4.3 The Method of Undetermined Coefficients

A particular solution Y of the nonhomogeneous nth order linear equation with constant coefficients

$$L[y] = \sum_{i=0}^{n} a_i y^{(n-i)} = g(t)$$

can be obtained by the method of undetermined coefficients, provided that $g(t)$ is of an appropriate form.

When the differential operator L is applied to a polynomial $A_0 t^m + A_1 t^{m-1} + \cdots + A_m$, an exponential function $e^{\alpha t}$, a sine function $\sin \beta t$, or a cosine function $\cos \beta t$, the result is a polynomial, an exponential function, or a linear combination of sine and cosine functions, respectively. If $g(t)$ is a sum of these functions, we can find a $Y(t)$ by choosing a suitable combination of these functions. The constants are determined by plugging in the assumed expression into the differential equation.

If $g(t)$ is a sum of several terms, it is sometimes easier to split it up and then compute the solutions separately and then add them together for the particular solution.

This method is easy, but it only works in specific cases (with constant coefficients).