components within the machine that could impact the experiments, although if any existed they were not impactful enough to be seen in the data.

3 Reaction Wheel Characterization

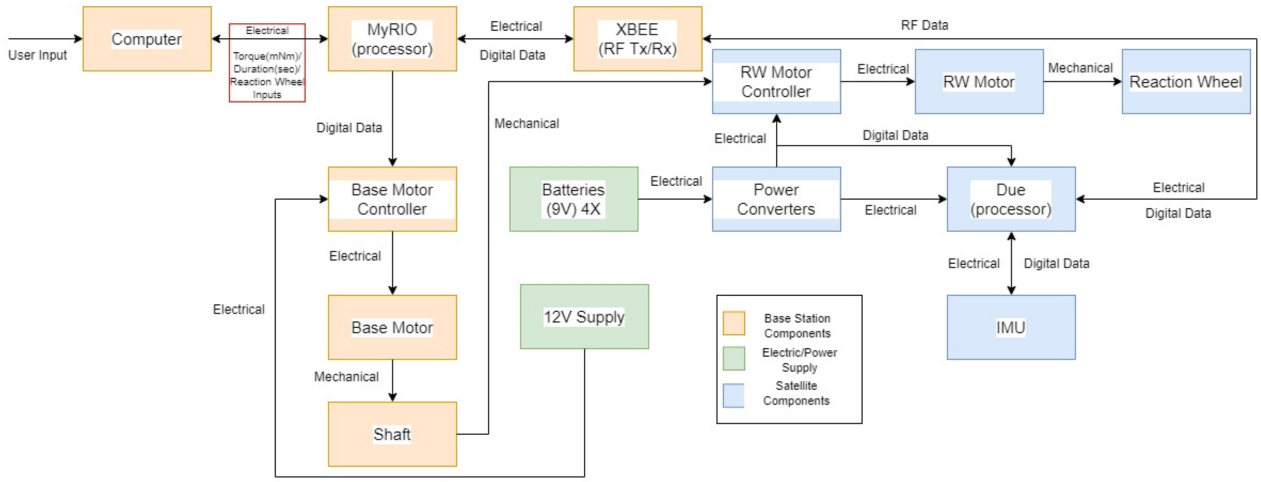

Figure 22: Functional Block Diagram Reaction Wheel

3.1 Theory

Knowing the product of the MOI matrix and angular velocity vector is equal to the angular momentum for a rigid body, the next step is to expand and take a derivative. This derivative gives the timed rate of change of the angular momentum, which is equal to the applied torque. Since the user decides what torque to apply, the equation can be rearranged to then solve for the moment of inertia since it can be obtained through the angular acceleration by plotting a line of best fit along the angular velocity, as measured by the sensors built into the system.

