University of Colorado Boulder

ASEN 3113: Thermodynamics and Heat Transfer

Heat Conduction Prelab

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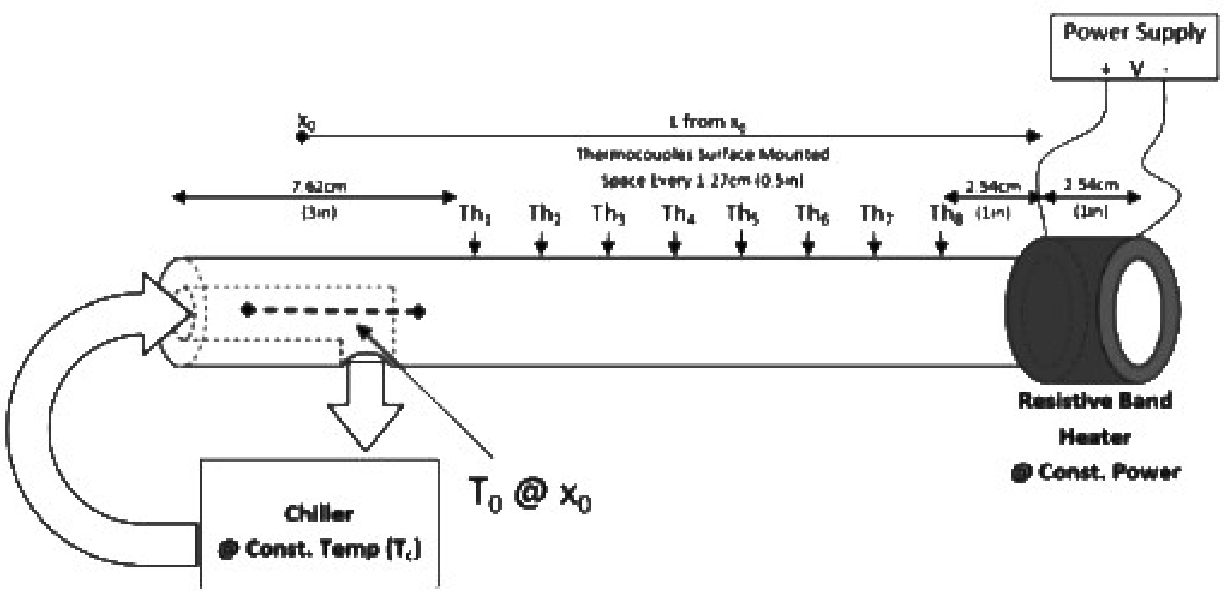

Due: Oct 17, 2022

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1 Question 1: Analytical Assumptions

Assumptions made when deriving the analytical solution/Associated Errors:

1. The initial temperature of the rod stays constant. Error: There could be variations in the surrounding temperature that could directly effect the initial temperature of the rod itself.
2. The rod has axial dimension. Error: this could effect the predicted rate of heat loss. Deformations are not accounted for as well.
3. Perfectly insulated. Error: Not accounting for possible changes in other thermal properties.

2 Question 2: Material Properties

	Aluminum 7075-T651	**Stainless Steel T-303**	**Brass C360**
Density ($\rho(\frac{kg}{m^3})$)	2810	8000	8490
Thermal Conductivity ($k(\frac{W}{m-K})$)	130	16.2	115
Specific Heat Capacity ($c_p(\frac{J}{kg \cdot K})$)	960	500	377
Thermal Diffusivity ($\alpha(\frac{m^2}{s})$)	4.8191 x 10^{-5}	4.05 x 10^{-6}	3.5929 x 10^{-5}

*Note Thermal Diffusivity is calculated by $k/(\rho * c_p)$.

