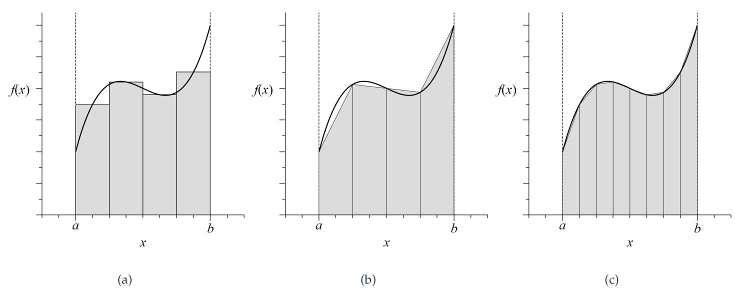
Chapter 5

Integrals and Derivatives

5.1 Fundamental Methods For Evaluating Integrals

To evaluate integrals numerically, we will use numerical approximations: the right Riemann sum, the left Riemann sum, or the trapezoidal rule.



5.1.1 The Trapezoidal Rule

Suppose we have a function $f(x)$, let

$$I(a,b) = \int_a^b f(x)dx$$

The trapezoidal rule is better than Riemann sums since it is closer to the correct area. Let's divide the interval from a to b into N equal steps so each slice has a width $h = (b-a)/N$. The left and right sides of the trapezoid are $a+(k-1)h$ and $a+kh$. So the area for slice k is

$$A_k = \frac{1}{2}h[f(a+(k-1)h) + f(a+kh)]$$

Now, our approximation of $I(a,b)$ becomes

$$\begin{aligned} I(a,b) \simeq \sum_{k=1}^{N} A_k = \frac{1}{2}h\sum_{k=1}^{N}[f(a+(k-1)h)+f(a+kh)] \\ = h[\frac{1}{2}f(a)+f(a+h)+f(a+2h)+\cdots+\frac{1}{2}f(b)] \\ = h[\frac{1}{2}f(a)+\frac{1}{2}f(b)+\sum_{k=1}^{N-1}f(a+kh)] \end{aligned}$$

This is an *extended trapezoidal rule*.