3.2 Analysis and Results

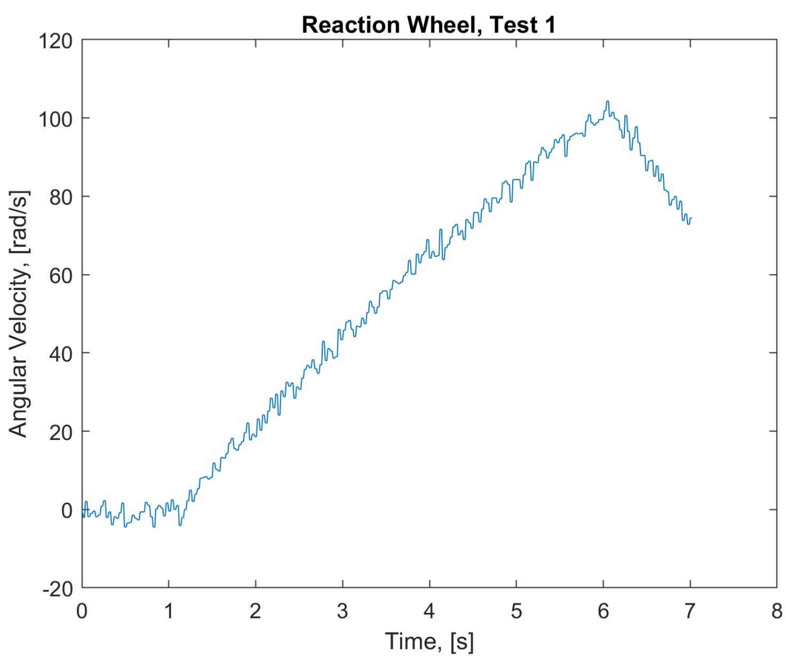

Figure 23: Reaction Wheel Test 1

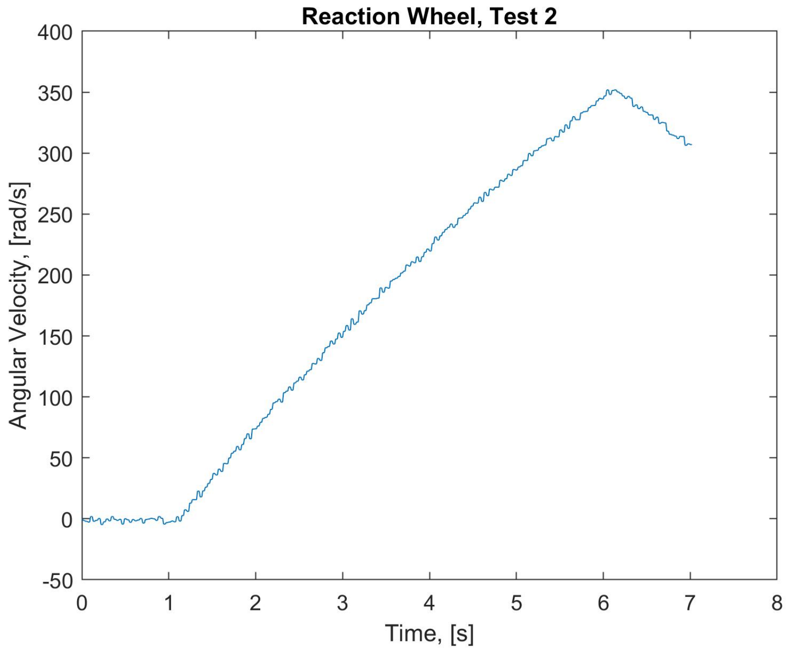

Figure 24: Reaction Wheel Test 2

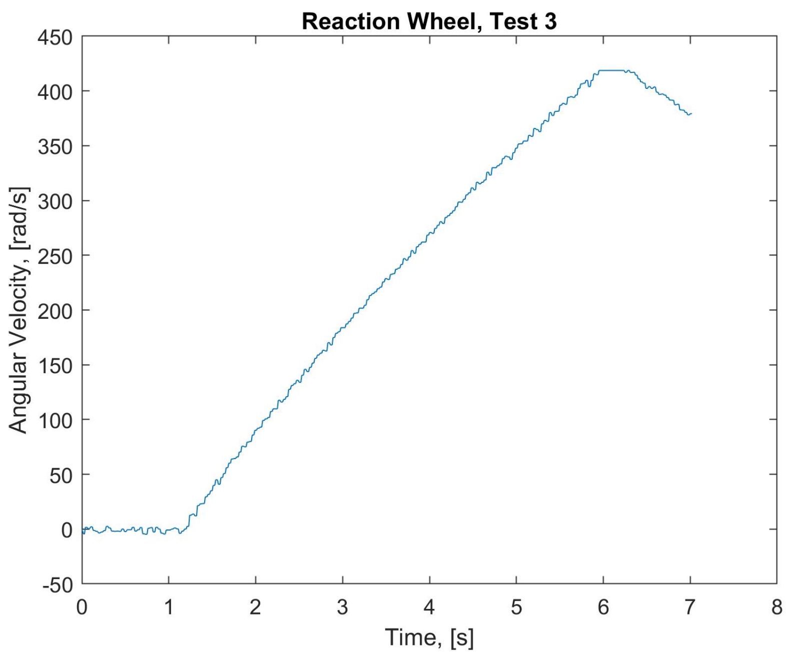

Figure 25: Reaction Wheel Test 3

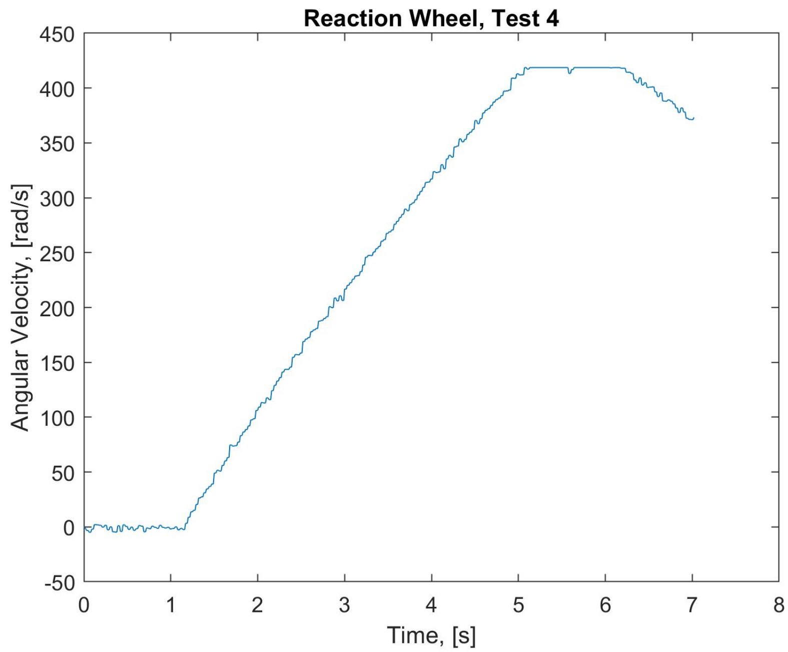

Figure 26: Reaction Wheel Test 4

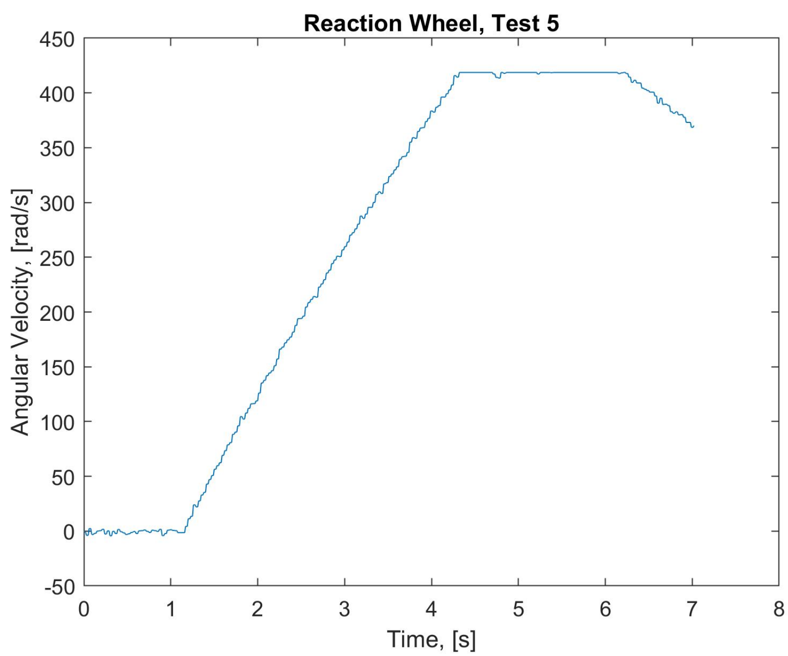

Figure 27: Reaction Wheel Test 5

Test Number	Torque Input [mNm]	Average Angular Acceleration [rad/s^2]	Average MOI [kgm^2]
Test 1	5	16.85	0.2967
Test 2	12	60.47	0.1984
Test 3	14	73.69	0.1899
Test 4	16	75.96	0.2106
Test 5	18	75.75	0.2376

Average MOI for all torques	Standard Deviation
0.2266	0.043

Initially, the moment of inertia would stay fixed, regardless of applied torques. In reality, the MOI actually changes, more specifically as you increase the applied torque, the MOI begins to become smaller. Looking at the equations, this makes sense as increasing the torque also increases the average angular acceleration. The MOI is inversely proportional to angular acceleration, so it will go down as the applied torques increase. However, because tests 4 and 5 contradict this as the MOI seems to increase, there must be some sort of error in our measurements. This error could possibly be traced to the fact that someone had to hold the spin module still as we were running the tests, and any sort of deviation from doing that could change the results the sensors were measuring.

Using the results and Matlab to answer the questions, it was found that the angular momentum capacity was 94.89 [Nm/s] and the time until the wheel exceeds the 4000 RPM limit was around 852 seconds.

4 Control Moment Gyro Experiment

Newton's Third Law plays an important role when conducting this experiment. For every action there is an equal and opposite reaction. When we spin the wheel, our group member stays in place and does not rotate. As soon as the person flips over the wheel, it results in the person to start spinning on the swivel board. When the member is off the swivel board, the same application is being applied but due to there being friction between the person's feet and ground, there is no rotation that occurs.

