

# Chapter 1

## A Review

### 1.1 Newton's Laws

Isaac Newton introduced the three basic laws of mechanics that are known as Newton's Laws.

#### Newton's Second Law

$$\vec{\mathbf{F}}_{net} = m\vec{\mathbf{a}}$$

Sometimes this equation is written as

$$\vec{\mathbf{F}}_{net} = \frac{d\vec{\mathbf{p}}}{dt}$$

to account for a changing mass, where  $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$ .

#### Newton's First Law

Newton's first law is a special case of the second law:

$$\text{If } \vec{\mathbf{F}}_{net} = \vec{\mathbf{0}}, \text{ the motion is uniform}$$

which means that velocity is constant and acceleration is 0.

#### Gravity

Acceleration caused by gravity is the same on every object. This seems contradictory to what we have seen from Newton's second law,  $\vec{\mathbf{a}} = \vec{\mathbf{F}}_{new}/m$ , but since the force of gravity is proportional to the mass, the acceleration is independent of the mass.

### Hooke's Law

The equation for spring force is  $F = -kx$ . So,

$$-kx = m \frac{d^2x}{dt^2}$$

The solution of this differential equation is

$$x(t) = A \sin(\omega t + \varphi)$$

where  $A$  is the *amplitude* of the motion, and  $\varphi$  is the *phase*. Both of these quantities are determined by the initial conditions.  $\omega$  is determined by the spring and the mass:

$$\omega = \sqrt{k/m}$$

where  $\omega$  is the *angular frequency*, which is related to the *period*  $T$  and *frequency*  $f$ .

$$T = \frac{1}{f}$$

so  $f = \omega/2\pi$ .

### Newton's Third Law

Every force has an equal and opposite reaction force.

$$\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$$

This can be restated in terms of momentum.

$$\frac{d\vec{\mathbf{p}}_1}{dt} = -\frac{d\vec{\mathbf{p}}_2}{dt} \text{ or } \frac{d}{dt}(\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2) = 0$$

So,

$$\vec{\mathbf{P}}_{tot} = \text{a constant vector}$$

## 1.2 Work, Energy, and the Conservation of Energy

For an object traveling in one dimension with a constant net force  $F$  and with velocity changing linearly with time ( $v \propto t$ ), we have

$$F \times (x_f - x_i) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

The quantity  $mv^2/2$  is called the **kinetic energy**  $K$ , while the left side is referred to as the **work**  $W$  done by the force. For forces that are not constant we have

$$W = \int_{x_i}^{x_f} F(x)dx$$

To extend to multiple dimensions

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}$$

With these new definitions, the first equation of this section can be written as

$$W_{net} = \Delta K = K_f - K_i$$

aka the **work-energy theorem**. A **conservative force** is a force which the work done by the force is path independent (it only depends on the start and end state).

$$\int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r} = f(\vec{r}_f) - f(\vec{r}_i)$$

We define potential energy for these forces as

$$U(\vec{r}) = - \int_{\vec{r}_0}^{\vec{r}} \vec{F} \cdot d\vec{r} + U(\vec{r}_0)$$

So  $U(\vec{r}_f) - U(\vec{r}_i) = - \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}$ . We also set **total energy** to

$$E \equiv U(\vec{r}) + K$$

### Important potential energies

$$\text{Local Gravity: } U(h) = mgh$$

$$\text{Local Gravity: } U(h) = mgh$$

$$\text{Gravitation: } U(r) = -\frac{Gm_1m_2}{r}$$

Given the potential energy, we can find the force using the gradient

$$\vec{F}(\vec{r}) = -\vec{\nabla}U$$

$$F(x) = -\frac{dU}{dx} \text{ (in one dimension)}$$

## 1.3 Rotations and the Center of Mass

The **angular velocity**  $\vec{\omega}$  is the rate at which the angle  $\theta$  changes. Its direction is determined by the right hand rule (Fingers in direction of motion, thumb is direction of  $\vec{\omega}$ ). The rate at which  $\omega$  changes is the **angular acceleration** vector  $\vec{\alpha}$ . The rotation equivalents for mass and force, are **rotational inertia**  $I$  and **torque**  $\vec{\tau}$ . So Newton's second law for rotation motion is

$$\vec{\tau} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$$

where  $\vec{\mathbf{L}}$  is **angular momentum**, which is defined as

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$$

Torque is similarly defined as

$$\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$$

The rotational inertia  $I$  is dependent on the way an objects mass is distributed. By breaking up the object of total mass

$$M = \sum_i \Delta m_i$$

into many discrete portions of mass, we can set

$$I \equiv \sum_i (\Delta m_i) r_i^2 = \int r^2 dm$$

Work one in a rotation rigid body is defined as

$$W = \int_{\theta_i}^{\theta_f} \tau d\theta$$

Kinetic energy of a rotating object is

$$K = \frac{1}{2} I \omega^2$$

The **center of mass** is defined as

$$\vec{\mathbf{R}} = \frac{\sum_i m_i \vec{\mathbf{r}}_i}{M}$$

For an external force moving the object, the mass will act as a point mass at  $\vec{\mathbf{R}}$ , so

$$\vec{\mathbf{F}}_{\text{net, external}} = M \vec{\mathbf{A}} = M \frac{d^2 \vec{\mathbf{R}}}{dt^2}$$

