

# *Study and Design of Target Tracking and AntiMissile Launching by Amplitude Comparison Monopulse Radar Tracking using Matlab*

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**Abstract**—The purpose of air defense guided missile systems is to reduce or minimize the attacks by detecting and destroying enemy aircraft or missiles approaching a important area. These systems must be capable of defending strategic areas against attack from high altitude, high speed enemy aircraft by performing precision bombing on the aircraft and on the missiles launched by the aircraft. So to destroy the targets, the system must be capable of predicting the trajectory of the targets before and after the missile launch. Our study aims to study the amplitude monopulse radar system and designing a missile launching system which is able to calculate the predicted trajectory of the target based on the amplitude monopulse radar system using MATLAB. This missile launching system also calculates the trajectory of the missile used for destroying the target. The speed, azimuth and elevation angle for launching the missile is also calculated. This missile launching system is designed for stationary and moving objects having uniform speed or acceleration.

**Keywords**—azimuth; elevation; monopulse; radar; antimissile; target; tracking; simulation; matlab; amplitude comparison; design;

## I. INTRODUCTION

Radar is a electromagnetic sensor used for the detection and location of targets. Radar is an acronym for radio and detection and ranging. It was first developed as a device to warn of the approach of hostile aircraft and for directing anti-aircraft weapons. All early radars used radio waves, but some modern radar today are based on optical waves and the use of lasers. It radiates electromagnetic energy from an antenna in the free space. Some of this energy is intercepted by a reflecting object i.e. a target [1]. The intercepted energy is reradiated by the target in many directions. Some of this reradiated energy is received by the antenna of the radar. This reradiated energy is amplified by a receiver antenna and after some signal processing, a decision is made at the output of the receiver as to whether a target echo is present or not [2].

Tracking radars have a pencil beam to receive echoes from a single and track the target in angle, range and it's velocity by Doppler Effect. A tracking radar system measures the coordinates of a target and provides data which may be used to determine the target path and to predict its future position. All or only part of the available radar data-range, elevation angle, azimuth angle, and Doppler frequency shift may be used in

predicting future position i.e. a radar might track in range, in angle or in Doppler, or with any combination [2].

The antenna beam in the continuous tracking radar is positioned in angle by a servomechanism which is put into action by an error signal. The various methods for generating the error signal may be classified as sequential lobing, conical scan, and simultaneous lobing or monopulse. The range and doppler frequency shift can also be continuously tracked, if desired, by a servo-control loop actuated by an error signal generated in the radar receiver. The information available from a tracking radar may be presented on a cathode-ray-tube (CRT) display for action by an operator, or may be supplied to an automatic computer which determines the target path and calculates the trajectory of the target [2].

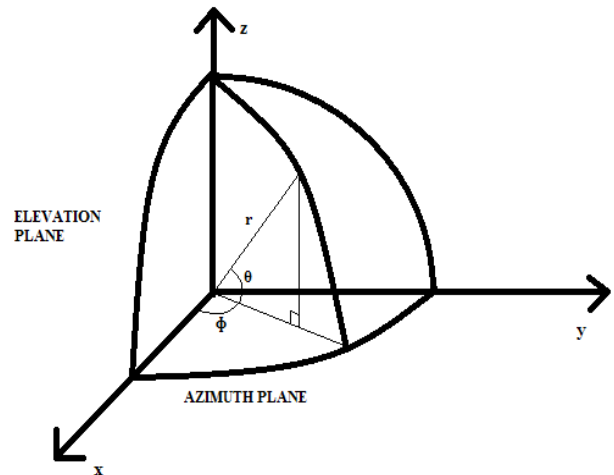


Figure 1. Geometrical representation on azimuth and elevation plane.

## II. AZIMUTH AND ELEVATION ERROR

In radian coordinates  $(r, \theta, \phi)$ , Azimuth plane is the plane where  $\phi$  is varying and  $\theta$  equals to constant. Whereas, Elevation plane is the plane where  $\phi$  is a constant and  $\theta$  is varying. Understanding azimuth and elevation planes makes it easy to understand azimuth and elevation errors. Azimuth error should make the radar rotate in the x-y plane. Whereas, an elevation error should make the radar rotate in the x-z or y-z plane.

