University of Colorado Boulder

ASEN 3112: Structures

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Lab 3: Vibrations

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This paper provides a structural analysis of a simplified model of an airplane vibrating under harmonic excitation by a shaker device as well as a comparison of the resonant frequencies with natural frequencies using Bernoulli-Euler beam finite element models.Through this lab we are able to show a better understanding of resonance on a flexible dynamic system using a form of applied excitation and the experimental technique used to identify points of local natural frequencies while determining resonance ranges.

1 Question 1: Experimental Results

The experimental data provided includes the accelerometer and displacement data of the system at various different points. The sensor is placed in between the shaker and the system to model the inputted acceleration/excitation. The other three sensors are placed at various points depending on the test to monitor and measure the system's response. The data was collected through a data acquisition system which led to a very large amount of data to filter. Because of this, sampling of the data was done to try to filter out noise in the system. Sampling is needed because of the large amounts of noise in the signal. To sample the data, the maximum value of 350 points at a time was taken to give an approximation of the data over the interval. This can be assumed because with 350 data points per sample the frequency change per step is around 0.05 Hz, so important resonance frequencies are not very likely to be missed. The acceleration response of each sensor was divided by the driven acceleration for each sampled interval leading to the "magnification factor" for the data. Once the data was plotted for each test case, the resonance frequencies were found visually by looking at distinct peaks in the graph. When peaks occur across multiple accelerometers, there can be high confidence that a resonance frequency occurs there.

Table 1: Sensor vs. Captured Resonance Frequency

Test	**Resonance Frequencies [Hz]**
"test 2min All 1"	12.53, 23.87, 40.04, 48.05
"test 2min Nose 1"	23.77, 40.33, 48.05
"test 2min Tail 1"	12.51, 44.63, 49.31
"test 2min Wings 1"	24.13, 41.48, 48.95
"test 5min All 1"	12.34, 24.42, 41.28

Table 2: Sensor vs. Captured Resonance Frequency

Test	**Resonance Frequency [Hz]**	**Mode Response 1 [Hz]**	**Mode Response 2 [Hz]**
test_2min_all_1	12.53, 48.05	12.53	48.05
test_2min_nose_1	48.05	N/A	48.05
test_2min_tail_1	12.51, 49.31	12.51	49.31
test_2min_wings_1	48.95	N/A	48.95
test_5min_all_1	12.34, -	12.34	N/A

Table 2 shows that in all test cases, there is a mode response of 2 near 48 Hz. The only tests that have a mode response of 1 are the "all" tests and the tail test, showing that the tail accelerometer determines if a mode response of 1 is seen. Table 1 shows that there are minor resonances that occur near 25 Hz frequency. This isn't a mode frequency, but it is a multiple of mode response 1. This might be a harmonic of that mode response, but further testing would need to be done.

The four main sensors used to capture resonance are the vibrometer and the accelerometers placed at the nose, tail and wings. To determine the shape of the model, experimental eigenmodes are used but note that additional accelerometers were not used in the actual experiment. This sensor would have been located somewhere in the middle of the structure, ideally the mid section of the fuselage. This change would result in being able to determine the eigenmode's actual shape whereas the current sensors only aid with determining the end states. When conducting a FEM analysis, each node is considered to represent an accelerometer and with the aid of more nodes possibly representing the middle of the structure, it allows the user to better understand the fuselage span's eigenmodes. Multiple nodes are used in the FEM analysis but without the additional sensor the mode shapes cannot be determined from the provided experimental data.

