University of Colorado Boulder

ASEN 3200: Orbital Mechanics

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Angular Momentum Lab Analysis

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This paper provides an analysis of how using different simulations and an object (in this case a bicycle wheel) being spun using external torque can help produce a better understanding of the effects of external torque, energy dissipation through spin stabilization, and the difference in spins for oblate and prolate bodies. We mainly used a bicycle wheel precession rate experiment to focus on collecting various different data points to better understand the rotational behavior of the bicycle wheel (similar to the attitude of a satellite). As applied torque increases, the angular velocity of the wheel increases. We found that, as a result, the precession rate of the body increases as well. Through this lab, we determined that there is a direct correlation between applied torque, angular velocity, and precession rate as each variable is dependent on one other and, in an ideal case, the relationship can be reduced down to a linear dependence. Due to external sources of error, the collected data did have values that were slightly off when compared to the predicted values.

1 Introduction

This lab aids with conceptualizing the behavior of a spinning satellite/spacecraft using analytical and experimental analysis. The rotational behavior can be summarized into the various different orientation cases given. The bicycle wheel precession rate experiment, on the other hand, aids the user in understanding how variables such as applied torque, angular velocity, and precession rate are all correlated and dependent on one another. When conducting the bicycle wheel precession rate experiment, the user applies torque until the wheel reaches the desired angular velocity. With the collected data, the user is able to predict the precession rate and overall rotational behavior of a rigid body. This information can be used when applying and predicting rotational motions of real-life applications such as satellites and spacecraft when it comes to aspects such as spin stabilization. Though satellites and spacecraft have much more complex structures with multiple moving components, using the bicycle wheel precession rate analysis, the user can apply it to each component individually and understand how each module plays a role in the entire system.

2 Spacecraft Animations

2.1 Preliminary Questions

1. What conditions are required for the angular momentum vector of a spacecraft to move as seen from an inertial frame?

   In order for the angular momentum vector to change as seen from the inertial frame, a torque has to be applied. If a torque is applied perpendicular to the angular momentum vector, then the direction of the angular momentum vector will change. If the torque is applied parallel to the angular momentum vector, the magnitude of the angular momentum will change.

2. What do the terms oblate and prolate mean? Explain what we mean by spin near the major or minor axis. How can these be distinguished by observing spacecraft motion?

   Oblate and prolate refer to the shape of the body. There are a few ways to differentiate between the two. To begin, a prolate body has moments of inertia $I_{xx} = I_{yy} > I_{zz}$ and an oblate body has moments of inertia $I_{xx} = I_{yy} < I_{zz}$. Additionally, if the nutation angle, θ, is greater than the wobble angle, γ, the object is considered prolate.

3. How does the angular velocity vector move in the body frame when the body is rigid (no internal dissipation)? How does energy dissipation change this motion?

   For a rigid body, the angular velocity traces out the intersection of the angular momentum sphere and the kinetic energy ellipse. This path remains constant when there is no energy dissipation in the system. If energy is being dissipated, then the angular velocity will decrease in magnitude and it will begin to change direction. This is due to the decrease in the size of the kinetic energy ellipse.

4. If a spacecraft is spinning, describe how thrust must be applied to move the angular momentum vector in a desired direction.

   If a spacecraft is spinning, it will have an angular momentum vector. To control the vector, a torque can be applied to move the angular momentum in the desired direction. To rotate the angular momentum, a torque needs to be applied in the desired direction that is perpendicular to the angular momentum. To increase or decrease the magnitude of the angular momentum, a torque needs to be applied parallel to the angular momentum vector (+ parallel to increase, - parallel to decrease).

2.2 Animation Analysis

2.2.1 Case A

For this case, a torque is applied and this is known because the angular momentum vector changes to match the angular velocity vector. Using the polhode plot it can be seen that the magnitude of the angular velocity doesn't change but the heading does. Because there is no change in the magnitude of the angular velocity, just a change in the direction, there is no energy dissipation. The object starts by spinning around the major axis with a precession angle, and throughout the simulation, the angular momentum vector leaves the major axis, passes by the intermediate axis, and ends back at the major axis.

