Modern Physics Notes

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Chapter 1

Introduction

1.1 Newton's Laws

Isaac Newton introduced the three basic laws of mechanics that are known as Newton's Laws.

Newton's Second Law

$$\vec{\mathbf{F}}_{net} = m\vec{\mathbf{a}}$$

Sometimes this equation is written as

$$\vec{\mathbf{F}}_{net} = \frac{d\vec{\mathbf{p}}}{dt}$$

to account for a changing mass, where $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$.

Newton's First Law

Newton's first law is a special case of the second law:

$$\text{If } \vec{\mathbf{F}}_{net} = \vec{\mathbf{0}}\text{, the motion is uniform}$$

which means that velocity is constant and accusation is 0.

Gravity

Acceleration caused by gravity is the same on every object. This seems contradictory to what we have seen from Newton's second law, $\vec{\mathbf{a}} = \vec{\mathbf{F}}_{new}/m$, but since the force of gravity is proportional to the mass, the acceleration is independent of the mass.

Hooke's Law

The equation for spring force is $F = -kx$. So,

$$-kx = m\frac{d^2x}{dt}$$

The solution of this differential equation is

$$x(t) = A\sin(\omega t + \varphi)$$

where A is the *amplitude* of the motion, and φ is the textitphase. Both of these quantities are determined by the initial conditions. ω is determined by the spring and the mass:

$$\omega = \sqrt{k/m}$$

where ω is the *angular frequency*, which is related to the *period* T and *frequency* f.

$$T = \frac{1}{f}$$

so $f = \omega/2\pi$.

Newton's Third Law

Every force has an equal and opposite reaction force.

$$\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$$

This can be restated in terms of momentum.

$$\frac{d\vec{\mathbf{p_1}}}{dt} = -\frac{d\vec{\mathbf{p_2}}}{dt} \text{ or } \frac{d}{dt}(\vec{\mathbf{p_1}} + \vec{\mathbf{p_2}}) = 0$$

So,

$$\vec{\mathbf{P}}_{tot} = \text{a constant vector}$$

1.2 Work, Energy, and the Conservation of Energy

For an object traveling in one dimension with a constant net force F and with velocity changing linearly with time ($v \propto t$), we have

$$F \times (x_f - x_i) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

The quantity $mv^2/2$ is called the **kinetic energy** K, while the left side is referred to as the **work** W done by the force. For forces that are not constant we have

$$W = \int_{x_i}^{x_f} F(x)dx$$

To extend to multiple dimensions

$$W = \int_{\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}_f} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$$

With these new definitions, the first equation of this section can be written as

$$W_{net} = \Delta K = K_f - K_i$$

aka the **work-energy theorem**. A **conservative force** is a force which the work done by the force is path independent (it only depends on the start and end state).

$$\int_{\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}_f} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = f(\vec{\mathbf{r}}_f) - f(\vec{\mathbf{r}}_i)$$

We define potential energy for these forces as

$$U(\vec{\mathbf{r}}) = -\int_{\vec{\mathbf{r}}_0}^{\vec{\mathbf{r}}} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} + U(\vec{\mathbf{r}}_0)$$

So $U(\vec{\mathbf{r}}_f) - U(\vec{\mathbf{r}}_i) = -\int_{\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}_f} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$. We also set **total energy** to

$$E \equiv U(\vec{\mathbf{r}}) + K$$

Important potential energies

$$\text{Local Gravity: } U(h) = mgh$$

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$$\text{Gravitation: } U(r) = -\frac{Gm_1m_2}{r}$$

Given the potential energy, we can find the force using the gradient

$$\vec{\mathbf{F}}(\vec{\mathbf{r}}) = -\vec{\boldsymbol{\nabla}}U$$

$$F(x) = -\frac{dU}{dx} \text{ (in one dimension)}$$

1.3 Rotations and the Center of Mass

The **angular velocity** $\vec{\omega}$ is the rate at which the angle θ changes. Its direction is determined by the right hand rule (Fingers in direction of motion, thumb is direction of $\vec{\omega}$). The rate at which ω changes is the **angular acceleration** vector $\vec{\alpha}$. The rotation eq uivalents for mass and force, are **rotational inertia** I and **torque** $\vec{\tau}$. So Newton's second law for rotation motion is