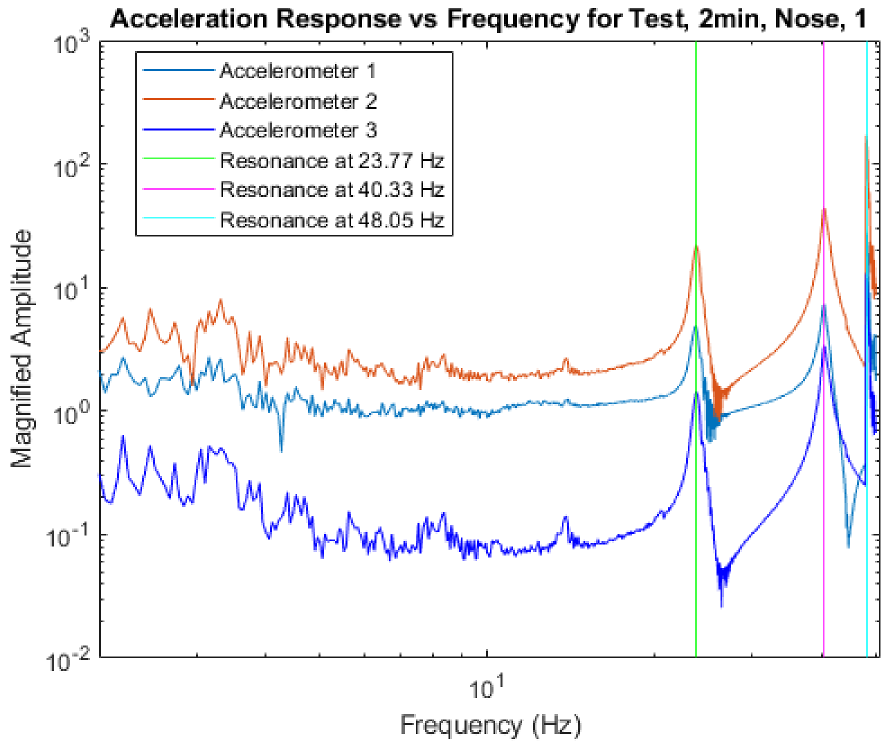

Figure 1: Acceleration Response 2min, Nose 1

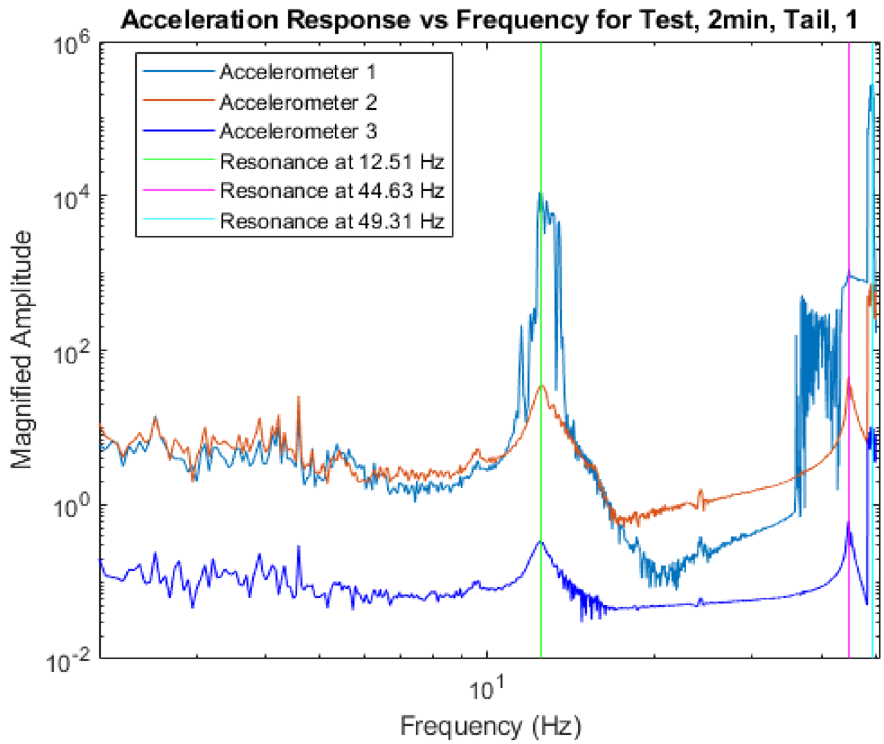

Figure 2: Acceleration Response 2min, Tail 1

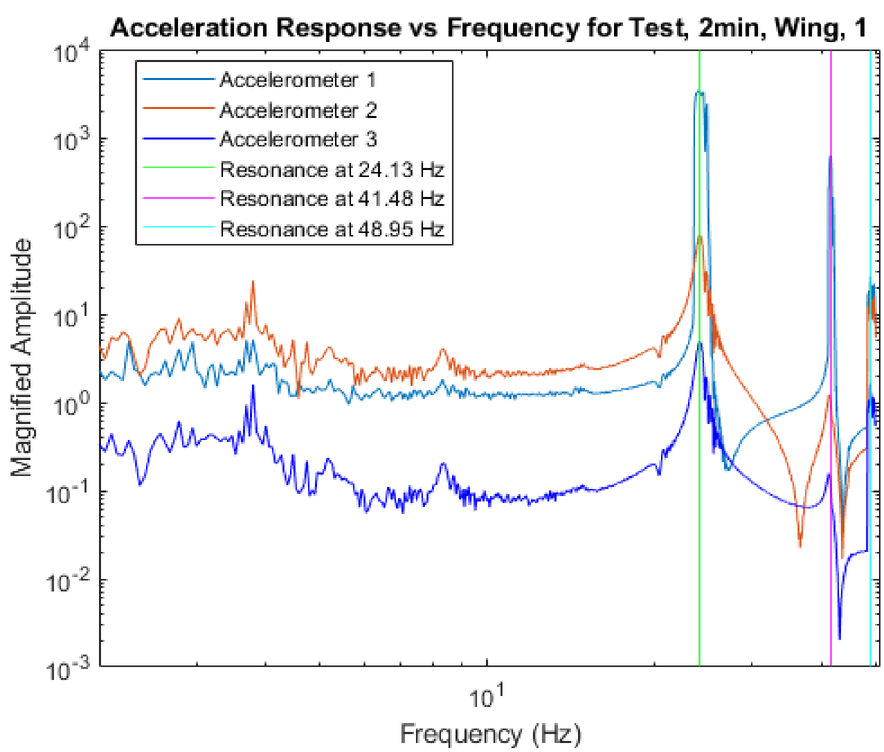

Figure 3: Acceleration Response 2min, Wing 1

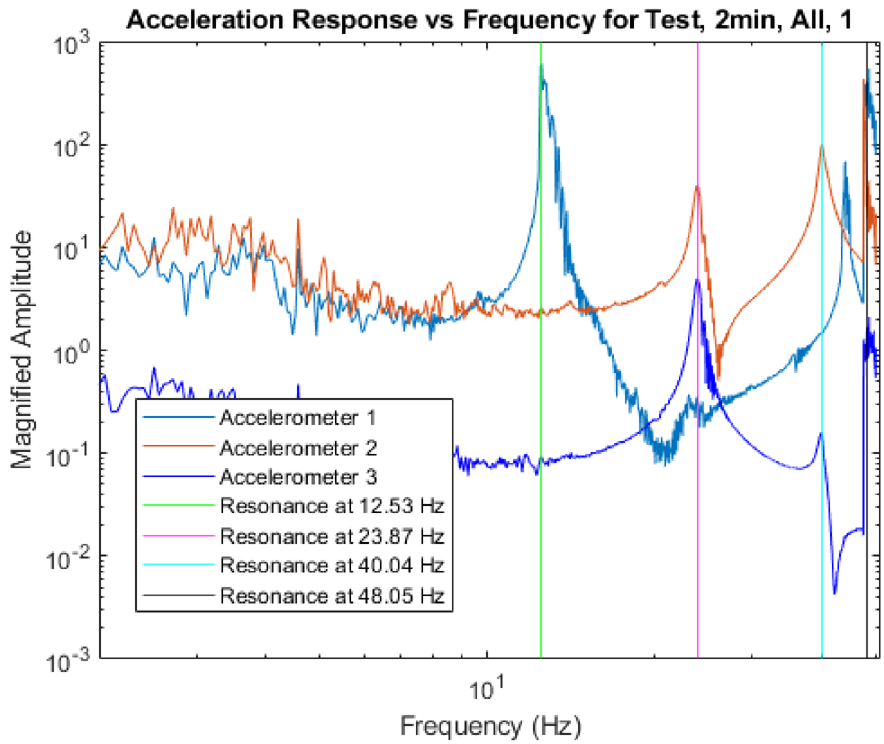

Figure 4: Acceleration Response 2min, All 1

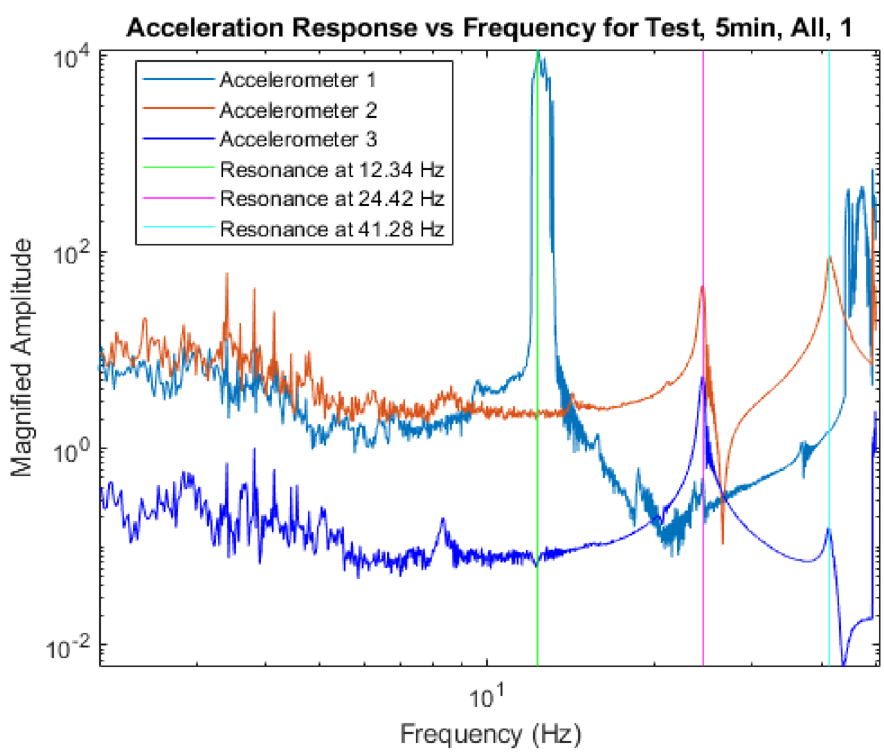

Figure 5: Acceleration Response, 5min, All 1

2 Question 2: FEM Results - Resonant Frequencies

Using the two element model, three more resonant frequencies were calculated. The first resonant frequency found was 12.02 Hz. Comparing it to the experimental data, there is an error of 4%. The second resonant frequency found through FEM was 51.02 Hz. Comparing that to the experimental value, there's an error of 13%. The FEM also provided one additional resonant frequency at 202.51 Hz. Because this go past the 50 Hz limit of the experimental data, we are unable to compare and determine error. Using the four element model, the first three resonant frequencies of the fuselage are 12.0294, 51.0824, 203.3665 Hz. Experimentally, the end of the tail has resonant frequencies at 12.51, 44.63, and 49.31 Hz. Overall, the calculated four element model has similar resonant frequencies to the actual fuselage, but did not match exactly. The first four element model resonant frequency is off from the actual first resonant frequency by 4%. The 44.63 Hz resonant frequency was determined by the four element model. The 49.31 Hz resonant frequency has a considerably larger error when compared to the FEM with an error of 3%. The reason for this difference in error can be explained by our method of finding the resonant frequency