### III. MONOPULSE TRACKING

The susceptibility of the earlier scanning and lobing techniques to echo amplitude fluctuations led to a need for developing a tracking radar that provides simultaneously all the necessary signals for angle-error sensing. Since the output from the lobes is compared simultaneously on a single pulse, any effect of time change of the echo amplitude due to noise is eliminated. The monopulse technique has very high precision as its feed structure is rigidly mounted i.e. no moving parts. This has made possible the development of pencil-beam tracking radars that meet missile range instrumentation radar requirements of  $0.003^\circ$  angle-tracking precision [2].

However, in practice the angular position of the target is obtained from multiple pulses in order to improve target detection probability and further improve angle measurement accuracy. If the echo amplitude changes, it changes in the same way in all receiver channels [3].

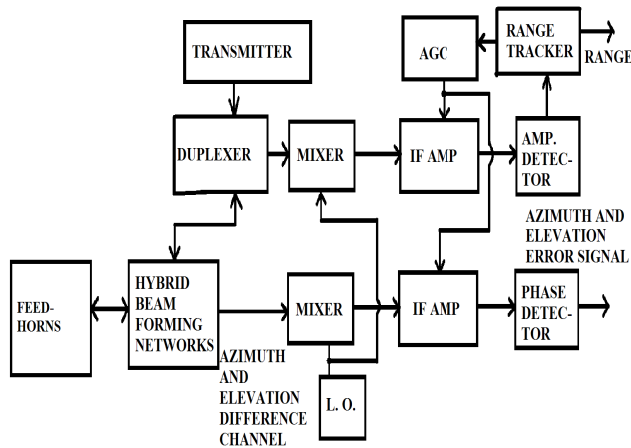


Figure 2. Simplified block diagram of a conventional monopulse radar system.

#### A. Amplitude Comparison Monopulse

An amplitude-comparison monopulse feed detects any displacement of the target from the center of the focal axis. A monopulse feed using the four-horn square would be centered at the focal point. The four horn square is actually four antennas having similar characteristics and the same radiation intensities [2]. Thus it provides symmetry. Thus, when the target axis is centered equal energy falls on each of the four horns. However, if the target axis isn't centered, there are different energies in the antennas. The radar senses this target displacement by comparing the amplitude of the echo signal excited in each of the antennas. This is done by the use of microwave hybrids to subtract outputs of pairs of horns and providing a sensitive device that gives a output when there is an unbalance caused by the target being off axis [3]. There are comparators which perform the addition and subtraction of the feedhorn outputs to obtain the monopulse sum and difference signals. As shown in Fig.3 the RF circuitry for a conventional four-horn square subtracts the output of the left pair (A,C) from the output of the right pair (B,D) of the four horns to sense any unbalance of energies in the azimuth direction i.e.  $(A+C)-(B+D)$ .

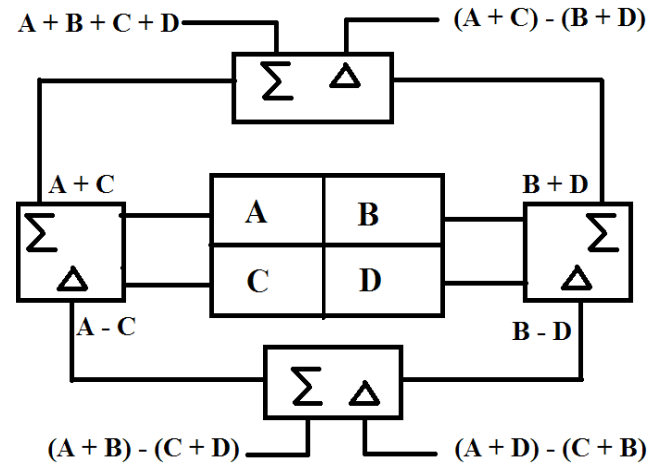


Figure 3. Block diagram of the RF comparator circuitry used in four horn monopulse.

This circuitry similarly subtracts the output of the top pair (A,B) from the bottom pair (C,D) for the unbalance in the elevation direction i.e.  $(A+B)-(C+D)$ . The difference signals are zero when the target is on focal point of the four horn square. The amplitude of the difference signal increases with the increase of displacement from the focal axis of the four horns. The sum of all four horn outputs provides a reference signal to allow angle-tracking sensitivity even though the target echo signal varies over a large dynamic range [3].

