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Q2) a)

Let image $f_1(x,y)$ be the actual image

Let image $f_2(x,y)$ be the same image $f_1(x,y)$ translated by coordinates (x_0, y_0).

$$\therefore f_2(x,y) = f_1(x-x_0, y-y_0)$$

Now, we know that $f(x-x_0, y-y_0) \overset{F}{\longleftrightarrow} e^{-j2\pi(\mu x_0 + \nu y_0)} F(\mu, \nu)$

Thus, we can say that

if $F_2(\xi, \eta)$ is the F.T. of $f_2(x,y)$

& $F_1(\xi, \eta)$ is the F.T. of $f_1(x,y)$,

hence $F_2(\xi,\eta) = e^{-j2\pi(\xi x_0 + \eta y_0)} * F_1(\xi, \eta)$.

Now, we compute the quantity -

$$\frac{F_2^*(\xi,\eta)\, F_1(\xi,\eta)}{|F_2(\xi,\eta)\, F_1^*(\xi,\eta)|} = e^{j2\pi(\xi x_0 + \eta y_0)}$$

cross power spectrum of images f_1 and f_2.

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We take the inverse transform of the above cross power spectrum we calculated for the 2 images.

Hence, we get an impulse, which is approximately 0 everywhere except at the displacement x_0, y_0 that matches the 2 images optimally.

The time complexity to get the DFT of an $N \times N$ image using fast fourier transforms is $O(N \log_2 N)$.

The time taken to calculate the cross power spectrum of these 2 fourier transforms is $O(N^2)$ (as we are multiplying matrices with N^2 elements each).

To take the IDFT, again we will need a time $O(N \log N)$.

Hence we get an overall time complexity of $O(N^2)$.

For finding the pixel by pixel comparison, we will need to compare each of the N^2 pixels of one image with N^2 pixels of the other image.

Hence we will require a time complexity $= O(N^4)$.

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b) Let us say we have an image with both rotation and translation with respect to an initial image.

We thus have,

$$f_2(x,y) = f_1(\underbrace{x\cos\theta_0 + y\sin\theta_0}_{\text{rotation}} - \underbrace{x_0}_{\text{translation}}, -x\sin\theta + y\cos\theta_2 - y_0).$$

now, by rotational property of Fourier transform,

we know that $\cancel{f_1(Ax+By) = AF_1(x)}$

$$F_2(\xi,\eta) = e^{-j2\pi(\xi x_0 + \eta y_0)} \times F_1(\xi\cos\theta + \eta\sin\theta, -\xi\sin\theta_0 + \eta\cos\theta_0)$$

if $M_2(\xi,\eta)$ is magnitude of $F_2(\xi,\eta)$, $M_1(\xi,\eta)$ is magnitude of $F_1(\xi,\eta)$.

we have, $M_2(\xi,\eta) = M_1(\xi\cos\theta_0 + \eta\sin\theta, -\xi\sin\theta + \eta\cos\theta)$.

Hence we have converted this into a pure rotation problem, where $M_2(\xi,\eta)$ is just a rotated version of $M(\xi,\eta)$.

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Now, we can express it similar to the original translation problem, if we convert it to polar coordinates.

Thus, when expressing it in polar form, ~~...~~ we get

$$M_1(\rho, \theta) = M_2(\rho, \theta - \theta_0).$$

(as the magnitude of Fourier transforms are rotated versions of each other).

Thus we take $M_1(\rho, \theta)$ as our f_1 in part (a), and $M_2(\rho, \theta \cancel{\theta_0})$ as our f_2 in part (b).

We do the same procedure as in part (a), to get the θ_0 translation between M_1 and M_2 in polar form.

Thus we find -

$$\mathrm{IDFT}\left(\frac{F(M_1(\rho,\theta))\,F^*(M_2(\rho,\theta))}{|F(M_1(\rho,\theta))|\,|F(M_2(\rho,\theta))|}\right), \text{ and}$$

~~Note~~ the ~~θ_0~~ the location where the above quantity is NOT approximately 0, gives us our rotation angle θ_0, between our 2 original images.