University of Colorado Boulder

ASEN 3112: Structures

Lab 1: Torsional Lab

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This paper provides an analysis on the shear strain of two specimens by using an extensometer to measure the response differences to varied applied torques. One of the specimens was a uniform cross section circular in shape, the closed-thin-wall specimen, while the other specimen was a circular cross section with a small opening/cut on the circumference of the specimen, the open-thin-wall specimen. After analyzing the data, we can determine that the closed-thin-wall specimen (CTW Specimen) was overall the more resistant specimen and responded better to varying applied torque when compared to the open-thin-wall specimen(OTW Specimen). For both the CTW and OTW specimens, we calculate torsional rigidity using two main methods. While utilizing the Extensometer Method, our torsional rigidity values for the CTW and OTW specimens resulted in 99784 $lb - in^2$ and 4379 $lb - in^2$ respectively. When performing the Twist Angle Method, the torsional rigidity values for the CTW and OTW specimens resulted in 5908 $lb - in^2$ and 33276 $lb - in^2$ respectively. Through this lab, we were able to gain a better understanding of how significant shape, dimensions and material specifications play a role in how different structures respond to external forces. We also learned the significance of error in models and measurements and how they compare to the true value of torsional rigidity.

1 Results and Analysis:

1.1 Analysis of the Closed Thin Wall Specimen

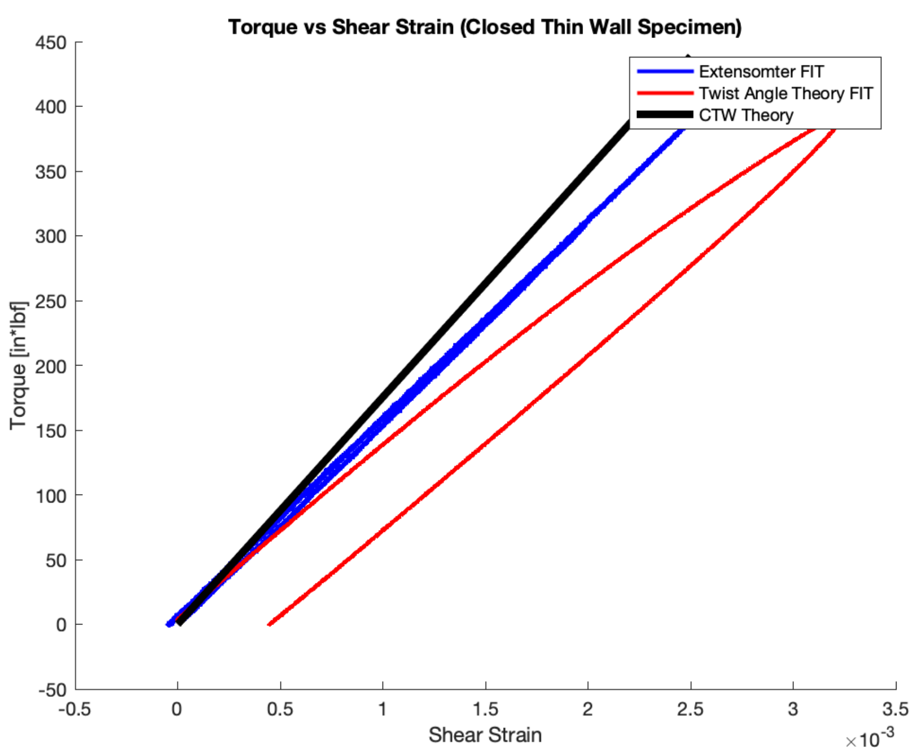

Figure 1: Closed Thin Wall Specimen

The plot above depicts the torque vs shear strain of the specimen using three different methods. The blue line represents shear strain formulated from the experimental data gathered by the extensometer during testing. By using a least squares fit, this method yielded a torsional rigidity of GJ = 99784.34 [lbin^2] and had an associated uncertainty of 1.7665 x 10-16 [lbin^2]. The second method we used to calculate the torsional rigidity of the specimen, represented as the red line, was twist angle theory. This method produced a torsional rigidity of GJ = 5907.94 [lbin^2] with a line fit uncertainty of 2.8932 x 10-16 [lbin^2]. The final method, shown as the black line, produced a theoretical value of GJ using the closed thin wall theory with a given shear modulus, G. The shear strain using closed thin wall theory is given as,

$$\gamma = \frac{TR}{GJ} \tag{1}$$

where T is the total torque applied, R is the radius of the specimen, and GJ is the torsional rigidity with

$$J = \frac{\pi}{2}(R_e^4 - R_i^4) \tag{2}$$

By rearranging the variables, we get the torsional rigidity GJ as a function of the shear strain and applied torque

$$GJ = \frac{TR}{\gamma} \tag{3}$$

and because T/γ is equal to the slope of the shear strain vs torque plot, the torsional rigidity can be calculated by multiplying radius R by the slope.

