EE 338: Digital Signal Processing

Filter Design Assignment

Student Name	*Roll Number*
Rishav Ranjan	180070045

Contents

1 Student Details

Name : Rishav Ranjan
Roll Number : 180070045
Filter Number : 56
Group Number : 2
Group Members :
1) Garaga V V S K Vamsi
2) Shubham Kar

2 Bandpass Filter

2.1 Un-normalized Discrete Time Filter Specifications

Filter number = 56
Corresponding value of $m = 56$
$\therefore \mathrm{q}(m) = \lfloor 0.1m \rfloor = 5$
$\therefore \mathrm{r}(m) = m - 10\,\mathrm{q}(m) = 6$
$B_L(m) = 25 + 1.7\,\mathrm{q}(m) + 6.1\,\mathrm{r}(m) = 70.1$
$B_H(m) = B_L(m) + 20 = 90.1$

The first filter is a **bandpass** filter with passband from $B_L(m)$ kHz to $B_H(m)$ kHz. Therefore the filter specifications are:

- **Passband**: 70.1 kHz to 90.1 kHz
- **Transition band**: 4 kHz on either side of passband.
- **Stopband**: 0–66.1 kHz and 94.1–165 kHz ($\because$ Sampling frequency = 330 kHz)
- **Tolerance**: 0.15 in magnitude in both passband and stopband
- **Passband nature**: Monotonic
- **Stopband nature**: Monotonic

2.2 Normalized Discrete Time Filter Specifications

$$\text{Sampling rate} = f_{\text{sample}} = 330\,\text{kHz}$$

Any analog frequency f upto half the sampling frequency can be represented on the normalized frequency axis by:

$$\omega = \frac{f}{f_{\text{sample}}} \cdot 2\pi$$

Therefore the normalized discrete time filter specifications can be given as follows:

- **Passband**: 1.3347 to 1.7154
- **Transition band**: 0.076 on either side
- **Stopband**: 0–1.2587 and 1.7914–π
- **Tolerance**: 0.15 in magnitude in both passband and stopband
- **Passband nature**: Monotonic
- **Stopband nature**: Monotonic

2.3 Analog Filter Specifications using the Bilinear Transform

The bilinear transform on normalized frequencies is given by:

$$\Omega = \tan\left(\frac{\omega}{2}\right)$$

Therefore the corresponding analog filter specifications for the same type of analog filter using the bilinear transformation are:

- **Passband**: 0.7879 (Ω_{1P}) to 1.1562 (Ω_{2P})
- **Transition band**: 0.7281 to 0.7879 and 1.1562 to 1.2491
- **Stopband**: 0 to 0.7281 (Ω_{1S}) and 1.2491 (Ω_{2S}) to ∞
- **Tolerance**: 0.15 in magnitude in both passband and stopband
- **Passband nature**: Monotonic
- **Stopband nature**: Monotonic

2.4 Frequency Transformation and Relevant Parameters

We want to design a bandpass analog filter by transforming an appropriately designed lowpass analog filter. For that we require the equivalent lowpass analog filter specifications. The lowpass analog filter frequency transformation is given by:

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega}$$

where:

$$B = \Omega_{2P} - \Omega_{1P} = 0.368$$
$$\Omega_0 = \sqrt{\Omega_{1P}\Omega_{2P}} = 0.9544$$

The critical points in this transformation are given in the Table 1.

Bandpass domain	Lowpass Domain
0^+	$-\infty$
$\Omega_{1S} = 0.7281$	$-\Omega_{1LS} = -1.4217$
$\Omega_{1P} = 0.7879$	-0.999
$\Omega_0 = 0.9544$	0
$\Omega_{2P} = 1.1562$	$+1$
$\Omega_{2S} = 1.2491$	$\Omega_{2LS} = 1.4079$
$+\infty$	$+\infty$

Table 1: Correspondence of critical points

2.5 Frequency Transformed Lowpass Analog Filter Specifications

Since both the passband and stopband are *monotonic* in nature, we need to use a *Butterworth filter*. The passband edge of this filter will be $\Omega_{LP} = 1$. When deciding the stopband edge for the lowpass filter, we need to ensure that the tolerance conditions are satisfied for both stopband edges in the bandpass filter. The Butterworth filter function is monotonically decreasing away from 0, so satisfying tolerance requirements at the stopband edge automatically satisfies tolerance requirements for all frequencies greater than it. Hence we should choose lowpass filter stopband edge $\Omega_{LS} := \min(\Omega_{1LS}, \Omega_{2LS}) = \Omega_{2LS} = 1.4079$.

Thus the Butterworth lowpass filter specifications are:

- Passband edge $= \Omega_{LP} = 1$
- Stopband edge $= \Omega_{LS} = 1.4079$

2.6 Analog Lowpass Filter Transfer Function

The squared magnitude of the Butterworth filter transfer function at frequency Ω_L is given by:

$$|H_{\text{analog,LPF}}(\Omega_L)|^2 = \frac{1}{1 + \left(\frac{\Omega_{LP}}{\Omega_C}\right)^{2N}}$$

We define two useful quantities related to the tolerance margins that are useful in deciding the filter parameters N and Ω_C:

$$D_1 = \frac{1}{(1-\delta_1)^2} - 1 = \frac{1}{(1-0.15)^2} - 1 = 0.3841$$

$$D_2 = \left(\frac{1}{\delta_2}\right)^2 - 1 = \frac{1}{(0.15)^2} - 1 = 43.4444$$

Then by using the tolerance constraints on the filter function, we get the following constraints on N and Ω_C:

$$N \geq \frac{\log\left(\sqrt{\frac{D_2}{D_1}}\right)}{\log\left(\frac{\Omega_{LS}}{\Omega_{LP}}\right)}$$

$$\frac{\Omega_{LP}}{D_1^{\frac{1}{2N}}} \leq \Omega_C \leq \frac{\Omega_{LS}}{D_2^{\frac{1}{2N}}}$$

As N has to be an integer, we choose $N = N_{\min}$ such that:

$$N_{\min} = \left\lceil \frac{\log\left(\sqrt{\frac{D_2}{D_1}}\right)}{\log\left(\frac{\Omega_{LS}}{\Omega_{LP}}\right)} \right\rceil = \lceil 6.9108 \rceil = 7$$

Hence we choose $N = 7$. For this value, the constraints on Ω_C are:

$$1.0707 \leq \Omega_C \leq 1.0754$$

Thus we can choose $\Omega_C = 1.075$. Now, the poles of the transfer function are the left half plane roots of the equation:

$$1 + \left(\frac{s_L}{j\Omega_C}\right)^{14} = 1 + \left(\frac{s_L}{j1.075}\right)^{14} = 0$$

The above equation gives the poles of $H_{\text{analog,LPF}}(s_L) \cdot H_{\text{analog,LPF}}(-s_L)$. We include only the left half poles for $H_{\text{analog,LPF}}(s_L)$ to ensure that the filter is stable. Solving the equation above using WolframAlpha, we get roots as shown in Figure 1.