in the experimental results. Finally, the 203.36 Hz resonant frequency determined by the four element model is not shown in the experimental results because it is outside of the range of tested frequencies. In general, the four element model tends to be more accurate. This makes sense because it essentially divides the beam into smaller sections rather than halves like the two element model. This allows us to get more specific values. It's also important to note that the FEM is a simplified system. In the FEM, the tail is modeled as lumped with the rest of the beam. Experimentally of course, that is not the case and the actual tail will create more complex behaviors during oscillation.

3 Question 3: FEM Results - Mode Shape

Below are three modal shape plots for the two element modal response and three for the four element modal response, all plotted in blue (Figures 6-11). Below those, figures 12-15 show the overlayed experimental modal shapes with the predicted modal shapes and the error between the two. As shown, the predicted data matches up almost exactly with the experimental data with an almost negligible amount of error. The frequencies tended plotted were 12.0294 Hz, 51.0824 Hz and 203.3665 Hz with lower frequencies showing significantly less error than the higher ones. This makes sense because the high experimental frequencies went over the expected max of 50 Hz, making the lower frequency plots more accurately within the predicted range. The small amount of misalignment could be due to human error and oversimplification of the two element response in the predicted modal response code. Miscalculations seem unlikely to have occurred due to the close match of the plot trends.

