Adaptive–Robust Control of Euler–Lagrange Systems With Linearly Parametrizable Uncertainty Bound

Spandan Roy, Sayan Basu Roy, and Indra Narayan Kar, *Senior Member, IEEE*

***Abstract*— This brief proposes a new adaptive–robust control (ARC) architecture for a class of uncertain Euler–Lagrange (EL) systems where the upper bound of the uncertainty satisfies linear in the parameters structure. Conventional ARC strategies either require structural knowledge of the system or presume that the overall uncertainties or its time derivative are norm bounded by a constant. Due to the unmodeled dynamics and modeling imperfection, true structural knowledge of the system is not always available. Furthermore, for the class of systems under consideration, prior assumption, regarding the uncertainties (or its time derivative) being upper bounded by a constant, puts a restriction on the states beforehand. Conventional ARC laws invite overestimation–underestimation problem of switching gain. Toward this front, adaptive switching-gain-based robust control (ASRC) is proposed, which alleviates the overestimation–underestimation problem of switching gain. Moreover, ASRC avoids any presumption of constant upper bound on the overall uncertainties and can negotiate uncertainties regardless of being linear or nonlinear in parameters. Experimental results of ASRC using a wheeled mobile robot note improved control performance in comparison with the adaptive sliding mode control.**

***Index Terms*— Adaptive–robust control (ARC), Euler–Lagrange (EL) systems, uncertainty, wheeled-mobile robot (WMR).**

I. INTRODUCTION

A. Background

THE controller design aspect for nonlinear systems subjected to parametric and nonparametric uncertainties has always been a challenging task. Adaptive control and robust control are the two popular control strategies to deal with uncertain nonlinear systems. In case of adaptive control, online computation of the unknown system parameters and controller gains for complex systems is significantly intensive [1]. On the other front, robust control reduces computation burden for complex systems compared with the adaptive control, while requiring a predefined upper bound on the uncertainties. However, in practice, it is not always possible to estimate a prior uncertainty bound due to the effect of unmodeled dynamics. Again, to increase the operating region of the controller, often higher uncertainty bounds are assumed. This, in turn, leads to overestimation of switching gain and high control effort [2].

Considering the individual constraints of adaptive and robust control, recently, global research is reoriented toward adaptive–robust control (ARC). The works [1], [3]–[11] regarding ARC estimate the individual uncertain system parameters through adaptive law, and robust control is utilized to negate the effect of external disturbances. These works utilize the projection operator in their respective adaptive laws, which necessitate the knowledge of lower and upper bound of individual uncertain system parameters. Adaptive sliding mode control (ASMC) is designed in [12] for parameter identification of mechanical servo systems with LuGre friction considering the uncertainties to be linear in parameters (LIP). In contrast, the controllers [13]–[20] assume that the overall uncertainty (or its time derivative) is bounded by some constant. Thereafter, that constant term is estimated by adaptive law, rather estimating individual uncertain system parameters. The adaptive laws in [17] and [18] involve a predefined threshold value; as a matter of fact, until the threshold value is achieved, the switching gain may still be increasing (resp. decreasing) even if the tracking error decreases (resp. increases) and thus creates the overestimation (resp. underestimation) problem of switching gain. While the underestimation problem compromises the controller accuracy by applying lower switching gain than the required amount, the overestimation problem causes larger gain and high control input requirement. The adaptive law reported in [19] requires predefined bound on the time derivative of the uncertainties. As observed in [20], the method in [19] also requires frequency characteristics of the perturbation to design the filter for equivalent control. However, the brief in [20] assumed that the time derivative of the uncertainties is bounded by an unknown constant.

B. Motivation

Let us consider the following system representing a chemostat operating under Monod kinetic [21]:

$$\dot{z}_1 = f_1(z_1, z_2) - Dz_1, \quad \dot{z}_2 = f_2(z_1, z_2) + S_0 - Dz_2 \tag{1}$$

where

$$f_1(z_1, z_2) = \frac{\delta_1\ z_1\ z_2}{\delta_2 + z_2}, \quad f_2(z_1, z_2) = \frac{-\delta_3\ z_1\ z_2}{\delta_2 + z_2}.$$

Here, $z_1 \geq 0, z_2 \geq 0\ \forall\ t \geq 0$ are states; δ_1, δ_2, and δ_3 are uncertain positive parameters; S_0 is a known constant, and D is the control input. For system (1), the following relations hold:

$$|f_1| \leq y_1 f(z), \quad |f_2| \leq y_2 f(z) \tag{2}$$

where

$$y_1 \triangleq |\delta_1|/|\delta_2|, \quad y_2 \triangleq |\delta_3|/|\delta_2|, \ \ f(z) \triangleq |z_1||z_2|.$$

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The authors are with the Department of Electrical Engineering, IIT Delhi, New Delhi 110016, India (e-mail: sroy002@gmail.com; sayanetce@gmail.com; ink@ee.iitd.ac.in).

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Inspection of the uncertainties f_1 and f_2 from (1) and (2) reveals that though f_1, f_2 are nonlinear in parameters (NLIP) but their upper bounds are LIP.

Similarly, EL systems can have uncertainties with the LIP or NLIP (e.g., system with nonlinear friction [22]) structure. However, the upper bound of the overall (or lumped) uncertainty for such systems has LIP property [23]. EL systems, in general, represent a large class of real world systems, such as robotic manipulators [24], [25], mobile robots [26], ship dynamics, aircraft, and pneumatic muscles [27]. These systems have immense applications in various domains, such as defense, automation industry, surveillance, and space missions. The controllers [13]–[18] assume that the overall uncertainties are upper bounded by some constant, while [19] and [20] assume the time derivative of the overall uncertainty to be bounded by some constant. Hence, for the aforementioned class of systems, consideration of such constant bound (known or unknown) restricts the system states *a priori*. Furthermore, the switching gain in [17] and [18] suffers from the overestimation and underestimation problems. In practice, it is also not always possible to have prior knowledge of the bounds for system parameters as required in [1] and [3]–[11] for projection operator.

C. Contribution

In view of the above-mentioned discussion and the importance of EL systems in real-life scenarios, it is imperative to formulate a dedicated ARC framework for uncertain EL systems. Toward this front, adaptive switching-gain-based robust control (ASRC) is presented in this brief for tracking control of uncertain EL systems. *The formulation of ASRC is insensitive toward the nature of the uncertainties, i.e., it can negotiate uncertainties that can be either LIP or NLIP. ASRC utilizes the LIP structure of the upper bound of uncertainty and does not presume the overall uncertainty (or its time derivative) to be upper bounded by a constant.* The adaptive law of ASRC prevents the switching gain from becoming a monotonically increasing function by allowing the switching gain to decrease within a finite time when tracking error decreases. Moreover, ASRC alleviates the overestimation–underestimation problem of switching gain. To realize the effectiveness, the performance of ASRC is compared with ASMC [17], [18] experimentally using PIONEER 3 wheeled-mobile robot (WMR).

