

In all these questions, my variables are  $x_0, x_1, x_2, \dots$  instead of  $x_1, x_2, x_3, \dots$ . So, the free variables in my output might look different from that in yours:)

3  
4  
1 -1 -1 3  
1 1 -2 1  
4 -2 4 1

```
PIVOT POSITION:
[(0, 0), (1, 1), (2, 2)]
RREF:
1.0  0.0  0.0  0.5
0.0  1.0  0.0 -1.5
0.0  0.0  1.0 -1.0

SOLUTION:
[0, 0, 0, 0]+x_3*[-0.5, 1.5, 1.0, 1]
```

3  
3  
3 5 -4  
-3 -2 4  
6 1 -8

```
PIVOT POSITION:
[(0, 0), (1, 1)]
RREF:
1.0  0.0  -1.3333333333333333
0.0  1.0  0.0
0.0  0.0  0.0

SOLUTION:
[0, 0, 0]+x_2*[1.3333333333333333, 0.0, 1]
```

3  
5  
1 -2 -1 3 0  
-2 4 5 -5 3

3 -6 -6 8 2

```
PIVOT POSITION:
[(0, 0), (1, 2), (2, 4)]
RREF:
1.0 -2.0 0.0 3.3333333333333335 0.0
0.0 0.0 1.0 0.3333333333333333 0.0
0.0 0.0 0.0 0.0 1.0

SOLUTION:
[0, 0, 0, 0, 0]+x_1*[2.0, 1, 0.0, 0, 0.0]+x_3*[-3.3333333333333335, 0, -0.3333333333333333, 1, 0.0]
```

4

3

1 0 0

0 2 0

0 0 1

0 1 3

```
PIVOT POSITION:
[(0, 0), (1, 1), (2, 2)]
RREF:
1.0 0.0 0.0
0.0 1.0 0.0
0.0 0.0 1.0
0.0 0.0 0.0

SOLUTION:
[0, 0, 0]
```

3

6

1 6 2 -5 -2 -4

0 0 2 -8 -1 3

0 0 0 0 1 7

```
PIVOT POSITION:
[(0, 0), (1, 2), (2, 4)]
RREF:
1.0 6.0 0.0 3.0 0.0 0.0
0.0 0.0 1.0 -4.0 0.0 5.0
0.0 0.0 0.0 0.0 1.0 7.0

SOLUTION:
[0, 0, 0, 0, 0, 0]+x_1*[-6.0, 1, 0.0, 0, 0.0, 0]+x_3*[-3.0, 0, 4.0, 1, 0.0, 0]+x_5*[0.0, 0, -5.0, 0, -7.0, 1]
```

4

5

0 0 2 0 4

0 8 0 0 1

8 0 9 0 3

80051

```
PIVOT POSITION:
[(0, 0), (1, 1), (2, 2), (3, 3)]
RREF:
1.0  0.0  0.0  0.0  -1.875
0.0  1.0  0.0  0.0  0.125
0.0  0.0  1.0  0.0  2.0
0.0  0.0  0.0  1.0  3.2

SOLUTION:
[0, 0, 0, 0, 0]+x_4*[1.875, -0.125, -2.0, -3.2, 1]
```

3

3

003

056

089

```
PIVOT POSITION:
[(0, 1), (1, 2)]
RREF:
0.0  1.0  0.0
0.0  0.0  1.0
0.0  0.0  0.0

SOLUTION:
[0, 0, 0]+x_0*[1, 0.0, 0.0]
```

5

7

003568-9

0562103

18-91570

08-91570

58-91570

```

PIVOT POSITION:
[(0, 0), (1, 1), (2, 2), (3, 3)]
RREF:
1.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  1.0  0.0  0.0  0.05556 -0.05556  0.83333
0.0  0.0  1.0  0.0  -0.34954 -0.6088  0.50694
0.0  0.0  0.0  1.0  1.40972  1.96528 -2.10417
0.0  0.0  0.0  0.0  0.0  0.0  0.0

SOLUTION:
[0, 0, 0, 0, 0, 0, 0]+x_4*[0.0, -0.05556, 0.34954, -1.40972, 1, 0, 0]+x_5*[0.0, 0.05556, 0.6088, -1.96528,
0, 1, 0]+x_6*[0.0, -0.83333, -0.50694, 2.10417, 0, 0, 1]

```

.....

5  
9  
0 0 3 5 6 8 -9 11 4  
0 0 6 2 1 0 3 2 1  
1 8 -9 1 5 7 0 -6 -1  
0 0 -9 1 5 7 0 14 -1  
5 0 -9 1 5 7 0 3 0

```

PIVOT POSITION:
[(0, 0), (1, 1), (2, 2), (3, 3), (4, 4)]
RREF:
1.0  0.0  0.0  0.0  0.0  0.0  0.0 -2.2  0.2
-0.0  1.0  -0.0  -0.0  -0.0  -0.0  -0.0 -2.225 -0.025
0.0  0.0  1.0  0.0  0.0  -0.95833  5.75  1.54167 -1.0
0.0  0.0  0.0  1.0  0.0  3.375 -23.25 -7.125  5.0
0.0  0.0  0.0  0.0  1.0  -1.0  15.0  7.0 -3.0

SOLUTION:
[0, 0, 0, 0, 0, 0, 0, 0, 0]+x_5*[0.0, 0.0, 0.95833, -3.375, 1.0, 1, 0, 0, 0]+x_6*[0.0, 0.0, -5.75, 23
.25, -15.0, 0, 1, 0, 0]+x_7*[2.2, 2.225, -1.54167, 7.125, -7.0, 0, 0, 1, 0]+x_8*[-0.2, 0.025, 1.0, -5
.0, 3.0, 0, 0, 0, 1]

```

.....

To test more test cases, you may use the following website:

<https://planetcalc.com/8328/>