The predicted four element plots also indicate that there was a strong modal response at around 12 Hz. This lines up with the results discussed and plotted in question 1. These acceleration vs frequency plots show peaks consistently around 12 Hz, leading to the conclusion that the predicted modal response plots shown are a relatively accurate portrayal of the experimental results.

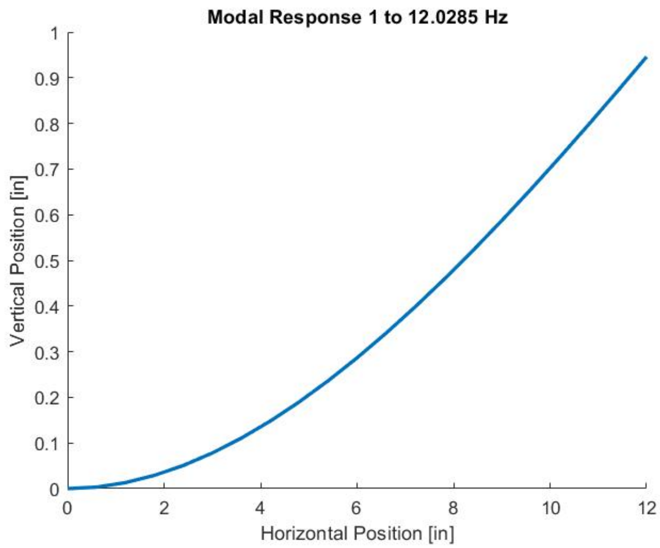

Figure 6: Two Element Model Response 1

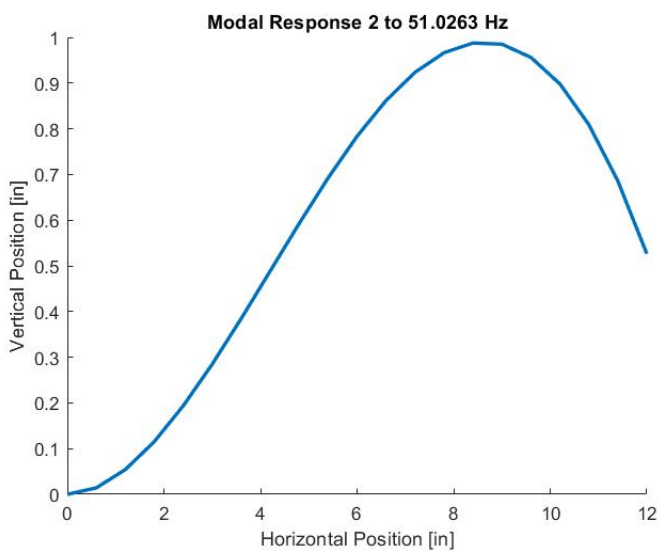

Figure 7: Two Element Model Response 2

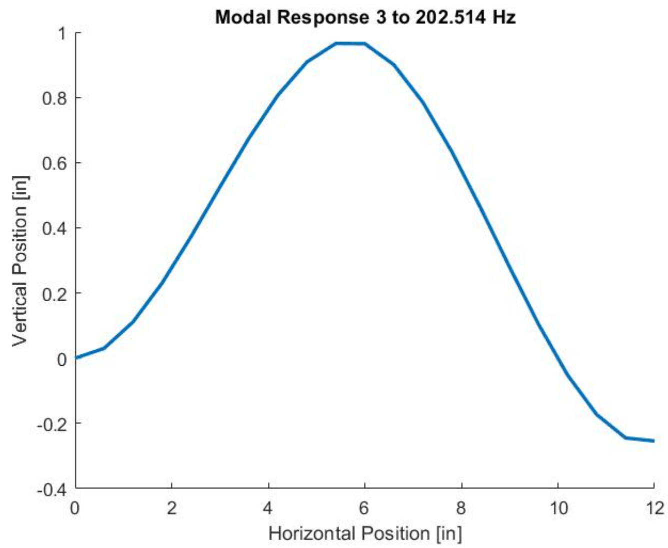

Figure 8: Two Element Model Response 3

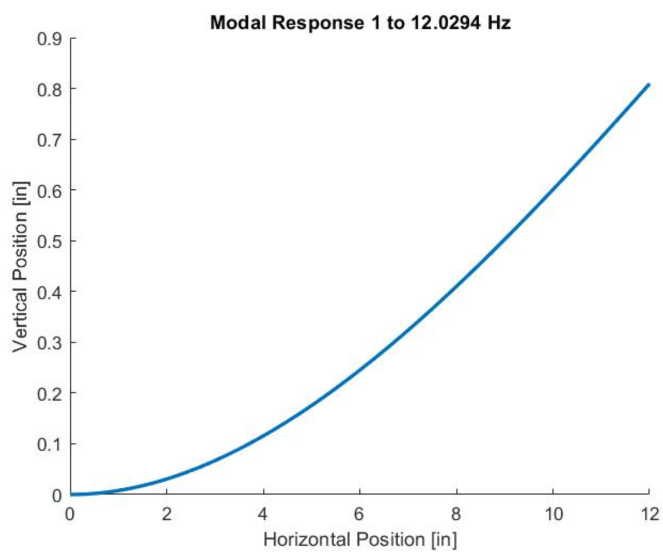

Figure 9: Four Element Model Response 1

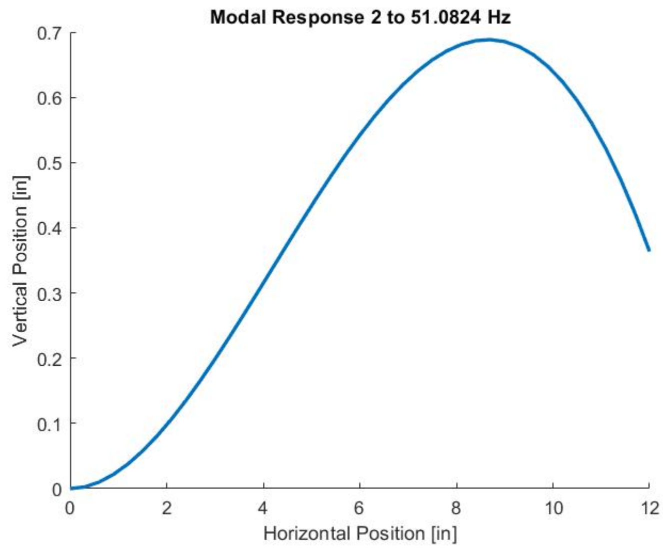

Figure 10: Four Element Model Response 2

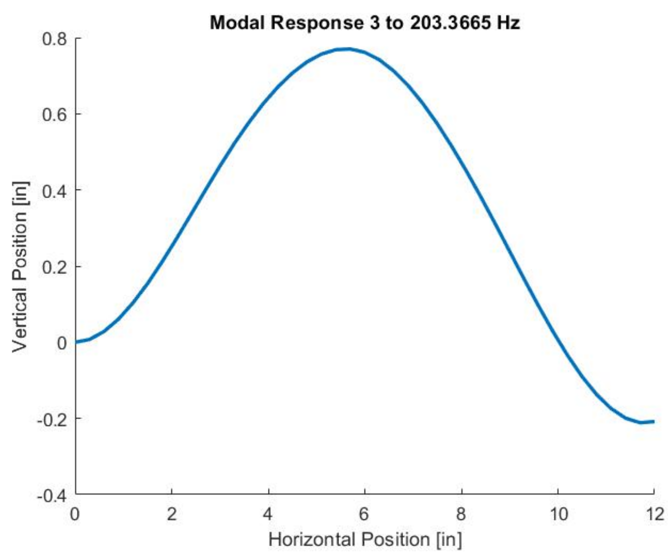

Figure 11: Four Element Model Response 3

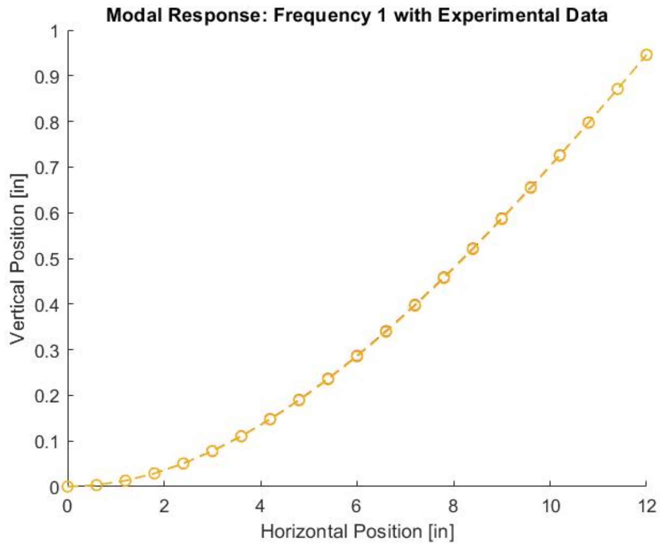

Figure 12: Two Element Modal Response, f=12.0294Hz

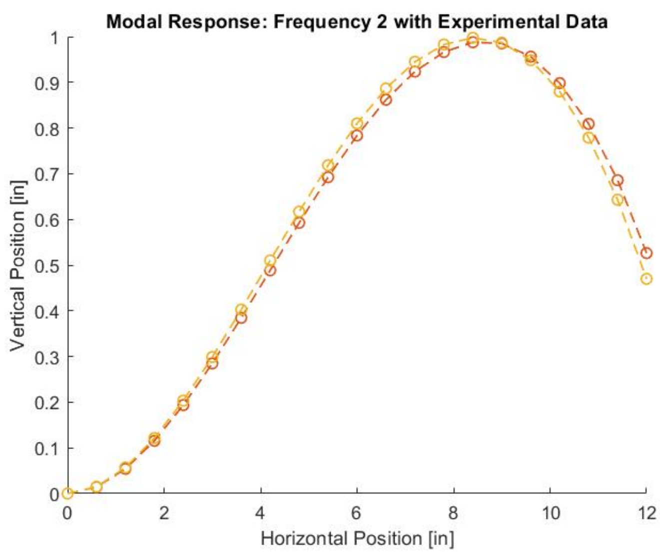

Figure 13: Two Element Modal Response, f=51.0824Hz

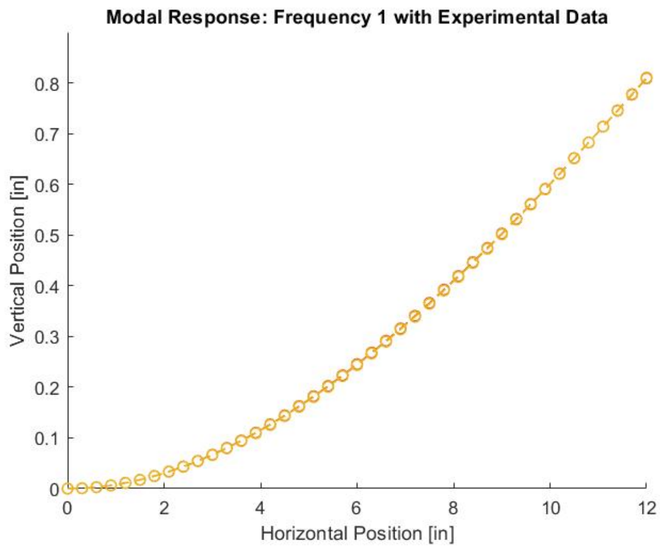

Figure 14: Four Element Modal Response, f=12.0294Hz

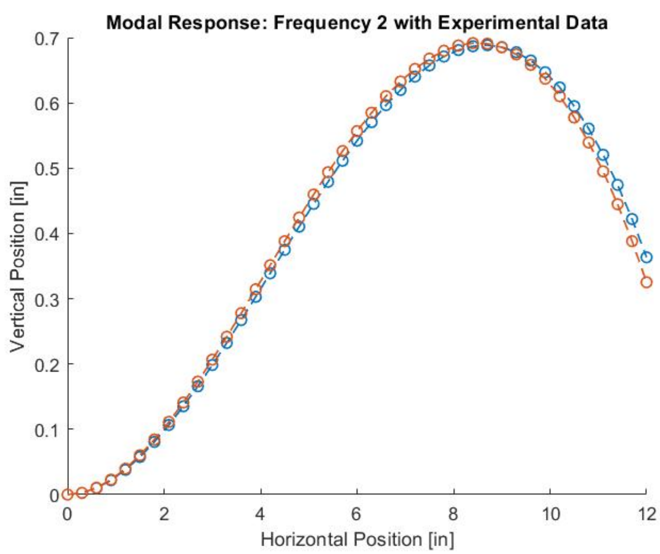

Figure 15: Four Element Modal Response, f=51.0824Hz

4 Acknowledgments

4.1 Member Contributions

Member	Contribution
Noah Abeson	100%
Nathan Evans	100%