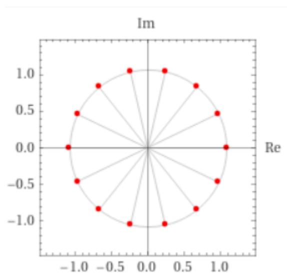


Figure 1: Poles of $H_{analog}(s_L)$ shown on the s plane

Thus the poles of $H_{\text{analog,LPF}}(s_L)$ are given by:

$$p_1 = -1.0747$$

$$p_2 = -0.9683 - 0.4663j$$
$$p_3 = -0.9683 + 0.4663j$$
$$p_4 = -0.6701 - 0.8403j$$
$$p_5 = -0.6701 - 0.8403j$$
$$p_6 = -0.2392 - 1.0478j$$
$$p_7 = -0.2392 + 1.0478j$$

Using these poles, we can write the analog lowpass filter transfer function as:

$$H_{\text{analog,LPF}}(s_L) = \frac{(\Omega_C)^7}{(s_L - p_1)(s_L - p_2)(s_L - p_3)(s_L - p_4)(s_L - p_5)(s_L - p_6)(s_L - p_7)}$$

$$= \frac{1.659}{(s_L + 1.0747)(s_L^2 + 1.3402\, s_L + 1.0747)(s_L^2 + 1.9366\, s_L + 1.0747)(s_L^2 + 0.4784\, s_L + 1.0747)}$$

By plotting the corresponding frequency response, we get Figure 2

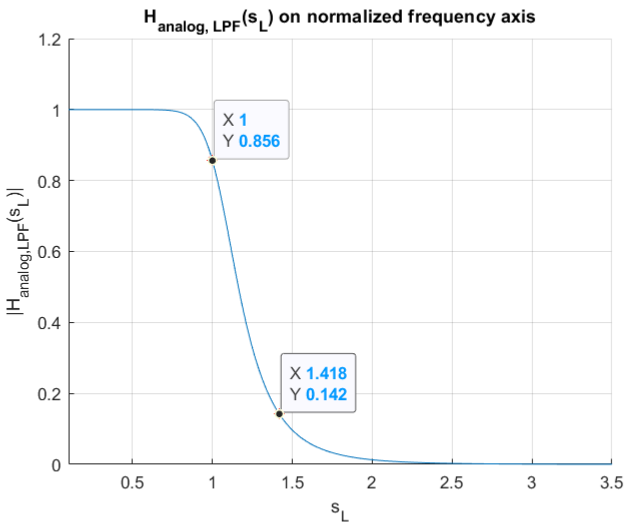


Figure 2: Low Pass Filter Frequency Response

2.7 Analog Transfer Function for Bandpass Filter

The lowpass to bandpass complex frequency transformation is given by:

$$s_L = \frac{s^2 + \Omega_0^2}{Bs}$$

Substituting the values for Ω_0 and B, we get:

$$s_L = \frac{s^2 + 0.9109}{0.368\, s}$$

Using this transformation on the lowpass filter transfer function, we can obtain the bandpass filter transfer function $H_{\text{analog,BPF}}(s) = \frac{N(s)}{D(s)}$ where $N(s)$ and $D(s)$ are polynomials, where:

$$N(s) = 0.001516\, s^7$$

and the coefficients of $D(s)$ are given in Table 2.

Degree	s^{14}	s^{13}	s^{12}	s^{11}	s^{10}
Coefficient	1	1.9524	8.1357	11.6103	25.7564

Degree	s^{9}	s^{8}	s^{7}	s^{6}	s^{5}
Coefficient	27.7923	41.9340	34.3209	38.2058	23.0702

Degree	s^{4}	s^{3}	s^{2}	s^{1}	s^{0}
Coefficient	19.4794	8.0002	5.1075	1.1167	0.5211

Table 2: Coefficients of $D(s)$

The frequency response of given bandpass filter is given in Figure 3

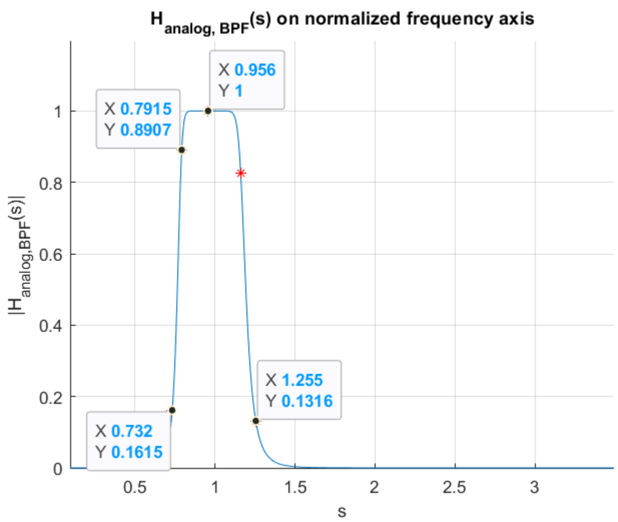


Figure 3: Frequency Response of H_{analog};BPF (s) on Normalized frequency axis

2.8 Discrete Time Bandpass Filter Transfer Function

Using the bilinear transform $s = \frac{1-z^{-1}}{1+z^{-1}}$, we get the discrete time filter transfer function as $H_{\text{discrete,BPF}}(z) = \frac{N(z^{-1})}{D(z^{-1})}$, where $N(z^{-1})$ and $D(z^{-1})$ are polynomials whose coefficients are given in Tables 3 and 4. Note that odd powers of z^{-1} in $N(z^{-1})$ have coefficients $= 0$, so they are not included in the table.

