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ASEN 3112: Structures

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Lab 4: Beam Buckling

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1 Question 1: Buckling Load

Two different test articles are given to analyze. One of them is an Aluminum square hollow cross section with outer dimensions 0.25 in x 0.25 in and a wall thickness of 0.0625 in. The other is an Aluminum rectangular solid cross section with outer dimensions of 0.125 in x 1 in. Both hold a Young's modulus of 10,000,000 psi and a yield stress of 35,000 psi. Using the Euler buckling of beam in equation 1, the experimental buckling loads can be determined for the given articles.

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{1}$$

The two parameters that will be varied are the length and the moment of inertia respective to the article. The length will vary between the minimum and maximum length which are 10.4 in and 12.4 in respectively. The moment of inertia of each of the beams are calculated using equations 2 and 3.

The Moment of Inertia of the square hollow cross section can be calculated using the following equation:

$$I_{square} = \frac{1}{12}h^3 w - \frac{1}{12}(h-2t)^3(w-2t) \tag{2}$$

$$I_{square} = 1.6276 * 10^{-4}\ lb\ in^2$$

The Moment of Inertia of the rectangular solid cross section can be calculated using the following equation:

$$I_{rectangle} = \frac{1}{12}h^3 w \tag{3}$$

$$I_{rectangle} = 3.051 * 10^{-4}\ lb\ in^2$$

The derivation of Moment of Inertia of the rectangular cross section and square hollow cross section can be viewed in the Appendix Section 5.1.

The table below shows the predicted vs experimental buckling load for each of the specimens (Note the length is set to 11.4 in).

	Predicted [lb]	Experimental [lb]	Error %
Rectangular	123.6	112.7	8.82
Square	231.8	213.5	7.89

Table 1: Predicted vs. Experimental buckling loads

After computing the predicted and experimental buckling loads, it was determined that the rectangular specimen had a predicted and experimental buckling load of 123.6 lbs and 112.7 lbs respectively while the square specimen had a predicted and experimental buckling load of 231.8 lbs and 213.5 lbs respectively. The rectangular cross section resulted in an error of 8.82% while the square hollow cross section had a slightly lower error of 7.89%. Overall the experimental critical buckling loads were lower when compared to the predicted loads. This is due to the predicted model not accounting for any deformities within the beam. Events such as the beam having slight bents as well not being completely aligned could all result in lower buckling loads. The significance of the imperfection/deformity also correlates with shape as the rectangular cross section had a much lower buckling load when compared to the square hollow cross section.

2 Question 2: Post-Buckling Behavior

The predicted lateral deflection corresponding to the initiation of plastic post-buckling was calculated using the lateral deflection from mode shape one:

$$v(x) = \delta sin(\frac{\pi x}{L}) \tag{4}$$

The strain in the beam is calculated by the equation:

$$\epsilon = \kappa(x)y \tag{5}$$

where the curvature $\kappa(x)$ is approximated as $\kappa(x) = v''(x)$ and y is the distance from where the load is applied to the neutral line of the specimen. The curvature is derived by differentiating the lateral deflection twice with respect to x.

$$v'(x) = \delta\frac{\pi}{L}cos(\frac{\pi x}{L}) \tag{6}$$

$$v''(x) = -\delta\frac{\pi^2}{L^2}sin(\frac{\pi x}{L}) \tag{7}$$

The yield strain can be calculated using the yield stress and the elastic modulus.

$$\epsilon_{max} = \frac{\sigma_{max}}{E} = 0.0035 \tag{8}$$

Plugging this value and Equation 7 into Equation 5 and rearranging yields and expression for the maximum deflection:

$$\delta = \frac{-\epsilon_{max}L^2}{\pi^2 y} \tag{9}$$

The neutral axis distances were measured for each specimen to be $y_{square} = 0.0625in$ and $y_{rectangle} = 0.125in$. The maximum deflection for three different lengths were calculated. The minimum length of the specimen, the maximum length of the specimen, and the measured length of the specimen used in the experiment.