Polly Fitton	100%
Tyler Hall	100%
Rishab Pally	100%
Nicole Rogers	100%
Tristian Seeley	100%

Signature of Approval: Noah Abeson, Nathan Evans, Polly Fitton, Tyler Hall, Rishab Pally, Nicole Rogers, Tristian Seeley

5 Appendix

5.1 Frequency vs. Magnified Amplitude / Resonance Matlab Derivation:

```
function plotFrequencyLab3(name,titleOf,ResonanceVec,ResonanceNames)
%Loading in Data
data_Full = load(name);
accel = data_Full(:,3:5);
shaker = data_Full(:,2);

%sampling data
sample_vec = 1:350:length(accel);

%Matrix setup
sampledAccel = [];
sampledShaker = [];


for i = 1:3  %"i" loop varies accelerometer
    for j = 1:length(sample_vec)-1  %"j" loop varies through rows

        %Takes 500 points at a time and takes the maximum value for the
        %data
        sampledAccel(j,i) = max(accel(sample_vec(j):sample_vec(j+1),i));
        sampledShaker(j) = max(shaker(sample_vec(j):sample_vec(j+1)));
    end

end
sampledShaker = sampledShaker';

%Frequency vector, rising linearly from 2 Hz to 50 Hz
frequencyVec = linspace(2,50,length(sample_vec)-1);

%Making a "magnification factor" based on the ratio of driven acceleration
%divided by the shakers acceleration
Ratio_Accel_1 = sampledAccel(:,1)./sampledShaker;
Ratio_Accel_2 = sampledAccel(:,2)./sampledShaker;
Ratio_Accel_3 = sampledAccel(:,3)./sampledShaker;

