

1.2 Work, Energy, and the Conservation of Energy

For an object traveling in one dimension with a constant net force F and with velocity changing linearly with time ($v \propto t$), we have

$$F \times (x_f - x_i) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

The quantity $mv^2/2$ is called the **kinetic energy** K , while the left side is referred to as the **work** W done by the force. For forces that are not constant we have

$$W = \int_{x_i}^{x_f} F(x)dx$$

To extend to multiple dimensions

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}$$

With these new definitions, the first equation of this section can be written as

$$W_{net} = \Delta K = K_f - K_i$$

aka the **work-energy theorem**. A **conservative force** is a force which the work done by the force is path independent (it only depends on the start and end state).

$$\int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r} = f(\vec{r}_f) - f(\vec{r}_i)$$

We define potential energy for these forces as

$$U(\vec{r}) = - \int_{\vec{r}_0}^{\vec{r}} \vec{F} \cdot d\vec{r} + U(\vec{r}_0)$$

So $U(\vec{r}_f) - U(\vec{r}_i) = - \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}$. We also set **total energy** to

$$E \equiv U(\vec{r}) + K$$

Important potential energies

$$\text{Local Gravity: } U(h) = mgh$$

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$$\text{Gravitation: } U(r) = - \frac{Gm_1m_2}{r}$$

Given the potential energy, we can find the force using the gradient

$$\vec{F}(\vec{r}) = -\vec{\nabla}U$$

$$F(x) = -\frac{dU}{dx} \text{ (in one dimension)}$$