1.2 Solutions of Some Differential Equations

Consider a differential equation of the form

$$\frac{dy}{dt} = ay - b$$

Solution:
First, we perform some basic operations:

$$\frac{dy/dt}{ay - b} = 1$$

$$\frac{dy/dt}{y - b/a} = a$$

Then by the chain rule we get:

$$\frac{d}{dt} \ln |y - b/a| = a$$

Then we integrate with respect to t

$$\int \frac{d}{dt} \ln |y - b/a| dt = \int a dt$$

$$ln|y - b/a| = at + C$$

Then through some basic manipulation

$$|y - b/a| = e^{at+C} = e^C e^{at}$$

$$y - b/a = \pm e^C e^{at}$$

Let $c = \pm e^C$

$$y - b/a = ce^{at}$$

$$y = b/a + ce^{at}$$

So, $y = b/a + ce^{at}$ is the **general solution** to $\frac{dy}{dt} = ay - b$. If you have an initial condition y_o, that is when $t = 0$, $y = y_o$, we can write c in terms of y_o. If we let $c = y_o - b/a$, then when $t = 0$, $y = y_o$.

$$y = b/a + (y_o - b/a)e^{at}$$