Static Test Stand Report

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ASEN 2004 - Section 304

This paper provides an analysis on the process for selecting usable data and aids in gaining a better understanding of specific impulse, I_{sp}, and rocket thrust. To analyze the data, we will use a combination of data conditioning and line fitting in order to accurately calculate a value for I_{sp}. We will also be calculating the mean and standard deviation for peak thrust as well as the total time of thrust while analyzing the significance of the standard error of the mean. When selecting usable data, we found the points that satisfied our desired condition. Then, we calculated the I_{sp} values for each data set by fitting a line to the conditioned data, integrating to find the total Impulse (I_t), and dividing this value by the product of the mass of the water and gravity. Simultaneously, we calculated the total time of thrust for each data set and the corresponding mean. This resulted in an average peak thrust value of $267.8N$ and time of thrust of $1.62s$. We acknowledge that the usable data still has sources of error within both the launch stand and data collection method which could effect the way we calculate I_{sp}. To account for this, we used the standard error of the mean to compute the confidence intervals for I_{sp} at 95%, 97.5%, and 99% confidence, respectively.

I. Nomenclature

SEM	=	Standard Error of the Mean
I_{sp}	=	Specific Impulse
I_t	=	Total Impulse
$F(t)$	=	Thrust force as a function of time
m_{prop}	=	Mass of propellant (water)
g_0	=	Acceleration due to gravity
ΔV	=	Change in velocity of rocket
z	=	Statistically derived parameter used to calculate Standard Error of the Mean
$\bar{X}$	=	Sampe mean

II. Introduction and Theory

The static test stand rocket thrust data is measured using two dynamic load cells that measure the force output of the rocket. This data is necessary to calculate the specific impulse, I_{sp}. Total impulse is the change in momentum of the rocket, or the force exerted by the thrust times the time it is applied. Specific impulse is defined as the ratio of total impulse to the mass of the propellant. This represents the "efficiency" of the rocket, or how much thrust can be generated per unit of mass. It is commonly used as a measure of rocket performance and efficiency and can be used to calculate ΔV. This can be used to help model how far a bottle rocket will fly, which is our final goal in this lab.

To calculate Specific Impulse, we must first find an expression for the Total Impulse by integrating the thrust of the rocket over time:

$$I = \int F(t)dt \tag{1}$$

Eq. (1) can then be used to solve for Specific Impulse, I_{sp}:

$$I_{sp} = \frac{I}{m_{prop}g_0} \tag{2}$$

The standard error of the mean (SEM) is an indicator of how accurate a model is. It provides a rough interval in which the sample mean is likely to fall. These intervals can be calculated for different confidence levels, but for this report, those were limited to 95%, 97.5%, and 99%.

III. Materials and Methods

The team used the following experimental setup, sourced from the Static Test Firing Procedure document, to test the thrust of the bottle rocket on the static test stand.* First, the testers verified that the LA procedure was completed correctly. When satisfied that the tank was pressurized, the test stand was set up appropriately, and the VI program (Fig. (1)) was connected to the rocket and running, the testers moved on with preparing the rocket. After adding 1 kg of water to the rocket using a beaker and measuring the exact weight to within 1kg $\pm 2g$, the testers followed steps to mount the rocket to the static test stand.

