Subject ______ Date: __/__/__

MON TUE WED THR FRI SAT SUN

Question 5

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dx\,dy \quad -(1)$$

To prove

$$F^*(u,v) = F(-u,-v) \quad | f(x,y) \text{ is real}$$

Now

Taking conjugate both side in (1)

$$F^*(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[f(x,y) e^{-j2\pi(ux+vy)} \right]^* dx\,dy$$

$$F^*(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) e^{+j2\pi(ux+vy)} \quad -(3)$$

$dx\,dy$ ($f(x,y)$ is real), (so $f^* = f$)

$$F^*(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) e^{-j2\pi((-u)x+(-v)y)} dx\,dy$$

$$F^*(u,v) = F(-u,-v) \quad -(2)$$

part 2 →

given $f(x,y) = f(-x,-y)$

To prove $F(u,v) = F(-u,-v)$

Subject

From eq 3

Now,
$$F^*(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{j2\pi(ux+vy)}\, dx\, dy$$

changing
$$x \to -x$$
$$y \to -y$$

$$F^*(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(-x,-y)\, e^{-j2\pi(ux+vy)}\, dx\, dy$$

Now since f is even so $f(-x,-y) = f(x,y)$

$$F^*(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{-j2\pi(ux+vy)}\, dx\, dy$$

$$F^*(u,v) = F(u,v)$$

Now from part 1 eq 2

$$F^*(u,v) = F(-u,-v) = F(u,v)$$

hence even

& since
$$F^*(u,v) = F(u,v)$$

hence real