Degree	z^{0}	z^{-2}	z^{-4}	z^{-6}
Coefficient	$6.153 * 10^{-6}$	$-4.307 * 10^{-5}$	$1.292 * 10^{-4}$	$-2.154 * 10^{-4}$

Degree	z^{-8}	z^{-10}	z^{-12}	z^{-14}
Coefficient	$2.154 * 10^{-4}$	$-1.292 * 10^{-4}$	$4.307 * 10^{-5}$	$-6.153 * 10^{-6}$

Table 3: Coefficients of $N(z^{-1})$

Degree	z^0	z^{-1}	z^{-2}	z^{-3}
Coefficient	1.0000	−0.56411	5.262	−2.5042

Degree	z^{-4}	z^{-5}	z^{-6}	z^{-7}	z^{-8}
Coefficient	11.8442	−4.6507	14.7831	−4.6244	11.0510

Degree	z^{-9}	z^{-10}	z^{-11}	z^{-12}	z^{-13}	z^{-14}
Coefficient	−2.5963	4.9470	−0.7802	1.2274	0.098	0.1301

Table 4: Coefficients of $D(z^{-1})$

This discrete time Filter Transfer specification is shown in Figure 4 where the phase response as well as the magnitude response is shown in the log scale in the normalized frequency of the discrete domain.

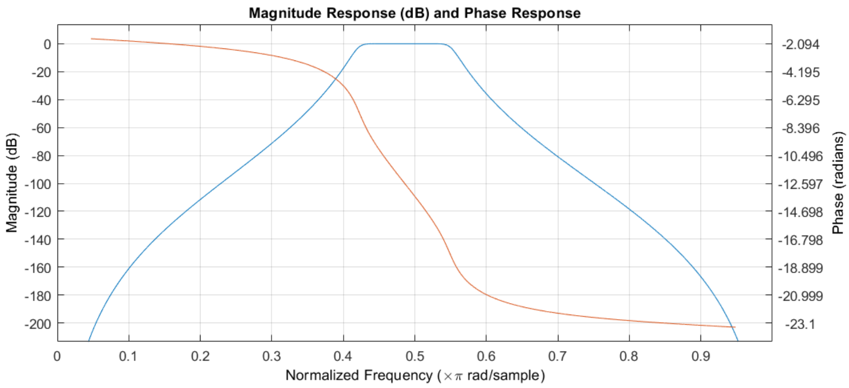


Figure 4: Magnitude and Phase response of the discrete BPF

The transfer function of the discrete filter with the unnormalized frequency specifications is shown in Figure 5.

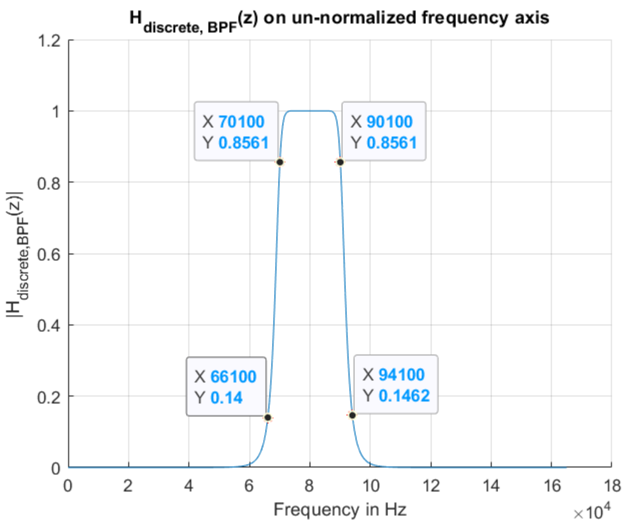


Figure 5: Magnitude response of the discrete BPF with unnormalized frequency

We can see from the Pole-Zero Plot of the discrete Band Pass filter in Figure 6 that all the poles are inside the unit circle, thus making the filter stable.

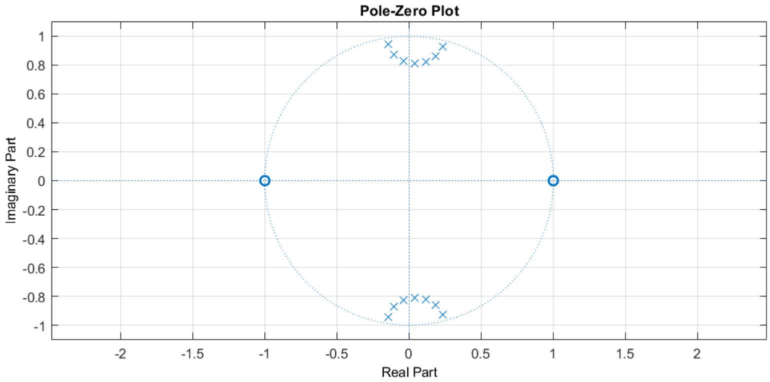


Figure 6: Pole-Zero plot of the discrete BPF

2.9 FIR Filter using Kaiser Window

A lowpass FIR filter is designed by multiplying the ideal lowpass filter impulse response, which is of infinite length, by a finite length window. Usually some of the best performance is obtained by using a Kaiser window. A bandstop filter can then be obtained by subtracting a bandpass filter impulse response (obtained as in the first filter) from the ideal all-pass filter impulse response. A Kaiser Window has two main adjustable parameters, the window length M and the shape parameter β. We can use empirical methods to select a value of β and to give a rough lower bound for the window length. First, we define the following constant:

$$A = -20\log_{10}\delta = 16.4782$$

As $A < 21$, the shape parameter β can be chosen to be zero, and the Kaiser window is simply a rectangular window. The rough lower bound for window length is given by:

$$M \geq 1 + \frac{A-8}{2.285\,\Delta\omega_T}$$

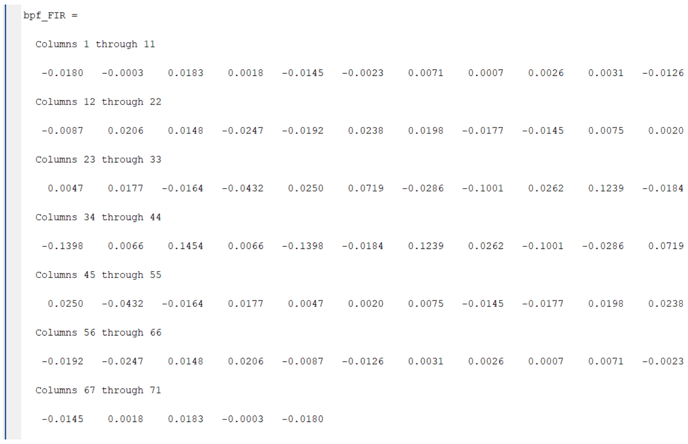
Here $\Delta\omega_T$ is the maximum transition band width in the normalized frequency domain. In our case, both transition bands are of equal widths (4 kHz), so:

$$\Delta\omega_T = \frac{4\times\pi}{165}$$

Hence the rough lower bound on M is given by:

$$2N_{min} + 1 \geq 1 + \frac{16.4782 - 8}{2.285 \times \frac{4\pi}{165}}$$

Hence we start with $N_{min} = 25$, and thus minimum length of kaiser window is 51 and keep increasing N until the tolerance requirements are met. It was found that a filter of length **71** was satisfactory. The time domain sequence values are written out in Figure 7.



```
bpf_FIR =

  Columns 1 through 11

   -0.0180   -0.0003    0.0183    0.0018   -0.0145   -0.0023    0.0071    0.0007    0.0026    0.0031   -0.0126

  Columns 12 through 22

   -0.0087    0.0206    0.0148   -0.0247   -0.0192    0.0238    0.0198   -0.0177   -0.0145    0.0075    0.0020

  Columns 23 through 33

    0.0047    0.0177   -0.0164   -0.0432    0.0250    0.0719   -0.0286   -0.1001    0.0262    0.1239   -0.0184

  Columns 34 through 44

   -0.1398    0.0066    0.1454    0.0066   -0.1398   -0.0184    0.1239    0.0262   -0.1001   -0.0286    0.0719

  Columns 45 through 55

    0.0250   -0.0432   -0.0164    0.0177    0.0047    0.0020    0.0075   -0.0145   -0.0177    0.0198    0.0238

  Columns 56 through 66

   -0.0192   -0.0247    0.0148    0.0206   -0.0087   -0.0126    0.0031    0.0026    0.0007    0.0071   -0.0023

  Columns 67 through 71

   -0.0145    0.0018    0.0183   -0.0003   -0.0180
```

Figure 7: Coeffcients of the FIR Band Pass Filter

These coefficients are obtained by multiplying the ideal BPF impulse response with the kaiser window impulse response. The impulse response of the FIR filter is shown in Figure 8.

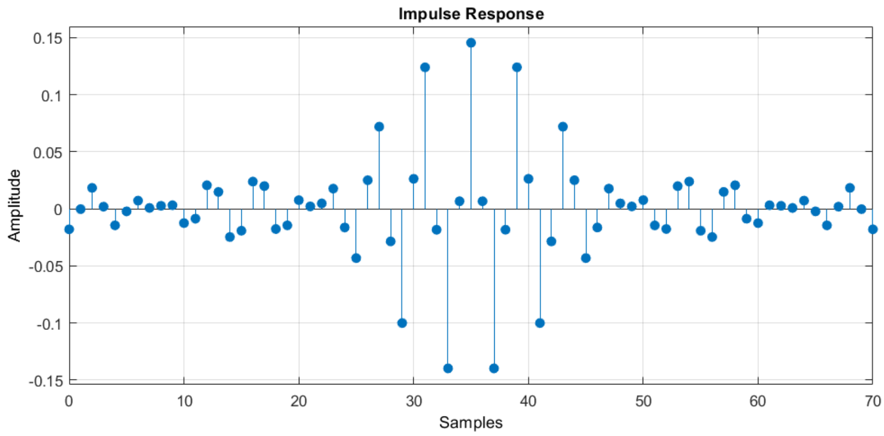


Figure 8: Impulse response of the FIR Band Pass Filter

The magnitude and the phase response of the FIR filter is shown in Figure 9.

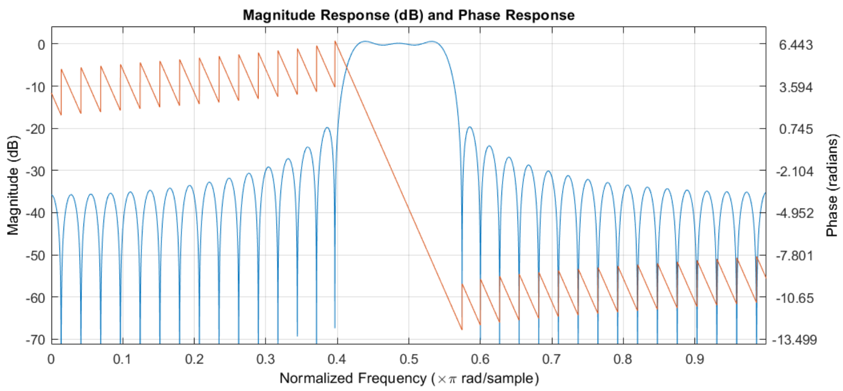


Figure 9: Magnitude and Phase response of the FIR Band Pass Filter

The magnitude response of the filter on un-normalized frequency is shown in Figure 10 where we can see that the discrete time filter specifications are indeed being met.

