1.5 Cantor's Theorem

Cantor's Diagonalization Method

Cantor initially published his discovery that **R** is uncountable in 1874, but in 1891 he offered another simpler proof that relies on decimal representations for real numbers.

THEOREM 1.5.1. *The open interval* $(0,1) = \{x \in \boldsymbol{R} : 0 < x < 1\}$ *is uncountable.*

Power Sets and Cantor's Theorem

Given a set A, the *power set* $P(A)$ refers to the collection of all subsets of A.
Example:

$$P(\{a,b\}) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$$

THEOREM 1.5.2 (Cantor's Theorem)**.** *Given any set A, there does not exist a function $f : A \to P(A)$ that is onto.*

Proof. For contradiction, assume that $f : A \to P(A)$ is onto. FINISH THIS PROOF □