Using the process described above, the torsional rigidity was found to be GJ = 60310.61 [lbin^2]. All three methods of determining GJ produced very different results as the difference between the extensometer value and the theoretical CWT value was 39,473.73 [lbin^2] (65.4% error), while the difference between the twist angle theory and extensometer was 93,876.4 [lbin^2] (94.1% error). When observing the plots, all three methods yielded similar results initially, but as the applied torque increased, so did the deviation between the methods. This leads us to believe that our models hold well for small applied torques but become less useful when used to analyze larger torques. Additionally, models such as the two used in this experiment cannot account for any version of "run time" deformation as well as any minor imperfections within the material as loads are being applied. The rotation angle of the bar was captured by the MTS machine via the frictional force that was holding the bar steady within the two clamps. Due to this, there is a possibility that slipping within the clamp could have attributed to the error found in utilizing the rotation angle to determine the Torque vs Strain slope and thus the accompanying torsional rigidity value.

1.2 Analysis of the Open Thin Wall Specimen

The plot for the open wall specimen depicts the torque vs. shear strain of the open wall tube. The three different lines describe the data measured by the extensometer, the data measured from the torque applying machine, and the predicted torque vs. shear strain from the open thin wall model. Using least squares fit for the experimental data of the extensometer, we calculated the value of the torsional rigidity to be GJ=4379.41 [lbin^2] with an uncertainty of 2.276e-17 [lbin^2] to the linear fit line. Using the least squares fit for the rotation angle, we calculated GJ=33276.53 [lbin^2] with an uncertainty of 2.561e-19 [lbin^2] from the linear fit. The theoretical value of GJ is calculated using the open thin wall theory using the shear modulus, G, and the J_β term.

$$J_\beta = \beta 2\pi R t^3 \tag{4}$$

where β is assumed to be $\frac{1}{3}$ (due to thickness being much less than length) and the radius is R=$\frac{R_e+R_i}{2}$

$$GJ = G\beta 2\pi \frac{R_e + R_i}{2} t^3 = (3.75 * 10^6[psi])(\frac{1}{3})2\pi \frac{\frac{3}{8}[in] + \frac{5}{16}[in]}{2}(\frac{1}{16}in)^3 \tag{5}$$

This calculation result in a torsional rigidity from the open thin wall model to be GJ=659.13 [lbin^2]. Using the extensometer data as the correct model for GJ, the percent error for the open thin wall assumption model is 84.9% which is quite large. The rigidity from using the rotation angle results in a value of about 33276.53 [lbin^2]. The percent error of the torsional rigidity calculated using the rotation angle is 659.9% using the

