1.3 Rotations and the Center of Mass

The **angular velocity** $\vec{\omega}$ is the rate at which the angle θ changes. Its direction is determined by the right hand rule (Fingers in direction of motion, thumb is direction of $\vec{\omega}$). The rate at which ω changes is the **angular acceleration** vector $\vec{\alpha}$. The rotation eq uivalents for mass and force, are **rotational inertia** I and **torque** $\vec{\tau}$. So Newton's second law for rotation motion is

$$\vec{\tau} = I\vec{\alpha} = \frac{d\vec{\mathbf{L}}}{dt}$$

where $\vec{\mathbf{L}}$ is **angular momentum**, which is defined as

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$$

Torque is similarly defined as

$$\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$$

The rotational inertia I is dependent on the way an objects mass is distributed. By breaking up the object of total mass

$$M = \sum_i \Delta m_i$$

into many discrete portions of mass, we can set

$$I \equiv \sum_i (\Delta m_i) r_i^2 = \int r^2 dm$$

Work one in a rotation rigid body is defined as

$$W = \int_{\theta_i}^{\theta_f} \tau d\theta$$

Kinetic energy of a rotating object is

$$K = \frac{1}{2} I\omega^2$$

The **center of mass** is defined as

$$\vec{\mathbf{R}} = \frac{\sum_i m_i \vec{\mathbf{r}}_i}{M}$$

For an external force moving the object, the mass will act as a point mass at $\vec{\mathbf{R}}$, so

$$\vec{\mathbf{F}}_{\text{net, external}} = M\vec{\mathbf{A}} = M\frac{d^2\vec{\mathbf{R}}}{dt^2}$$