AGC keeps the gain of the angle-tracking loops constant for stable automatic angle tracking. The local oscillators and mixers are used to convert the signals into intermediate frequency (IF). The IF is detected and provides the input to the range tracker. The range tracker determines the time of arrival of the desired target echo and provides gate pulses which switches on portions of the radar only for the time when the desired target echo is expected. The range tracker also provides input to the AGC for gain control and also maintains constant angle-tracking sensitivity. The sum signal at the IF output also provides a reference signal to phase detectors which derive angle-tracking-error voltages from the difference signal [3]. The phase detectors perform a dot-product to produce the output voltage given by the formula as reported in [3].

$$e = \frac{|\Delta|}{|\Sigma|} \cos \theta \quad (1)$$

Where  $|\Delta|$  is the magnitude of difference signal.  $|\Sigma|$  is the magnitude of sum signal and  $\theta$  is the phase angle between sum and difference signals. Normally,  $\theta$  is either  $0^\circ$  or  $180^\circ$  when the radar is properly adjusted.

### IV. TYPES OF TARGETS

#### A. Point Targets

A point target is physically small enough that no significant smearing or spreading in time occurs in received pulses. Many aircraft, satellites, small boats, can be considered as point targets [1].

### B. Extended Targets

Targets that are very large are called extended targets. Such kind of targets can cause smearing in received pulses and an loss in performance. e.g. large buildings, hills, some large ships [1].

### C. Distributed Targets

Still larger targets are called distributed targets. There are two types of distributed targets. They are area targets and volume targets. Farms, oceans, etc are area targets. Rain, smoke, fog, hail, etc are volume targets [1].

### D. Moving Targets

Moving targets are those having motion relative to the radar. Moving targets such as missiles, jet aircraft, satellites, and cannon shells are often fast enough to shift the spectrum of the received signal by a significant amount (Doppler) in frequency relative to the transmitted signal. When the radar is not stationary, such as on an aircraft, all targets in the field of the radar are affected by the radar's motion as though they were moving and the radar was stationary [1].

We are going to have a detailed study of point targets and moving targets as missiles and aircrafts are usually point and moving targets. Thus there is no smearing or spreading of the echo signals in the time domain of the echo pulses [1].

## V. PREDICTION OF THE TRAJECTORY OF THE TARGET(MISSILE) AND THE ANTI MISSILE

### A. Prediction of trajectory of the target

The monopulse method study shows that the radar aligns the antenna axis with the target axis so that the azimuth error and the elevation error becomes zero [2]. Thus, the antenna points in the direction of the target at any given point of time. The antenna is rotated in the azimuth and the elevation plane with the help of two motors. So, the azimuth angle i.e.  $\phi$  and the elevation angle i.e.  $\theta$  is already known.

To predict the trajectory of the target and the missile to be launched for destroying the target we make use of kinematic equations. Since kinematic equations can only be applied to linear coordinates [4]. So we convert the range(r), azimuth angle ( $\phi$ ) and the elevation angle ( $\theta$ ) into Cartesian coordinates by using the following formulas.

$$x = r \times \cos\phi \times \cos\theta \quad (2)$$

$$y = r \times \sin\phi \times \cos\theta \quad (3)$$

$$z = r \times \sin\theta \quad (4)$$

Where x, y, and z are the coordinates of the target in the Cartesian coordinate system. Let the monopulse radar tracker generate values for three echo pulses at a time interval of T secs between the echo signals. Let (x1,y1,z1), (x2,y2,z2), and (x3,y3,z3) be the coordinates of the target at obtained at 1st, 2nd and 3rd echo signal respectively.

$$ux2 = (x2 - x1) / T \quad (5)$$

$$ux3 = (x3 - x2) / T \quad (6)$$

$$ax3 = (ux3 - ux2) / T \quad (7)$$

Where, ux2 and ux3 are the velocities of the target in the direction of the x-axis at the time of reception of the 2nd and the 3rd echo signal respectively. ax3 is the acceleration of the target in the direction of the x-axis at the time of the reception of the 3rd echo signal. The displacement of a target in the x axis after a certain amount of time t is given by formula as reported in [4].

$$sx = (u \times t) + (ax3 \times t^2) / 2 \quad (8)$$

Where sx is the displacement of the target along x-axis after time t. This equation takes care of all targets no velocity, uniform velocity or uniform acceleration [4]. Therefore, the target predicted x coordinate (x4) of the target will be

$$x4 = x3 + sx \quad (9)$$

Similarly we can calculate the uy3, ay3, y4, uz4, az4 and z4 in the y and z coordinates.

