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Roll NO- 2020569

SML Assignment-2 (Theory)

Part B

Date

DELTA Pg No

Ques 1

(a)

$$\text{Dataset} = \begin{bmatrix} 1 & 3 \\ 4 & 7 \end{bmatrix}$$

$$X^T = \begin{bmatrix} 1 & 4 \\ 3 & 7 \end{bmatrix}$$

$$\text{Mean} = \begin{bmatrix} (1+4)/2 \\ (3+7)/2 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 5 \end{bmatrix}$$

~~Cov(x, x) = (1-2.5)^2 + (4-2.5)^2~~

$$Cov(x, x) = \frac{(1-2.5)^2 + (4-2.5)^2}{2-1}$$

$$= 2.25 + 2.25 = 4.5$$

$$Cov(y, y) = \frac{(3-5)^2 + (7-5)^2}{2-1}$$

$$= 4 + 4 = 8$$

$$Cov(x, y) = Cov(y, x) = \frac{(1-2.5)(3-5) + (1-2.5)(7-5) + (4-2.5)(3-5) + (4-2.5)(7-5)}{2-1}$$

$$= \cancel{3} + \cancel{(-3)} + \cancel{(-3)} + \cancel{(3)} = 0$$

$$\text{Cov Matrix, } C = \begin{bmatrix} cov(x,x) & cov(x,y) \\ cov(y,x) & cov(y,y) \end{bmatrix}$$

$$C = \begin{bmatrix} 4.5 & 0 \\ 0 & 8 \end{bmatrix}$$

Scanned with CamScanner

$$Cx = \lambda x$$

$$Cx - \lambda x = 0$$

$$(C - \lambda I)x = 0$$

Let $C - \lambda I = A$

$$Ax = 0$$

$$A = \begin{bmatrix} 4.5 & 0 \\ 0 & 8 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4.5-\lambda & 0 \\ 0 & 8-\lambda \end{bmatrix}$$

<u>Characteristic Eq^n :</u>

$$|A| = 0$$

$$(4.5 - \lambda)(8 - \lambda) = 0$$

$4.5 - \lambda = 0$ or $8 - \lambda = 0$

$\lambda = 4.5$ $\quad$ $\lambda = 8$

So, value $\lambda = 4.5, 8$

Scanned with CamScanner

when $\lambda = 4.5$

$$\begin{bmatrix} 4.5 - 4.5 & 0 \\ 0 & 8 - 4.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 3.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x_1 \rightarrow$ free $\Rightarrow$ $x_1 = x_1$

$3.5 x_2 = 0$ $\quad x_2 = 0 . x_1$

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} x_1$$

~~eigen vector~~ ↳ eigen vector corresponding to $\lambda = 4.5$

when $\lambda = 8$

$$\begin{bmatrix} 4.5 - 8 & 0 \\ 0 & 8 - 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3.5 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$3.5 x_1 = 0$ $\Rightarrow$ $x_1 = 0 . x_2$

$x_2 \rightarrow$ free $\quad x_2 = x_2$

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} x_2$$

↳ eigen vector corresponding to $\lambda = 8$

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pca weight matrix $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Ques 2 x_1 and x_2 are given data matrices

$y_1 = \omega^T x_1$
$y_2 = \omega^T x_2$

$y = [y_1 \ y_2] \in \{0,1\}^n$

$P(y|\theta) = \prod_{i=1}^{N} \theta^{y_i}(1-\theta)^{1-y_i}$

$\ln P(y|\theta) = \sum_i y_i \ln\theta + (1-y_i)\ln(1-\theta)$

$= \sum_i \omega^T x_i \ln\theta + (1-\omega^T x_i)\ln(1-\theta)$

x_i is the i^{th} column of matrix $X = [x_1 \ x_2]$

Priors on ω →

$p(\omega) \propto e^{-\omega^T\omega/2}$

For MAP :

$\max_{\omega} \sum_i \omega^T x_i \ln\theta + (1-\omega^T x_i)\ln(1-\theta) - \frac{1}{2}\omega^T\omega$

taking derivative of above $[\because \omega^T\mu_1 = \omega^T\mu_2]$

$$\sum_i \{x_i \ln\theta - x_i \ln(1-\theta)\} - w - \lambda(\mu_1-\mu_2) = 0$$

$$w = \sum_i x_i \ln\left(\frac{\theta}{1-\theta}\right) - \lambda(\mu_1-\mu_2)$$

$$w = \frac{N}{2}(\mu_1+\mu_2)\ln\left(\frac{\theta}{1-\theta}\right) - \lambda(\mu_1-\mu_2)$$

$$\left[\because \sum_i x_i = x_1 + \dots + x_{N/2} + x_{N/2} + \dots + x_N\right.$$

$$= N/2\left[\frac{(x_1+\dots+x_{N/2})}{N/2} + \frac{(x_{N/2+1}+\dots+x_N)}{N/2}\right]$$

$$= N/2\,[\mu_1+\mu_2]$$

$$\boxed{w = \frac{N}{2}(\mu_1+\mu_2)\ln\left(\frac{\theta}{1-\theta}\right) - \lambda(\mu_1-\mu_2)}$$

Ques 3 Given $p(y|\omega_2) = \frac{1}{z} e^{-1/2 (y-\mu^T U)^T (y-\mu^T U)}$

$p(y|\omega_3) = \frac{1}{z} . e^{-1/2 (y+\mu^T U) (y+\mu^T U)}$

prior $\cdot p(y) = \frac{1}{z} . e^{-y^T y}$

Goal : $\max_U p(y|\omega_2) \cdot p(y|\omega_3) p(y)$

$= \max_U \ln p(y|\omega_2) p(y|\omega_3) p(y)$

$= \max_U \left[-\frac{1}{2}(y-\mu^T U)^T (y-\mu^T U) - \frac{1}{2}(y+\mu^T U)^T (y+\mu^T U) - y^T y \right]$

Now, Applying FDA condition

$\max_U U^T S_B U$ and $U^T U = I$

$\max_U (y-\mu^T U)^T (y-\mu^T U) - (y+\mu^T U)^T (y+\mu^T U) - y^T y + U^T S_B U - \lambda (U^T S_\omega U - 1)$

$= \max_U U^T (S_B - \mu \mu^T) U - \lambda (U^T S_\omega U - 1)$

Take Derivative of above

$\frac{\partial F(U)}{\partial U} = 0$

$\Rightarrow (S_B - \mu \cdot \mu^T) U - \lambda S_\omega U = 0$

$(S_B - \mu \mu^T) U = \lambda S_\omega U$