The remainder of this brief is organized as follows. The proposed ASRC framework for a second-order EL system is detailed in Section II. This is followed by the experimental results of ASRC and its comparison with ASMC [17], [18] in Section III. Section IV presents concluding remarks.

The following notations are used in this brief: $\lambda_{\min}(\bullet)$ and $||\bullet||$ represent the minimum eigenvalue and Euclidean norm of $(\bullet)$, respectively; I denotes identity matrix with appropriate dimension.

II. Controller Design

A. Problem Formulation

In general, an EL system with the second-order dynamics can be written as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + f(\dot{q}) + d_s = \tau \tag{3}$$

where $q \in \mathbb{R}^n$ denotes system state, $\tau \in \mathbb{R}^n$ denotes the vector of generalized control input forces, $M(q) \in \mathbb{R}^{n\times n}$ represents mass/inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ denotes Coriolis, centripetal terms, $g(q) \in \mathbb{R}^n$ denotes gravity vector, $f(\dot{q}) \in \mathbb{R}^n$ represents the vector of slip, damping, and friction forces, and $d_s(t)$ denotes the bounded external disturbances. System (3) possesses the following properties [23].

Property 1: The matrix $(\dot{M} - 2C)$ is skew symmetric.

Property 2: $\exists\ g_b, f_b, \bar{d} \in \mathbb{R}^+$, such that $||g(q)|| \le g_b$, $||f(\dot{q})|| \le f_b||\dot{q}||$, and $||d_s(t)|| \le \bar{d}$.

Property 3: The matrix $M(q)$ is uniformly positive definite and there exist two positive constants μ_1, μ_2, such that

$$0 < \mu_1 I \le M(q) \le \mu_2 I. \tag{4}$$

Property 4: $\exists\ C_b \in \mathbb{R}^+$, such that $||C(q,\dot{q})|| \le C_b||\dot{q}||$.

Let $q^d(t)$ be the desired trajectory to be tracked, and it is selected, such that $q^d, \dot{q}^d, \ddot{q}^d \in \mathcal{L}_\infty$. Let $e(t) \triangleq q(t) - q^d(t)$ be the tracking error and e_f be the filtered tracking error

$$e_f \triangleq \dot{e} + \Omega e \Rightarrow e_f = \Gamma\xi \tag{5}$$

where $\Gamma \triangleq [\Omega\ I]$, $\xi \triangleq [e^T\ \dot{e}^T]^T$, and $\Omega \in \mathbb{R}^{n\times n}$ is a positive definite matrix. Multiplying the time derivative of (5) by M and using (3) yields

$$M\dot{e}_f = M(\ddot{q} - \ddot{q}^d + \Omega\dot{e}) = \tau - C(q,\dot{q})e_f + \sigma \tag{6}$$

where $\sigma \triangleq -(C(q,\dot{q})\dot{q} + g(q) + f(\dot{q}) + d_s + M\ddot{q}^d - M\Omega\dot{e} - C(q,\dot{q})e_f)$ represents the overall uncertainty. Furthermore, $\xi = [e^T\ \dot{e}^T]^T$ implies $||\xi|| \ge ||e||, ||\xi|| \ge ||\dot{e}||$.

1) Characterization of the Upper Bound of σ: Relation (5) and system Property 4 yield

$$\begin{aligned} &||Ce_f - C\dot{q}|| \\ &\quad = ||C(\dot{e} + \Omega e) - C\dot{q}|| = || - C\dot{q}^d + C\Omega e|| \\ &\quad \le C_b||\dot{e} + \dot{q}^d||||\dot{q}^d|| + C_b||\dot{e} + \dot{q}^d||||\Omega||||\xi|| \\ &\quad \le C_b\{||\xi||||\dot{q}^d|| + ||\dot{q}^d||^2 + ||\xi||^2||\Omega|| + ||\dot{q}^d||||\Omega||||\xi||\}. \end{aligned} \tag{7}$$

Furthermore, system Properties 2 and 3 provide the following:

$$\begin{aligned} &||g(q) + f(\dot{q}) + d_s + M\ddot{q}^d - M\Omega\dot{e}|| \\ &\quad \le g_b + f_b||\dot{e} + \dot{q}^d|| + \bar{d} + \mu_2||\ddot{q}^d|| + \mu_2||\Omega||||\xi|| \\ &\quad \le g_b + f_b||\xi|| + f_b||\dot{q}^d|| + \bar{d} + \mu_2||\ddot{q}^d|| + \mu_2||\Omega||||\xi||. \end{aligned} \tag{8}$$

Since $q^d, \dot{q}^d, \ddot{q}^d \in \mathcal{L}_\infty$, it can be verified using (7) and (8) that $\exists\ \theta_i^* \in \mathbb{R}^+\ i = 0,1,2$, such that the upper bound of σ holds the following LIP structure [23]:

$$||\sigma|| \le \theta_0^* + \theta_1^*||\xi|| + \theta_2^*||\xi||^2 \triangleq Y(\xi)^T\Theta^* \tag{9}$$

where $Y(\xi) = [1\ ||\xi||\ ||\xi||^2]^T$ and $\Theta^* = [\theta_0^*\ \theta_1^*\ \theta_2^*]^T$.

Let $\bar{\Theta} \triangleq \{\Theta \in \mathbb{R}^3 : \theta_i \ge \theta_i^*\ \forall\ i = 0,1,2\}$, such that the following condition always holds from (9):

$$||\sigma|| \le Y(\xi)^T\Theta, \quad \forall\ \Theta \in \bar{\Theta}. \tag{10}$$

A robust controller for system (3) can be designed as [23]

$$\tau = -e - Ge_f - \Delta\tau, \quad \Delta\tau = \begin{cases} \rho\dfrac{e_f}{||e_f||}, & \text{if } ||e_f|| \ge \varpi \\ \rho\dfrac{e_f}{\varpi}, & \text{if } ||e_f|| < \varpi \end{cases} \tag{11}$$

$$\rho = Y(\xi)^T\Theta \tag{12}$$

where $\Delta\tau$ provides robustness against σ through the switching gain ρ; $\varpi \in \mathbb{R}^+$ is a small scalar used for chattering removal; $G \in \mathbb{R}^{n\times n}$ is a positive definite matrix.

