1.2 Work, Energy, and the Conservation of Energy

For an object traveling in one dimension with a constant net force F and with velocity changing linearly with time ($v \propto t$), we have

$$F \times (x_f - x_i) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

The quantity $mv^2/2$ is called the **kinetic energy** K, while the left side is referred to as the **work** W done by the force. For forces that are not constant we have

$$W = \int_{x_i}^{x_f} F(x)dx$$

To extend to multiple dimensions

$$W = \int_{\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}_f} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$$

With these new definitions, the first equation of this section can be written as

$$W_{net} = \Delta K = K_f - K_i$$

aka the **work-energy theorem**. A **conservative force** is a force which the work done by the force is path independent (it only depends on the start and end state).

$$\int_{\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}_f} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = f(\vec{\mathbf{r}}_f) - f(\vec{\mathbf{r}}_i)$$

We define potential energy for these forces as

$$U(\vec{\mathbf{r}}) = -\int_{\vec{\mathbf{r}}_0}^{\vec{\mathbf{r}}} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} + U(\vec{\mathbf{r}}_0)$$

So $U(\vec{\mathbf{r}}_f) - U(\vec{\mathbf{r}}_i) = -\int_{\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}_f} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$. We also set **total energy** to

$$E \equiv U(\vec{\mathbf{r}}) + K$$

Important potential energies

$$\text{Local Gravity: } U(h) = mgh$$

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$$\text{Gravitation: } U(r) = -\frac{Gm_1m_2}{r}$$

Given the potential energy, we can find the force using the gradient

$$\vec{\mathbf{F}}(\vec{\mathbf{r}}) = -\vec{\boldsymbol{\nabla}}U$$

$$F(x) = -\frac{dU}{dx} \text{ (in one dimension)}$$