1.1 Newton's Laws

Isaac Newton introduced the three basic laws of mechanics that are known as Newton's Laws.

Newton's Second Law

$$\vec{\mathbf{F}}_{net} = m\vec{\mathbf{a}}$$

Sometimes this equation is written as

$$\vec{\mathbf{F}}_{net} = \frac{d\vec{\mathbf{p}}}{dt}$$

to account for a changing mass, where $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$.

Newton's First Law

Newton's first law is a special case of the second law:

$$\text{If } \vec{\mathbf{F}}_{net} = \vec{\mathbf{0}}, \text{ the motion is uniform}$$

which means that velocity is constant and accusation is 0.

Gravity

Acceleration caused by gravity is the same on every object. This seems contradictory to what we have seen from Newton's second law, $\vec{\mathbf{a}} = \vec{\mathbf{F}}_{new}/m$, but since the force of gravity is proportional to the mass, the acceleration is independent of the mass.

Hooke's Law

The equation for spring force is $F = -kx$. So,

$$-kx = m\frac{d^2x}{dt}$$

The solution of this differential equation is

$$x(t) = A\sin(\omega t + \varphi)$$

where A is the *amplitude* of the motion, and φ is the textitphase. Both of these quantities are determined by the initial conditions. ω is determined by the spring and the mass:

$$\omega = \sqrt{k/m}$$

where ω is the *angular frequency*, which is related to the *period* T and *frequency* f.

$$T = \frac{1}{f}$$

so $f = \omega/2\pi$.

Newton's Third Law

Every force has an equal and opposite reaction force.

$$\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$$

This can be restated in terms of momentum.

$$\frac{d\vec{\mathbf{p_1}}}{dt} = -\frac{d\vec{\mathbf{p_2}}}{dt} \text{ or } \frac{d}{dt}(\vec{\mathbf{p_1}} + \vec{\mathbf{p_2}}) = 0$$

So,

$$\vec{\mathbf{P}}_{tot} = \text{a constant vector}$$