2) Evaluation of Switching Gain: Evaluation of ρ, such as (12), is conservative in nature and evidently requires the knowledge of Θ^*, which is not always possible in the face of uncertain parametric variations and external disturbances. The control laws developed in [13]–[20] assume that σ and $\dot{\sigma}$ are upper bounded by a constant, respectively. Exploring the structure of $||\sigma||$ from (9), it can be inferred that such *constant bound assumption on the uncertainties, whether known or unknown, puts a restriction on the states a priori*. Moreover, the switching gain in [17] and [18] suffers from the overestimation–underestimation problem.

B. Adaptive Switching-Gain-Based Robust Control

The major aims of the proposed ASRC framework are as follows.

1) To compensate the uncertainties that can be either LIP or NLIP. However, the upper bound of the uncertainties satisfies the LIP property (9).
2) To alleviate the overestimation–underestimation problem of the switching gain.

The control input of the proposed ASRC is designed as

$$\tau = -e - Ge_f - \Delta\tau, \quad \Delta\tau = \begin{cases} \hat{\rho}\dfrac{e_f}{||e_f||}, & \text{if } ||e_f|| \geq \varpi \\ \hat{\rho}\dfrac{e_f}{\varpi}, & \text{if } ||e_f|| < \varpi \end{cases} \tag{13}$$

$$\hat{\rho} = \hat{\theta}_0 + \hat{\theta}_1||\xi|| + \hat{\theta}_2||\xi||^2 + \gamma \triangleq Y(\xi)^T\hat{\Theta} + \gamma \tag{14}$$

where $\Delta\tau$ provides robustness against σ through $\hat{\rho}$; $\hat{\Theta} = [\hat{\theta}_0\ \hat{\theta}_1\ \hat{\theta}_2]^T$ is the estimate of Θ^*; γ is an auxiliary gain. The importance of γ will be explained later. The gains $\gamma, \hat{\theta}_i$, $i = 0, 1, 2$ are evaluated using the following adaptive laws.

1) For $||e_f|| \geq \varpi$

$$\dot{\hat{\theta}}_i = \begin{cases} \alpha_i||\xi||^i||e_f||, & \text{if } \{e^T\dot{e} > 0\} \cup \{\bigcup_{i=0}^2 \hat{\theta}_i \leq 0\} \\ & \quad \cup\{\gamma \leq \beta\} \\ -\alpha_i||\xi||^i||e_f||, & \text{otherwise} \end{cases} \tag{15}$$

$$\dot{\gamma} = \begin{cases} \alpha_3||e_f||, & \text{if } \{e^T\dot{e} > 0\} \cup \{\bigcup_{i=0}^2 \hat{\theta}_i \leq 0\} \\ & \quad \cup\{\gamma \leq \beta\} \\ -\varsigma\alpha_3||\xi||^4, & \text{otherwise.} \end{cases} \tag{16}$$

2) For $||e_f|| < \varpi$

$$\dot{\hat{\theta}}_i = 0, \quad \dot{\gamma} = 0, \tag{17}$$

$$\text{with } \hat{\theta}_i(t_0) > 0, \quad i = 0, 1, 2, \ \gamma(t_0) > \beta. \tag{18}$$

Here, t_0 is the initial time and $\beta, \varsigma, \alpha_0, \alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}^+$ are user defined scalars. Substituting (13) into (6), the closed-loop system is formed as

$$M\dot{e}_f = -e - Ge_f - \Delta\tau - Ce_f + \sigma. \tag{19}$$

Remark 1: For $||e_f|| \geq \varpi$, it can be noticed from the adaptive laws (15) and (16) that the gains $\hat{\theta}_i, \gamma$ increase if the error trajectories move away from $||e|| = 0$ (governed by $e^T\dot{e} > 0$) and decrease if error trajectories do not move away from $||e|| = 0$ [governed by the "otherwise" condition in (15) and (16), which implies $\{e^T\dot{e} \leq 0\} \cap \{\bigcup_{i=0}^2 \hat{\theta}_i > 0\} \cap \{\gamma > \beta\}$]. Hence, the proposed law certainly does not make the switching gain a monotonically increasing function and thus alleviates the overestimation problem.

Remark 2: For $||e_f|| < \varpi$, the tracking error remains bounded inside the ball $B_\varpi \triangleq \{||\Gamma\xi|| < \varpi\}$ using the relation $e_f = \Gamma\xi$. This implies that the switching gains are sufficient enough to keep the error within B_ϖ. Hence, the gains are kept unchanged for $||e_f|| < \varpi$. One can choose small ϖ to improve tracking accuracy (as B_ϖ gets reduced) as long as the value of ϖ does not invite chattering.

Remark 3: The initial condition of the gains is selected as $\hat{\theta}_i(t_0) > 0$, $\gamma(t_0) > \beta$. Furthermore, for $||e_f|| \geq \varpi$, the adaptive laws (15) and (16) force the gains to increase if either of the gains attempt to breach their respective lower bounds (governed by $\{\bigcup_{i=0}^2 \hat{\theta}_i \leq 0\} \cup \{\gamma \leq \beta\}$). This ensures that $\gamma(t) \nleq \beta$, $\hat{\theta}_i(t) \nleq 0\ \forall\, i = 0, 1, 2$ when $||e_f|| \geq \varpi$. Again, the gains remain unchanged for $||e_f|| < \varpi$. Hence, combination of the conditions mentioned earlier implies

$$\hat{\theta}_i(t) \geq 0 \quad \forall\, i = 0, 1, 2 \text{ and } \gamma(t) \geq \beta \quad \forall\, t \geq t_0. \tag{20}$$

Condition (20) is later exploited in the stability analysis.

To guarantee the alleviation of the overestimation problem of switching gain, it is necessary that $\hat{\theta}_i, \gamma$ decrease within a finite time. This is shown through Theorem 1.