$$\vec{\tau} = I\vec{\alpha} = \frac{d\vec{\mathbf{L}}}{dt}$$

where $\vec{\mathbf{L}}$ is **angular momentum**, which is defined as

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$$

Torque is similarly defined as

$$\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$$

The rotational inertia I is dependent on the way an objects mass is distributed. By breaking up the object of total mass

$$M = \sum_i \Delta m_i$$

into many discrete portions of mass, we can set

$$I \equiv \sum_i (\Delta m_i) r_i^2 = \int r^2 dm$$

Work one in a rotation rigid body is defined as

$$W = \int_{\theta_i}^{\theta_f} \tau d\theta$$

Kinetic energy of a rotating object is

$$K = \frac{1}{2} I \omega^2$$

The **center of mass** is defined as

$$\vec{\mathbf{R}} = \frac{\sum_i m_i \vec{\mathbf{r}}_i}{M}$$

For an external force moving the object, the mass will act as a point mass at $\vec{\mathbf{R}}$, so

$$\vec{\mathbf{F}}_{\text{net, external}} = M\vec{\mathbf{A}} = M\frac{d^2\vec{\mathbf{R}}}{dt^2}$$

1.4 Elastic Media and Waves

Internal forces that resemble the force in Hooke's law lead to wave motion. This means that if $h(x,t)$ is some variation from equilibrium (one dimension but could be generalized to $\vec{\mathbf{r}}$) then the variable h must satisfy the wave equation.

$$\frac{\partial^2 h}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 h}{\partial t^2}$$

This equation arises from applying Newton's second law to the elastic medium. This equation is linear so any linear combination of two known solutions is also a solution (**superposition principle**). v is the **wave speed** and is

$$v = \sqrt{\frac{\text{restoring force factor}}{\text{inertial factor}}}$$

The solution to the wave equation is $h(x, t) = g(x - vt)$ for any function g. h is a **traveling wave**, and it represents some shape described by g moving in the x-direction with velocity v. If g takes a sinusoidal form, h is a **harmonic wave**.

$$h(x, t) = H \sin(k(x - vt)) = H \sin(kx - \omega t)$$

H is the **amplitude**, the maximum departure from equilibrium. k and ω the repeat length and time. The **wavelength** λ (repeat length), **period** T, and frequency $f = 1/T$ are specified by the **wave number** k and **angular frequency** ω.

$$\omega = 2\pi f = \frac{2\pi}{T} \text{ and } k = \frac{2\pi}{\lambda}$$

and wave speed links wavelength and frequency.

$$v = \lambda f$$

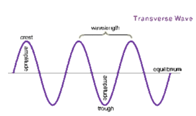
Fourier's theorem states that any wave can be written as a linear combination of harmonic waves.

Another solution for the wave equation is for **standing waves**, with an equation

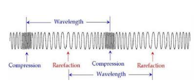
$$h(x, t) = H \sin(kx + \varphi) \cos(\omega t + \delta)$$

These waves oscillate in place rather than traveling. Frequency is still $f = \omega/2\pi$. The wavelength is given by $\lambda = 2L/n$ where n is the number of nodes minus 1.

Waves can either be **transverse** (the displacement is perpendicular to the motion of the wave),



or **longitudinal** (the displacement is parallel to the motion of the wave).



Power and Energy in Waves

The **power** of a wave is the rate at which energy is delivered through a unit area perpendicular to the waves motion. For a harmonic wave propagating on a string of mass density μ,

$$P = \mu v \omega^2 H^2 \cos^2(kx - \omega t)$$

The energy density in the harmonic wave is P/v. Both power and energy density are both traveling waves as well and are proportional to the amplitude squared and the frequency squared.

Reflection and Refraction

Waves that reach boundaries between different media reflect back into the original medium and refract into the new one. The refracted wave is called the **transmitted** wave. The amplitudes of reflected and refracted waves are constrained by the requirement that energy must be conserved.