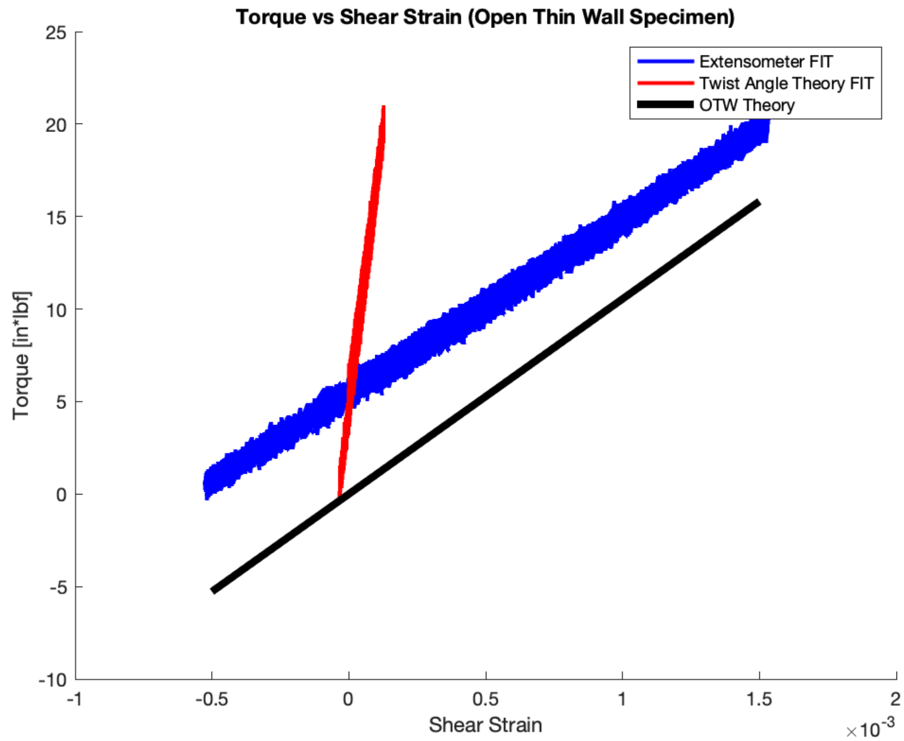

Figure 2: Open Thin Wall Specimen

extensometer as the true value. The sources of error for this measurement include the slipping of the torsion machine when applying a torque as well as the assumptions we make about the specimen being an open wall thin tube. The torque machine also has an error associated with the angle measurement it records. The discrepancy in the open thin wall theory can be described by the assumption that the cut in the specimen has a negligible width compared to the cross sectional radial dimension. The cut in the tube does impact the properties of the tube that the equations do not model. The cut also does not span the entire length of the bar. This in turn causes the model and calculated torsional rigidity to have a large error. All of these factors produce error that propagates through calculations when using these measured values and result in 3 very different values for the torsional rigidity of the open wall tube.

1.3 Importance of the Extensometer:

The Extensometer plays an important role when comparing between torsional rigidity of the OTW specimen measurements to the testing machine. This is due to the torsional rigidity of the OTW specimen being much lower while also accounting for errors that occur at the ends. The Extensometer method does not apply as much to the CWT specimen as the values determined are not too different from the Twist Angle Method due to there possibly being higher torsional rigidity.

The Extensometer provides the user with feedback regarding the shear strain of the shaft being used. Some readings may be inaccurate due to the bar slipping but in most cases we can assume that the Extensometer is connected at two individual points which result in the user needing to measure the center of the specimen. Measuring in the center of the OTW and CTW specimens is very significant when determining the values as there could be deformations that occur at the ends which are not accounted for. Overall, using an Extensometer we are able to model things like torsional rigidity and further calculate shear strain for both specimens as well. Not only that, but due to the high accuracy of this device, the data collected was treated as something to compare and validate our other various methods with.

1.4 Plastic Deformation:

If the specimens are pushed beyond their elastic regime during testing, they will begin to exhibit a different type of material behavior and will no longer be able to return to their original configuration. This behavior is illustrated in the Torque vs Shear Strain plot below.

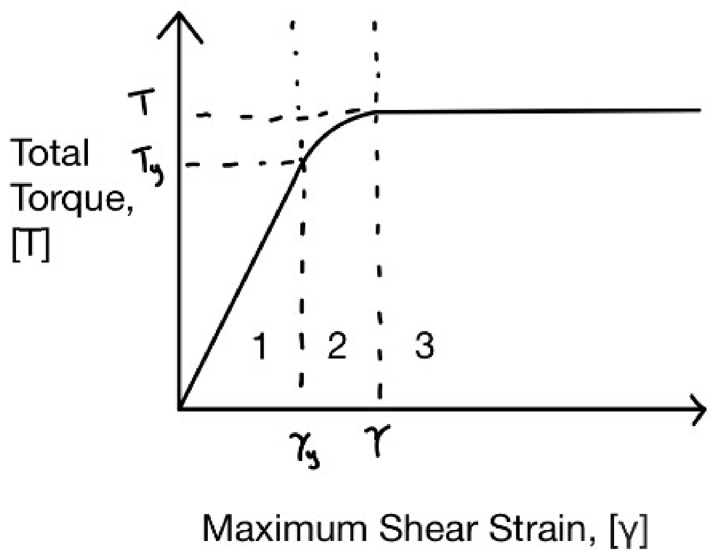

Figure 3: Expected Material Behavior Undergoing Plastic Deformation

The plot above has been divided into three separate regions, each representing a different stage of the specimen's behavior as it is plastically deformed. The three regions are:

1. The linear elastic region - any deformation experienced in this region is fully reversible, relationship between torque and shear strain is linear.
2. Partially plastic / Transition region - the specimen has reached its yield point and plastic deformation begins, however, some portion of the deformation can still be recovered
3. Fully plastic region - any deformation in this region is permanent and can't be recovered. All additional torques will result in more deformation.