Cross Section	Deflection Min L [in]	Deflection Max L [in]	Deflection L=11.4 [in]
Square	-.6137	-.8724	-.7374
Rectangle	-.3068	-.4362	-.3687

Table 2: *Lateral Deflection for Initiation of Plastic Post-Buckling*

As the model used for Figures 1 and 2 theoretical analysis assumes elastic deformation, the calculated applied load vs. deflection continued to increase past the point of buckling that was seen experimentally. In reality, the response of the beam transitions from elastic to plastic at some point. This location was calculated and the predicted value is shown on the graph with the dotted vertical line.

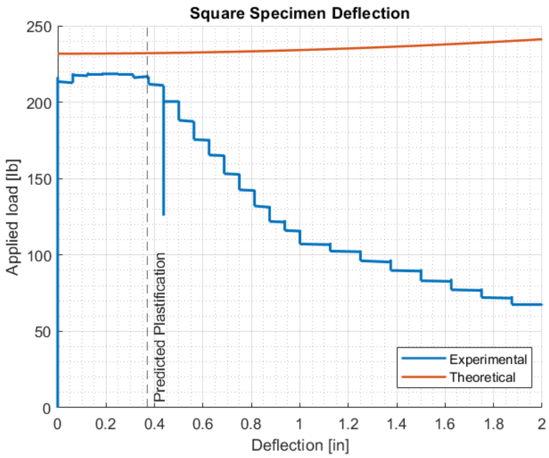

Figure 1: Square Specimen Deflection

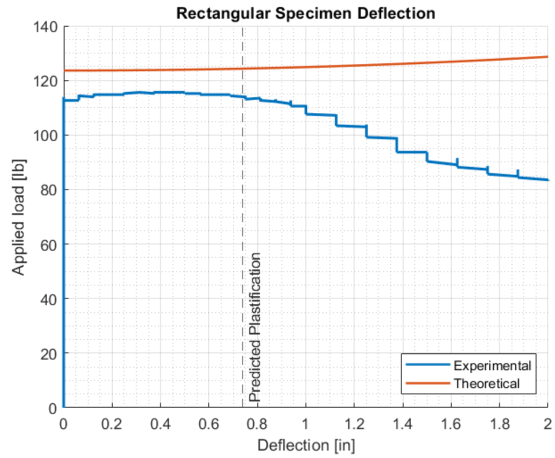

Figure 2: Rectangular Specimen Deflection

3 Question 3: Design Study

The buckling load as a function of the length was calculated and plotted for both the case of a simply supported-simply supported beam as well as a fixed-fixed beam. It can be seen in figures three and four that as the length of the beam increases, the buckling load decreases.

A shorter beam will fail due to axial yield stress, while a longer beam will fail due to buckling. This can be seen when the yield load due to axial stress is plotted on top of the figures. At any given length, the beam will fail at the lower of the two applied forces. Because the yield force is less than the buckling force for a shorter beam but greater than the buckling force for a longer beam, a shorter beam will fail at yield stress while the longer beam will fail due to buckling. For the simply supported beam, the transition between a short and a long beam occurs at 4.32 inches for the square cross-section and 1.84 inches for the rectangular cross-section. For the fixed-fixed boundary condition, the transition is at 8.48 inches for the square cross-section and 3.72 inches for the rectangular cross-section.

$$P_{cr} = \frac{\pi^2 EI}{L_{eff}^2} = \frac{\pi^2 EI}{(kL)^2} \tag{10}$$

Equation 10 was used to plot the critical load for each aluminum beam. For the simply supported - simply supported beam, the value of k is 1, while for the fixed-fixed beam, the k value is 0.5.

