x spells before combat, y spells after combat, constraint: $x+y=z$ ← given constant ← x and y are all instants and sorceries

Let s be the number of non instant or sorcery cast. Let these be cast post combat since that will yield more life gain.

$\ell = f(x,y,s) = f(z,s)$ is the total amount of life gained this turn.

Pre combat:

x spells cast pre combat so we gained ℓ_1 life so far where

$$\ell_1 = \sum_{i=1}^{x} i = \frac{x(x+1)}{2} \leftarrow \text{sum formula} = \frac{1}{2}x^2 + \frac{1}{2}x$$

Combat:

Now we get $(x+1)$ more Aetherflux Reservoirs. We now have $(x+2)$ of them.

Post Combat:

We will now cast $y+s$ more spells, triggering $x+2$ Reservoirs and we already cast x spells this turn

$$\ell_2 = (x+2)\left[\sum_{i=1}^{y+s}(i+x)\right] = (x+2)\left[\sum_{i=1}^{y+s} i + \sum_{i=1}^{y+s} x\right] = (x+2)\left[\frac{(y+s)(y+s+1)}{2} + (y+s)x\right]$$

$$= (y+s)(x+2)\left[\frac{1}{2}(y+s+1)+x\right] = \frac{1}{2}(x+2)(y+s)(y+s+1) + (y+s)(x+2)x$$

$$= \frac{1}{2}(x+2)(y^2+ys+y+sy+s^2+s) + (yx+2y+sx+2s)x = \left(\frac{1}{2}x+1\right)(y^2+s^2+2ys+y+s) + (yx^2+sx^2+2yx+2sx)$$

$$= \frac{1}{2}(y^2x+s^2x+2xys+yx+sx) + (y^2+s^2+2ys+y+s) + (yx^2+sx^2+2yx+2sx)$$

$$= \frac{1}{2}y^2x + \frac{1}{2}s^2x + ysx + \underline{\frac{1}{2}yx + \frac{1}{2}sx} + y^2+s^2+2ys+y+s + yx^2+sx^2 \underline{+2yx+2sx}$$

$$= ysx + yx^2 + sx^2 + \frac{1}{2}y^2x + \frac{1}{2}s^2x + \frac{5}{2}yx + \frac{5}{2}sx + y^2 + s^2 + 2ys + y + s$$

So the total function for ℓ is

$$\ell = f(x,y,s) = \ell_1 + \ell_2 = \frac{1}{2}x^2 + \frac{1}{2}x + ysx + yx^2 + sx^2 + \frac{1}{2}y^2x + \frac{1}{2}s^2x + \frac{5}{2}yx + \frac{5}{2}sx + y^2 + s^2 + 2ys + y + s$$

under the constaint $x+y=z$ given z and s

Optimize

$$\ell = f(x,y) = \frac{1}{2}x^2 + \frac{1}{2}x + ysx + yx^2 + sx^2 + \frac{1}{2}y^2x + \frac{1}{2}s^2x + \frac{5}{2}yx + \frac{5}{2}sx + y^2 + s^2 + 2ys + y + s$$

under $g(x,y) = x + y = z$ where z and s are given constants

Using Lagrangian optimization, we need to solve

$$\nabla f(x,y) = \begin{bmatrix} \partial f/\partial x \\ \partial f/\partial y \end{bmatrix} = \lambda \nabla g(x,y) = \begin{bmatrix} \lambda \\ \lambda \end{bmatrix} \qquad \nabla g(x,y) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; \quad \lambda \nabla g(x,y) = \begin{bmatrix} \lambda \\ \lambda \end{bmatrix}$$

Now,

$$\frac{\partial f}{\partial x} = x + \frac{1}{2} + ys + 2yx + 2sx + \frac{1}{2}y^2 + \frac{1}{2}s^2 + \frac{5}{2}y + \frac{5}{2}s = \lambda \qquad \text{AND} \quad x + y = z$$

$$\frac{\partial f}{\partial y} = sx + x^2 + yx + \frac{5}{2}x + 2y + 2s + 1 = \lambda$$

Setting $\partial f/\partial x - \partial f/\partial y = \lambda - \lambda = 0$

$$-x^2 + \frac{1}{2}y^2 + [2-1]yx + \left[1 + 2s - s - \frac{5}{2}\right]x + \left[s + \frac{5}{2} - 2\right]y + \left[\frac{1}{2} + \frac{1}{2}s^2 + \frac{5}{2}s - 2s - 1\right] = 0$$

$$-x^2 + \frac{1}{2}y^2 + yx + \left(s - \frac{3}{2}\right)x + \left(s + \frac{1}{2}\right)y + \left[\frac{1}{2}s^2 + \frac{1}{2}s + \frac{1}{2}\right] = 0 \quad \times 2$$

$$-2x^2 + y^2 + 2yx + (2s-3)x + (2s+1)y + [s^2 + s - 1] = 0 \quad y = z - x$$

$$-2x^2 + (z-x)^2 + 2(z-x)x + (2s-3)x + (2s+1)(z-x) + [s^2+s-1] = 0$$

$$-2x^2 + \underline{z^2} + x^2 \cancel{-2zx} \cancel{+2zx} - 2x^2 + (2s-3)x + (-2s-1)x + z(2s+1) + [s^2+s-1] = 0$$

$$-3x^2 + [\cancel{2s} - 3 \cancel{-2s} - 1]x + [z^2 + z(2s+1) + (s^2+s-1)] = 0$$

$$-3x^2 - 4x + c = 0 \qquad \text{where} \quad c = z^2 + z(2s+1) + (s^2+s-1)$$

Using quadratic formula,

$$x = \frac{4 \pm \sqrt{16 - 4(-3)c}}{2(-3)} = \frac{1}{6}\left[-4 \mp 2\sqrt{4+3c}\right] = \frac{1}{3}\left[-2 \mp \sqrt{4+3c}\right] \qquad \text{AND} \quad y = z - x$$

$$\hookrightarrow x = \frac{1}{3}\left[\sqrt{4+3c} - 2\right] \leftarrow \text{for } x \text{ to be positive}$$

Final Results:

Given z, s the optimal play is

$$x = \text{round}\left\{\frac{1}{3}\left[\sqrt{4+3c} - 2\right]\right\} \quad \text{where} \quad c = z^2 + z(2s+1) + (s^2+s-1)$$

$$y = z - x$$

Which will gain you ℓ life

$$\ell = f(x,y) = \frac{1}{2}x^2 + \frac{1}{2}x + ysx + yx^2 + sx^2 + \frac{1}{2}y^2x + \frac{1}{2}s^2x + \frac{5}{2}yx + \frac{5}{2}sx + y^2 + s^2 + 2ys + y + s$$