In terms of Conservation of Angular Momentum, the system can be set using the term L. When the wheel is spinning clockwise, the whole system including the wheel, the group member and the swivel board start at angular momentum of -L. When the wheel is flipped over, the angular momentum of the spinning wheel is +L and the group member starts to spin in the clockwise direction. Due to Conservation of Angular Momentum the system needs to hold an angular momentum of -L meaning the group member and the swivel board produce an angular momentum of -2L. Therefore the system which combines the angular momentum of the wheel and group member plus the swivel board result in an angular momentum of -L (+L(Spinning Wheel) - 2L (group member and swivel board)).

Some general observations through the experiment are that after each flip, the spin was in the direction of the original spin of the wheel. When you flip back the wheel stopped spinning. It was also difficult to keep it stable so when flipping the wheel, the ending point was slightly different every time, therefore was not a perfect system but overall the Conservation of Angular Momentum holds true.

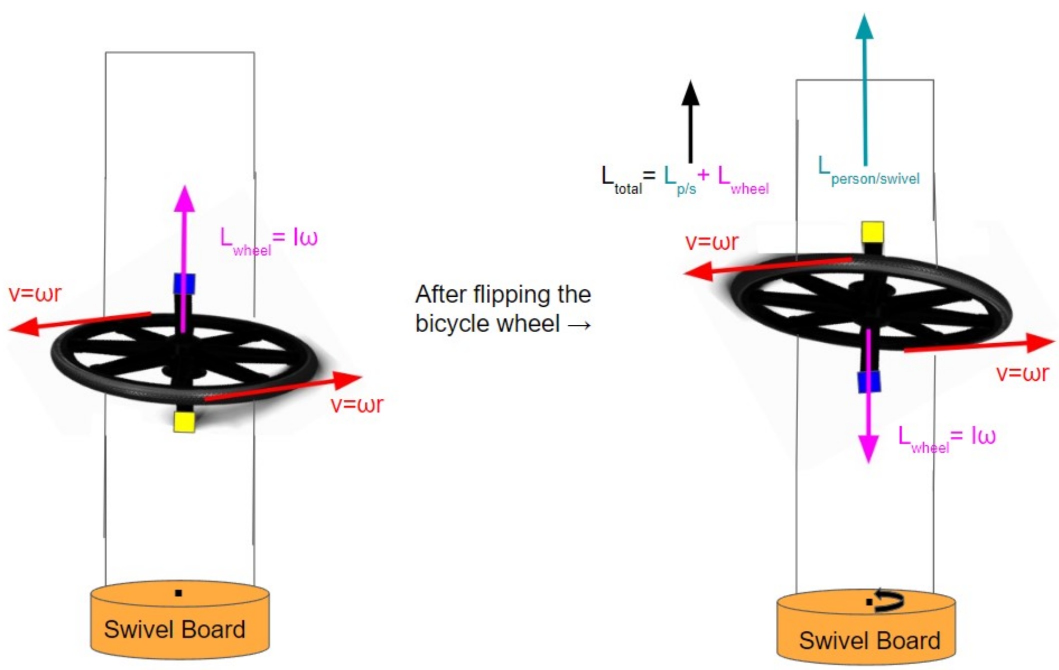

Figure: Control Moment Gyro Momentum Vector Free Body Diagram

5 Physical Rate Gyro

The Physical Rate Gyro was a small hollow box with a spinning wheel assembly inside the box. After turning it on and after the wheel comes up to speed, the experiment of rotating it in different directions began. Some of the collected data was that if you rotated it towards +y then the Rate Gyro would resist your motion, while if you rotate it towards the +x side then nothing would happen. Finally, it was noted if you rotate it towards +z the Rate Gyro reacts by resisting and aiding the motion in a very confusing way, almost as if the Rate Gyro wobbled as the motion continued. The block inside rotated about the +y axis in a counter clockwise direction, at least on the unit we acquired. In order to get a better understanding of how the gyro works, a coordinate frame was decided:

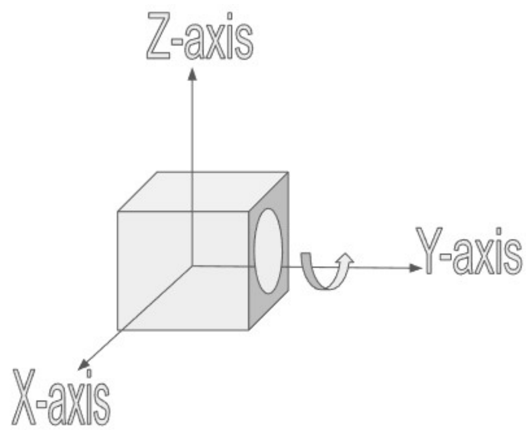

Figure: Coordinated Frame Physical Rate Gyro

This makes sense as total angular momentum within the system has to be conserved, and when there are added forces to the system such as the angular momentum from the hand, the system will resist the movement from other angles and do nothing from others. In terms of Euler kinematics, multiple sequences can be performed. To start off, if a 3-2-1 Euler Sequence is performed, it would yaw about the z-axis, pitch about the y-axis and roll about the x-axis. Then using the angles measured, the rotation matrix can be computed using rotation matrix M1 times M2 times M3. If a 3-1-3 Euler Sequence is performed, the rotation matrix can be similarly calculated using rotation matrix M3 times M1 times M3. Using Euler equations and the respective rotation matrices, the attitude of the object can be determined. Overall this experiment showed that the spinning wheel tried to maintain its direction while the outside box is rotated. This can be extremely useful outside this lab, especially when designing parts that have to maintain their orientation. This application aided the understanding small scale angular momentum. It was interesting seeing the spinner try to rotate to keep its angular momentum when we were testing the rotation from different angles. We learned that some forces can be completely negated through the use of an internal spinner. This of course applies in a lot of ways, such as small satellites keeping their orientation constant. We would recommend increasing the power of the spinner inside to make the different scales of forces and how the spinner itself reacts much more evident to the tester, as the low power made it difficult to determine what is actually happening.

6 Conclusions and Recommendations

Through this experiment, the group members understood the significance of Conservation of Momentum in different modules and how they aid in stabilization of spacecrafts and satellites. This was done by analyzing various different attitude sensors and actuators such as the MEMS Rate Gyro, Reaction Wheel Actuator, Control Moment Gyro and Physical Rate gyro. Overall this lab did a great job helping users investigate how different attitude sensors and actuators work. The main recommendation to help improve this experiment is to do testing using a Physical Rate Gyro with a stronger motor to better understand movements in the box. The Rate Gyro worked by resisting the motion but movements made in the box were inconsiderable at times making it hard to observe what axis it was rotating on. Another recommendation would be showing a few example graphs/plots or a range of what values we should expect, so work can be checked accordingly. In general, the lab was very understandable and interesting.

7 Acknowledgments

7.1 Member Contributions

Deliverable	**Ian Mccarty**	**Rishab Pally**	**Liam Schulz**
Abstract	0%	100%	0%
Preliminary Questions	0%	0%	100%
MEMS Rate Gyro Characterization	0%	0%	100%
Reaction Wheel Characterization	100%	0%	0%
Control Moment Gyro	0%	100%	0%
Physical Rate Gyro	0%	100%	0%
Conclusions/Recommendations	0%	100%	0%
Style and Clarity	33%	33%	33%
TOTAL	**33.3%**	**33.3%**	**33.3%**

Signature of Approval: Ian Mccarty, Rishab Pally, Liam Schulz

8 References

[1] Heidrich, C., Hodgkinson, R., "ASEN 3200 LAB A-1 Attitude Sensors and Actuators"

[2] Heidrich, C., Hodgkinson, R., "Smead AES Labs Spin Modules/ Software"

[3] MITK12Videos. "MIT Physics: Spinning Bike Wheel and Conservation of Angular Momentum." YouTube, 16 Aug. 2017,
https://www.youtube.com/watch?v=iaauRiRX4do.

[4] Hodgkinson, R., Schwartz, T., "An Apparatus for Exploring Small-satellite Estimation and Control", July 26-29, 2021