## 1.4 Elastic Media and Waves

Internal forces that resemble the force in Hooke's law lead to wave motion. This means that if  $h(x, t)$  is some variation from equilibrium (one dimension but could be generalized to  $\vec{\mathbf{r}}$ ) then the variable  $h$  must satisfy the wave equation.

$$\frac{\partial^2 h}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 h}{\partial t^2}$$

This equation arises from applying Newton's second law to the elastic medium. This equation is linear so any linear combination of two known solutions is also a solution (**superposition principle**).  $v$  is the **wave speed** and is

$$v = \sqrt{\frac{\text{restoring force factor}}{\text{inertial factor}}}$$

The solution to the wave equation is  $h(x, t) = g(x - vt)$  for any function  $g$ .  $h$  is a **traveling wave**, and it represents some shape described by  $g$  moving in the  $x$ -direction with velocity  $v$ . If  $g$  takes a sinusoidal form,  $h$  is a **harmonic wave**.

$$h(x, t) = H \sin(k(x - vt)) = H \sin(kx - \omega t)$$

$H$  is the **amplitude**, the maximum departure from equilibrium.  $k$  and  $\omega$  the repeat length and time. The **wavelength**  $\lambda$  (repeat length), **period**  $T$ , and frequency  $f = 1/T$  are specified by the **wave number**  $k$  and **angular frequency**  $\omega$ .

$$\omega = 2\pi f = \frac{2\pi}{T} \text{ and } k = \frac{2\pi}{\lambda}$$

and wave speed links wavelength and frequency.

$$v = \lambda f$$

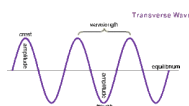
Fourier's theorem states that any wave can be written as a linear combination of harmonic waves.

Another solution for the wave equation is for **standing waves**, with an equation

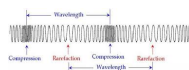
$$h(x, t) = H \sin(kx + \varphi) \cos(\omega t + \delta)$$

These waves oscillate in place rather than traveling. Frequency is still  $f = \omega/2\pi$ . The wavelength is given by  $\lambda = 2L/n$  where  $n$  is the number of nodes minus 1.

Waves can either be **transverse** (the displacement is perpendicular to the motion of the wave),



or **longitudinal** (the displacement is parallel to the motion of the wave).



## Power and Energy in Waves

The **power** of a wave is the rate at which energy is delivered through a unit area perpendicular to the waves motion. For a harmonic wave propagating on a string of mass density  $\mu$ ,

$$P = \mu v \omega^2 H^2 \cos^2(kx - \omega t)$$

The energy density in the harmonic wave is  $P/v$ . Both power and energy density are both traveling waves as well and are proportional to the amplitude squared and the frequency squared.

## Reflection and Refraction

Waves that reach boundaries between different media reflect back into the original medium and refract into the new one. The refracted wave is called the **transmitted** wave. The amplitudes of reflected and refracted waves are constrained by the requirement that energy must be conserved.

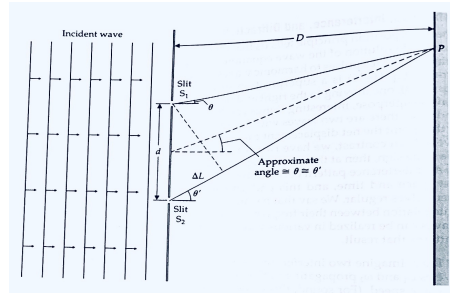
## Coherence, Interference, and Diffraction

**Interference** is essential the superposition of waves. If you have two waves with equal  $h$ -values and opposite signs, the superposition of the two waves is destructive interference. If two waves have equal  $h$ -values and the same sign, it will be constructive interference. Harmonic waves are **coherent** if there is a definite relation between their frequencies and phases.

Two waves with slightly different angular frequencies  $\omega_1$  and  $\omega_2$  are interfering in the same medium (with the same speed and amplitude:

$$H \sin(k_1 x - \omega_1 t) + H \sin(k_2 x - \omega_2 t) = 2H \sin(Kx - \Omega t) \cos\left(\frac{\delta k}{2}x - \frac{\delta \omega}{2}t\right)$$

where  $K = (k_1 + k_2)/2$ ,  $\Omega = (\omega_1 + \omega_2)/2$ ,  $\delta k = k_1 - k_2$ , and  $\delta \omega = \omega_1 - \omega_2$ . This results in a product of two waves, one of which has a small angular frequency and wave number  $\delta k/2$ . The part with the small frequency is the **beat**.



With the setup shown in the figure above, one wave is being split by two slits.  $\Delta L$  is the difference in the distance required to travel by each "source". The condition for constructive interference is  $\Delta L = n\lambda$ ; the condition for destructive interference is  $\Delta L = (n+1/2)\lambda$ .  $n$  is any integer. Using the approximation ( $\theta' = \theta$ ),  $\Delta L = d \sin \theta$ , so  $d \sin \theta = n\lambda$  shows a pattern of constructive interference.

**Gratings** are similar to the setup shown in the figure except instead of two sources there are  $N$  sources to sharpen the interference pattern.

A wave front can also interfere with itself. Wave front propagation can be thought of as a continual regeneration of "wavelets" that interfere with

each other to create the straight-line propagation. When there is a barrier, the constructive interference is no longer present so the wave front will bend. This is called **diffraction**.

### **The Doppler Shift**

This is how the movement of the receiver or emitter of the wave or medium affects the frequency or wavelength of the wave.