If we consider a close-walled specimen of length L, outer radius R_e, inner radius R_i, and thickness t = R_e-R_i, we can express the shear strain γ that corresponds to the transition between regions as a function of the shear strain at yield γ_y and the geometry of the sample. Using equation 7.4 from the lecture notes, we get

$$\gamma = \frac{\rho}{R}\gamma_{max} \tag{6}$$

$$\gamma_y = \frac{R_i}{R_e}\gamma_{max} \tag{7}$$

where γ_{max} is the maximum shear strain experienced in the cross section and the shear at yield γ_y is a function of the ratio of the inner to outer radii. Rearranging the equation yields

$$\gamma_{max} = \frac{R_e}{R_i}\gamma \tag{8}$$

Given that the outer radius is larger than the inner radius, the ratio above will always be positive, causing the transition shear strain to be slightly larger than the shear strain at yield, which we would expect for a thin-walled cylinder.

2 Conclusion:

Using MTS Torsional Testing Machine, we were able to apply different levels of torque to the two specimens being tested. Using the Extensometer data, Twist Angle Method, and Thin Wall Theory, we were able to determine the difference in response to varied applied torque and find their respective torsional rigidity values. By applying two different methods when solving for the torsional rigidity values, we were able to see how much more accurate the Extensometer and Thin Wall Method are as there is a much larger disparity in values for the Twist Angle Method. When calculating the torsional rigidity, we can see a clear correlation of how shape, dimensions such as radius, length, cross-section, and material specifications like the shear modulus and factor of safety affect the overall limits of the structure. These aspects are very critical in designing any kind of structure or when looking for any kind of structural integrity.

3 Acknowledgments

3.1 Member Contributions

Member	**Contribution**
Matthew Chen	100%
Manas Katragadda	100%
Rishab Pally	100%
William Steinfort	100%
Zachary Zerr	100%

Signature of Approval: Matthew Chen, Manas Katragadda, Rishab Pally, William Steinfort, Zachary Zerr

3.2 Member Contributions Notes

Matthew Chen and William Steinfort did a great job on the OTW and CTW analysis. Rishab Pally showed the importance of the extensometer, understanding of the MTS machine and how it can be used in real life applications. Manas Katraggada and William Steinfort did an exceptional analysis on Plastic Deformation. Zachary Zerr provided the group with the Matlab component and aided with OTW and CTW analysis as well.

4 Appendix A

4.1 Equations

$$\begin{gathered} ExactTheory: \\ \gamma_{OTW} = \frac{\phi t}{L} \\ \gamma_{CTW} = \frac{\phi R_e}{L} \end{gathered}$$

$$\begin{gathered} ThinWallTheory: \\ \gamma_{OTW} = \frac{Tt}{GJ} \qquad -> \qquad J = \beta \times b \times t^3 \\ \gamma_{CTW} = \frac{TR}{GJ} \qquad -> \qquad J = \frac{\pi}{2} \times (R_e^4 - R_i^4) \end{gathered}$$

$$\begin{gathered} TorsionalRigidity(GJ): \\ OTW \qquad -> \qquad GJ = \frac{Tt}{\gamma} \\ CTW \qquad -> \qquad GJ = \frac{TR}{\gamma} \end{gathered}$$

$$\begin{gathered} LeastSquaresUncertainty: \\ \sqrt{\frac{\sum x_i - x_{lineBestFit}}{N}} \end{gathered}$$

4.2 MATLAB Code

CTW/OTW MatLab Analysis continued on next page

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- Calculate Torsional Ridgidity For Each Method
- Calc Error

ASEN 3112 - Torsion Analysis - Main

Create Torsion vs Strain plots and examine how Closed Thin Wall and Open Thin Wall specimens hold up to applied torsion.