Theorem 1: Let $t = t_{\text{in}}$ be any time instant when gains start increasing. Then, there exist finite times t_1, t_2, t_3, and δt, such that the gains $\hat{\theta}_0, \hat{\theta}_1, \hat{\theta}_2$, and γ decrease for $t \geq (t_{\text{in}} + T)$ where $T \leq (\bar{t} + \delta t)$, $\bar{t} \triangleq \max\{t_1, t_2, t_3\}$. These times are obtained as

$$t_1 \leq \frac{\theta_0^*}{(\alpha_0 + \alpha_3)\varpi}, \quad t_2 \leq \frac{\theta_1^*||\Gamma||}{\alpha_1\varpi^2}, \quad t_3 \leq \frac{\theta_2^*||\Gamma||^2}{\alpha_2\varpi^3} \tag{21}$$

$$\delta t \leq (1/\varrho)\ln\{2V(t_{\text{in}} + \bar{t})/(||e(t_{\text{in}} + \bar{t})||^2)\} \tag{22}$$

where

$$V = \frac{1}{2}e_f^TMe_f + \frac{1}{2}e^Te, \quad \varrho \triangleq \frac{\min\{\lambda_{\min}(G), \lambda_{\min}(\Omega)\}}{\max\{\mu_2, 1\}}.$$

Proof: Here, t_{in} can be any time when the gains start increasing and it is solely used for analysis. The objective of Theorem 1 is to find when the gains start to decrease. Furthermore, it is to be noted from the laws (15)–(17) that the gains increase only when $||e_f|| \geq \varpi$. So, it is sufficient to investigate the condition when all the gains increase and $||e_f|| \geq \varpi$. Moreover, using $e_f = \Gamma\xi$ from (5), one has

$$\varpi \leq ||e_f|| \leq ||\Gamma||||\xi|| \Rightarrow ||\xi|| \geq (\varpi/||\Gamma||). \tag{23}$$

So, the first laws of (15) and (16) and the condition (23) yield

$$\dot{\hat{\theta}}_0 \geq \alpha_0\varpi, \dot{\hat{\theta}}_1 \geq (\alpha_1\varpi^2)/||\Gamma||, \quad \dot{\hat{\theta}}_2 \geq (\alpha_2\varpi^3)/||\Gamma||^2, \ \dot{\gamma} \geq \alpha_3\varpi. \tag{24}$$

Let V be a Lypaunov function. Using (19) and the relation $e^T \dot{e} = e^T(e_f - \Omega e)$ [from (5)], the time derivative of V yields

$$\begin{aligned}\dot{V} &= e_f^T M \dot{e}_f + (1/2) e_f^T \dot{M} e_f + e^T \dot{e} \\ &= e_f^T(-e - G e_f - \Delta\tau + \sigma) + (1/2) e_f^T (\dot{M} - 2C) e_f \\ &\quad + e^T(e_f - \Omega e). \end{aligned} \tag{25}$$

Furthermore, substituting (13) into (25) and using Property 1 [implying $e_f^T(\dot{M} - 2C)e_f = 0$], $\dot{V}$ is simplified as

$$\begin{aligned}\dot{V} &= -e_f^T G e_f - e^T \Omega e + e_f^T(-\hat{\rho}(e_f/||e_f||) + \sigma) \\ &\le -e_f^T G e_f - e^T \Omega e - (Y(\xi)^T \hat{\Theta} + \gamma)||e_f|| + Y(\xi)^T \Theta^* ||e_f|| \\ &\le -\lambda_{\min}(G)||e_f||^2 - \lambda_{\min}(\Omega)||e||^2 \\ &\quad - \{(\hat{\theta}_0 + \gamma - \theta_0^*) + (\hat{\theta}_1 - \theta_1^*)||\xi|| \\ &\qquad + (\hat{\theta}_2 - \theta_2^*)||\xi||^2\}||e_f||. \end{aligned} \tag{26}$$

Thus, the sufficient condition to achieve $\dot{V} < 0$ would be

$$\hat{\theta}_0 + \gamma \ge \theta_0^*, \quad \hat{\theta}_1 \ge \theta_1^* \text{ and } \hat{\theta}_2 \ge \theta_2^*. \tag{27}$$

Let the system (3) does not possess finite time escape [28]. Then, integrating both sides of the inequalities in (24) and using those results in (27) lead to the expressions of t_1, t_2, t_3 in (21). So, for $t \ge t_{\text{in}} + \bar{t}$ and taking $\varrho_m \triangleq \min\{\lambda_{\min}(G), \lambda_{\min}(\Omega)\}$

$$\begin{aligned}\dot{V} &\le -\lambda_{\min}(G)||e_f||^2 - \lambda_{\min}(\Omega)||e||^2 \\ &\le -\varrho_m(||e_f||^2 + ||e||^2). \end{aligned} \tag{28}$$

Furthermore, the definition of V yields

$$V \le \varrho_M(||e_f||^2 + ||e||^2) \tag{29}$$

where $\varrho_M \triangleq \max\{\mu_2, 1\}$. Substituting (29) into (28) and using the comparison Lemma [29] yield

$$\dot{V} \le -\varrho V \Rightarrow V(t) \le V(t_{\text{in}} + \bar{t}) e^{-\varrho(t - \bar{t})} \quad \forall\, t \ge t_{\text{in}} + \bar{t} \tag{30}$$

where $\varrho \triangleq \varrho_m/\varrho_M$. Here, $\hat{\theta}_i > 0, \gamma > \beta$ as gains were increasing. So, to ensure the "*otherwise*" condition (i.e., $\{e^T \dot{e} \le 0\} \cap \{\bigcup_{i=0}^{2} \hat{\theta}_i > 0\} \cap \{\gamma > \beta\}$), the condition $e^T \dot{e} \le 0$ (i.e., $||e(t)||$ does not increase) needs to take place. From the definition of V, the upper bound of e follows:

$$V(t) \ge (1/2)||e(t)||^2 \Rightarrow ||e(t)|| \le \sqrt{2V(t)} \quad \forall\, t \ge t_0. \tag{31}$$

Let $||e(t_{\text{in}} + \bar{t})|| = \psi$, which implies $V(t_{\text{in}} + \bar{t}) \ge \psi^2/2$ from (31). As $V(t)$ decreases exponentially $\forall\, t \ge t_{\text{in}} + \bar{t}$ following (30), there exists a finite time $\delta t = t - (t_{\text{in}} + \bar{t})$, such that $V(t_{\text{in}} + \bar{t} + \delta t) = \psi^2/2$, implying $||e\,(t_{\text{in}} + \bar{t} + \delta t)|| \le \psi$. So, $\{e^T \dot{e} \le 0\} \cap \{\bigcup_{i=0}^{2} \hat{\theta}_i > 0\} \cap \{\gamma > \beta\}$ would occur at $t \ge t_{\text{in}} + T$ where $T \le \bar{t} + \delta t$ and $\hat{\theta}_i, \gamma$ would start decreasing. The time δt is found from (30)

$$\begin{aligned}\psi^2 &\le 2V(t_{\text{in}} + \bar{t}) e^{-\varrho \delta t}, \quad \forall\, t \ge t_{\text{in}} + \bar{t} \\ &\Rightarrow \delta t \le (1/\varrho)\ln\{2V(t_{\text{in}} + \bar{t})/\psi^2\}. \end{aligned} \tag{32}$$

Remark 4: The increment and decrement of $\hat{\theta}_i, \gamma$ can occur several times depending on the error incurred by the system. However, time interval Δt between any two successive decrements will always satisfy $\Delta t \le \bar{t} + \delta t$. Moreover, high values of $\alpha_0, \alpha_1, \alpha_2$, and α_3 help to reduce $\bar{t}$ and achieve faster adaptation.

