Q2. Consider the barbara256.png image from the homework folder. Implement the following in MATLAB: (a) an ideal low pass filter with cutoff frequency D ∈ {40, 80}, (b) a Gaussian low pass filter with σ ∈ {40, 80}. Show the effect of these on the image and display all filtered images in your report. Display the frequency response (in log absolute Fourier format) of all filters in your report as well. Comment on the differences in the outputs. Also display the log absolute Fourier transform of the original and filtered images. Comment on the differences in the outputs. Make sure you perform appropriate zero-padding while doing the filtering!

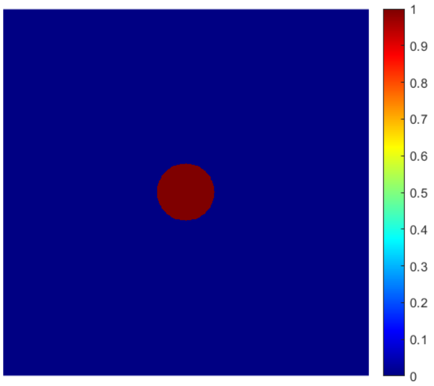
Ans.

a) For part a), Ideal Low pass filter was created by defining u and v (512x1 with zero at 256th entry) and creating its mesh grid. Then filter was created by making all positions in the mesh grid where $u^2+v^2 \leq D^2$ (here D is 0) 1 else 0. Image was read and zero padded to double its size (512x512). Then Fourier transform was calculated and shifted to make its center as zero. This was multiplied by the ideal low pass filter and inverse Fourier transform was calculated after shifting it accordingly. Finally, the middle 256x256 was taken as the final image.
The original and the final image is given below:

Original

Ideal Low Pass filter with D = 40

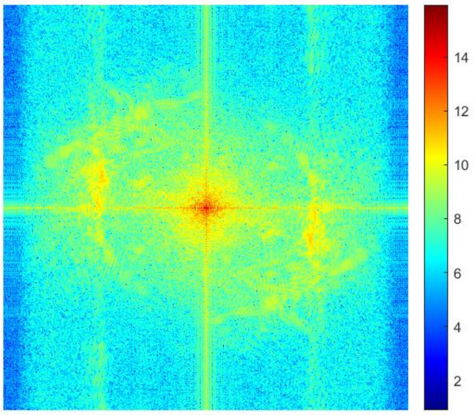
We see that blurring effecct has been achieved by Ideal low pass filter. Also, there are ringing artifacts in the final image if we use Ideal low pass filter.
The Fourier transform of the ideal low pass filter is given:

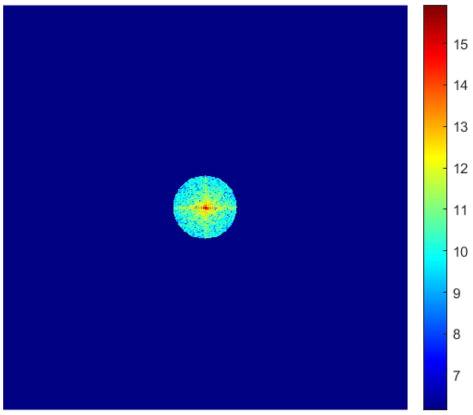


Ideal Low Pass filter with D = 40

As we expect, $u^2+v^2 \leq D^2$ is an equation of a circle, which is what we see in the frequency domain.

The Fourier transform of these images are given as well:





Original

Ideal Low Pass filter with D = 40

As we expect, multiplying the Image with ideal low pass filter in frequency domain retains the original image only in the circle of $u^2+v^2 \leq D^2$.

b) Same as before but D = 80.

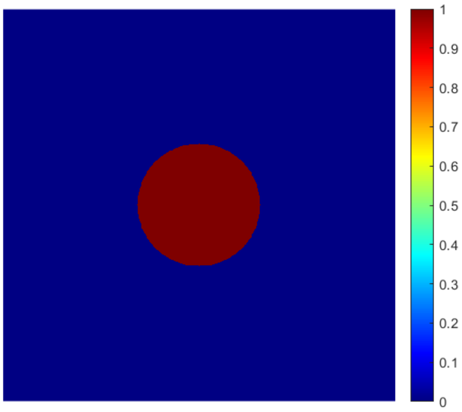
The original and the final image is given below:

Original

Ideal Low Pass filter with D = 80

We see that the blurring effect is reduced, which means blurring is inversely proportional to D. The ringing artifacts are not removed however.

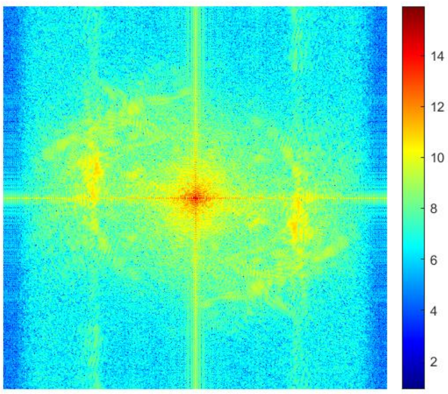
The Fourier transform of the ideal low pass filter is given:



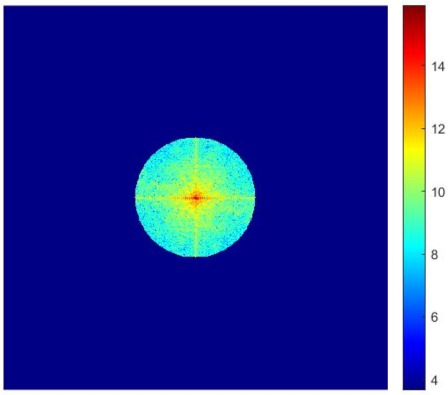
Ideal Low Pass filter with D = 80

The circle here is bigger as D is 80 here instead of 40.