### B. Prediction of trajectory of the antimissile

Since the missile can be of any type. Our motto is to make it reach the destination without making use of its on flight maneuvering skills, so as to save time and money. So we consider it as a free falling body i.e. under gravitational pull with no flight control. Therefore the missile is considered as a body under a constant acceleration of gravitational force only in the z direction. Therefore there is no force acting in x and y direction. Assuming negligible air drag.

Therefore the formulas as reported in [4] are

$$uz = x4 / t \quad (10)$$

$$uy = y4 / t \quad (11)$$

$$vz^2 = uz^2 + 2as \quad (12)$$

We also don't want any z direction velocity after the missile reaches the target in the minimum possible energy, if possible. Therefore we put the final velocity in the z- direction (vz) of the object to be zero in (12). Therefore, g is the gravitational acceleration acting on the antimissile in the z direction. Therefore the equation (12) becomes

$$uz = \sqrt{2 \times g \times z4} \quad (13)$$

This condition is only possible if

$$t = \sqrt{2/g} \quad (14)$$

Where t is the time required for target to the predicted position [4]. If this condition is not satisfied then the system calculates uz as

$$uz = z4 \times t + (1/2) \times g \times t \quad (15)$$

Thus, the resultant speed (U) of the antimissile will be equal to

$$U = \sqrt{ux^2 + uy^2 + uz^2} \quad (16)$$

Where U is the effective speed of a missile in a certain direction. This speed should be lesser than the maximum attainable speed of the antimissile (Umax). The azimuth and the elevation angle for the antimissile launch can be calculated by

$$\theta = \tan^{-1} \frac{uy}{ux} \quad (17)$$

$$\phi = \tan^{-1} \frac{uz}{\sqrt{ux^2 + uy^2}} \quad (18)$$

Where  $\theta$  and  $\phi$  are in radians.

## VI. SIMULATION OF THE TRACKING LOGIC USING MATLAB

The tracking logic of the monopulse radar was used to design a control system for launching of an antimissile. This control system calculates the trajectory of the target and also calculates its own trajectory for it to destroy the target. This system was implemented using MATLAB.

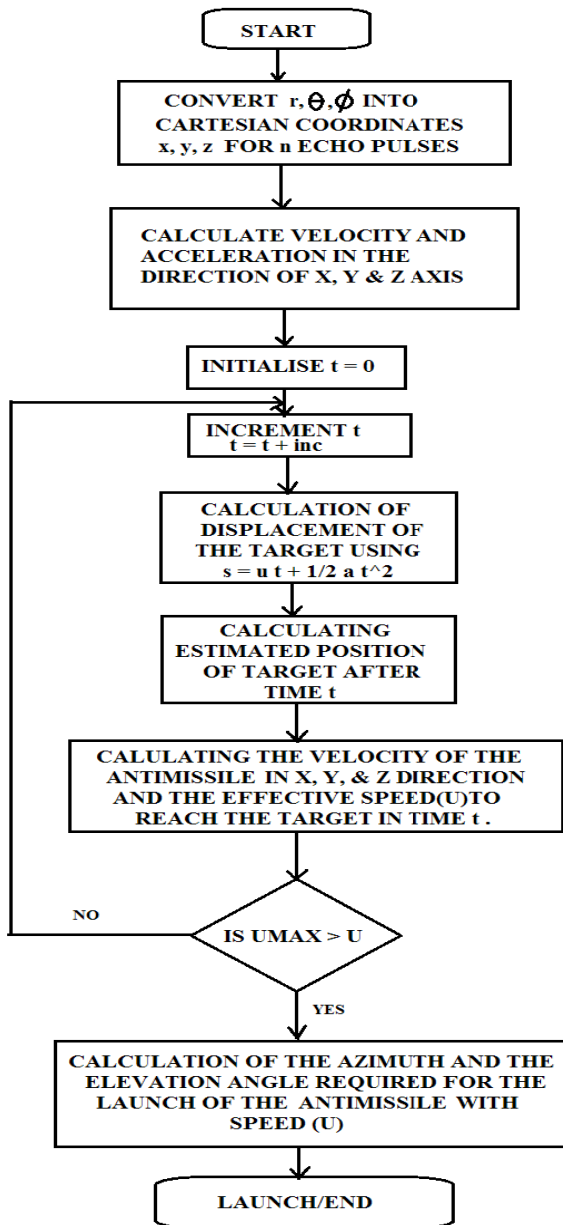


Figure 4. Flowchart of the antimissile control system.