Names = ["Accelerometer 1" "Accelerometer 2" "Accelerometer 3" ResonanceNames];
colors = ["g","m","c","k"];
%Plotting
figure

```

```
loglog(frequencyVec,Ratio_Accel_1,frequencyVec,Ratio_Accel_2,frequencyVec,Ratio_Accel_3,"b")
hold on
for i = 1:length(ResonanceVec)
    xline(ResonanceVec(i),colors(i))
end
xlabel("Frequency (Hz)")
ylabel("Magnified Amplitude")
title("Acceleration Response vs Frequency for " + titleOf )
legend(Names,Location="best")
xlim([2 51])


%Preset function calls used to make plots for write up

%plotFrequencyLab3('test_2min_all_1','Test, 2min, All, 1',[12.52 23.87 40.04 ...
    48.05],["Resonance at 12.53 Hz" "Resonance at 23.87 Hz" "Resonance at 40.04 Hz" ...
    "Resonance at 48.05 Hz"])

%plotFrequencyLab3('test_2min_nose_1','Test, 2min, Nose, 1',[23.77 40.33 ...
    48.05],["Resonance at 23.77 Hz" "Resonance at 40.33 Hz" "Resonance at 48.05 Hz"])

%plotFrequencyLab3('test_2min_tail_1','Test, 2min, Tail, 1',[12.51 44.63 ...
    49.31],["Resonance at 12.51 Hz" "Resonance at 44.63 Hz" "Resonance at 49.31 Hz"])

%plotFrequencyLab3('test_2min_wing_1','Test, 2min, Wing, 1',[24.13 41.48 ...
    48.95],["Resonance at 24.13 Hz" "Resonance at 41.48 Hz" "Resonance at 48.95 Hz"])

%plotFrequencyLab3('test_5min_all_1','Test, 5min, All, 1',[12.34 24.42 41.28],["Resonance ...
    at 12.34 Hz" "Resonance at 24.42 Hz" "Resonance at 41.28 Hz"])
end
```