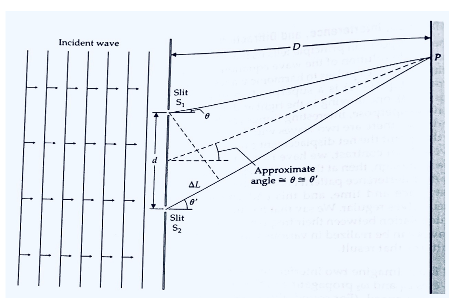
Coherence, Interference, and Diffraction

Interference is essential the superposition of waves. If you have two waves with equal h-values and opposite signs, the superposition of the two waves is destructive interference. If two waves have equal h-values and the same sign, it will be constructive interference. Harmonic waves are **coherent** if there is a definite relation between their frequencies and phases.

Two waves with slightly different angular frequencies ω_1 and ω_2 are interfering in the same medium (with the same speed and amplitude:

$$H\sin(k_1x-\omega_1t)+H\sin(k_2x-\omega_2t)=2H\sin(Kx-\Omega t)\cos\left(\frac{\delta k}{2}x-\frac{\delta\omega}{2}t\right)$$

where $K=(k_1+k_2)/2$, $\Omega=(\omega_1+\omega_2)/2$, $\delta k=k_1-k_2$, and $\delta\omega=\omega_1-\omega_2$. This results in a product of two waves, one of which has a small angular frequency and wave number $\delta k/2$. The part with the small frequency is the **beat**.



With the setup shown in the figure above, one wave is being split by two slits. ΔL is the difference in the distance required to travel by each "source". The condition for constructive interference is $\Delta L=n\lambda$; the condition for destructive interference is $\Delta L=(n+1/2)\lambda$. n is any integer. Using the approximation ($\theta'=\theta$), $\Delta L=d\sin\theta$, so $d\sin\theta=n\lambda$ shows a pattern of constructive interference.

Gratings are similar to the setup shown in the figure except instead of two sources there are N sources to sharpen the interference pattern.

A wave front can also interfere with itself. Wave front propagation can be thought of as a continual regeneration of "waveletes" that interfere with each other to create the straight-line propagation. When there is a barrier, the constructive interference is no longer present so the wave front will bend. This is called **diffraction**.

The Doppler Shift

This is how the movement of the receiver or emitter of the wave or medium affects the frequency or wavelength of the wave.

1.5 Thermal Phenomena

Thermal systems have a **temperature** and **thermal contact** allows the system to reach **thermal equilibrium**, or a common temperature. The **volume** and **pressure** of a gas are closely related to temperature.

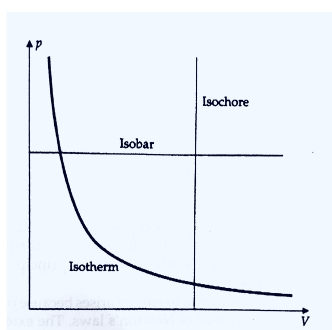
$$T = \lim_{p \to 0} \frac{p}{p_{tp}} (273.16K)$$

p_{tp} is the pressure of water at its triple point: when ice, steam, and liquid water can coexist in equilibrium. The **ideal-gas** equation of state relates thermodynamic variables:

$$pV = nRT = NkT$$

where n is the number of moles of gas, N is the number of individual gas molecules. $n = N \times N_A$ where N_A is **Avogadro's numbers** (6.23×10^{23}). R and k are the **universal gas constant** (8.314 J/(mol $\cdot$ K)) and **Boltzmann's constant** ($R \times N_A$).

Thermodynamic transformations occur when thermodynamic variables change under outside influences. A transformation can either by a **reversible transformation** (a piston pushed into a cylinder) or an **irreversible transformation** (gas freely escaping from a container), which are often spontaneous. There are three types of reversible transformations: isothermal, isobaric, and isochoric, in which the temperature, pressure, and volume, respectively, remain fixed.



In the case of a piston being pushed into a cylinder containing gas, we can look at the work done by the gas on the piston. $dW = F\ dx = pA\ dx = p\ dV$, so

$$W = \int_{V_i}^{V_f} p\ dV$$

or the area under a curve of a p-V diagram. This is a path dependent quantity.