C. Stability Analysis

Exploring the structures of the adaptive laws (15)–(17), three possible scenarios are identified—Case 1): $\dot{\hat{\theta}}_i, \dot{\gamma}$ increase and $||e_f|| \ge \varpi$; Case 2): $\dot{\hat{\theta}}_i, \dot{\gamma}$ decrease and $||e_f|| \ge \varpi$; Case 3) $\dot{\hat{\theta}}_i = 0, \dot{\gamma} = 0$ when $||e_f|| < \varpi\ \forall\, i = 0, 1, 2$.

Theorem 2: The closed-loop system (19) with control input (13)–(17) guarantees $e(t), e_f(t), \tilde{\theta}_i(t), \gamma\ (t)$ to be uniformly ultimately bounded (UUB), where $\tilde{\theta}_i \triangleq (\hat{\theta}_i - \theta_i^*)$, $i = 0, 1, 2$.

Proof: The stability analysis of the overall system is carried out for the three cases mentioned earlier using the following common Lyapunov function:

$$V_1 = V + \sum_{i=0}^{2} (1/2\alpha_i)\tilde{\theta}_i^2 + (1/2\alpha_3)\gamma^2. \tag{33}$$

Case 1): $\dot{\hat{\theta}}_i, \dot{\gamma}$ increase $\forall\, i = 0, 1, 2$ and $||e_f|| \ge \varpi$.

Note that $\sum_{i=0}^{2} 1/\alpha_i \tilde{\theta}_i \dot{\hat{\theta}}_i = Y(\xi)^T(\hat{\Theta} - \Theta^*)||e_f||$. Then, using (15) and (16) and following the procedure in (26), one obtains:

$$\begin{aligned}\dot{V}_1 &\le -e_f^T G e_f - e^T \Omega e + e_f^T(-\hat{\rho}(e_f/||e_f||) + \sigma) \\ &\quad + Y(\xi)^T(\hat{\Theta} - \Theta^*)||e_f|| + \gamma\,||e_f|| \\ &\le -e_f^T G e_f - e^T \Omega e - (Y(\xi)^T \hat{\Theta} + \gamma)||e_f|| \\ &\quad + Y(\xi)^T \Theta^* ||e_f|| + Y(\xi)^T(\hat{\Theta} - \Theta^*)||e_f|| + \gamma\,||e_f|| \\ &\le -\lambda_{\min}(G)||e_f||^2 - \lambda_{\min}(\Omega)||e||^2 \le 0. \end{aligned} \tag{34}$$

From (34), it can be inferred that $V_1(t) \in \mathcal{L}_\infty$. Thus, the definition of V_1 yields $\gamma(t), \tilde{\theta}_i(t), e(t), e_f(t) \in \mathcal{L}_\infty \Rightarrow \xi(t), \hat{\theta}_i \in \mathcal{L}_\infty$. Thus, for this case, the closed-loop system remains stable.

The gains $\gamma, \hat{\theta}_i$, $i = 0, 1, 2$ remain bounded for Case 1), decrease for Case 2), and remain constant for Case 3). Hence, $\exists\ \bar{\gamma}, \bar{\theta}_i \in \mathbb{R}^+$, such that

$$\hat{\theta}_0(t) \le \bar{\theta}_0, \quad \hat{\theta}_1(t) \le \bar{\theta}_1, \ \hat{\theta}_2(t) \le \bar{\theta}_2, \ \gamma(t) \le \bar{\gamma} \ \forall\, t \ge t_0. \tag{35}$$

Case 2): $\dot{\hat{\theta}}_i, \dot{\gamma}$ decrease $\forall\, i = 0, 1, 2$ and $||e_f|| \ge \varpi$.

Using $\gamma \ge \beta$ [from (20)] and $||e_f|| \le ||\Gamma||||\xi||$ yield

$$\begin{aligned}\dot{V}_1 &\le -e_f^T G e_f - e^T \Omega e + e_f^T(-\hat{\rho}(e_f/||e_f||) + \sigma) \\ &\quad - Y(\xi)^T(\hat{\Theta} - \Theta^*)||e_f|| - \gamma\,\varsigma\,||\xi||^4 \\ &\le -e_f^T G e_f - e^T \Omega e - (Y(\xi)^T \hat{\Theta} + \gamma)||e_f|| \\ &\quad + Y(\xi)^T \Theta^* ||e_f|| - \varsigma\beta||\xi||^4 - Y(\xi)^T(\hat{\Theta} - \Theta^*)||e_f|| \\ &\le -\varsigma\beta||\xi||^4 + 2||\Gamma||\{\theta_0^* + \theta_1^*||\xi|| + \theta_2^*||\xi||^2\}||\xi|| \\ &\quad - \lambda_{\min}(G)||e_f||^2 - \lambda_{\min}(\Omega)||e||^2. \end{aligned} \tag{36}$$

Since $0 \le \hat{\theta}_i\ (t) \le \bar{\theta}_i$, $\beta \le \gamma \le \bar{\gamma}$ [from (20) and (35)], the definition of V_1 in (33) yields

$$V_1 \le \varrho_M(||e_f||^2 + ||e||^2) + \zeta \tag{37}$$

where $\zeta \triangleq \sum_{i=0}^{2} \frac{1}{\alpha_i}(\theta_i^{*2} + \bar{\theta}_i^2) + \frac{1}{\alpha_3}\bar{\gamma}^2$. Thus, using (37)

$$-\lambda_{\min}(G)||e_f||^2 - \lambda_{\min}(\Omega)||e||^2 \le -\varrho V_1 + \varrho\zeta. \tag{38}$$

Substitution of (38) into (36) yields

$$\dot{V}_1 \le -\varrho V_1 + f_p(||\xi||) \tag{39}$$

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where $f_p(||\xi||) = -\varsigma\beta||\xi||^4 + 2||\Gamma||\{\theta_0^*||\xi|| + \theta_1^*||\xi||^2 + \theta_2^*||\xi||^3\} + \varrho\zeta$ and it has exactly one positive real root owing to the constant term $\varrho\zeta \in \mathbb{R}^+$ (can be verified using Descartes' rule of sign change [31] and Bolzano's Theorem [32]). Let $\iota \in \mathbb{R}^+$ be the positive real root of $f_p(||\xi||)$. As $\varsigma, \beta \in \mathbb{R}^+$, the leading coefficient of $f_p(||\xi||)$ (the coefficient of the highest degree term $||\xi||^4$) is negative. Therefore, $f_p(||\xi||) \leq 0$ when $||\xi|| \geq \iota$ [33] (detailed in the supplementary [37]). Hence, the overall system would be UUB [29] for this case.

