## 1.2 Solutions of Some Differential Equations

Consider a differential equation of the form

$$\frac{dy}{dt} = ay - b$$

Solution:

First, we perform some basic operations:

$$\frac{dy/dt}{ay-b} = 1$$

$$\frac{dy/dt}{y - b/a} = a$$

Then by the chain rule we get:

$$\frac{d}{dt}\ln|y - b/a| = a$$

Then we integrate with respect to t

$$\int \frac{d}{dt} \ln|y - b/a| dt = \int a dt$$

$$ln|y - b/a| = at + C$$

Then through some basic manipulation

$$|y - b/a| = e^{at+C} = e^C e^{at}$$
  
 $y - b/a = \pm e^C e^{at}$ 

Let  $c = \pm e^C$ 

$$y - b/a = ce^{at}$$
$$y = b/a + ce^{at}$$

So,  $y = b/a + ce^{at}$  is the **general solution** to  $\frac{dy}{dt} = ay - b$ . If you have an initial condition  $y_o$ , that is when t = 0,  $y = y_o$ , we can write c in terms of  $y_o$ . If we let  $c = y_o - b/a$ , then when t = 0,  $y = y_o$ .

$$y = b/a + (y_o - b/a)e^{at}$$