3 Question 3: Experimental Steady State Solution

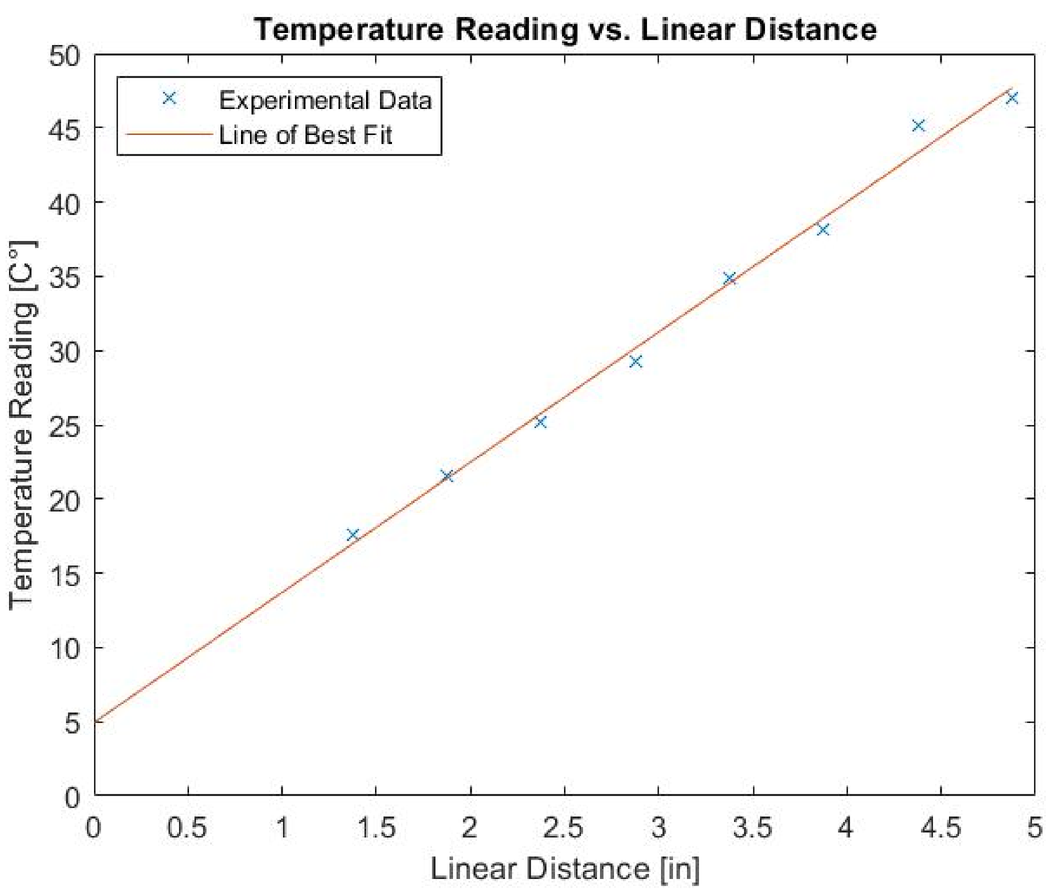

Figure 1

Initial Temperature:
T_0 = 4.9333 °C

Slope:
H = 8.773

4 Question 4: Analytical Transient Derivation

$$b_n = \frac{8HL(-1)^n}{\pi^2(2n-1)^2}$$

Work for the derived equation can be viewed in the Appendix Section 8.1

5 Question 5: Analytical Solution Convergence

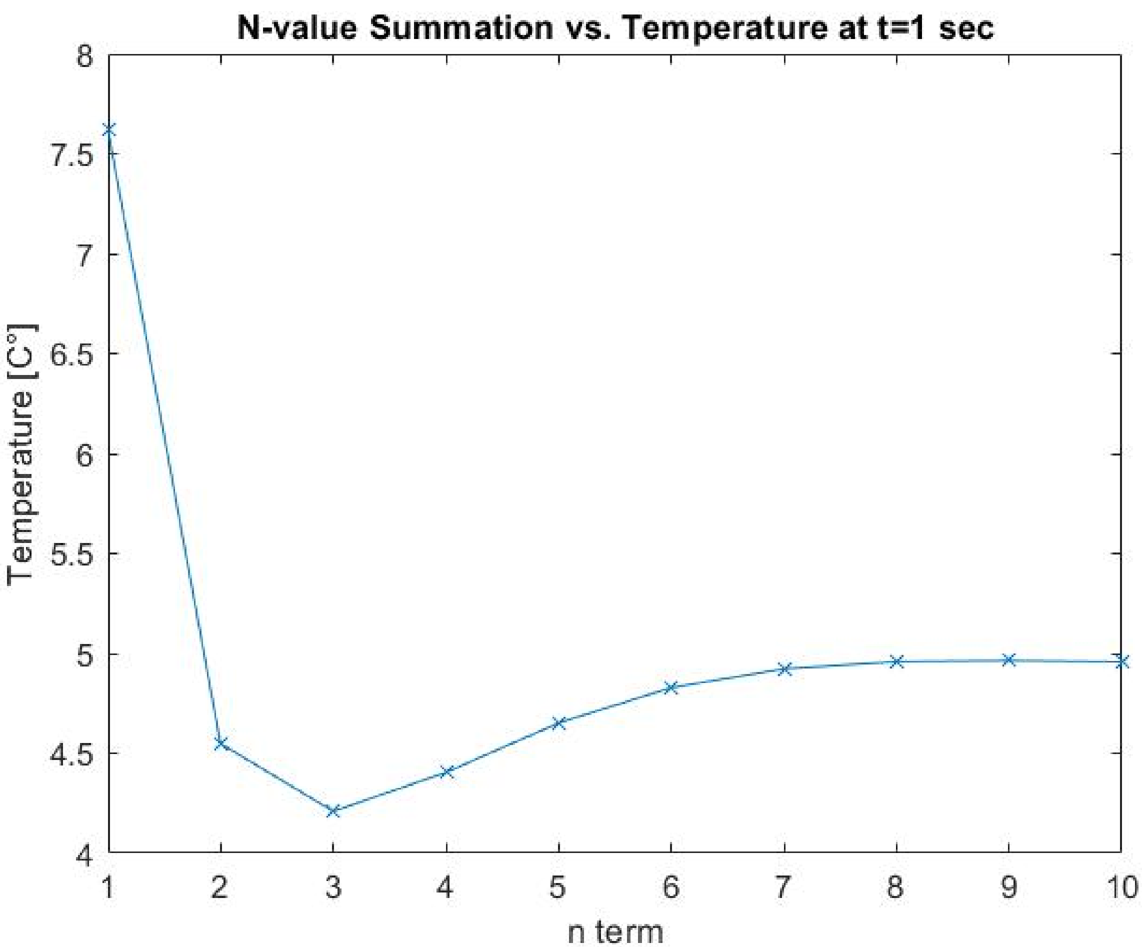

Figure 2

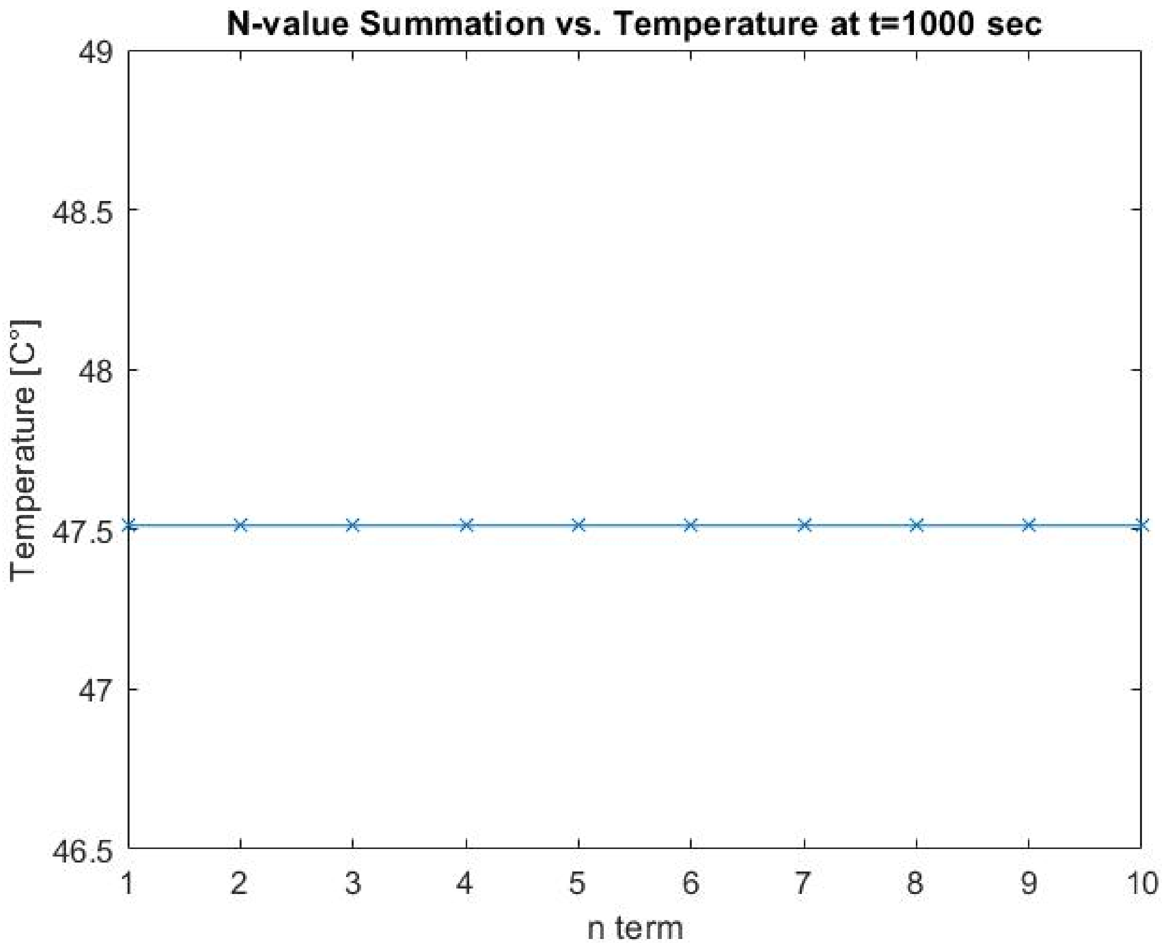

Figure 3

Fourier Number:

$$Fo = \frac{\alpha t}{L^2}$$

The Fourier number for Aluminum 7075-T651 at t=1 sec is .002171.
The Fourier number for Aluminum 7075-T651 at t=1000 sec is 2.171.

The plots above show the temperature of the last thermalcouple. Here we varied the n value and time variable. In Figure 2, time is set to 1 second and in Figure 3 time is set to 1000 seconds. When comparing the plots, we can determine that with higher time values, less terms are required to converge due to the pipe approaching a steady state. Therefore one term is sufficient for the experiment only when considering higher time duration values.