2.2.2 Case B

In case B, the initial motion is torque-free around the minor axis, H_1. A short torque is then applied in the $+H_2$ axis. We know this because the angular momentum vector begins to change direction temporarily. The angular momentum then returns to a constant direction and magnitude indicating a torque-free motion. The final motion is a rotation around the major axis, H_3. In this case, there is no energy dissipation because the magnitude of the angular velocity vector does not change. The direction does change, but this does not cause a change in the energy of the system.

2.2.3 Case C

The body is continuously processing about the positive z_N axis, with the precession increasing as time passes. It is determined that one external torque exists even though the angular momentum vector does not move. This is determined by the energy ellipsoid in the polhode plot getting larger. From this, we can deduce that, for the energy to increase, and for the external force to exist, then the magnitude of the angular momentum vector would have to increase. We should note it spins near the smaller inertia. To achieve this, the external torque would have to be applied parallel to the angular momentum vector. Angular velocity originally acts in the same direction as angular momentum resulting in an increase in the magnitude of the velocity vector.

2.2.4 Case D

In this case, the body will originally process about the positive intermediate inertial axis y_N before flipping and processing about the negative y_N axis. There is no external torque applied so there is no energy dissipation or increase, and the angular momentum vector in the inertial frame does not move.

2.2.5 Case E

The procession begins about the positive z_N and continues to process about the same axis but at a larger angle. No external torque is applied since, in the inertial frame, the angular momentum vector does not move. Note it is initially near the minor axis, then heads towards the major. There is energy dissipation though, as the polhode shows it initially beginning in a maximum energy state before decreasing close to a minimum energy state.

2.2.6 Analysis

3 Bicycle Procession

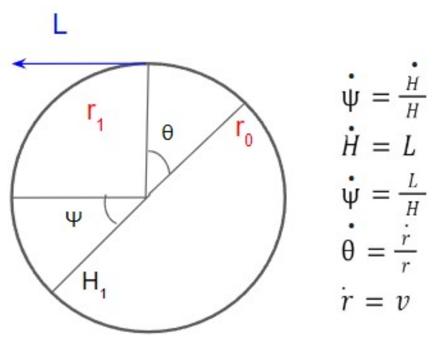

Figure 1: Bicycle Wheel Experiment View 1

To find the procession rate as a function of the spin rate, we will use the concept of torque which is a product of the moment of inertia and the angular acceleration of the wheel. We begin with the equation for angular momentum about the center of the wheel:

$$\mathbf{H_c} = [I_c] \cdot \vec{\omega}_{W/N} \tag{1}$$
$$= I_{xx}\vec{\omega}_s\hat{g}_1 + I_{zz}\vec{\omega}_p\hat{g}_3 \tag{2}$$

where

$$\vec{\omega}_{W/N} = \vec{\omega}_{W/G} + \vec{\omega}_{G/N} \tag{3}$$
$$= \vec{\omega}_s\hat{g}_1 + \vec{\omega}_p\hat{g}_2 \tag{4}$$

Then, taking the time derivative of the angular momentum yields:

$$\dot{\mathbf{H}}_\mathbf{c} = \frac{d^G}{dt}(\mathbf{H_c}) + \vec{\omega}_{G/N} \times \mathbf{H_c} = \mathbf{L_c} \tag{5}$$
$$\frac{d^G}{dt}(\mathbf{H_c}) = I_{xx}\dot{\omega}_s\hat{g}_1 + I_{zz}\dot{\omega}_p\hat{g}_3 \tag{6}$$
$$= 0 \tag{7}$$
$$\vec{\omega}_{G/N} \times \mathbf{H_c} = I_{xx}\vec{\omega}_s\vec{\omega}_p\hat{g}_2 \tag{8}$$

The time rate of change of the angular momentum is zero since we are assuming the wheel spins and precesses at a constant rate.
Next, we can some forces and torques around point C, the center of the wheel to find $\mathbf{L_c}$:

$$\sum \mathbf{L_c} = F_N \times D \tag{9}$$
$$= m_w g \cdot D\hat{g}_2 \tag{10}$$