The flowchart of the antimissile control system is shown in the Fig.4. The control system logic is same as explained earlier. This control system calculates the least time required for the

launch of the antimissile by taking into consideration the maximum attainable speed of the missile as seen earlier. We have also introduced a logic for calculating the trajectory of the antimissile such that it requires the least amount of energy for reaching the target (if possible), as well as reaches the target in the minimum amount of time depending upon its hardware limitation on its attainable speed.

It was also seen that when the Umax isn't sufficient enough for the missile to launch then the system enters a never ending for loop and doesn't launch the missile. This can be overcome easily by giving a break command.

This control system was tested for three types of point targets depending upon their motions.

### A. Stationary Targets

These targets have no velocity in the x, y or z direction.

For e.g. Let us assume that the coordinates obtained by the tracking radar is (3,6,8) in Cartesian coordinates. The maximum attainable velocity of the antimissile is assumed to be 15 m/s.

As the target is stationary with respect to the radar, its coordinates remain constant over time when the echo pulses are received. The code was simulated in MATLAB. The displacement along x, y and z axis was plotted against time.

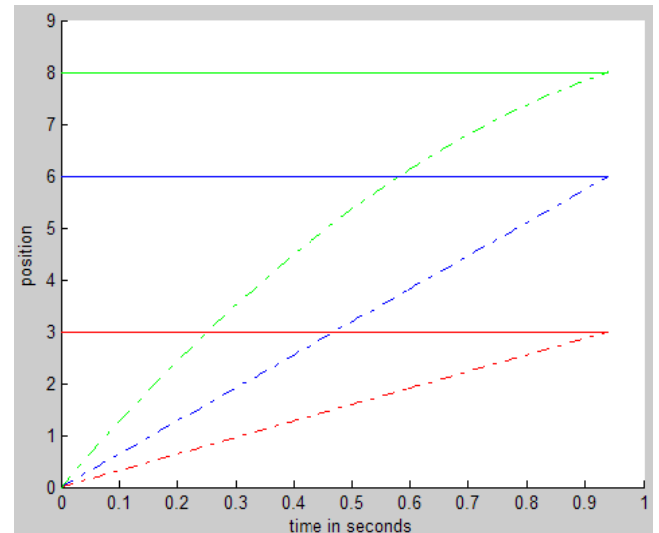


Figure 5. Position vs time plot of the target and the antimissile.

In Fig.5, the dotted lines indicate predicted position vs time of the antimissile, whereas the solid lines indicate the position of the target. Red lines indicate x coordinates, Blue lines indicate y coordinates and green lines indicate z coordinates. As observed in Fig.5 the position of the antimissile and the target becomes same for all x, y and z position after a certain amount of time. This shows that the antimissile will hit the target.

The effective speed (Ueff or U) of the antimissile is calculated and displayed on the command window for the minimum amount of time, depending on the maximum attainable speed of the missile.

```

velocity =
    3.1844    6.3688   13.2022
ueff =
    14.9999
time =
    0.9421
theta =
    63.4349
phi =
    61.6602

```

Figure 6. Command window output for the antimissile launcher.

The first line of Fig.6 indicates the velocity matrix. It has the values of the velocity with which it has to be projected in the x, y and z direction. time or t is the minimum possible time required for the reaching the target. theta ( $\theta$ ) and phi ( $\phi$ ) are the angles(in radians) for the antimissile launch so that the desired velocities are achieved in each direction.

The 3D plot the antimissile is also generated using Matlab using plot3 command. The red line indicates the trajectory of the antimissile

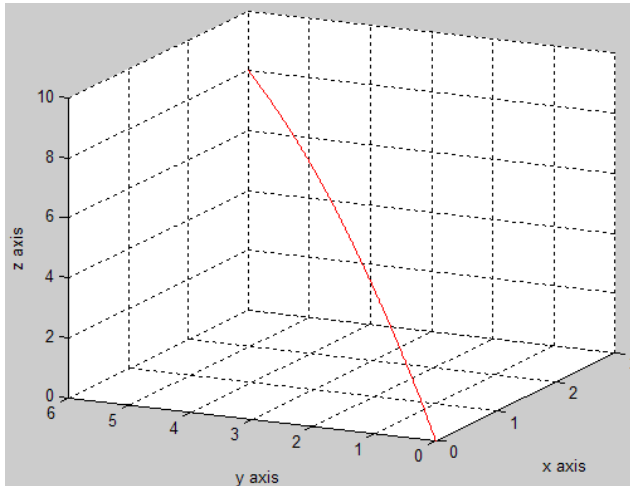


Figure 7. 3-D plot of the predicted trajectory of the antimissile.