9 Appendix: Matlab Derivations

Matlab MEMS Rate Gyro and Reaction Wheel Analysis continued on next page

```
%% ASEN 3200 Lab A-1
% Author: Liam Schulz
% Group 17, Section 13: Liam, Rishab, Ian
% 2/3/23

%% Housekeeping
clc; clear; close all

%% Data Intake
% Torque Data
Nm06 = load("2023_02_03_013_UNIT01_TORQ6_05");
%Nm10 = load("2023_02_03_013_UNIT01_TORQ1_10"); data nonexistent
Nm12 = load("2023_02_03_013_UNIT01_TORQ2_12");
Nm14 = load("2023_02_03_013_UNIT01_TORQ3_14");
Nm16 = load("2023_02_03_013_UNIT01_TORQ4_16");
Nm18 = load("2023_02_03_013_UNIT01_TORQ5_18");

% Gyro Data - Manual
manual1 = load("2023_23_01_013_UNIT2_RATEGYRO_MAN");
manual2 = load("2023_27_01_013_UNIT2__RATEGYRO_MAN_2");

% Gyro Data - Auto
auto1 = load("2023_27_01_013_UNIT2_RATEGYRO_AUTO_F0.2C0.5");
auto2 = load("2023_27_01_013_UNIT2_RATEGYRO_AUTO_F0.5C0.3");
auto3 = load("2023_27_01_013_UNIT2_RATEGYRO_AUTO_F0.4C0.75");

%% PART 1 - RATE GYRO MEASUREMENTS

[K1,s1] = plotting(auto1,"Raw Input/Output for F0.2, C0.5","Best Fit for F0.2, C0.5","Calil
% = plotting(manual1);
[K2, s2] = plotting(auto2,"Raw Input/Output for F0.5, C0.3","Best Fit for F0.5, C0.3","Cal
[K3, s3] = plotting(auto3,"Raw Input/Output for F0.4, C0.75","Best Fit for F0.4, C0.75","C

% Mean/Std Calculations
K_mat = [K1, K2, K3];
bias_mat = [s1, s2, s3];
K_mean = (K1+K2+K3)/3;
bias_mean = (s1+s2+s3)/3;
K_std = std(K_mat);
bias_std = std(bias_mat);





%% Data Function
function [K,bias,K_mean,bias_mean] = plotting(data,Title1,Title2,Title3,Title4,Title5,Titl
data_time = data(2:end,1) - data(2,1);
data_output = data(2:end, 2);
data_input = data(2:end, 3);

% RPM Conversion
data_inputrad = data_input .* ((2*pi) / 60);
```

```
 % Manual Data Plot
 figure
 hold on
 grid on
 plot(data_time, data_inputrad)
 plot(data_time, data_output, 'r')
 xlabel("Time [s]")
 ylabel("Rates [rad/s]")
 title(Title1)
 legend("Input","Output", "Location", "best")

% Calibraton Plot
p = polyfit(data_output,data_inputrad,1);
poly = polyval(p,data_inputrad);
figure
hold on
grid on
plot(data_output, data_inputrad, ".", 'MarkerSize', 3)
plot(data_inputrad, poly, 'r', "LineWidth",1.1)
xlabel("Gyro Input [rad/s]")
ylabel("Gyro Output [rad/s]")
title(Title2)
legend("Data","Best Fit Line",'Location', "best")
K = p(1)
bias = p(2)

% Application Plot
figure
hold on
grid on
calib_output = data_output .* K + bias;
plot(data_time, data_inputrad,"k","LineWidth",1)
plot(data_time, calib_output, 'm',"LineStyle",'--',"LineWidth",1.5)
xlabel("Time [s]")
ylabel("Rates [rad/s]")
title(Title3)
legend("Input","Calib. Output","Location","Best")


% Angular Rate Error
figure
hold on
grid on
plot(data_time, calib_output - data_inputrad)
xlabel("Time [s]")
ylabel('Rate Error [rad/s]')
title(Title4)

% True + Measured Angular Position
delta_time = data_time(3) - data_time(2);
poschange_true = zeros(1,length(data_inputrad)-1);
poschange_meas = zeros(1,length(calib_output)-1);
for i = 2:length(data_inputrad)
    poschange_true(i) = poschange_true(i-1) + (delta_time * data_inputrad(i));
```

```
    poschange_meas(i) = poschange_meas(i-1) + (delta_time * calib_output(i));
end

% Position Graph
figure
hold on
grid on
plot(data_time, poschange_true,'k',"LineWidth",1.25)
plot(data_time, poschange_meas,"color","#4CBB17","LineWidth",1.5,"LineStyle",'--')
xlabel("Time [s]")
ylabel("Angular Positions [rad]")
title(Title5)
legend("True Pos.","Measured Pos.","Location","best")

% Angular Position Error
figure
hold on
grid on
plot(data_time, poschange_meas - poschange_true)
xlabel("Time [s]")
ylabel("Error [rad]")
title(Title6)

end
```