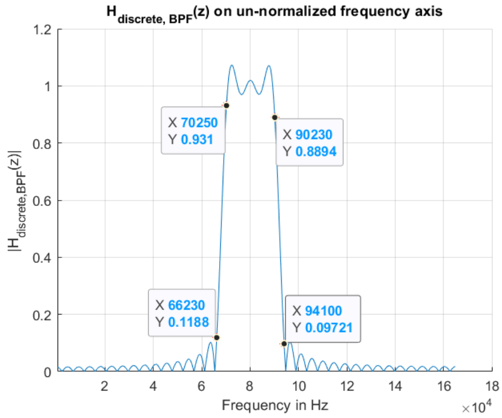


Figure 10: Magnitude response of the BPF filter on un-normalized frequencies

2.10 Comparison between FIR and IIR realizations

We note the following points between the 2 types of filters:

- The phase response of the FIR filter is piece-wise linear whereas that of the IIR filter is far from linear. Even the Butterworth filter which is supposed to give a better phase generates a non-linear phase response for the Band Pass Filter.
- The number of delay lines required to realize the IIR Band Pass Filter is (7 + 14) = 21 whereas we require 70 delay lines for realizing the FIR filter. This implies that FIR filter requires more hardware than the IIR filters at the cost of providing linear phase response to the filters.

3 Bandstop Filter

3.1 Un-normalized Discrete Time Filter Specifications

Filter number = 56
Corresponding value of $m = 56$
$\therefore \mathrm{q}(m) = \lfloor 0.1m \rfloor = 5$
$\therefore \mathrm{r}(m) = m - 10\,\mathrm{q}(m) = 6$
$B_L(m) = 25 + 1.9\,\mathrm{q}(m) + 4.1\,\mathrm{r}(m) = 59.1$
$B_H(m) = B_L(m) + 55 = 79.1$

The second filter is a **bandstop** filter with stopband from $B_L(m)$ kHz to $B_H(m)$ kHz. Therefore the filter specifications are:

- **Stopband**: 59.1 kHz to 79.1 kHz
- **Transition band**: 4 kHz on either side of stopband.
- **Passband**: 0–55.1 kHz and 83.1–130 kHz ($\because$ Sampling frequency = 260 kHz)
- **Tolerance**: 0.15 in magnitude in both passband and stopband
- **Passband nature**: Equiripple
- **Stopband nature**: Monotonic

3.2 Normalized Discrete Time Filter Specifications

$$\text{Sampling rate} = f_{\text{sample}} = 260\,\text{kHz}$$

Any analog frequency f upto half the sampling frequency can be represented on the normalized frequency axis by:

$$\omega = \frac{f}{f_{\text{sample}}} \cdot 2\pi$$

Therefore the normalized discrete time filter specifications can be given as follows:

- **Stopband**: 1.4282 to 1.9115
- **Transition band**: 0.0966 on either side
- **Passband**: 0 to 1.3315 and 2.008 to π
- **Tolerance**: 0.15 in magnitude in both passband and stopband
- **Passband nature**: Equiripple
- **Stopband nature**: Monotonic

3.3 Analog Filter Specifications using the Bilinear Transform

The bilinear transform on normalized frequencies is given by:

$$\Omega = \tan\left(\frac{\omega}{2}\right)$$

Therefore the corresponding analog filter specifications for the same type of analog filter using the bilinear transformation are:

- **Stopband**: 0.8667 (Ω_{1S}) to 1.4155 (Ω_{2S})
- **Transition band**: 0.7853 to 0.8667 and 1.4155 to 1.571
- **Passband**: 0 to 0.7853 (Ω_{1P}) and 1.571 (Ω_{2P}) to ∞
- **Tolerance**: 0.15 in magnitude in both passband and stopband
- **Passband nature**: Equiripple
- **Stopband nature**: Monotonic

3.4 Frequency Transformation and Relevant Parameters

We want to design a bandstop analog filter by transforming an appropriately designed lowpass analog filter. For that we require the equivalent lowpass analog filter specifications. The lowpass analog filter frequency transformation is given by:

$$\Omega_L = \frac{B\Omega}{\Omega_0^2 - \Omega^2}$$

where:

$$B = \Omega_{2P} - \Omega_{1P} = 0.7857$$
$$\Omega_0 = \sqrt{\Omega_{1P}\Omega_{2P}} = 1.1107$$

The critical points in this transformation are given in the Table 5.

Bandpass domain	Lowpass Domain
0^+	0^+
$\Omega_{1P} = 0.7853$	$+1$
$\Omega_{1S} = 0.8667$	$\Omega_{1LS} = 1.4113$
$\Omega_0^- = 1.1107^-$	$+\infty$
$\Omega_0^+ = 1.1107^+$	$-\infty$
$\Omega_{2S} = 1.4155$	$-\Omega_{2LS} = -1.4443$
$\Omega_{2P} = 1.571$	-0.999
$+\infty$	0^-

Table 5: Correspondence of critical points

3.5 Frequency Transformed Lowpass Analog Filter Specifications

Since the passband is *equiripple* and stopband is *monotonic*, we need to use a *Chebyschev filter*. The passband edge of this filter will be $\Omega_{LP} = 1$. When deciding the stopband edge for the lowpass filter, we need to ensure that the tolerance conditions are satisfied for both stopband edges in the bandpass filter. The Chebyschev filter function is monotonically decreasing beyond $+1$, so satisfying tolerance requirements at the stopband edge automatically satisfies tolerance requirements for all frequencies greater than it. Hence we should choose lowpass filter stopband edge $\Omega_{LS} := \min(\Omega_{1LS}, \Omega_{2LS}) = \Omega_{2LS} = 1.4113$.

Thus the Chebyschev lowpass filter specifications are:

- Passband edge $= \Omega_{LP} = 1$
- Stopband edge $= \Omega_{LS} = 1.4113$

3.6 Analog Lowpass Filter Transfer Function

The squared magnitude of the Chebyschev filter transfer function at frequency Ω_L is given by:

$$|H_{\text{analog,LPF}}(\Omega_L)|^2 = \frac{1}{1 + \epsilon^2 C_N^2\left(\frac{\Omega_L}{\Omega_P}\right)}$$

We define two useful quantities related to the tolerance margins that are useful in deciding the filter parameters ϵ and N:

$$D_1 = \frac{1}{(1-\delta_1)^2} - 1 = \frac{1}{(1-0.15)^2} - 1 = 0.3841$$

$$D_2 = \left(\frac{1}{\delta_2}\right)^2 - 1 = \frac{1}{(0.15)^2} - 1 = 43.4444$$