6 Question 6: Sensitivity Study on Diffusivity

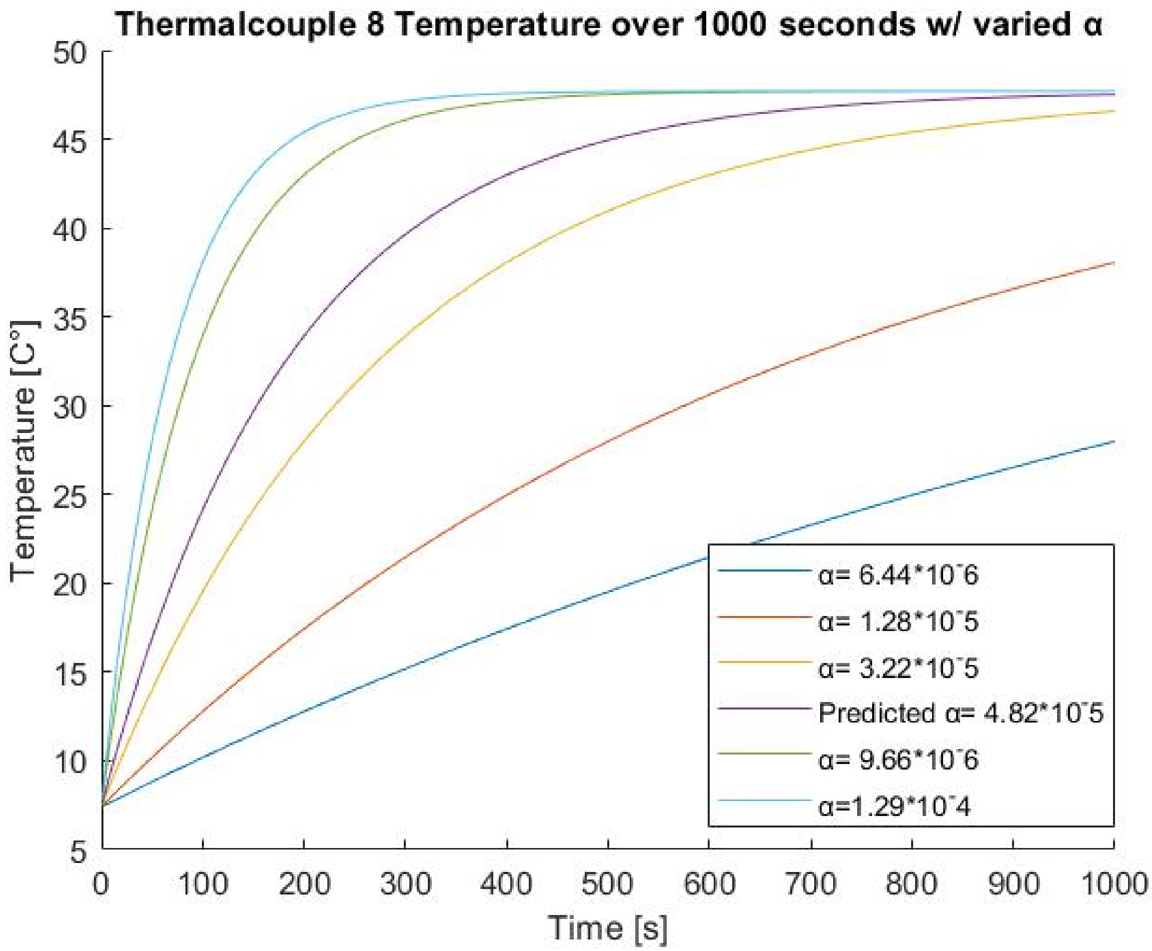

Figure 4

To better understand the shape of the transient model, we varied thermal diffusivity values from $6.44 * 10^{-6} \frac{m^2}{s}$ to $1.29 * 10^{-4} \frac{m^2}{s}$. When analyzing the figure above, we can clearly see that as thermal diffusivity decreases, the transient model tends to become more linear. Whereas for higher thermal diffusivity values, the rod approaches the steady state much faster.

7 References

[1] "The Online Materials Information Resource." MatWeb, https://www.matweb.com/search/DataSheet.aspx?MatGUID=4f19a42be94546b686bbf43f79c51b7dckck=1

[2] "The Online Materials Information Resource." MatWeb, https://www.matweb.com/search/QuickText.aspx?SearchText=Stainless+Steel+T-303+Annealed.

[3] "The Online Materials Information Resource." MatWeb, https://www.matweb.com/search/QuickText.aspx?SearchText=Brass

[4] Edge, Engineers. "Thermal Diffusivity Table." Engineers Edge - Engineering, Design and Manufacturing Solutions,

https://www.engineersedge.com/heat$_{t}ransfer/thermal_{d}iffusivity_{t}able_{1}3953.htm.$

8 Appendix

8.1 Analytical Transient Derivation Work:

$$b_n = \frac{-2H}{L}\int_0^L x\sin(\lambda_n x)\,dx$$

Integration by Parts

