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MON TUE WED THU FRI SAT SUN

Q2) $V(x,y) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} x^i y^j$

Now, we have intensities of the 16 closest neighbours of that point.

Let the 4 closest neighbours be at (0,0), (0,1), (1,0) and (1,1).

$\rightarrow$ $V(0,0) = a_{00}$ = intensity at (0,0).
$V(0,1)$ = intensity at $(0,1) = a_{00} + a_{01} + a_{02} + a_{03}$
$V(1,0) = \ldots$
$V(1,1) = \ldots$
Let f be my intensity at those points.

Let us find the gradient f_x at (0,0), (0,1)...
$\left(f_x(a,b) = \frac{f(a+1,b) - f(a-1,b)}{2}\right)$

We get fixed values for every gradient as we know our intensities.

$\rightarrow$ $f_x(0,0) = \frac{\partial V(0,0)}{\partial x}$ $\longrightarrow$ 1 equation

similarly for $f_x(0,1)$ etc and $f_y(0,0), f_y(0,1)$ etc.

Thus we get 8 equations using the first derivatives f_x & f_y.

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→ Now let us look at the mixed derivatives

(mixed derivative $f_{xy}(a,b) = \frac{f_x(a,b+1) - f_x(a,b-1)}{2}$)

Thus $f_{xy}(0,0) = \frac{\partial^2 v(0,0)}{\partial x \partial y}$ → 1 equation

similarly for $f_{xy}(0,1)$, $f_{xy}(1,0)$ & $f_{xy}(1,1)$.

Thus we now have a total of $4+8+4 = 16$ equations.

Hence if these equations have coefficients for a_{00}, a_{01}, a_{10} etc as $a_1, b_1, c_1, a_2, b_2, c_2$ etc,

we have:

$$\underbrace{\begin{bmatrix} a_1 & b_1 & c_1 & \cdots & p_1 \\ a_2 & b_2 & c_2 & \cdots & p_2 \\ \vdots & & & & \vdots \\ a_{16} & b_{16} & c_{16} & \cdots & p_{16} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a_{00} \\ a_{01} \\ a_{10} \\ \vdots \end{bmatrix}}_{V} = \underbrace{\begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_{16} \end{bmatrix}}_{\text{all known}} \to K$$

We can get V as -

$$AV = K$$

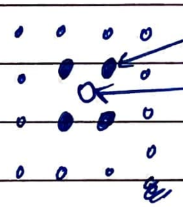
$$\therefore V = A^{-1}K.$$

Hence we can find all 16 coefficients by this process.

Why do we need 16 points ?

We need the 4 nearest neighbours intensity for the ~~the~~ first 4 equations.

We also need the derivatives of the intensity (i.e. f_x, f_y, f_{xy}) at all 4 neighbouring points. In order to calculate this, we need all of the neighbours of these 4 points too. Hence we need a total of 16 points (the 4 neighbouring points and their neighbours) to carry out cubic interpolation.



Also, as our equation has 16 unknown coefficients, we need 16 equations to solve for all ~~of~~ them, hence we need intensity of 16 points.