The Fourier transform of these images are given as well:



Original



Ideal Low Pass filter with D = 80

As we expect, everything in the circle $u^2+v^2 \leq D^2$ is retained, rest is 0.

c) Here, a gaussian filter was implemented as $e^{\frac{-(u^2+v^2)}{2\sigma^2}}$ (here $\sigma = 40$). Rest is same as before.

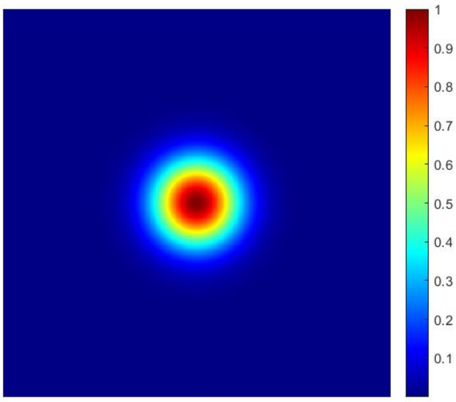
The original and the final image is given below:

Original

Gaussian FIlter with σ = 40

We see that effect of blurring has been achieved and the ringing artifacts are also gone. Ringing artifacts were due to non-differentiability of ideal low pass filter, which is now removed in the Gaussian Filter.

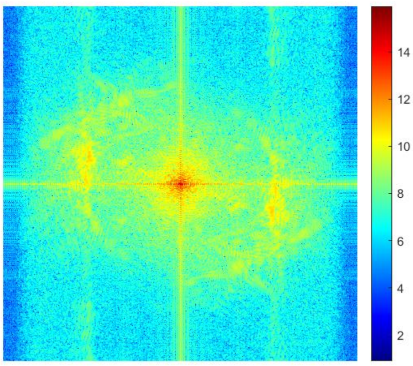
The Fourier transform of the Gaussian filter is given:



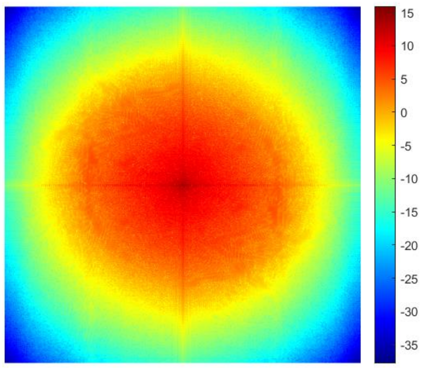
Gaussian FIlter with σ = 40

This filter in frequency domain does not abruptly jump from 1 to 0 as it was in the case of ideal low pass filter. IT instead gradually dies down to zero with higher u and v.

The Fourier transform of these images are given as well:



Original



Gaussian FIlter with σ = 40

We see that Gaussian filter retains the image outside the circle, but the magnitude gradually dies down to zero.

d) Same as before with σ = 80.

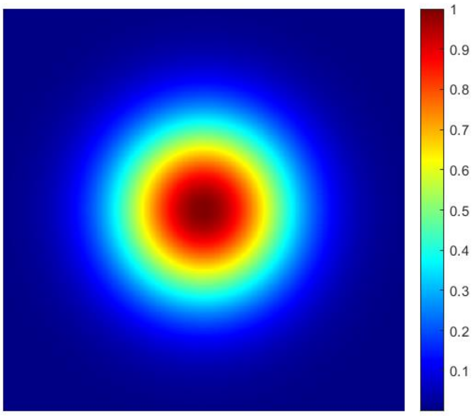
The original and the final image is given below:

Original

Gaussian Filter with σ = 80

We see that the blurring effect has reduced. Hence, blurring effect is also inversely proportional to σ.

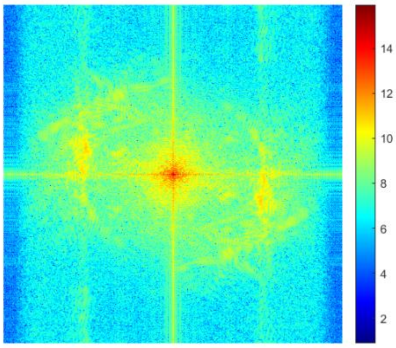
The Fourier transform of the ideal low pass filter is also given:

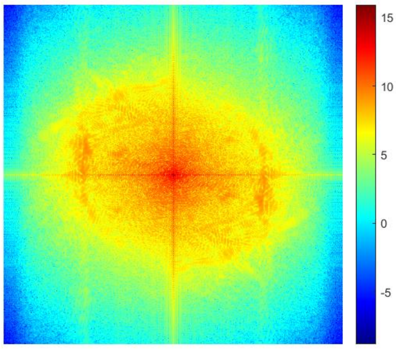


Gaussian Filter with σ = 80

As we expect, the spread here is bigger than what it was with σ = 40 as Gaussian flter dies down to zero more slowly.

The Fourier transform of these images are given as well:





Original

Gaussian Filter with σ = 80

We see that the spread of the resultant is more according to the scale, which is what we expect as the spread of the Gaussian filter is more for σ = 80.