Case 3): $\dot{\hat{\theta}}_i = 0, \dot{\gamma} = 0, \forall\, i = 0, 1, 2$ when $||e_f|| < \varpi$. Similar to Case 1), $\dot{V}_1$ can be simplified for this case as

$$\begin{aligned}\dot{V}_1 &\leq -e_f^T G e_f - e^T \Omega e + e_f^T(-\hat{\rho}(e_f/\varpi) + \sigma)\\ &\leq -e_f^T G e_f - e^T \Omega e - (Y(\xi)^T\hat{\Theta} + \gamma)(||e_f||^2/\varpi)\\ &\quad + Y(\xi)^T\Theta^*||e_f||\\ &\leq -\lambda_{\min}(G)||e_f||^2 - \lambda_{\min}(\Omega)||e||^2 + Y(\xi)^T\Theta^*||e_f||. \quad (40)\end{aligned}$$

For $||e_f|| < \varpi$, the system remains bounded inside the ball $B_\varpi \triangleq \{||\Gamma\xi|| < \varpi\}$ as $e_f = \Gamma\xi$. This implies that $Y(||\xi||) \in \mathcal{L}_\infty$. Hence, for $||e_f|| < \varpi$, $\exists\, \vartheta \in \mathbb{R}^+$ such that

$$||Y(\xi)^T\Theta^*||\,||e_f|| \leq \varpi\vartheta. \quad (41)$$

Let us define a scalar z as $0 < z < \lambda_{\min}(G)$. Then, using (37) and (41), (40) is modified as

$$\begin{aligned}\dot{V}_1 &\leq -\lambda_{\min}(\Omega)||e||^2 - \{\lambda_{\min}(G) - z\}||e_f||^2 - z||e_f||^2 + \varpi\vartheta\\ &\leq -\varrho_1 V_1 - z||e_f||^2 + \varrho_1\zeta + \varpi\vartheta \quad (42)\end{aligned}$$

where $\varrho_1 \triangleq \{\min\{\,(\lambda_{\min}(G) - z), \lambda_{\min}(\Omega)\}\}/\varrho_M$. Hence, the system would be UUB when

$$||e_f|| = ||\Gamma\xi|| \geq \sqrt{(\varrho_1\zeta + \varpi\vartheta)/z}. \quad (43)$$

Since the closed-loop system remains UUB for both the cases $||e_f|| \geq \varpi$ and $||e_f|| < \varpi$ using the common Lyapunov function (33), the overall closed-loop system also remains UUB [34].

Remark 5: It is noteworthy that condition (20) is necessary for the stability of the system. Moreover, high values of ς help to reduce ι, which consequently can improve controller accuracy. However, one needs to be careful that too high value of ς may excite the condition $\gamma \leq \beta$, leading to the increment in all the gains $\gamma, \hat{\theta}_i$, $i = 0, 1, 2$. Furthermore, *the scalar terms* $z, \vartheta, \psi, \mu_2, \zeta, \bar{\theta}_i, \theta_i^*$, *and* $\bar{\gamma}$ *are only used for the purpose of analysis and not used to design the control law*.

Remark 6: The importance of the auxiliary gain γ can be realized from Theorems 1 and 2. It can be observed from (21) that t_1 gets reduced due to the presence of α_3 (contributed by $\dot{\gamma} > 0$), which leads to faster adaptation. Moreover, the negative fourth degree term $-\varsigma\beta||\xi||^4$ in $f_p(||\xi||)$ (contributed by $\dot{\gamma} < 0$) ensures system stability for Case 2) by making $f_p(||\xi||) \leq 0$ for $||\xi|| \geq \iota$. This also indicates the reason for selecting $\beta > 0$ while lower bounds of other gains $\hat{\theta}_i$ $i = 0, 1, 2$ are selected as zero.

1) Special Case: The quadratic term $||\xi||^2$ in uncertainty bound (9) is contributed by the matrix $C(q, \dot{q})$ [through Property 4 in (7)]. EL systems, such as robotic manipulator, underwater vehicles, and ship dynamics, include $C(q, \dot{q})$. However, there also exist the second-order EL system (e.g., reduced order WMR system), which does not have the term $C(q, \dot{q})$. For such systems, the following LIP structure would hold:

TABLE I
ASRC ALGORITHM FOR VARIOUS SYSTEM STRUCTURES

System Structure		LIP structure of $\|\|\sigma\|\|$	Control law
(3)	$C(q,\dot{q}) \neq 0$	(9)	(13) - (18)
	$C(q,\dot{q}) = 0$	(44)	(13), (45) - (49)

$$||\sigma|| \leq \theta_0^* + \theta_1^*||\xi|| \triangleq Y(\xi)^T\Theta^* \quad (44)$$

where $Y(\xi) = [1\ \ ||\xi||]^T$ and $\Theta^* = [\theta_0^*\ \ \theta_1^*]^T$. Hence, following the switching gain laws (14)–(17), the control laws for uncertainty structure (44) are modified as:

$$\hat{\rho} = \hat{\theta}_0 + \hat{\theta}_1||\xi|| + \gamma := Y(\xi)^T\hat{\Theta} + \gamma. \quad (45)$$

1) For $||e_f|| \geq \varpi$

$$\dot{\hat{\theta}}_i = \begin{cases}\alpha_i||\xi||^i||e_f||, & \text{if } \{e^T\dot{e} > 0\} \cup \{\bigcup_{i=0}^{1}\hat{\theta}_i \leq 0\}\\ & \cup\{\gamma \leq \beta\}\\ -\alpha_i||\xi||^i||e_f||, & \text{otherwise}\end{cases} \quad (46)$$

$$\dot{\gamma} = \begin{cases}\alpha_3||e_f||, & \text{if } \{e^T\dot{e} > 0\} \cup \{\bigcup_{i=0}^{1}\hat{\theta}_i \leq 0\}\\ & \cup\{\gamma \leq \beta\}\\ -\varsigma\alpha_3||\xi||^3, & \text{otherwise.}\end{cases} \quad (47)$$

