Q1) Convolution mask — $(w_0, w_1, w_2, \dots, w_6)$.

let f is f_1, f_2, f_3 for example.

Thus convolution will be —

$$
\begin{array}{cccccccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & f_1 & f_2 & f_3 & 0 & 0 & 0 & 0 & 0 \\
w_6 & w_5 & w_4 & w_3 & w_2 & w_1 & w_0 & & & & & & & w_0 f_1 \\
& w_6 & w_5 & w_4 & w_3 & w_2 & w_1 & w_0 & & & & & & w_1 f_1 + w_0 f_2 \\
& & w_6 & w_5 & w_4 & w_3 & w_2 & w_1 & w_0 & & & & & w_2 f_1 + w_1 f_2 + w_0 f_3 \\
& & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & & & & \\
& & & & & & & & w_6 & w_5 & w_4 & w_3 & w_2 \; w_1 & w_0
\end{array}
$$

Thus, if our vector was f_1, f_2, f_3, the convolution we multiply is —

$$
\text{Let } A = \begin{bmatrix}
w_0 & w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & 0 & 0 & \boxed{0} \\
0 & w_0 & w_1 & w_2 & w_3 & \cdots & \cdots & w_6 & 0 & \boxed{0} \\
0 & 0 & w_0 & w_1 & w_2 & \cdots & \cdots & \cdots & w_6 & \boxed{0} \\
0 & 0 & 0 & w_0 & w_1 & \cdots & \cdots & \cdots & \vdots & \boxed{\vdots} \\
0 & 0 & 0 & 0 & w_0 & \cdots & \cdots & \cdots & \vdots & \boxed{\vdots} \\
0 & 0 & 0 & 0 & 0 & w_0 & \cdots & \cdots & \vdots & \boxed{\vdots} \\
0 & 0 & 0 & 0 & 0 & 0 & w_0 & w_1 & w_2 & \boxed{w_3}
\end{bmatrix}
$$

This is not necessary as all these columns are 0

Then our convolution is given by-

$$A \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \text{our desired convolution}$$

Thus A is our desired matrix.

Properties of the matrix -

- It has ~~the~~ 7 rows (as w_6 to w_0) and $6+n$ columns, where n is the number of elements in the image vector f

Date :...............................
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Applications -

This matrix is used in the concept of Fast Fourier transforms.

Fast fourier tranform algorithm is used to find the discrete fourier tranform of an input signal very efficiently in time $O(n \log n)$