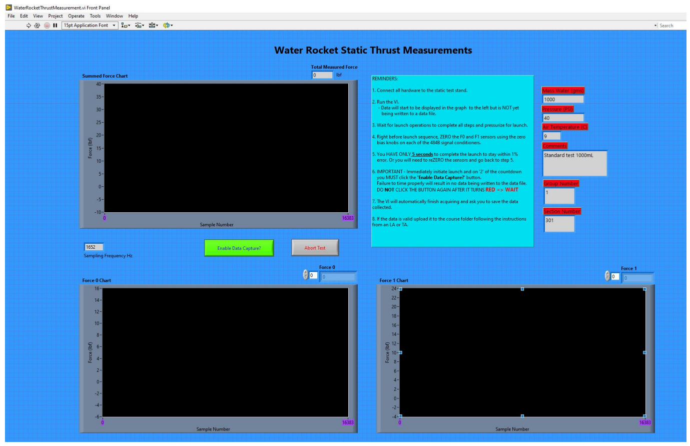


Fig. 1 Screenshot of VI data capture program

The testers adjusted the desired pressure in the air tank regulator, and then raised the clamps on the test stand, removed the center plate on the test stand, and screwed the center plate onto the neck of the bottle (Fig. (2)). They installed the release mechanism and stopper, coating the stopper with soap to ensure a low-friction seal and plugged it into the nozzle of the bottle rocket. They then installed the release bracket and re-attached the bottle and center plate to the test stand, securing it with latches. The bottle was then pressurized to no more than 40 psi using the air tank regulator, checking to ensure the actual pressure was within 1 psi of the desired pressure.

After bubbling in the rocket stopped, the testers cut off and disconnected the air supply, and verified the VI console was working. They continued to follow a checklist, ensuring that safety glasses were worn, pressure was good, and that the load cells were zeroed. The testers performed a final countdown, enabling the VI data capture at $T = -2s$ to ensure the VI was completely zeroed, and pulled the launch cord at $T = 0s$. When finished, the testers cleaned the area and our team prepared to analyze the gathered data.

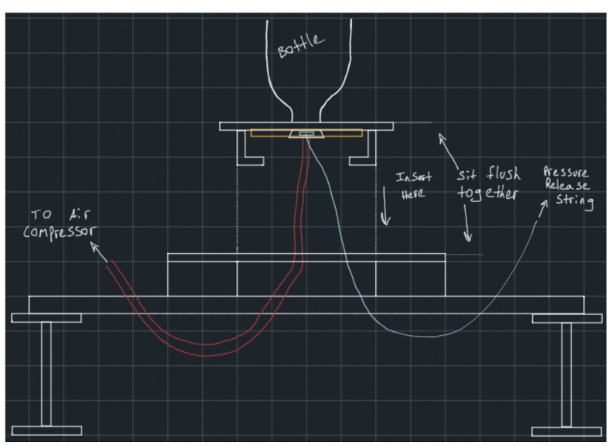


Fig. 2 Sketch of Rocket Model and Components

*Note that this test was performed for us.

Analyzing the data, the thrust is observed to be changing on the VI console before the rocket is launched. This occurrence can be attributed to the experimental setup process, in which the rocket is mounted on the test stand. During this time, while the console is recording, but before the rocket is launched, human inputs on the rocket, for example adjusting the rocket on the test stand and holding the release cord, can be detected by the load cells on the test stand and interpreted as thrust. After the rocket is fired, the thrust curve noticeably fails to return to zero. This is because, at the beginning of the test, the load cells are zeroed out with the bottle rocket full of water. As the rocket expels the water inside, the weight decreases, and, at the end of the test, the missing weight of the water is interpreted as thrust by the load cells. This is corrected for in our analysis. Additionally, the data consistently drops and spikes for all of the tests performed. This is due to vibrations throughout the test stand that occur when the chord is pulled to release the bottle stopper. Since the load cells cannot move horizontally, the abrupt chord pull motion causes vibrations. The load cells then interpret these vibrations as a form of thrust causing the data to have a large number of spikes and dips.