Then by using the tolerance constraints on the filter function, we get the following constraints on ϵ and N:

$$\epsilon^2 \leq D_1$$

$$N \geq \frac{\cosh^{-1}\left(\sqrt{\frac{D_2}{\epsilon^2}}\right)}{\cosh^{-1}\left(\frac{\Omega_{LS}}{\Omega_{LP}}\right)}$$

We want N to be as small as possible, so we must choose ϵ as large as possible. Hence we choose $\epsilon = \sqrt{D_1} = 0.6197$. As N has to be an integer, we choose $N = N_{\min}$ such that:

$$N_{\min} = \left\lceil \frac{\cosh^{-1}\left(\sqrt{\frac{D_2}{\epsilon^2}}\right)}{\cosh^{-1}\left(\frac{\Omega_{LS}}{\Omega_{LP}}\right)} \right\rceil = \lceil 3.4819 \rceil = 4$$

Hence we choose $N = 4$. Now, the poles of the transfer function are the left half plane roots of the equation:

$$1 + \epsilon^2 \cos^2\left(4\cos^{-1}\left(\frac{s_L}{j}\right)\right) = 1 + 0.3841 \cdot \cos^2\left(4\cos^{-1}\left(\frac{s_L}{j}\right)\right) = 0$$

The above equation gives the poles of $H_{\text{analog,LPF}}(s_L) \cdot H_{\text{analog,LPF}}(-s_L)$. We include only the left half poles for $H_{\text{analog,LPF}}(s_L)$ to ensure that the filter is stable. Solving the equation above using WolframAlpha, we get roots as shown in Figure 11.

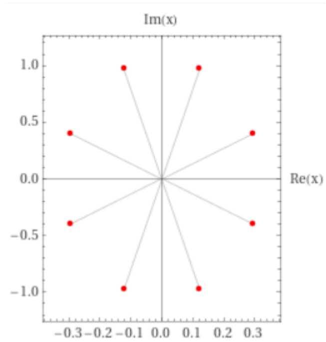


Figure 11: Poles of $H_{analog}(s_L)$ shown on the s plane

Thus the poles of $H_{\text{analog,LPF}}(s_L)$ are given by:

$$p_1 = -0.2949 - 0.4017j$$
$$p_2 = -0.2949 + 0.4017j$$
$$p_3 = -0.1222 + 0.9698j$$
$$p_4 = -0.1222 - 0.9698j$$

Using these poles, and using the fact that for odd order filters the DC gain is 1, we can write the analog lowpass filter transfer function as:

$$H_{\text{analog,LPF}}(s_L) = \frac{p_1\, p_2\, p_3\, p_4}{\sqrt{1 + D_1}(s_L - p_1)(s_L - p_2)(s_L - p_3)(s_L - p_4)}$$
$$= \frac{0.2017}{(s_L^2 + 0.5898\, s_L + 0.2483)(s_L^2 + 0.2444\, s_L + 0.9554)}$$

Plotting it on the s_L axis we get the frequency response of the LPF as shown in Figure 12.

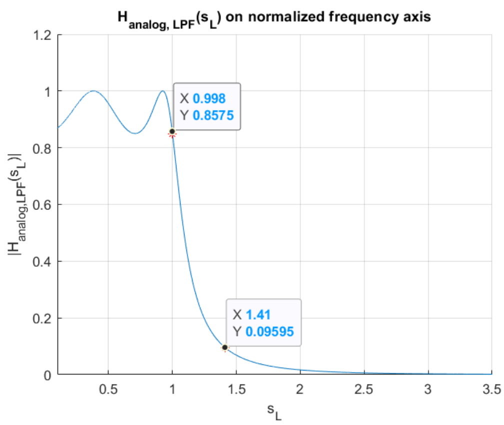


Figure 12: Low Pass Filter Frequency Response

3.7 Analog Transfer Function for Bandstop Filter

The lowpass to bandstop complex frequency transformation is given by:

$$s_L = \frac{Bs}{s^2 + \Omega_0^2}$$

Substituting the values for Ω_0 and B, we get:

$$s_L = \frac{0.7857\, s}{s^2 + 1.2336}$$

Using this transformation on the lowpass filter transfer function, we can obtain the bandpass filter transfer function $H_{\text{analog,BPF}}(s) = \frac{N(s)}{D(s)}$ where $N(s)$ and $D(s)$ are polynomials, whose coefficients are given in Tables 6 and 7. Note that odd powers of $N(s)$ have coefficients = 0, so they are not included in the table.

Degree	s^8	s^6	s^4	s^2	s^0
Coefficient	0.85	4.1966	7.7698	6.3935	1.9729

Table 6: Coefficients of $N(s)$

Degree	s^8	s^7	s^6	s^5	s^4	s^3
Coefficient	1.0000	2.0688	8.4509	9.3735	19.4292	11.5696

Degree	s^2	s^1	s^0
Coefficient	12.8748	3.8902	2.3210

Table 7: Coefficients of $D(s)$

The frequency response of the Band Stop Filter is shown in Figure 13.

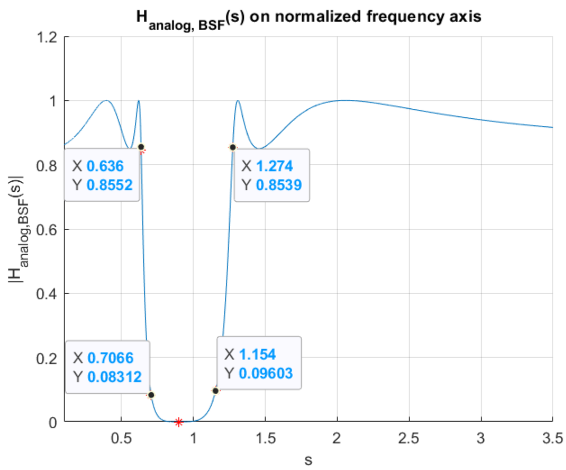


Figure 13: Frequency Response of H_{analog};BSF (s) on Normalized frequency axis

3.8 Discrete Time Bandstop Filter Transfer Function

Using the bilinear transform $s = \frac{1-z^{-1}}{1+z^{-1}}$, we get the discrete time filter transfer function as $H_{\text{discrete,BSF}}(z) = \frac{N(z^{-1})}{D(z^{-1})}$, where $N(z^{-1})$ and $D(z^{-1})$ are polynomials whose coefficients are given in Tables 8 and 9.