Temperature can also be added through thermal contact: adding heat to the system. The differential amount of heat dQ added is proportional to the temperature change dT, so that

$$dQ = C\ dT$$

where C is the **heat capacity**, a quantity that depends on how the temperature change has been made. C_V is when volume is constant. Q, heat, is also a path dependent quantity and if no external work is done then heat must be conserved. A transformation with no thermal contact is called **adiabatic**.

Thermal energy U, aka internal energy, is

$$dU = dQ - dW$$

where dQ is the heat added to the system and dW is the work done by the system. This is known as the **first law of thermodynamics**. For constant volume,

$$U(T) = \int dQ - 0 = \int^T C_V\ dT = C_V T$$

The **second law of thermodynamics** states that no engine can be perfectly efficient.

$$\text{maximum efficiency} = 1 - T_c/T_h$$

Entropy is a measure of chaos in a system. In an isolated system, entropy has the property that it increases when irreversible processes occur. The stable equilibrium state is the state of maximum entropy.

Kinetic Theory

Temperature can be understood in terms of the **kinetic theory of gases**. Consider this situation, gas is applying pressure to a wall. The internal energy of the gas is the sum of all the kinetic energies of the particles ($U = NK_{av}$). We can assume that all the particles are moving at the same speed.

Now consider a single molecule with initial velocity $\vec{\mathbf{v}}$ bouncing elastically from the wall with area A in the yz-plane. The change in momentum of the particle is $2mv_x$.

Let $n(\vec{\mathbf{v}})$ be the distribution function: the number of particles with velocity $\vec{\mathbf{v}}$ per unit area. Let's look at the molecules with velocities ranging from $\vec{\mathbf{v}} = (v_x, v_y, v_z)$ to $\vec{\mathbf{v}} + d\vec{\mathbf{v}} = (v_x + dv_x, v_y + dv_y, v_z + dv_z)$. $n(\vec{\mathbf{v}})d^3\vec{\mathbf{v}}$ is the number of molecules within this range per unit volume since it is a uniformly random distribution.

For the particle to hit the wall it must be within $v_x dt$ of the wall and it must be in the area A. So it must be in the prism with height $v_x dt$ and base area A (the wall).

So the total change in momentum is

$$dp_{tot} = (2mv_x)n(\vec{\mathbf{v}})d^3\vec{\mathbf{v}}(v_x dtA) = 2mdtAv_x^2 n(\vec{\mathbf{v}})d^3\vec{\mathbf{v}}$$

$$dF = \frac{dp_{tot}}{dt} = 2mAv_x^2 n(\vec{\mathbf{v}})d^3\vec{\mathbf{v}}$$

$$dP = \frac{dF}{A} = 2mv_x^2 n(\vec{\mathbf{v}})d^3\vec{\mathbf{v}}$$

To find total pressure P (for all particles not only in velocity range) we must integrate through all v_y, v_z, and all $v_x > 0$ (must be going towards wall).

$$P = 2m\int v_x^2 n(\vec{\mathbf{v}})d^3\vec{\mathbf{v}}\text{for all } v_x > 0 = 2m\frac{1}{2}\int v_x^2 n(\vec{\mathbf{v}})d^3\vec{\mathbf{v}}$$

since $v_x > 0$ will occur about half the times since direction uniformly random.

Since all directions are uniformly distributed ($n(\vec{\mathbf{v}}) = n(v)$),

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$$

and $v^2 = v_x^2 + v_y^2 + v_z^2$ so on average $v_x^2 = v^2/3$. Since v is a constant for all molecules, it can be taken out of the integral.

$$\int n(\vec{\mathbf{v}})d^3\vec{\mathbf{v}} = N/V$$

the number of molecules per unit volume since $n(\vec{\mathbf{v}})$ is for the range from $\vec{\mathbf{v}}$ to $\vec{\mathbf{v}} + d\vec{\mathbf{v}}$ and the integral is for all $\vec{\mathbf{v}}$.