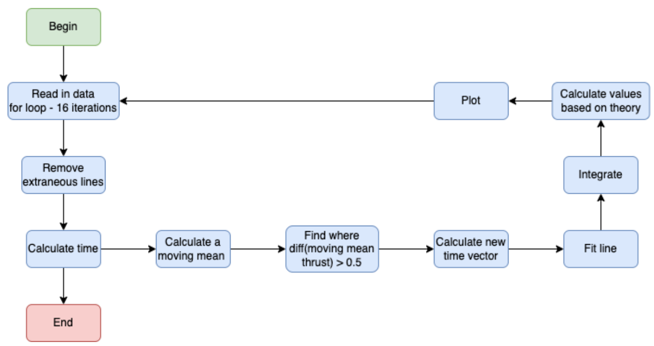


Fig. 3 Seudo code for I_{sp}

IV. Results

A. Representative Force Curve

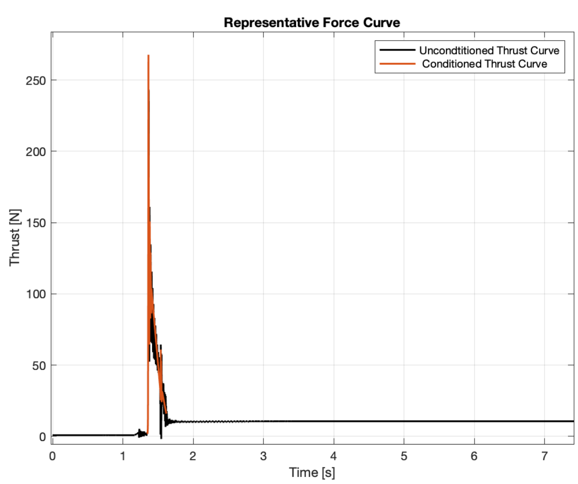


Fig. 4 Initial thrust data and conditioning.

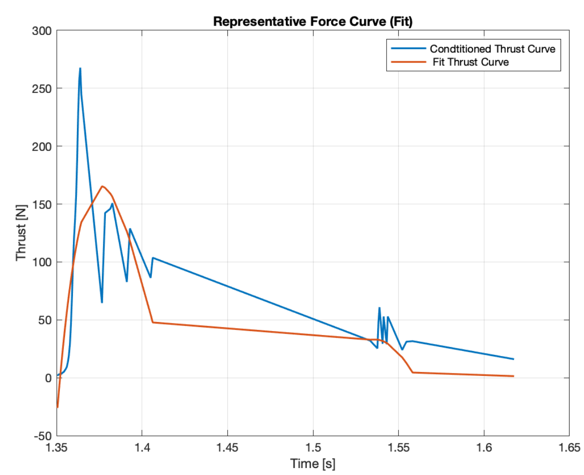


Fig. 5 Final conditioned and fit data.

Figures (3) and (4) show our representative force curves both prior to, during, and after cleaning. Figure (3) on the left shows the approximate points in which we chose to bound the integration (red line). We chose to condition the data such that the points we integrate satisfy the condition that the difference in thrust from one point to the next is greater than 0.5N. We found that this gave us a good estimate as to where the thrust starts and stops. Figure (4) shows the completed process in which we fit a line to the conditioned data in order to get a better numerical approximation for the integration.

Table 1 Calculated Results (abbrev.)

Statistics	I_{sp} [s]	Peak Thrust [N]	Total Thrust Time [s]
Average	1.30	222	1.56
Standard Deviation	0.545	62.2	0.766

Note that we chose to eliminate the data from Trial 12 in our calculations. This data was an outlier with a significant negative thrust that impacted the I_{sp}. This heavily impacted the mean I_{sp} as well. For full table of calculated values, see Appendix A.

B. SEM Analysis

Standard error of the mean essentially shows the accuracy or precision of the data. It describes how accurately the data represents the true mean. SEM provides confidence intervals in which the given percent of the data lies. For example, a 95% confidence interval means that 95% of our data lies within the specified range. This is calculated with the following equation:

$$\bar{X} \pm z \cdot SEM \tag{3}$$

Confidence intervals can be calculated for different ranges of % confidence, which is determined by z. This is useful because the confidence interval shows us an interval that likely contains a large amount of our data. We know that our water bottle rocket likely will not produce numbers exactly in this range, so having a confidence interval allows us to have an idea of how many of our experimental I_{sp}s lie within the desired range.

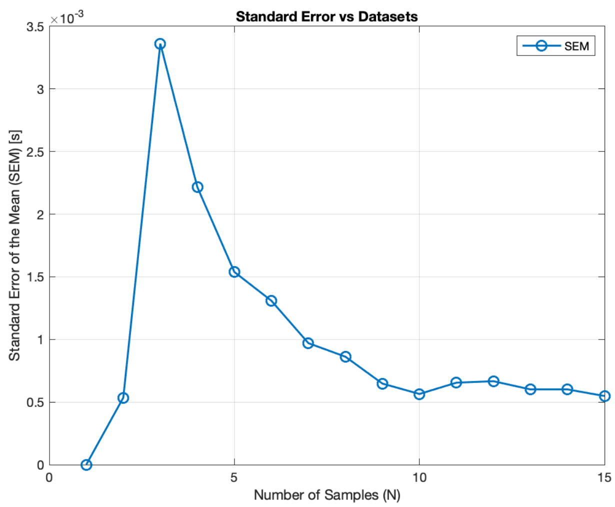


Fig. 6 Standard error of the mean compared to number of data files

Fig. (6) above shows that the SEM decreases as the number of datasets (N) increases. This aligns with general intuition, since more data will generally give a more accurate estimate, as well as mathematically, as seen in the equation for SEM:

$$SEM = \frac{\sigma}{\sqrt{N}} \tag{4}$$

In Eq. (4), σ is standard deviation of I_{sp} and N is the number of samples. Using this equation, we calculated our confidence intervals for 95%, 97.5%, and 99%. They are as follows:

Table 2 Calculated Confidence Intervals

Confidence	z	Interval
95%	1.96	[1.3008 1.3030]
97.5%	2.24	[1.3007 1.3031]
99%	2.58	[1.3005 1.3033]

From the table, we can see that as our confidence increases, the size of our confidence interval also increases. This makes sense because there is a higher chance that the mean of our data lies in a larger range and thus we are more confident in that interval.

In order for the sample mean I_{sp} to be within 0.1 seconds of the of the real mean I_{sp} within 95%, 97.5%, and 99% confidence, 62, 81, and 107 test firings would need to be completed, respectively. To accomplish the same confidence intervals within 0.01 seconds of the real mean I_{sp}, 6147, 8029, and 10651 test firings would need to be completed, respectively. These numbers were calculated using a rearranged version of the latter half of Eq. (3):

$$N = \left(\frac{z\sigma}{r}\right)^2 \tag{5}$$

where z is the statistically derived parameter, σ is the standard deviation, and r is the desired range. This shows that as the number of test fires increases, the error will decrease providing a better estimate, further validating the claim that the SEM will decrease as the number of samples increases.

V. Conclusion

After calculating different statistical values from our data, we found that our average Specific Impulse lies safely within the expected range. Our standard deviation and error values agree both mathematically and logically, and we can be confident in our model to a certain degree. Additionally, we calculated the number of test firings required in order to have a confidence interval of 95% in the specific impulse to be 62 test firings. Since only 16 test firings were actually done, we cannot say that 95% of our data lies within the given confidence interval. In order to achieve that, we would need to complete more tests and gather additional data and therefore we cannot have a very high certainty in our rocket model.

Appendix

A. Table of Values

Table 3 Calculated Results

Trial #	I_{sp} [s]	Peak Thrust [N]	Total Thrust Time [s]
1	1.24	268.8	1.62
2	1.16	245.5	2.01
3	2.21	209.1	2.95
4	1.26	237.1	2.33
5	1.26	218.0	2.69
6	0.95	217.1	1.89
7	1.00	224.6	1.54
8	0.88	230.0	1.36
9	0.97	233.1	0.89
10	1.51	272.7	1.39
11	2.25	250.4	1.53
12	-11.6	252.2	8.47
13	1.75	240.7	1.72
14	1.47	263.3	1.70
15	1.78	222.0	0.34
16	1.15	224.2	1.10

Note that trial 12 was not included in the computation of the mean, standard deviation, and SEM as it is an outlier.

B. Matlab Code

MATLAB code continued on next page

Table of Contents


```
close all
clear
clc

format longG
```

Constants

```
m_prop = 1; %.06852; % [slugs] = 1kg
g0 = 9.81; %32.2; % [ft/s^2]
```

Load data

data1 = load ('test1');