Degree	z^0	z^{-1}	z^{-2}	z^{-3}
Coefficient	0.2984	0.2504	1.2725	0.7621

Degree	z^{-4}	z^{-5}	z^{-6}	z^{-7}	z^{-8}
Coefficient	1.9487	0.7621	1.2725	0.2504	0.2984

Table 8: Coefficients of $N(z^{-1})$

Degree	z^0	z^{-1}	z^{-2}	z^{-3}
Coefficient	1.0000	0.6141	2.0022	0.9666

Degree	z^{-4}	z^{-5}	z^{-6}	z^{-7}	z^{-8}
Coefficient	1.9131	0.6193	0.8317	0.1824	0.2420

Table 9: Coefficients of $D(z^{-1})$

This discrete time Filter Transfer specification is shown in Figure 14 where the phase response as well as the magnitude response is shown in the log scale in the normalized frequency of the discrete domain.

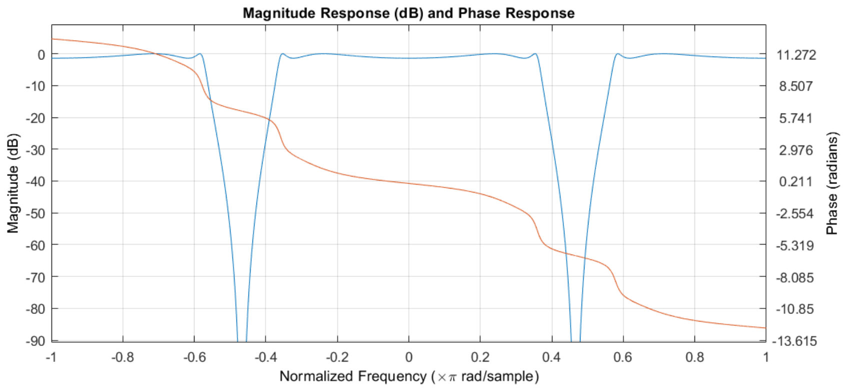


Figure 14: Magnitude and Phase response of the discrete BSF

The transfer function of the discrete filter with the un-normalized frequency specifications is shown in Figure 15.

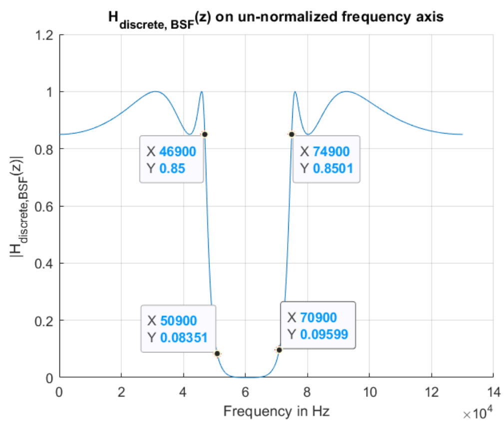


Figure 15: Magnitude response of the discrete BSF with un-normalized frequency

We can see from the Pole-Zero Plot of the discrete Band Pass filter in Figure 16 that all the poles are inside the unit circle, thus making the filter stable.

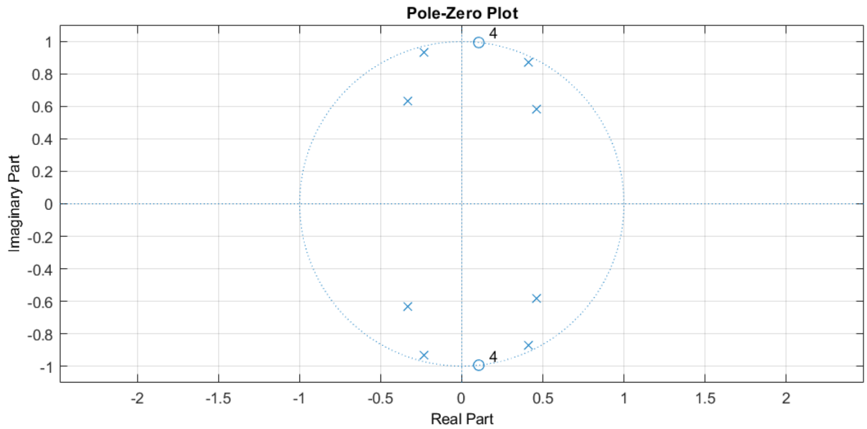


Figure 16: Pole-Zero plot of the discrete BSF

3.9 FIR Filter using Kaiser Window

A lowpass FIR filter is designed by multiplying the ideal lowpass filter impulse response, which is of infinite length, by a finite length window. Usually some of the best performance is obtained by using a Kaiser window. A bandstop filter can then be obtained by subtracting a bandpass filter impulse response (obtained as in the first filter) from the ideal all-pass filter impulse response. A Kaiser Window has two main adjustable parameters, the window length M and the shape parameter β. We can use empirical methods to select a value of β and to give a rough lower bound for the window length. First, we define the following constant:

$$A = -20\log_{10}\delta = 16.4782$$

As $A < 21$, the shape parameter β can be chosen to be zero, and the Kaiser window is simply a rectangular window. The rough lower bound for window length is given by:

$$M \geq 1 + \frac{A-8}{2.285\,\Delta\omega_T}$$

Here $\Delta\omega_T$ is the maximum transition band width in the normalized frequency domain. In our case, both transition bands are of equal widths (4 kHz), so:

$$\Delta\omega_T = 0.0966$$

Hence the rough lower bound on M is given by:

$$2N_{min} + 1 \geq 1 + \frac{16.4782 - 8}{2.285 \times 0.0966}$$

Hence we start with $N_{min} = 20$, and thus minimum length of kaiser window is 41 and keep increasing N until the tolerance requirements are met. It was found that a filter of length **55** was satisfactory. The time domain sequence values are written out in Figure 17.

```
bsf_FIR =

  Columns 1 through 9

    0.0106   -0.0197   -0.0129    0.0120    0.0079   -0.0016    0.0051   -0.0059   -0.0223

  Columns 10 through 18

    0.0065    0.0363   -0.0006   -0.0395   -0.0066    0.0276    0.0066   -0.0025    0.0084

  Columns 19 through 27

   -0.0279   -0.0409    0.0521    0.0866   -0.0601   -0.1346    0.0475    0.1710   -0.0180

  Columns 28 through 36

    0.8154   -0.0180    0.1710    0.0475   -0.1346   -0.0601    0.0866    0.0521   -0.0409

  Columns 37 through 45

   -0.0279    0.0084   -0.0025    0.0066    0.0276   -0.0066   -0.0395   -0.0006    0.0363

  Columns 46 through 54

    0.0065   -0.0223   -0.0059    0.0051   -0.0016    0.0079    0.0120   -0.0129   -0.0197

  Column 55

    0.0106
```

Figure 17: Coefficients of the FIR Band Stop Filter

The impulse response of the FIR filter is shown in Figure 18.

