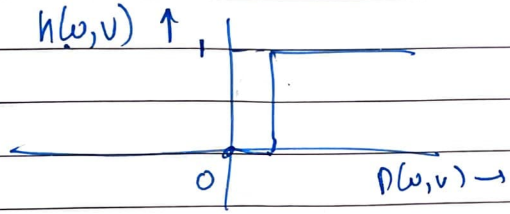
Note:- rect[x] is discrete rectangular function
sinc([x]) is $\frac{\sin[x]}{[x]}$ in the discrete domain.

Q7)

I.D.F.T

The I.D.F.T of the ideal low pass filter described by



has a peak at its center. We need to explain why.

Let us take any one straight line passing through the centre of the low pass filter, and find its IDFT. We can do so, as IDFT and DFT preserves rotation.

We know that the DFT of the rect function (Fig 1) is a sinc function (Fig 2).

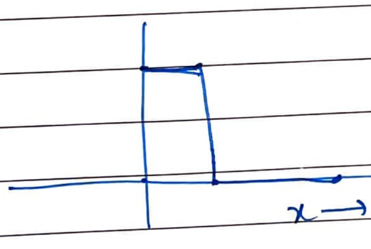
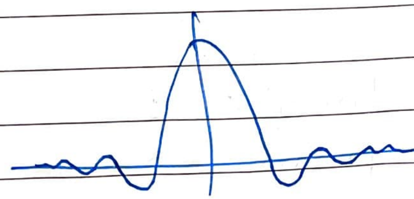
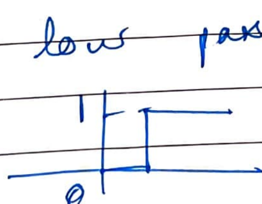


Fig 1



Our low pass filter is described as



Thus it is 1 − graph in Fig 1
$= 1 - rect(x)$.

Now, IDFT and DFT both are linear functions.

∴ $IDFT(-1 \times rect(f)) = -(sinc(\,)) \times \underbrace{k}_{\text{some constant}}$

$IDFT(1 - rect(f))$ =

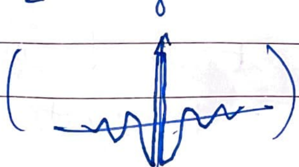
$= IDFT(1) + IDFT(-rect(f))$.

Now, we know that the IDFT of a constant unit signal in the fourier domain is the dirac delta function in the time domain.

∴ ~~IDFT(low pass filter~~

∴ $IDFT(\underbrace{1 - rect(f)}_{\text{low pass filter description on a line passing through center}}) = \delta(x) - k\,sinc(x)$

= Figure 4.53a given in the question



Thus the spike at $x=0$ comes due to the $\delta(x)$ term.

Similarly, the IDFT of a regular Gaussian or butterworth graph would be continuous. But our ~~also~~ low pass filters are ~~1-gaussian~~ discribed as (1 − gaussian in f) for gaussian filter and (1 − butterworth in f) for butterworth filter.

Hence we get a $\delta(x)$ term in the IDFT of those low pass filters too causing a peak at the center in the spacial domain.

~~We got a spike at the center of a single for IDFT of line on~~

The IDFT of a single line through the center gave us a spike at the center of the corresponding single line in the spatial domain. We can take IDFT of all other such lines passing through the center, to get the entire spatial domain picture of the ~~figure~~ figure in 2D, but as they would be superposed on each other, the spike at the center of the image will still remain.