Calculate time and frequency

```
 freq = 1.652 * 1000; % [Hz]
% time = (1 / freq) .* linspace(0, length(data1), length(data1))';


% ------------------------------------------ %
% ------------------GOOD TEST--------------- %
% ------------------------------------------ %
```

fit

```
% means = movmean(data1(:, 3), 10);
% points = find(diff(means) > .05);
% time_new = time(points);
%
% % Polyfit to clean up line
% c1 = polyfit(time(points), data1(points, 3), 8);
% f1 = c1(1) .* time(points).^8 + c1(2) .* time(points).^7 + c1(3) .*
 time(points).^6 + c1(4) .* time(points).^5 + c1(5) .* time(points).^4
 + c1(6) .* time(points).^3 +  c1(7) .* time(points) .^2 + c1(8) .*
 time(points) + c1(9);
%
% % calc slope
```

```
% slope = (data1(points(1), 3) - data1(points(end), 3)) /
 (time(points(1)) - time(points(end)));
% y = slope .* time(points) - slope .* time(points(end)) +
 data1(points(end), 3);
%
% fx = f1 - y;
%
% fitobject = fit(time_new, fx, 'cubicinterp');
%
% I = trapz(time(points), fx);
% I2 = integrate(fitobject, time_new(end), time_new(1));
%
% I_sp = I / (m_prop * g0);
% I_sp2 = I2 / (m_prop * g0);
```

Plotting

plot(time(points), data1(points, 3)); hold on plot(time(points), zeros(1, length(points))); plot(time(points), f1) plot(time(points), y) xlabel('Time [s]') ylabel('Thrust [lbf]')

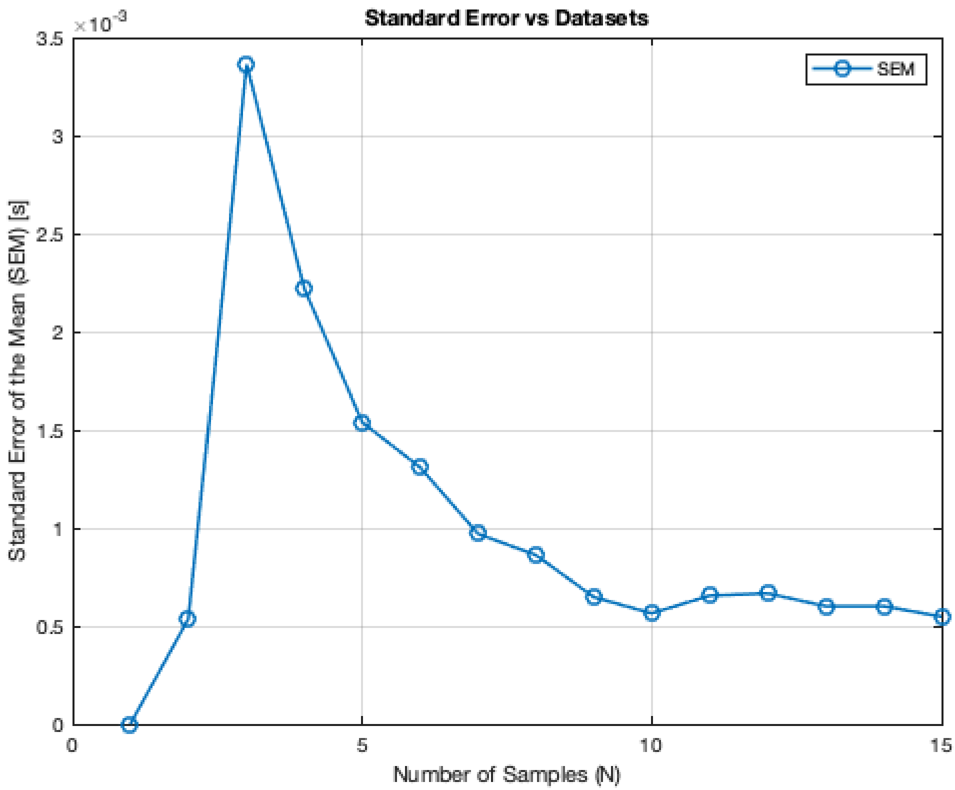
```
% ------------------------------------------ %
% ------------------ALL FILES--------------- %
% ------------------------------------------ %
```