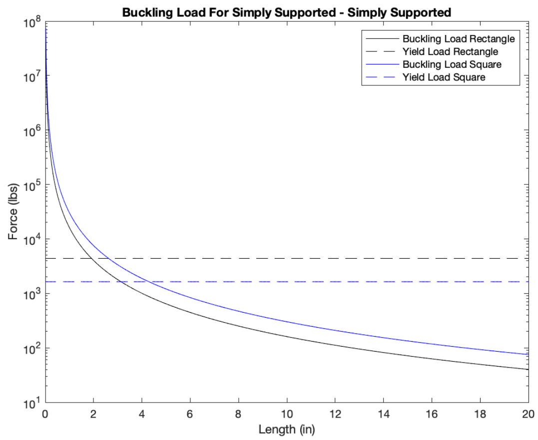

Figure 3: Simply Supported Critical Loads

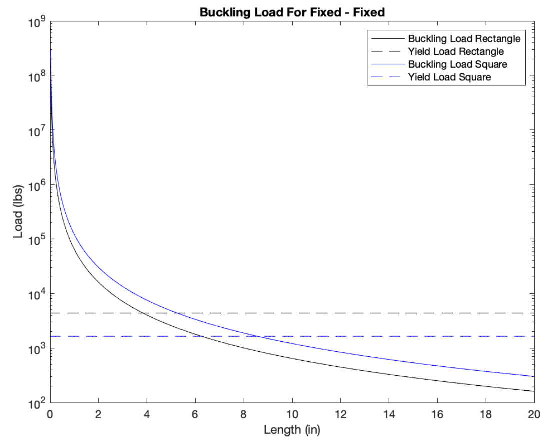

Figure 4: Fixed-Supported Critical Loads

4 Acknowledgments

4.1 Member Contributions

Member	**Contribution**
Nick Barbattini	100%
Matthew Chen	100%
Madison Lin	100%
Rishab Pally	100%
Justin Travis	100%

Signature of Approval: Nick Barbattini, Matthew Chen, Madison Lin, Rishab Pally, Justin Travis

5 Appendix

5.1 Moment of Inertia: Rectangle vs. Square derivation:

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$I_{rectangle} = \frac{1}{12} h^3 w = \frac{1}{12}(.125)^3(1) = \boxed{1.6276 \times 10^{-4}\ lb\ in^2}$$

$$I_{square} = \frac{1}{12} h^3 w - \frac{1}{12}(w-2t)(h-2t)^3$$

$$= \frac{1}{12}(.125)^3(.125) - \frac{1}{12}(.125-2(.0625))(.125-2(.0625))$$

$$= \boxed{3.051 \times 10^{-4}\ lb\ in^2}$$

	Rectangular P_{cr}	Square P_{cr}
Max L	104.4732	195.887
Min L	148.518	278.47

	11.4in
Square	231.76
Rectangular	123.605

5.2 Max Deflection Derivation:

$$V(x) = \delta \sin\left(\frac{\pi x}{L}\right)$$

$$V'(x) \quad \frac{\delta \pi}{L} \cos\left(\frac{\pi x}{L}\right)$$

$$V''(x) = -\frac{\delta \pi^2}{L^2} \sin\left(\frac{\pi x}{L}\right)$$

$$\varepsilon_{max} = -\frac{\delta \pi^2}{L^2} \sin\left(\frac{\pi x}{L}\right) y$$

$$-\frac{\varepsilon_{max} L^2}{\pi^2 \sin\left(\frac{\pi x}{L}\right) y} = \delta$$

$y_{rect} = .125$ [in] $\quad$ $L_{min} = 10.4$

$y_{square} = .0625$ [in] $\quad$ $L_{max} = 12.4$

$$w/\ x = \frac{L}{2}, \quad \delta = -\frac{\varepsilon_{max} L^2}{\pi^2 y}$$

$\varepsilon = .0035$

$\delta_{rect,min} = -.3068$ [in] $\quad$ $\delta_{square,min} = -.6137$ [in]

$\delta_{rect,max} = -.4362$ [in] $\quad$ $\delta_{square,max} = -.8724$ [in]

$\delta_{rect,11.4=L} = -.3687$ [in] $\quad$ $\delta_{square,11.4=L} = -.7374$ [in]