Fig.7 shows that the target was reached after time t. The predicted trajectory of target cannot be seen in Fig.7 as it is stationary and it's located at a single point (3,6,8).

### B. Moving Targets

Moving targets are of many types too. Targets having uniform velocity or uniform acceleration or a combination of both can be classified as moving target. Moving targets can be stationary along one axis, having uniform velocity along the other and having uniform acceleration along the third. They can be classified as follows

1) *Free falling targets*: These targets are the ones which are launched by an aircraft but have no speed except the one of the aircraft when dropped. e.g. bombs dropped by an aircraft [4].

2) *Uniform velocity targets*: These are targets which have flight stabilization. So, they have uniform velocity in x, y and z direction as they are not affected by air drag coefficients [4].

3) *Uniform acceleration targets*: These targets are those whose speed is increasing linearly with respect to time by a certain factor [4].

For checking our designed system for all these targets, we consider a target having uniform velocity along one of its axis, uniform acceleration along other and stationary with respect to the third. This is a target having hybrid motion in the coordinate system. If the system is able to predict the trajectory of the target and the antimissile accurately, it would be able to destroy all kinds of targets. For the sake of understanding, we consider each one of the coordinate axis one at a time.

For e.g. Let the three input coordinates where (1,2,6), (2,4,6), and (3,8,6) which are submitted by the radar at a interval of 1s between the echo signals. We can observe that the coordinates are selected such that the target has uniform velocity of 1m/s along x axis, uniform acceleration of 2m/s along y axis and is stationary with respect to the z axis.

```

velocity =
    3.7589   12.4444   10.9547
ueff =
    17.0000
time =
    1.0874
theta =
    73.1929
phi =
    40.1206

```

Figure 8. Command window output for the antimissile launcher for moving object.

We observe from Fig. 8, that the time required to reach the target by the antimissile is 1.0874 s. The velocity is indicated by the velocity matrix, and the  $\theta$  and  $\phi$  are given in radians.

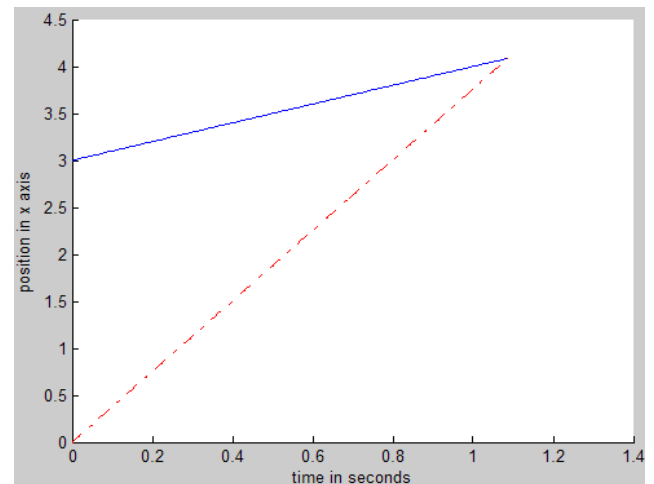


Figure 9. x-position vs time plot of the target and the antimissile.



In all position vs time plots, the blue line indicates the position of the target and the red dotted line indicates the position of the antimissile after the 3<sup>rd</sup> echo. We observe from Fig. 9, that the velocity of the target is 1m/s with respect to x axis.

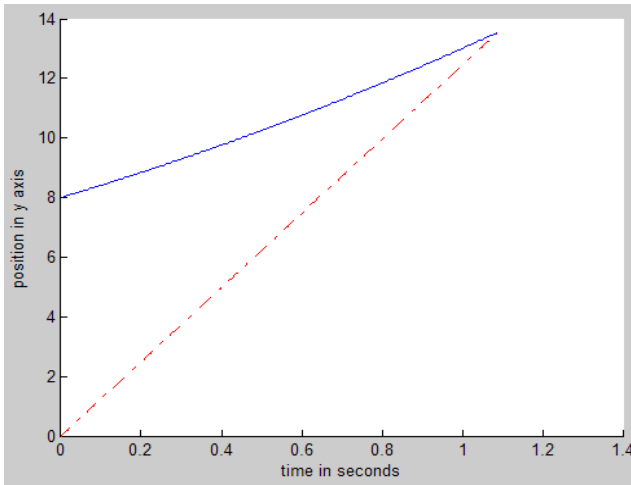


Figure 10. y-position vs time plot of the target and the antimissile.