MTS Torsional Testing Machine CTW & OTW (Stock Aluminum Tube)

Author: {Matt Chen, Manas Katragadda, Zachary Zerr, Rishab Pally, Will Steinfort} Collaborators: None Date: {09/15/22}

```
clc, clear, close all
```

Load In Data

```
fileIDo = fopen('20inlbf_002.txt','r');
fileIDc = fopen('400inlbf_01.txt','r');
formatSpec = '%s %s %s %s %s';
frewind(fileIDo)
frewind(fileIDc)
otw = textscan(fileIDo,formatSpec);
otwData(:,1) = otw{1,1};
otwData(:,2) = otw{1,2};
otwData(:,3) = otw{1,3};
otwData(:,4) = otw{1,4};
otwData(:,5) = otw{1,5};
otwData = otwData(12:end,:);
ctw = textscan(fileIDc,formatSpec);
ctwData(:,1) = ctw{1,1};
ctwData(:,2) = ctw{1,2};
ctwData(:,3) = ctw{1,3};
ctwData(:,4) = ctw{1,4};
ctwData(:,5) = ctw{1,5};
ctwData = ctwData(5:end,:);
          % Data Formal
          %[Time, Torsional Angle, Torsional Epsilon, Torsional Torque, Axial Epsilon]
          %[s   ,deg             ,deg               ,in*lbf           ,in            ]
fclose(fileIDo);
fclose(fileIDc);
```

```
Error using fseek
Invalid file identifier. Use fopen to generate a valid file identifier.

Error in frewind (line 16)
status = fseek(fid, 0, -1);

Error in Stuctures_Lab1_Main (line 18)
frewind(fileIDo)
```

Vaiables

Both

```
          t = 1/16; %[in]
          R_e = 3/8; %[in]
          R_i = R_e - t; %[in]
          G = 3.75 * 10^6; %[psi]
          R = (R_e + R_i)/2; %[in]
          L = 10; %[in]
          beta = 1/3;
          alpha = 1/3;
          J_beta = beta*(2*pi*R)*t^3;             % OTW J
          J = (pi/2)*(R_e^4 - R_i^4);             % CTW J

          % OTW
```

```
        T_otw = otwData(:,4);
        angle_otw = otwData(:,2);
        realShear_otw = otwData(:,3);
        TorsRidg_otw = G*J_beta;                        % OTW GJ
        TorsRidgFunc_otw = @(T) (T*t/TorsRidg_otw);     % OTW strain "By Thin Wall Theory

        % CTW
        T_ctw = ctwData(:,4);
        angle_ctw = ctwData(:,2);
        realShear_ctw = ctwData(:,3);
        TorsRidg_ctw = G*J;                             % CTW GJ
        TorsRidgFunc_ctw = @(T) (T*R/TorsRidg_ctw);     % CTW strain "By Thin Wall Theory
```

Equations for intermedidiate values

```
for i = 1:28703
    calcShear_otw(1,i) = deg2rad(str2double(cell2mat(angle_otw(i,1))) - str2double(cell2mat(angle_otw(1,1)))) * t / L;    % OTW strain "By Exact Theory"
end


for i = 1:61294
    calcShear_ctw(1,i) = deg2rad(str2double(cell2mat(angle_ctw(i,1))) - str2double(cell2mat(angle_ctw(1,1)))) * R_e / L;  % CTW strain "By Exact Theory"
end
```

Plots

House Keeping

```
        for i = 1:28703
            T_otwm(i) = str2double(cell2mat(T_otw(i,1)));
            realShear_otwm(i) = deg2rad(str2double(cell2mat(realShear_otw(i,1))));
        end

        % House Keeping
        for i = 1:61294
            T_ctwm(i) = str2double(cell2mat(T_ctw(i,1)));
            realShear_ctwm(i) = deg2rad(str2double(cell2mat(realShear_ctw(i,1))));
        end
```

Rigidity Calculations (Using Least Squares) "Line Best Fit"