5.3 Buckling Load Matlab Derivation:

```
% Experimental Data Analysis
clear; clc;
close all;

%% Load in data
data1 = load("Rectangular Specimen.mat");
data2 = load("Square Specimen.mat");

Rect = data1.data;
Sq = data2.data;

L = 11; % [in]

%% Process Data
% Convert voltage to loading
Rect(:, 2) = Rect(:, 2) / 0.00237; % Convert to lb
Sq(:, 2) = Sq(:, 2) / 0.00237;

%% Plot Load vs Deflection
% Rectangular Specimen
figure(1);
```

```
hold on;
grid on;
grid minor;
plot(Rect(:, 3), Rect(:, 2), 'LineWidth', 1.5);
title("Rectangular Specimen Deflection");
xlabel("Deflection [in]");
ylabel("Applied load [lb]");
hold off;

% Square Specimen
figure(2);
hold on;
grid on;
grid minor;
plot(Sq(:, 3), Sq(:, 2), 'LineWidth', 1.5);
title("Square Specimen Deflection");
xlabel("Deflection [in]");
ylabel("Applied load [lb]");
hold off;

%% Plot with predicted values
% Calculated critical loadings
PcrRec = 123.6;
PcrSq = 231.76;

%fRec = @(P) sqrt(((P/PcrRec)-1) * (8*L^2 / (pi^2)));
%fSq = @(P) sqrt(((P/PcrSq)-1) * (8*L^2 / (pi^2)));

% Function declaration
fRec = @(v) PcrRec .* (((v.^2 .* pi.^2)./(8 .* L.^2)) + 1);
fSq = @(v) PcrSq .* (((v.^2 .* pi.^2)./(8 .* L.^2)) + 1);

% Set up deflection vector
defl = linspace(0, 2, 33)';

% Calculate associated loadings
PRec = fRec(defl);
PSq = fSq(defl);

% Plot onto respective plots
figure(1);
hold on;
plot(defl, PRec, "LineWidth", 1.5);
xline(0.7374, '--', 'Predicted Plastification', 'LabelVerticalAlignment', 'bottom');
legend("Experimental", "Theoretical", "");
hold off;

figure(2);
hold on;
plot(defl, PSq, "LineWidth", 1.5);
xline(0.3687, '--', 'Predicted Plastification', 'LabelVerticalAlignment', 'bottom');
legend("Experimental", "Theoretical", "");
hold off;
```

5.4 Force on Beam Matlab Derivation:

```
clear; clc; close all;

%Critical Load Predictions

E = 10000000; %psi
Irec = 1.6276*10^-4;
Isq = 3.051*10^-4;
L = linspace(0,20,1000);
Arec = 0.125; %in^2
Asq = 0.046875; %in^2

%Pcrit Pinned Pinned

P_critPPRec = pi^2*E*Irec./(L.^2);
P_critPPSq = pi^2*E*Isq./(L.^2);
YieldRec = 35000*Arec;
YieldSq = 35000*Asq;

figure(1)
a = semilogy(L,P_critPPRec,'k');
hold on
b = semilogy(L, YieldRec*ones(length(L)),'--','Color','k');
hold on
c = semilogy(L,P_critPPSq,'-','Color','b');
hold on
d = semilogy(L, YieldSq*ones(length(L)),'--','Color','b');
title('Buckling Load For Simply Supported - Simply Supported')
legend([a(1) b(1) c(1) d(1)], 'Buckling Load Rectangle','Yield Load Rectangle','Buckling Lo
ylabel('Force (lbs)')
xlabel('Length (in)')


%Pcrit Fixed Fixed

P_critFFRec = pi^2 * E * Irec ./ ((1/2.*L).^2);
P_critFFSq = pi^2 * E * Isq ./ ((1/2.*L).^2);

figure(2)
e = semilogy(L,P_critFFRec,'k');
hold on
f = semilogy(L, YieldRec*ones(length(L)),'--','Color','k');
hold on
g = semilogy(L,P_critFFSq,'b');
hold on
h = semilogy(L, YieldSq*ones(length(L)),'--','Color','b');
title('Buckling Load For Fixed - Fixed')
legend([e(1) f(1) g(1) h(1)], 'Buckling Load Rectangle','Yield Load Rectangle','Buckling Lo
ylabel('Load (lbs)')
xlabel('Length (in)')
```