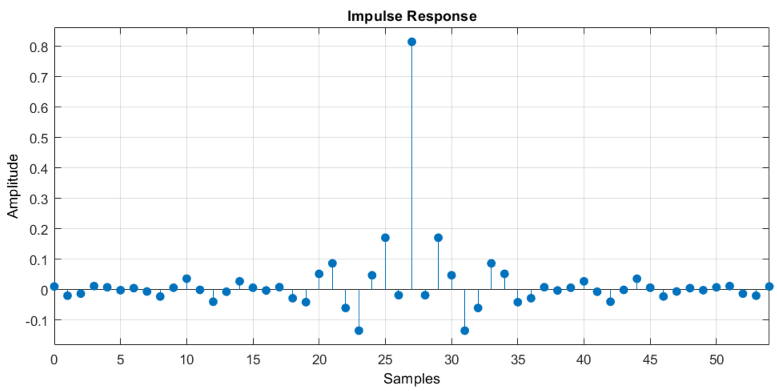


Figure 18: Impulse response of the FIR Band Stop Filter

The magnitude and phase response of FIR filter is shown in Figure 19

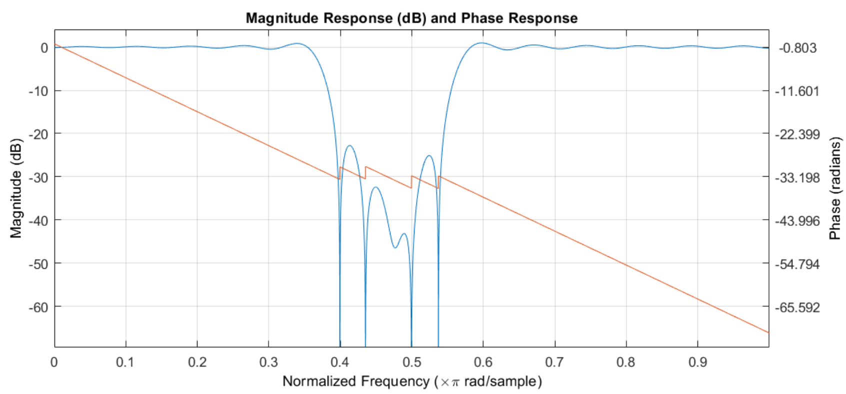


Figure 19: Magnitude and Phase response of the FIR Band Stop Filter

The magnitude response of the filter on un-normalized frequency is shown in Figure 20 where we can see that the discrete time filter specifications are indeed being met.

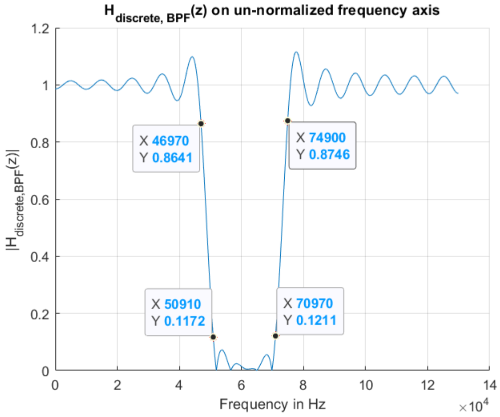


Figure 20: Magnitude response of the BPF filter on un-normalized frequencies

3.10 Comparison between FIR and IIR realizations

- As discussed in the class, we can see that the FIR filter has linear / Pseudo linear phase in the Pass band where as IIR filter has non-linear phase.
- The number of delay lines required for IIR filter is 8+8 = 16, where as for FIR filter we need 54 delay lines. Hence it is very evident that for the same Filter specifications the FIR filters need a lot more hardware.

4 Review Report

4.1 Detailed corrections and improvements done on the report

- **Issue**: Error in Butterworth filter magnitude response plot pointed out by Shubham Kar.
  Status: resolved . There was a minor error in the MATLAB code used for plotting the magnitude response
- **Issue**: Typo in H_{analog}lpf(s) formula pointed out by Garaga Vamsi.
  Status: resolved
- **Issue**: Error in positioning of images pointed out by Garaga Vamsi.
  Status: resolved

4.2 Detailed review for group member's report

Detailed review performed for Shubham Kar's report. Suggestions were made regarding the format of images and typos . All suggestions were taken and all minor corrections are made.

- All Specifications were correctly chosen.
- All the frequency response specifications for all filters are met.
- All the mandatory parts were completed.

Detailed review performed for Garaga Vamsi's report. Suggestions were made regarding the typos . All suggestions were taken and all minor corrections are made.

- All Specifications were correctly chosen.
- All the frequency response specifications for all filters are met.
- All the mandatory parts were completed.

5 Conclusion

We have observed that Butterworth filters have a much more linear phase response than the Chebyshev Filters and the Chebyshev filters much more than the Equiripple filters.We also observe the marked difference in the phase response of the IIR from the FIR filters. We see linear phase response of the FIR filters in comparison to non-linear phase response of the IIR filters. Linear phase response of the FIR filters however, come at a cost as we see more number of delay lines are to be utilized for the same specifications for FIR filters rather than the IIR filters.

6 Appendix

MATLAB codes written for carrying out this assignment and all the resultant images are uploaded in the following **GitHub Public Repository**. Readers are encouraged to have a look for the same.