5.2 Two Element Model Response Matlab Derivation:

```
clc; clear; close all;

L = 12; %cantilever span S'B' [in]
L_E = 4.5; %elevator san [in]
L_R = 5; %rudder span [in]
w = 1; %width of all members [in]
h = 1/8; %thickness of fuselage memeber [in]
h_E = 1/4; %thickness of elevator [in]
h_R = 0.040; %thickness of rudder member [in]
E = 10175000; %material elastic modulus [lb/in^2]
rho = 0.0002505; %material mass density [lb-sec^2/in^4]
M_T = 1.131*rho; %mass of tail assembly
S_T = 0.5655*rho; %first mass-moment of tail assembly wrt B'
I_T = 23.124*rho; %second mass-moment of tail assembly wrt B'

A = w*h;
Izz = (w*h^3)/12;

cM2 = rho*A*L/100800;
cK2 = 4*E*Izz/L^3;

M2 = cM2*[19272 1458*L 5928 -642*L 0 0;...
           1458*L 172*L^2 642*L -73*L^2 0 0;...
           5928 642*L 38544 0 5928 -642*L;...
           -642*L -73*L^2 0 344*L^2 642*L -73*L^2;...
           0 0 5928 642*L 19272 -1458*L;...
           0 0 -642*L -73*L^2 -1458*L 172*L^2]...
         +[0 0 0 0 0 0;...
           0 0 0 0 0 0;...
```

```
            0 0 0 0 0 0;...
            0 0 0 0 0 0;...
            0 0 0 0 M_T S_T;...
            0 0 0 0 S_T I_T];

 K2 = cK2*[24 6*L -24 6*L 0 0;...
             6*L 2*L^2 -6*L L^2 0 0;...
             -24 -6*L 48 0 -24 6*L;...
             6*L L^2 0 4*L^2 -6*L L^2;...
             0 0 -24 -6*L 24 -6*L;...
             0 0 6*L L^2 -6*L 2*L^2];

 M2(1,:) = [];
 M2(1,:) = [];
 M2(:,1) = [];
 M2(:,1) = [];
 K2(:,1) = [];
 K2(:,1) = [];
 K2(1,:) = [];
 K2(1,:) = [];

 M2_hat = M2;
 K2_hat = K2;

[eigen_values, eigen_vectors, w_n, f_n] = Eigen_values(M2_hat, K2_hat);


%plotting
plot_Eigen_vectors(1, f_n(4,4), L, [0; 0; eigen_vectors(:, 4)], 2, 10, 1, 1)
plot_Eigen_vectors(2, f_n(3,3), L, [0; 0; eigen_vectors(:, 3)], 2, 10, 1, 2)
plot_Eigen_vectors(3, f_n(2,2), L, [0; 0; eigen_vectors(:, 2)], 2, 10, 1, 3)


mat_2_1 = K2_hat - (12.51*2*pi)^2.*M2_hat; %omega is the frequencey that is closest to the ...
    first predicited resonant frequecny (convert to rad)
mat_2_2 = K2_hat - (49.31*2*pi)^2.*M2_hat;

[eigen_vector_e1,eigen_values_e1] = eig(mat_2_1);
[eigen_vector_e2,eigen_values_e2] = eig(mat_2_2);

plot_Eigen_vectors(4, 12.51, L, [0; 0; eigen_vector_e1(:, 1)], 2, 10, 1, 1)
plot_Eigen_vectors(5, 49.31, L, [0; 0; eigen_vector_e2(:, 2)], 2, 10, 1, 2)

%plotting
plot_Eigen_vectors(1, f_n(4,4), L, [0; 0; eigen_vectors(:, 4)], 2, 10, 1, 1)
plot_Eigen_vectors(2, f_n(3,3), L, [0; 0; eigen_vectors(:, 3)], 2, 10, 1, 2)
plot_Eigen_vectors(1, 12.51, L, [0; 0; eigen_vector_e1(:, 1)], 2, 10, 1, 1)
plot_Eigen_vectors(2, 49.31, L, [0; 0; eigen_vector_e2(:, 2)], 2, 10, 1, 2)


function plot_Eigen_vectors(fig, freq, L, ev, ne, nsub, scale, responseNum)
    nv=ne*nsub+1; Le=L/ne; dx=Le/nsub; k=1;
    x=zeros(1,nv); % declare and set to zero plot arrays
    v=zeros(1,nv); % declare and set to zero plot arrays
    for e = 1:ne
        xi=Le*(e-1); vi=ev(2*e-1); qi=ev(2*e); vj=ev(2*e+1); qj=ev(2*e+2);
        for n = 1:nsub
            xk=xi+dx*n; X=(2*n-nsub)/nsub;
            vk=scale*(0.125*(4*(vi+vj)+2*(vi-vj)*(X^2-3)*X+Le*(X^2-1)*(qj-qi+(qi+qj)*X)));
            k = k+1; x(k)=xk; v(k)=vk;
        end
    end


    %plots
```

```
    f = figure(fig);
    hold on;
    plot(x, v,'--o', 'LineWidth', 1);
    hold on
    xlabel('Horizontal Position [in]');
    ylabel('Vertical Position [in]');
    title("Modal Response: Frequency " + string(responseNum) + " with Experimental Data");
end


function [eigen_values, eigen_vectors, w_n, f_n] = Eigen_values(M2, K2)
[eigen_vectors, eigen_values] = eig(M2\K2);
w_n = abs(eigen_values).^(1/2);
f_n = w_n./(2*pi);

end
```