Combine for all files

```
trials = 16;
N = 0;

for v = 1:trials

    % Set to make sure file 12 is not accounted for in calculations
    if v == 12
         I_sp_mean(12) = NaN;
         SEM_I_sp(12) = NaN;
        continue
    end

    %Read in data file v
    filename = sprintf('%s%d','LA_Test_FixedMass_Trial',v);
    inData = readmatrix(filename);

    % Take out first 4 rows
    inData = inData(4:end, :);
    inData(:,3) = inData(:,3) .* 4.4482216; % Converting to N

    time = (1 / freq) .* linspace(0, length(inData), length(inData))';

    means = movmean(inData(:, 3), 20);
    points = find(diff(means) > .5);
    time_new = time(points);

    % Fitting line to thrust data
```

```
   c1 = polyfit(time(points), inData(points, 3), 5);
   f1 = c1(1) .* time(points).^5 + c1(2) .* time(points).^4 +
c1(3) .* time(points).^3 + c1(4) ...
       .* time(points).^2 + c1(5) .* time(points) + c1(6);

   % calculate slope
   slope = (inData(points(1), 3) - inData(points(end), 3)) /
(time(points(1)) - time(points(end)));
   y = slope .* time(points) - slope .* time(points(end)) +
inData(points(end), 3);

   % Zero out line
   fx = f1 - y;

   % Second integration test
   fitobject = fit(time_new, fx, 'cubicinterp');

   % Calculate Impulse, I
   I(v) = trapz(time(points), fx);
   I2(v) = integrate(fitobject, time_new(end), time_new(1));

   % Calculate specific impulse
   I_sp(v) = I(v) / (m_prop * g0);
   I_sp2(v) = I2(v) / (m_prop * g0);

   % I_sp Statistical data
   I_sp_mean = mean(I_sp);
   I_sp_mean2 = mean(I_sp2);

   peak_thrust(v) = max(inData(:,3));
   thrust_time(v) = time_new(end);

   peak_thrust_mean = mean(peak_thrust);
   peak_thrust_std = std(peak_thrust);

   thrust_time_mean = mean(thrust_time);
   thrust_time_std = std(thrust_time);

   % Number of entries considered
   N = N + length(time_new);

   % Standard error of the mean
   SEM_I_sp(v) = std(I_sp(1:v)) / N;

   % Confidence Interval
   CI1 = [I_sp_mean - 1.96 * SEM_I_sp(end), I_sp_mean + 1.96 *
SEM_I_sp(end)]; % 95% CI
   CI2 = [I_sp_mean - 2.24 * SEM_I_sp(end), I_sp_mean + 2.24 *
SEM_I_sp(end)]; % 97.5% CI
   CI3 = [I_sp_mean - 2.58 * SEM_I_sp(end), I_sp_mean + 2.58 *
SEM_I_sp(end)]; % 99% CI

   % Remove 12th Index from plot
   for i = 12:length(SEM_I_sp) - 1
```

```
        SEM_I_sp(i) = SEM_I_sp(i + 1);
    end


      % Manipulate this section depending on desired plots

%     figure()
%     %plot(time, inData(:, 3),'k', 'LineWidth', 1.5)
%     hold on
%     plot(time_new, inData(points, 3), 'LineWidth', 1.5);
%     hold on
%     % plot(time_new, y, 'LineWidth', 1.5);
%     plot(time_new, fx, 'LineWidth', 1.5);
%     xlabel('Time [s]')
%     ylabel('Thrust [N]')
%     legend('Uncondtitioned Thrust Curve', ' Conditioned Thrust
 Curve')
%     % legend('Thrust Force (Unconditioned)', 'Thrust Force
 (Conditioned)')
%     title('Representative Force Curve')
%     grid on
%     hold off


end

dataset = 1:1:15;

% Sem plot
plot(dataset, SEM_I_sp(1:15), '-o', 'MarkerSize', 8, 'LineWidth', 1.5)
grid on
xlabel('Number of Samples (N)')
ylabel('Standard Error of the Mean (SEM) [s]')
title('Standard Error vs Datasets')
legend('SEM')
```



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VI. References

[1] https://www.grc.nasa.gov/www/k-12/airplane/specimp.html, "*Specific Impulse*". 2021

[2] https://www.investopedia.com/ask/answers/042415/what-difference-between-standard-error-means-and-standard-deviation.asp, "*Standard Error of the Mean vs. Standard Deviation: The Difference*". 2022

[3] Mah, J., Rafi, M., Clark, T., Schwartz T., "*Water Rocket Static Test Stand Operating Procedure*". 2022