Using these properties,

$$P = m\int v_x^2 n(\vec{\mathbf{v}})d^3\vec{\mathbf{v}} = \frac{mv^2}{3}\int n(\vec{\mathbf{v}})d^3\vec{\mathbf{v}} = \frac{mv^2}{3}(\frac{N}{V}) = \frac{2}{3}K_{av}(\frac{N}{V})$$

$$PV = nRT = NkT = \frac{2}{3}K_{av}N = \frac{2}{3}U$$

$$U = \frac{3}{2}NkT \text{ and } kT = \frac{2}{3}K_{av}$$

1.6 The Atomic Structure of Matter

Bulk matter comes in three states: gaseous, liquid, and solid. Each of these correspond to different organizations of atoms or molecules that make up matter. In gas, particles are spread apart; in liquid, the atoms are closely bunched; in solid, the atoms are closely bunched and ordered in their arrangement. Electromagnetic forces caused by the presence of **electric charge** cause these particles to be grouped differently. Charge can either be negative or positive, and like charges repel while different charges attract. The force also depends on the distance between particles.

Rutherford's experiment consisted of firing **alpha particles** ($+2e$), aka helium nuclei, at a thin foil. Most of the particles went straight through but a few deflected off the nucleus (when it passes at a distance R away). It feels a force due to the positive charge Ze:

$$F = \frac{(2e)(Ze)}{4\pi\epsilon_0 R^2}$$

We can estimate the deflection angle (since it is small $\theta \approx \sin\theta \approx \tan\theta$. The time the particle spends near the charge is approximately $2R/v$

$$\frac{\Delta p}{p} = \frac{F\Delta t}{M_\alpha v} = \frac{F(2R/v)}{M_\alpha v} = \frac{4Ze^2/4\pi\epsilon_0 R}{M_\alpha v^2}$$

Rutherford's explanation for the small number of deflections was that the radius of the positive charge distribution is 10^4 times smaller than the 10^{-10} m from the previous model. The atom was still 10^{-10} m, but most of it is electron orbitals while the nucleus only takes up a small portion, which orbit like planets since the coulomb force varies as $1/r^2$ just like gravity.

1.7 Electricity and Magnetism

Electrical forces occur between electric charges; **magnetic forces** occur between moving electric charges, or, electric currents. As a charge moves through space, it traces **electric and magnetic fields**, which are vector fields: vectors that have a value for every point in space. $\vec{\mathbf{E}}(\vec{\mathbf{r}})$ is electric field, and $\vec{\mathbf{B}}(\vec{\mathbf{r}})$. Electric charges are associated with electric fields and electric currents are associated with magnetic fields. The electric force is a conservative force and have a potential energy. By dividing out the charge that is feeling the electric field (test charge), we get something that only depends on the source charge distribution. This called **electric potential**.

$$V(\vec{\mathbf{r}}) = -\int_{\vec{\mathbf{r}}_0}^{\vec{\mathbf{r}}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} + V(\vec{\mathbf{r}}_0)$$

Gauss' law is an equation that relates a field F to its source S:

$$\Phi_F = \iint \vec{\mathbf{F}} \cdot d\vec{\mathbf{A}} = \text{Constant} \cdot S$$

The most important fundamental laws of electricity and magnetism are known as the four **Maxwell's equations**.

Maxwell's Equations

1. Gauss' law for electric fields

$$\Phi_E = \iint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q}{\epsilon_0}$$

where Q is the net electric charge enclosed by the surface which he electric field is integrated. Gauss' law is equivalent to **Coulomb's Law** in static situations for the force between charges:

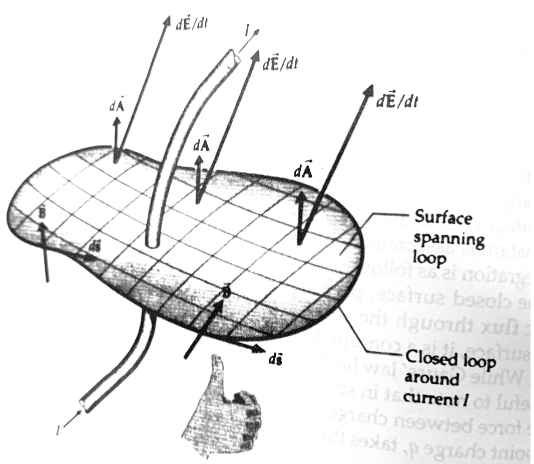
$$\vec{\mathbf{E}} = \frac{q}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}}$$

where r is the distance from the point charge
2. Gauss' law for magnetic fields

$$\Phi_B = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

This shows that there are no magnetic monopoles and the magnetic fields trace out closed loops instead of just pointing radially outwards.
3. The generalized Ampere's law
This relates magnetic fields to the currents and changing electric fields that produce them by the equation:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I + \mu_0\epsilon_0 \frac{d}{dt}\iint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$



The double integral is the flux going through any surface that spans the loop. The first integral is a line integral around a closed loop enclosing the current I.
4. Faraday's law:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

Faraday's law demonstrates how changing magnetic flux can generate an electric field. The minus sign shows that the secondary / generated magnetic flux tends to oppose the original change in magnetic flux. This is known as **Lenz's law**.