5.3 Four Element Model Response Matlab Derivation:

```

clear;
close all;



%4 modes
E = 10175000;
rho = 0.0002505;
L = 12; %in
w = 1; %in
h_fw = 0.125;
%h_e = .25;
%h_r = 0.04;
A = w*h_fw;
c_M4 = rho*A*L/806400;
I_zz = w*(h_fw^3)/12;
c_K4 = 8*E*I_zz/(L^3);
M_T = rho*1.131;
S_T = rho*0.5655;
I_T = rho*23.124;




M4 = c_M4*[77088 2916*L 23712 -1284*L 0 0 0 0 0 0; 2916*L 172*L^2 1284*L -73*L^2 0 0 0 0 0 ...
    0; 23712 1284*L 154176 0 23712 -1284*L 0 0 0 0;-1284*L -73*L^2 0 344*L^2 1284*L ...
    -73*L^2 0 0 0 0;0 0 23712 1284*L 154176 0 23712 -1284*L 0 0;0 0 -1284*L -73*L^2 0 ...
    344*L^2 1284*L -73*L^2 0 0;0 0 0 0 23712 1284*L 154176 0 23712 -1284*L;0 0 0 0 -1284*L ...
    -73*L^2 0 344*L^2 1284*L -73*L^2;0 0 0 0 0 0 23712 1284*L (77088+(M_T/c_M4)) ...
    (-2916*L+(S_T/c_M4));0 0 0 0 0 0 -1284*L -73*L^2 (-2916*L+(S_T/c_M4)) ...
    (172*L^2+(I_T/c_M4));];

K4 = c_K4*[96 12*L -96 12*L 0 0 0 0 0 0;12*L 2*L^2 -12*L L^2 0 0 0 0 0 0;-96 -12*L 192 0 ...
    -96 12*L 0 0 0 0;12*L L^2 0 4*L^2 -12*L L^2 0 0 0 0;0 0 -96 -12*L 192 0 -96 12*L 0 0;0 ...
    0 12*L L^2 0 4*L^2 -12*L L^2 0 0;0 0 0 0 -96 -12*L 192 0 -96 12*L;0 0 0 0 12*L L^2 0 ...
    4*L^2 -12*L L^2;0 0 0 0 0 0 -96 -12*L 96 -12*L; 0 0 0 0 0 0 12*L L^2 -12*L 2*L^2];


M4_reduced = M4(3:end,3:end);
K4_reduced = K4(3:end,3:end);


```

```
[eigen_values, eigen_vectors, w_n, f_n] = Eigen_values(M4_reduced, K4_reduced);




%mode shapes for expiriment
mat_4_1 = K4_reduced - (12.51*2*pi)^2.*M4_reduced; %omega is the frequency that is ...
    closest to the first predicited resonant frequecny (convert to rad)
mat_4_2 = K4_reduced - (49.31*2*pi)^2.*M4_reduced;

[eigen_vector_e1,eigen_values_e1] = eig(mat_4_1);
[eigen_vector_e2,eigen_values_e2] = eig(mat_4_2);

plot_Eigen_vectors(1, 12.51, L, [0; 0; eigen_vector_e1(:, 1)], 4, 10, 1, 1)
plot_Eigen_vectors(5, 49.31, L, [0; 0; eigen_vector_e2(:, 2)], 4, 10, 1, 2)

%plotting
plot_Eigen_vectors(1, f_n(8,8), L, [0; 0; eigen_vectors(:, 8)], 4, 10, 1, 1)
plot_Eigen_vectors(1, 12.51, L, [0; 0; eigen_vector_e1(:, 1)], 4, 10, 1, 1)

plot_Eigen_vectors(2, f_n(7,7), L, [0; 0; eigen_vectors(:, 7)], 4, 10, 1, 2)
plot_Eigen_vectors(2, 49.31, L, [0; 0; eigen_vector_e2(:, 2)], 4, 10, 1, 2)



plot_Eigen_vectors(3, f_n(6,6), L, [0; 0; eigen_vectors(:, 6)], 4, 10, 1, 3)


function plot_Eigen_vectors(fig, freq, L, ev, ne, nsub, scale, responseNum)
    nv=ne*nsub+1; Le=L/ne; dx=Le/nsub; k=1;
    x=zeros(1,nv); % declare and set to zero plot arrays
    v=zeros(1,nv); % declare and set to zero plot arrays
    for e = 1:ne
        xi=Le*(e-1); vi=ev(2*e-1); qi=ev(2*e); vj=ev(2*e+1); qj=ev(2*e+2);
        for n = 1:nsub
            xk=xi+dx*n; X=(2*n-nsub)/nsub;
            vk=scale*(0.125*(4*(vi+vj)+2*(vi-vj)*(X^2-3)*X+Le*(X^2-1)*(qj-qi+(qi+qj)*X)));
            k = k+1; x(k)=xk; v(k)=vk;
        end
    end


    %plots
    f = figure(fig);
    hold on;
    plot(x, v, '--o',  'LineWidth', 1);
    xlabel('Horizontal Position [in]');
    hold on
    ylabel('Vertical Position [in]');
    title("Modal Response: Frequency " + string(responseNum) + " with Experimental Data");

end


function [eigen_values, eigen_vectors, w_n, f_n] = Eigen_values(M4, K4)
[eigen_vectors, eigen_values] = eig(M4\K4);
w_n = abs(eigen_values).^(1/2);
f_n = w_n./(2*pi);

end
```

```
function [eigen_values, eigen_vectors, w_n, f_n] = Eigen_values(M4, K4)
[eigen_vectors, eigen_values] = eig(M4\K4);
w_n = abs(eigen_values).^(1/2);
f_n = w_n./(2*pi);

end
```