Equating the two $\hat{g}_2$ components and solving for $\vec{\omega}_p$ gives the precession rate as a function of the spin rate:

$$\vec{\omega}_p = \frac{m_w g \cdot D}{I_{xx}\vec{\omega}_s}\hat{g}_2 \tag{11}$$
$$I_{xx} = mR_w^2 \tag{12}$$
$$\vec{\omega}_p = \frac{g \cdot D}{\vec{\omega}_s R_w^2}\hat{g}_2 \tag{13}$$

3.1 Experiment

To start the experiment, the necessary measurements were taken which include: wheel diameter, the length between the center of mass and support point, and the mass of the wheel. This would be used to calculate the moment of inertia for the final precession rate equation. A motor was used to spin the wheel in the clockwise direction and data for 6 velocities were taken. Once spun to the desired velocity, the wheel was let go and the precession time was measured for one period. Multiple bicycle wheel views can be seen below:

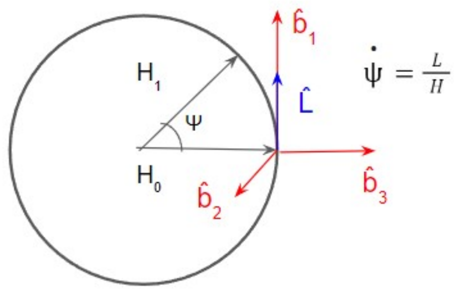

Figure 2: Bicycle Wheel Experiment View 2

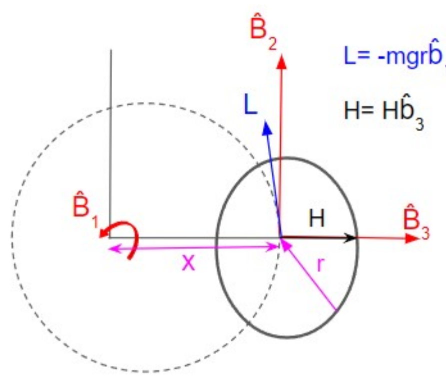

Figure 3: Bicycle Wheel Experiment View 3

Speed[km/h]	Period[s]	Calc. Procession[rad/s]	Exp. Procession[rad/s]	Precession Error[%]
20	3.91	-.7373	-1.6070	117.95%
21.5	4.05	-0.6859	-1.5514	126.18%
29.5	5.56	-0.4999	-1.1301	126.02%
30	6.62	-0.4916	-0.9491	93.06%
38	6.92	-0.3881	-0.9080	133.96%