Lorentz force law: the force on a charge q moving with velocity $\vec{\mathbf{v}}$ in an electric field $\vec{\mathbf{E}}$ and magnetic field $\vec{\mathbf{B}}$

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$$

so $\vec{\mathbf{F}} = \vec{\mathbf{F}}_E + \vec{\mathbf{F}}_B = q\vec{\mathbf{E}} + q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$. $\vec{\mathbf{F}}_B$ can sometimes be written as a force on the length of a wire $d\vec{\ell}$ carrying a current in a magnetic field.

$$d\vec{\mathbf{F}}_B = I\vec{\ell} \times \vec{\mathbf{B}}$$

This can be integrated to find $\vec{\mathbf{F}}_B$.

If a loop carrying current is placed in a uniform magnetic field $\vec{\mathbf{B}}$, theres no net force, but there is a torque that tends to twist the loop.

$$\vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}}$$

where $\vec{\mu}$ is the magnetic moment which is $I\vec{\mathbf{A}}$, where I is the current carried in the loop and $\vec{\mathbf{A}}$ is the area of the loop with direction determined by the right hand rule (fingers following current and thumb at $\vec{\mathbf{A}}$). Torque tends to rotate the loop so that $\vec{\mu}$ is in the same direction as $\vec{\mathbf{B}}$. When it is placed in the field, the loop has a minimum potential energy when $\vec{\mu}$ and $\vec{\mathbf{B}}$ are aligned.

$$U = -\vec{\mu} \times \vec{\mathbf{B}}$$

Electric and magnetic fields themselves contain energy. This is given by the energy density u, or energy per unit volume:

$$u = \frac{1}{2\mu_0}B^2 + \frac{1}{2}\epsilon_0 E^2$$

Insulators have the property that electric charges cannot move through them very easily. **Conductors** have the property that electrons can freely move within them. **Semiconductors** lie somewhere in between.

The atoms of an insulator in an electric field align so that the electric field within the material is diminished. The effects of insulators, aka dielectrics, can be summarized by replacing ϵ_0 with $\epsilon = \kappa\epsilon_0$, where κ is the *dielectric constant* of the material. The charges within conductors in an external electric field move until the electric field within is canceled. **Ferromagnetic** materials produce a magnetic field on their own by the alignment of its atoms which act like tiny magnets.

Ohm's law describes how currents in a piece of conducting material are formed when an electric field is maintained across the material.

$$V = IR$$

where V is the potential difference from end to end, R is the resistance, and I is the current. When R is independent of V, the material is *ohmic* and Ohm's law applies.

1.8 Electromagnetic Waves and Lights

Refraction is the bending of light as it propagates from one medium to another. **Diffraction** is the bending of light as it goes past a barrier. These are wave like properties.

Newton had a theory, in which indivisible particles of light moved in straight lines and are subject to gravity. Later, Young's famous experiment, essentially the double slit experiment with light, showed that light must be described as a wave since light showed diffraction and interference to form an interference pattern, just like waves. Through this experiment Young was even able to find the wavelengths λ.