2) For $||e_f|| < \varpi$

$$\dot{\hat{\theta}}_i = 0, \dot{\gamma} = 0 \quad (48)$$
$$\text{with } \hat{\theta}_i(t_0) > 0, \quad i = 0, 1, \ \gamma(t_0) > \beta. \quad (49)$$

System stability employing (45)–(48) can be analyzed exactly, such as Theorem 2, using the following Lyapunov function:

$$V_1 = V + \sum_{i=0}^{1}(1/2\alpha_i)\tilde{\theta}_i^2 + (1/2\alpha_3)\gamma^2. \quad (50)$$

One can verify that the cubic polynomial $2||\Gamma||\{\theta_0^* + \theta_1^*||\xi|| + \theta_2^*||\xi||^2\}||\xi||$ in $f_p\ (||\xi||)$ of Case 2) would be modified into a quadratic polynomial $2||\Gamma||\{\theta_0^* + \theta_1^*||\xi||\}||\xi||$ using (44) and (50). Hence, following the argument in Remark 6, it can be noticed that a cubic term $-\varsigma\alpha_3||\xi||^3$ is selected in the adaptive law (48) for system stability.

Thus, with EL system (3), only two structures are possible for $||\sigma||$: 1) $Y(\xi) = [1\ ||\xi||\ ||\xi||^2]^T$, $\Theta^* = [\theta_0^*\ \theta_1^*\ \theta_2^*]^T$ and 2) $Y(\xi) = [1\ ||\xi||]^T$, $\Theta^* = [\theta_0^*\ \theta_1^*]^T$. Both these situations are covered here. For better inference, the ASRC algorithm is summarized in Table I for various system structures.

2) Comparison With Existing Adaptive–Robust Law: To elaborate the advantage of the proposed adaptive law,

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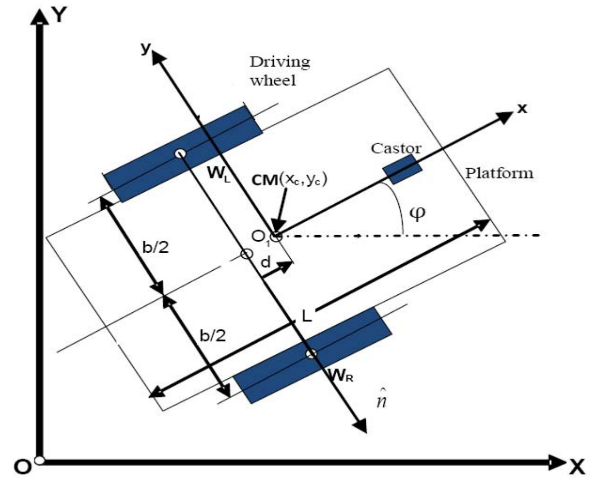

Fig. 1. Schematic of a WMR.

consider the adaptive law of ASMC [17], [18] for switching gain K as

$$\dot{K}(t) = \begin{cases} \bar{K}||s||\operatorname{sgn}(||s|| - \epsilon), & \text{if } K > \beta \\ \beta, & \text{if } K \le \beta \end{cases} \tag{51}$$

where $\epsilon, \bar{K} \in \mathbb{R}^+$ are user-defined scalars and s is the sliding surface. It can be observed from (51) that when $||s|| \ge \epsilon$, the switching gain K increases monotonically even if error trajectories move close to $||s|| = 0$. This gives rise to the overestimation problem of switching gain. Again, even if K is sufficient to keep $||s||$ within ϵ, it decreases monotonically when $||s|| < \epsilon$. Thus, at a certain time, K would become insufficient and error will increase again. However, K will not increase (rather it keeps on decreasing) until $||s|| > \epsilon$, which creates underestimation problem. Low (resp. High) value of ϵ may force K to increase (resp. decrease) for longer duration when $||s|| \ge \epsilon$ (resp. $||s|| < \epsilon$), resulting in escalation of the overestimation (resp. underestimation) problem of ASMC.

Whereas, ASRC allows its gains to decrease when error trajectories move toward $||e|| = 0$ and $||e_f|| \ge \varpi$ (overcoming overestimation problem) and keeps the gains unchanged when they are sufficient to keep the error within the ball B_ϖ (overcoming underestimation problem). Since the overestimation–underestimation problems are alleviated by ASRC for any ϖ, one can, in fact, reduce ϖ for better tracking accuracy as long as chattering does not occur.

III. Application: Nonholonomic WMR

Nonholonomic WMR, which has vast applications in transportation, planetary exploration, surveillance, security, and human–machine interfaces, provides a unique platform to test the proposed control law. Hence, the performance of the proposed ASRC is verified using a commercially available "PIONEER 3" WMR in comparison to ASMC [17], [18]. The ASMC law is detailed in [17] and [18], while it follows the adaptive law (51).

The EL formulation of a nonholonomic WMR (Fig. 1) is given as [26], [35]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = L\tau - A^{*T}\lambda^* \tag{52}$$

where

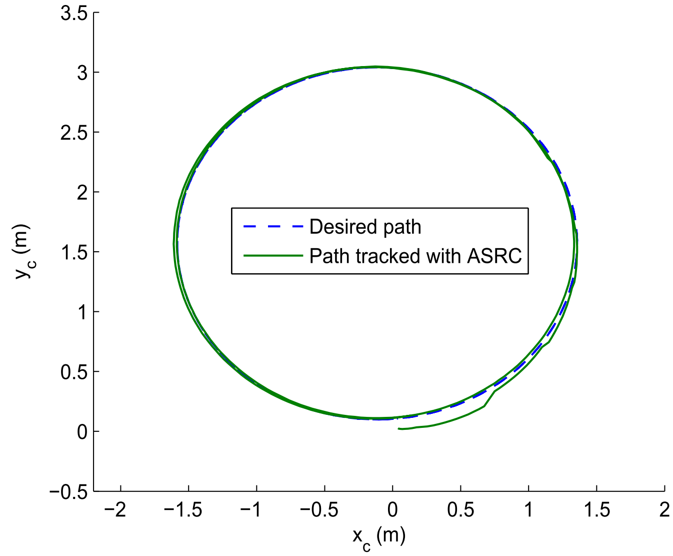

Fig. 2. Circular path tracking with ASRC.

$$M = \begin{bmatrix} m & 0 & md\sin\varphi & 0 & 0 \\ 0 & m & -md\cos\varphi & 0 & 0 \\ md\sin\varphi & -md\cos\varphi & \bar{I} & 0 & 0 \\ 0 & 0 & 0 & I_w & 0 \\ 0 & 0 & 0 & 0 & I_w \end{bmatrix}$$

$$L = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C(q,\dot{q})\dot{q} = \begin{bmatrix} md\dot{\varphi}^2\cos\varphi \\ md\dot{\varphi}^2\sin\varphi \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \tau = \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix}.$$