Table 1: Experimental Data

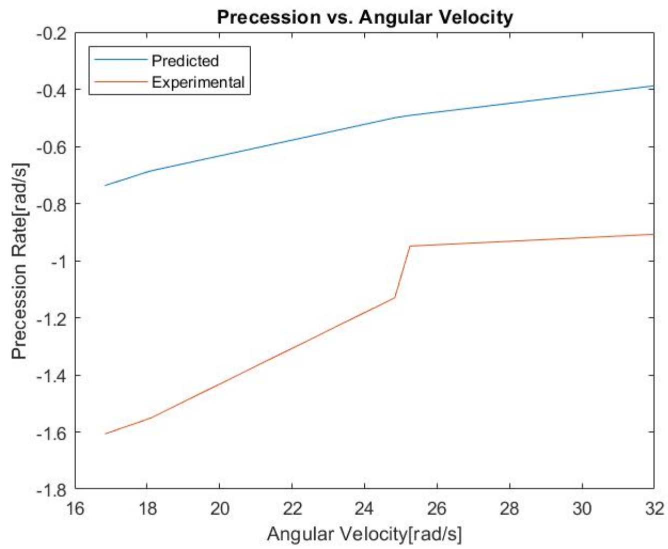

Figure 4: Precession vs. Angular Velocity

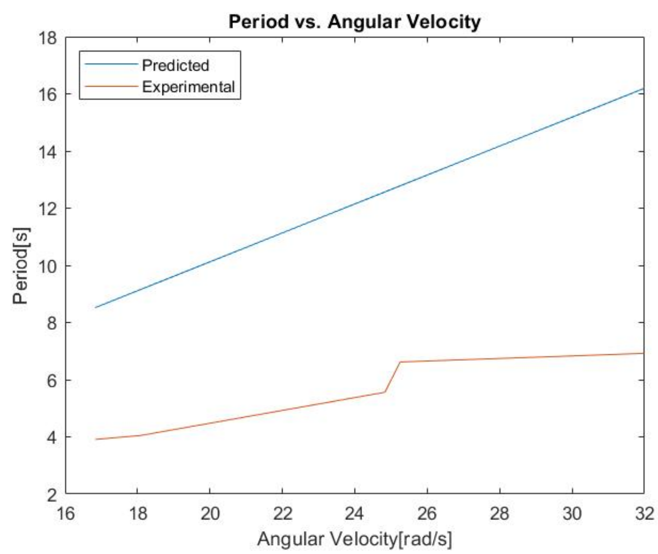

Figure 5: Period vs. Angular Velocity

3.2 Analysis

4 Conclusions and Recommendations

Using a bicycle wheel, the user is able to identify different variables such as angular velocity and time taken to finish one rotation to determine applied external torque, precession rate and precession period. After plotting the data collected, a direct correlation between the wheel's angular velocity and precession rate/period can be concluded. As the wheel's spin rate increases, the precession rate decreases and precession period increases. After further analysis, the user can also determine that there is a direct correlation between angular velocity and angular momentum of the body being tested. Note, in this experiment, external torque to move/shift the wheel needs to be added in order to move/shift any related vectors. This lab overall helps the user better understand the significance of variables like angular velocity and precession rates/periods when understanding spin stabilization which can be further applied in real-life applications such as satellites and spacecraft.

5 Acknowledgments

5.1 Member Contributions

Deliverable	**Matt Bechtel**	**Rishab Pally**	**Tristan Seeley**
Abstract	0%	100%	0%
Introduction	0%	100%	0%
Preliminary Questions	100%	0%	0%
Spacecraft Analysis	0%	0%	100%
Bicycle Wheel Precession Analysis	50%	50%	0%
Conclusions/Recommendations	0%	100%	0%
Matlab Component	0%	100%	0%
Style and Clarity	33%	33%	33%
TOTAL	**33%**	**33%**	**33%**

Signature of Approval: Matt Bechtel, Rishab Pally, Tristan Seeley

6 References

7 Appendix: Matlab Derivations

```
% Authors: Matthew Betchel, Rishab Pally, Tristan Seeley
%LAB A2
close all
clear
clc

mass= 2.98;
%unit conversions to cm to meters
diameter= (33E-2)*2;
x_position= .1778;
gravity_constant=9.81;

Inertia_1= 0.5*mass*(diameter/2)^2;
Inertia_3= mass*(diameter/2)^2 +mass*x_position^2;

%Unit conversion %km/h to m/s
vt= [20, 21.5, 29.5, 30, 38]'.*0.277778;
w3= vt./(diameter/2);


I= [Inertia_1,0,0;0,Inertia_3,0;0,0,Inertia_3];
H3= Inertia_3*w3;
P= (mass*-gravity_constant*x_position)./(H3);

torque= mass*-gravity_constant*x_position;

Torque_v= ((2*pi)./P);
```

```
T_matrix= [-3.91, -4.05, -5.56, -6.62, -6.92]; %[s]
P_matrix= (2*pi)./T_matrix;

P_zero= zeros(1,5);
T_zero= zeros(1,5);
for i=1:5

    P_zero(i)= ((P_matrix(i)-P(i))./P(i))*100;
    T_zero(i)= (T_matrix(i)-Torque_v(i))./Torque_v(i);
end
P
P_matrix

figure
plot(w3,P)
hold on
xlabel('Angular Velocity[rad/s]')
ylabel('Precession Rate[rad/s]')
title('Precession vs. Angular Velocity')
plot(w3,P_matrix)
hold off
legend('Predicted','Experimental','location','northwest')

figure
plot(w3,-1*Torque_v)
hold on
xlabel('Angular Velocity[rad/s]')
ylabel('Period[s]')
title('Period vs. Angular Velocity')
plot(w3,-1*T_matrix)
hold off
legend('Predicted','Experimental','location','northwest')
```