$$= \frac{-2H}{L}\left(\frac{-x\cos(\lambda_n x)}{\lambda_n} - \int -\frac{\cos(\lambda_n x)}{\lambda_n}\right)$$

$$= \frac{-2H}{L}\left(\frac{-x\cos(\lambda_n x)}{\lambda_n} + \frac{\sin(\lambda_n x)}{\lambda_n^2}\Bigg]_0^L\right)$$

$$= \frac{-2H}{L}\left(\frac{-L\cos(\lambda_n L)}{\lambda_n} + \frac{\sin(\lambda_n L)}{\lambda_n^2} - 0\right)$$

$$= \frac{-2H}{L}\left(0 + \frac{(-1)^n}{(2\pi n-\pi)^2/4L^2}\right)$$

$$= \frac{-2H}{L}\left(\frac{4L^2(-1)^{n-1}}{(2\pi n-\pi)^2}\right)$$

$$= \frac{-8HL(-1)^{n-1}}{(2\pi n-\pi)^2}$$

$$= \frac{8HL(-1)^n}{(2n-1)^2\pi^2}$$

$$\boxed{b_n = \frac{8HL(-1)^n}{\pi^2(2n-1)^2}}$$

$$\int u\,dv = uv - \int v\,du$$

$u = x$ $\quad du = 1\,dx$

$dv = \sin(\lambda_n x)$

$$v = -\frac{\cos(\lambda_n x)}{\lambda_n}$$

$$\lambda_n = \frac{(2n-1)\pi}{2L}$$

$$\cos\left(\frac{(2n-1)\pi}{2L}L\right) = \cos\left(\pi n - \frac{\pi}{2}\right)$$

$$\sin\left(\frac{(2n-1)\pi}{2L}L\right) = \sin(\pi n - \pi/2)$$

$\sin(\alpha-\beta) = \cos(\beta)\sin(\alpha) - \cos(\alpha)\sin(\beta)$

$\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$

$\cos(\pi n - \pi/2) = \sin(\pi n) = 0$

$\sin(\pi n - \pi/2) = -\cos(\pi n) = (-1)^{n-1}$

$$\lambda_n^2 = \left(\frac{(2n-1)\pi}{2L}\right)^2 = \frac{((2n-1)\pi)^2}{4L^2} = \frac{(2\pi n-\pi)^2}{4L^2}$$