OTW Extensometer

```
                  line_otw_real = polyfit(-T_otwm,-realShear_otwm,1);
                  line_otw_real = polyval(line_otw_real,-T_otwm);
                  % Calculated
                  line_otw_calc = polyfit(-T_otwm,-calcShear_otw,1);
                  line_otw_calc = polyval(line_otw_calc,-T_otwm);
        % CTW
                  % Extensometer
                  line_ctw_real = polyfit(-T_ctwm,-realShear_ctwm,1);
                  line_ctw_real = polyval(line_ctw_real,-T_ctwm);
                  % Calculated
                  line_ctw_calc = polyfit(-T_ctwm,-calcShear_ctw,1);
                  line_ctw_calc = polyval(line_ctw_calc,-T_ctwm);
```

Plots

```
        % T-Strain (OTW)
        figure(1);
        hold on
        plot(-T_otwm,-realShear_otwm,'b',"LineWidth",2);                    % Extensometer
        %plot(-T_otwm,line_otw_real,'b',"LineWidth",3)
        plot(-T_otwm,-calcShear_otw,'r',"LineWidth",2);                     % Exact Theory
        %plot(-T_otwm,line_otw_calc,'r',"LineWidth",3)
        fplot(TorsRidgFunc_otw,[0 20],'k',"LineWidth",4);                    % Thin Wall Theory
        title("Torque vs Shear Strain (Open Thin Wall Specimen)")
        legend("Extensometer FIT","Twist Angle Theory FIT","OTW Theory");
        xlabel("Torque [in*lbf]");
        ylabel("Shear Strain");
        hold off


        % T-Strain (CTW)
        figure(2);
        hold on
        plot(-T_ctwm,-realShear_ctwm,'b',"LineWidth",2);                    % Extensometer
        %plot(-T_ctwm,line_ctw_real,'y',"LineWidth",3)
        plot(-T_ctwm,-calcShear_ctw,'r',"LineWidth",2);                     % Exact Theory
```

```
%plot(-T_ctwm,line_ctw_calc,'g',"LineWidth",3)
fplot(TorsRidgFunc_ctw,[0 400],'k',"LineWidth",4);                  % Thin Wall Theory
title("Torque vs Shear Strain (Closed Thin Wall Specimen)")
legend("Extensomter FIT","Twist Angle Theory FIT","CTW Theory");
xlabel("Torque [in*lbf]");
ylabel("Shear Strain");
hold off
```

Calculate Torsional Ridgidity For Each Method

```
GJ_otw(1) = TorsRidg_otw;                                       % Thin Wall Therory
GJ_otw(2) = t/(line_otw_calc(20)/(-T_otwm(20)));                % Exact Theory
GJ_otw(3) = t/(line_otw_real(20)/(-T_otwm(20)));                % Extensometer

GJ_ctw(1) = TorsRidg_ctw;                                       % Thin Wall Therory
GJ_ctw(2) = R/(line_ctw_calc(20)/(-T_ctwm(20)));                % Exact Theory
GJ_ctw(3) = R/(line_ctw_real(20)/(-T_ctwm(20)));                % Extensometer

fprintf("Torsional Ridgidity OTW : ThinWall -> %f, Exact -> %f, Extensometer -> %f\n",GJ_otw(1),GJ_otw(2),GJ_otw(3))
fprintf("Torsional Ridgidity CTW : ThinWall -> %f, Exact -> %f, Extensometer -> %f\n",GJ_ctw(1),GJ_ctw(2),GJ_ctw(3))
```

Calc Error

OTW

```
summ = 0;
summm = 0;
for i = 1:length(realShear_otwm)
    summ = summ + (-realShear_otwm(i) - line_otw_real(i));
    summm = summm + (-calcShear_otw(i) - line_otw_calc(i));
end
U_real_otw = sqrt((summ^2)/(length(realShear_otwm)));
U_calc_otw = sqrt((summm^2)/(length(realShear_otwm)));

% CTW
summ = 0;
summm = 0;
for i = 1:length(realShear_ctwm)
    summ = summ + (-realShear_ctwm(i) - line_ctw_real(i));
    summm = summm + (-calcShear_ctw(i) - line_ctw_calc(i));
end
U_real_ctw = sqrt((summ^2)/(length(realShear_ctwm)));
U_calc_ctw = sqrt((summm^2)/(length(realShear_ctwm)));

fprintf("OTW : Extensometer Uncertanty = %d, Exact Uncertanty = %d\n",U_real_otw,U_calc_otw);
fprintf("CTW : Extensometer Uncertanty = %d, Exact Uncertanty = %d\n",U_real_ctw,U_calc_ctw);
```

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