We observe from Fig. 10, that the acceleration of the target is 2m/s with respect to y axis by using (5), (6), (7), (8) and (9).

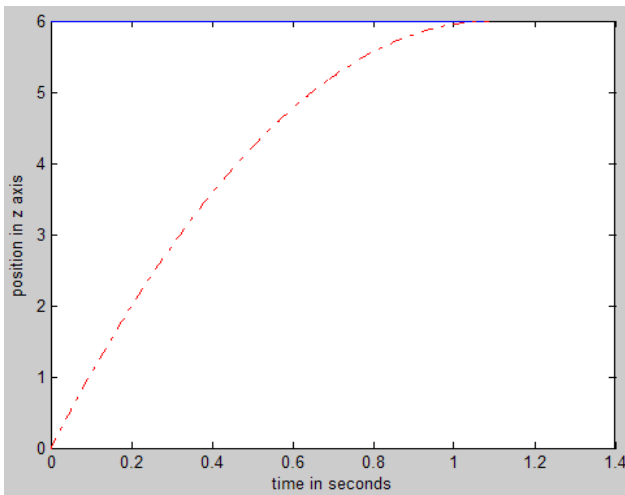


Figure 11. z-position vs time plot of the target and the antimissile.

We can observe from Fig. 11, that the object is stationary hence it's position over time will be constant with respect to z axis. In all the position vs time plots we see the antimissile's position coinciding with that of the target's after time t. This shows that the antimissile can successfully destroy the target after the launch.

To understand it more clearly plot3 command in Matlab is used. In Fig. 12, the red line indicates the predicted trajectory of the antimissile, whereas the blue line indicates the predicted trajectory of the missile after the end of third echo signal. Fig. 12 is the actual 3-D space representation of the predicted trajectories of the target and the antimissile.

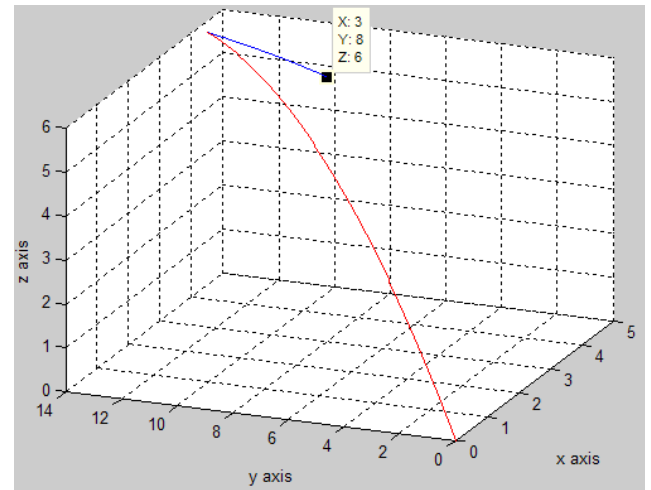


Figure 12. 3-D plot of the predicted trajectory of the antimissile and the target.

This system can be further be used for continuous tracking of the target where the system checks for any deviation of the target from its estimated path and adjusts the velocity of the antimissile accordingly by using thrusters in x, y and z direction respectively.

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#### VII. CONCLUSION

Thus we have studied and designed a antimissile system using amplitude comparison monopulse tracker for destroying the target by predicting the target's trajectory. It was observed that the code used for target and antimissile trajectory prediction can be used for most of the types of point moving targets e.g. stationary, uniform velocity, uniform accelerating, as well as targets having the combination of these. The control system also calculates the speed of the antimissile in a direction, which is specified in terms of azimuth and elevation angle. This control system calculates the trajectory of the antimissile such that minimum amount of time is required by the antimissile to reach the target depending on its maximum attainable velocity. It is also observed that the effective velocity of the antimissile tends to reach its maximum attainable velocity depending upon the size of increment (inc) as shown in Fig.4.

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