8.2 Matlab Derivation:

8.2.1 Question 3

```
%% Given
tr= [17.60, 21.61, 25.13, 29.22, 34.92, 38.10, 45.21, 47.01];
```

```
ld= [1.375 1.875 2.375 2.875 3.375 3.875 4.375 4.875];
%% polyfit
ht= polyfit(ld,tr,1);
x= [0 1.375 1.875 2.375 2.875 3.375 3.875 4.375 4.875];
y= polyval(ht,x);
initial_temp= y(1);
H= (y(2)-y(1))/(x(2)-x(1));

fprintf('T_0: %0.3f \n',initial_temp)
fprintf('H: %0.3f \n',H)

figure
plot(ld,tr,'x')
hold on
plot(x,y)
title('Temperature Reading vs. Linear Distance')
ylabel('Temperature Reading [C ]')
xlabel('Linear Distance [in]')
legend('Experimental Data','Line of Best Fit')
hold off
```

8.2.2 Question 5 Plot 1

```
t= 1;
L= 5.875;
x= 4.875;
H= 8.773;
initial_temp= 4.933;
eqconstant=8*H*L;
thermal_diff= 0.075;
term=((2*n-1)*pi)/(2*L);
bn=(eqconstant*(-1)^(n))/((2*n-1)*pi)^2;

sum_function= zeros(1,10);
for i= 1:10
    total= 0;
    for j= 1:i
        sineq= sin(term(j)*x);
        lsquare= term(j)^2;
        exponent= exp(-(lsquare)*thermal_diff*t);
        newbn= bn(j);
        product= newbn*sineq*exponent;
        total= total+ product;
    end
    sum_function(i)= total;
end
function_temperature= zeros(1,10);
for i= 1:10
    slope= H*x;
    function_temperature(i)= slope+ sum_function(i)+ initial_temp;
end

figure
plot(1:10,function_temperature, '-x')
hold on
```

```
xlabel('n term')
ylabel('Temperature [C ]')
title('N-value Summation vs. Temperature at t=1 sec')
```

8.2.3 Question 5 Plot 2

```
t= 1000;
L= 5.875;
x= 4.875;
H= 8.773;
initial_temp= 4.933;
eqconstant=8*H*L;
thermal_diff= 0.075;
term=((2*n-1)*pi)/(2*L);
bn=eqconstant*(-1)^(n))/((2*n-1)*pi)^2;

sum_function= zeros(1,10);
for i= 1:10
    total= 0;
    for j= 1:i
        sineq= sin(term(j)*x);
        lsquare= term(j)^2;
        exponent= exp(-(lsquare)*thermal_diff*t);
        newbn= bn(j);
        product= newbn*sineq*exponent;
        total= total+ product;
    end
    sum_function(i)= total;
end
function_temperature= zeros(1,10);
for i= 1:10
    slope= H*x;
    function_temperature(i)= slope+ sum_function(i)+ initial_temp;
end

figure
plot(1:10,function_temperature, '-x')
hold on
xlabel('n term')
ylabel('Temperature [C ]')
title('N-value Summation vs. Temperature at t=1000 sec')
```

8.2.4 Question 6

```
L= 5.875;
x= 4.875;
H= 8.773;
initial_temp= 4.933;
eqconstant=8*H*L;
thermal_diff= [.01, .02, .05, .075, .15, 0.2];
term= pi/(2*L);
bn=(eqconstant*(-1)^(n))/((2*n-1)*pi)^2;


array= cell(1,3,1);
```

```
  sum_function= zeros(1,1000);
  for i= 1:6
     alpha= thermal_diff(i);
     for t= 1:1000
         sineq= sin(term*x);
         lsquare= term^2;
         exponent= exp(-(lsquare)*alpha*t);
         bnvalue= bn(1);
        total= bnvalue*sineq*exponent;
        slope= H*x;
        sum_function(t)= slope+ total+ initial_temp;
     end
     array(1,i,1)= mat2cell(sum_function,1,1000);
 end

x=cell2mat(array(1,1,1));
y=cell2mat(array(1,2,1));
z=cell2mat(array(1,3,1));
m=cell2mat(array(1,4,1));
n=cell2mat(array(1,5,1));
b=cell2mat(array(1,6,1));

figure
hold on
plot(1:1000,x)
plot(1:1000,y)
plot(1:1000,z)
plot(1:1000,m)
plot(1:1000,n)
plot(1:1000,b)
ylabel('Temperature [C  ]')
xlabel('Time [s]')
title('Thermalcouple 8 Temperature over 1000 seconds w/ varied   ')
legend('  = 6.44*10^-6', '  = 1.28*10^-5', '  = 3.22*10^-5', 'Predicted   = 4.82*10^-5', '
```