Using Maxwell's equations, for an electric field, the presence of a set of charges oscillating in the x-direction allows us to generate a wave equation of the form

$$\frac{\partial^2}{\partial z^2}E_x = \mu_0\epsilon_0\frac{\partial^2}{\partial t^2}E_x$$

This equation represents a wave propagating in the z-direction with a speed of $v = 1/\sqrt{\mu_0\epsilon_0}$, which is the speed of light c.

$$\frac{1}{\sqrt{\mu_0\epsilon_0}} = c$$

This showed that light was indeed a wave. The same conditions held true for the equation for the magnetic field (in the y-direction)

$$\frac{\partial^2}{\partial z^2}B_y = \mu_0\epsilon_0\frac{\partial^2}{\partial t^2}B_y$$

There wave equations have general solutions of the form

$$E_x = E_0\cos(kz - \omega t + \varphi) \text{ and } B_y = B_0\cos(kz - \omega t + \varphi)$$

φ is the same for each equation because Maxwell's equations require this condition. For this wave, the wavelength $\lambda = 2\pi/k$ and the frequency $f = \omega/2\pi$ are related by $\lambda f = v - c$.

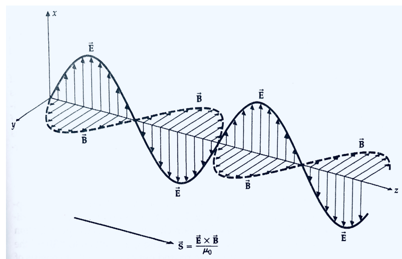
• Nothing restricts the wavelength in the **electromagnetic spectrum**. Visible light only contains a limited range of wavelengths, but there are infinitely many.

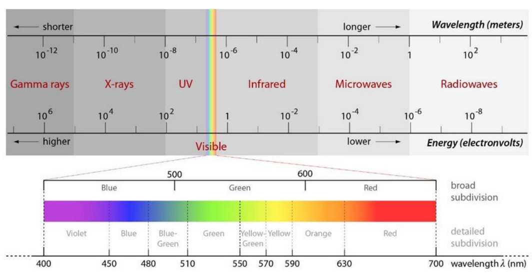
• Accelerating charges are necessary to start an electromagnetic wave, but the wave equation does not require any further charges to be present. It does not require a medium, but it can propagate in dielectric media. If the media does not destroy the waves, it is transparent. The only changes are that ϵ_0 must be switched to ϵ and μ_0 must be switched to μ. So the speed of light in this media is $c' = c/n$ where n is the **index of refraction**. Transparent materials have $\mu \approx \mu_0$ so $n \approx \sqrt{\kappa}$

• Since the electric and magnetic fields have the same phase, when one is at a maximum so is the other; they oscillate together.

• In electromagnetic waves $E = vB = cB$ to balance the fields.

• The electric and magnetic fields are perpendicular to the direction of wave propagation and to each other. The electromagnetic waves are *transverse*





Energy and Momentum Transport

The energy density of electromagnetic waves is

$$u = \frac{1}{2}\epsilon_0(E^2 + c^2B^2) = \epsilon_0 E^2$$

since $E = cB$. The energy density is split evenly between the electric and magnet parts of the wave. Also, the energy itself is a wave since it contains $\cos^2(kz - \omega t + \varphi)$ and it propagates with speed c. In other words electromagnetic waves transport energy. The rate at which energy arrives at a surface perpendicular to the direction of wave propagation is the **energy flux** and it is given by cu. The energy flux can be more fully characterized by a vector (the **Poynting vector**) in the direction of $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$ and with magnitude cu.

$$\vec{\mathbf{S}} = (\vec{\mathbf{E}} \times \vec{\mathbf{B}})/\mu_0$$

Intensity I is the time average of the energy flux. Since the average of cosine squared is $1/2$, $I = S/2$.

The electromagnetic fields also exert a net force in the direction of propagation on charged particles, which means the wave also transports momentum to the charge on collision. The momentum per unit volume, or momentum density, is $\vec{\mathbf{S}}/c^2$. One consequence of this is that the wave exerts pressure on surfaces called *radiation pressure* which is given by $2u$ for a perfectly reflecting surface and u for a perfectly absorbing surface.

Polarization

Although the electric and magnetic fields lies in the plane perpendicular to the direction of propagation, the direction in that plane can be varied. That direction specifies the **polarization** of the wave. It is possible to measure and filer this direction. A filter that only allows light at a certain angle will reduce the intensity from I to $I\cos^2\theta$ where θ is the angle between the original polarization and the final polarization. This also reduces the electric field since the filter only uses the component of electric field in the specified direction. The intensity relationship is known as *Malus's law*.

Chapter 2

The Basics of Relativity