Here, $q \in \mathbb{R}^5 = \{x_c, y_c, \varphi, \theta_r, \theta_l\}$ is the generalized coordinate vector of the system; (x_c, y_c) are the coordinates of the center of mass (CM) of the system and φ is the heading angle; (θ_r, θ_l) and (τ_r, τ_l) are rotation and torque inputs of the right and left wheels, respectively; $m, \bar{I}, I_w, r_w$, and b represent the system mass, system inertia, wheel inertia, wheel radius, and robot width, respectively; d is the distance to the CM from the center of the line joining the two wheel axis; A^* and λ^* represent the constraint matrix and vector of constraint forces (Lagrange multipliers), respectively. The expressions of A^* and $\bar{I}$ can be found in [26] and [35].

It is noteworthy that system (52) has only two control inputs although having five generalized coordinates and one can only control wheel positions (θ_r, θ_l) explicitly rather than (x_c, y_c, φ). Furthermore, the ASRC is formulated for fully actuated systems. Therefore, the system dynamics is represented as a combination of a reduced order dynamics and kinematic model for efficient controller design [30], [35]

$$M_R\ddot{q}_R + C_R\dot{q}_R = \tau, \quad \dot{q} = S(q)\dot{q}_R \tag{53}$$

$$S = \begin{bmatrix} \frac{r_w}{b}\left(\frac{b}{2}\cos(\varphi) - d\sin(\varphi)\right) & \frac{r_w}{b}\left(\frac{b}{2}\cos(\varphi) + d\sin(\varphi)\right) \\ \frac{r_w}{b}\left(\frac{b}{2}\sin(\varphi) + d\cos(\varphi)\right) & \frac{r_w}{b}\left(\frac{b}{2}\sin(\varphi) - d\cos(\varphi)\right) \\ \frac{r_w}{b} & -\frac{r_w}{b} \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

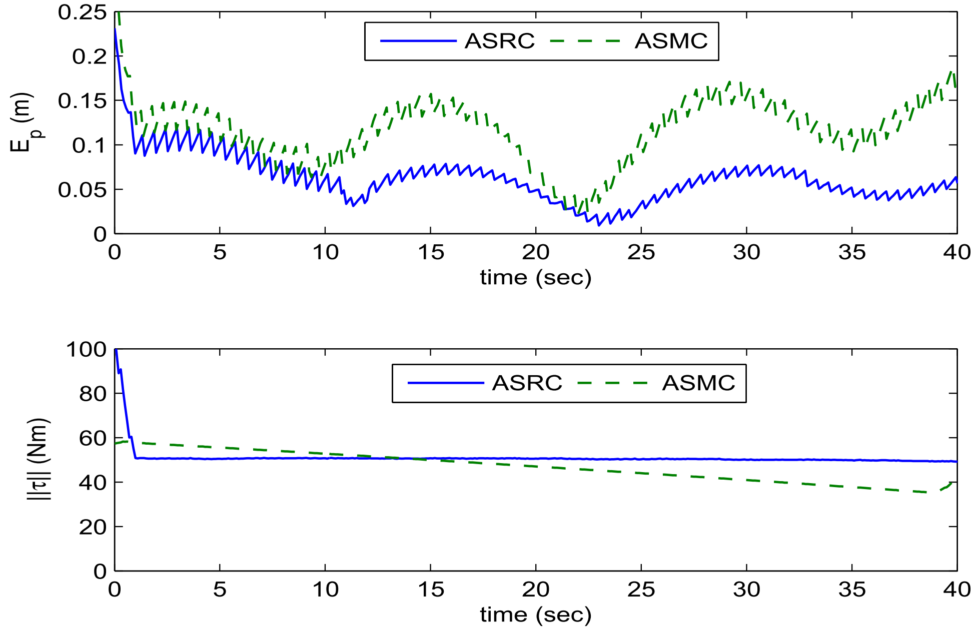

Fig. 3. Performance comparison between ASRC and ASMC.

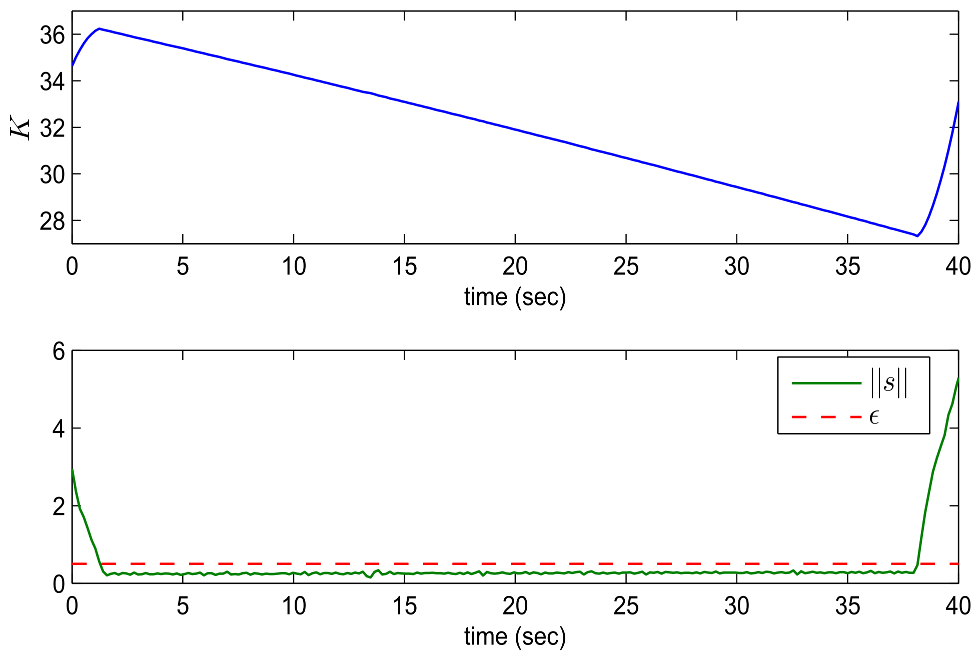

Fig. 4. Switching gain evaluation of ASMC.

where

$$M_R = S^T M S = \begin{bmatrix} k_1 & k_2 \\ k_2 & k_1 \end{bmatrix}$$
$$k_1 = I_w + \{\bar{I} + m\ (b^2/4 - d^2)\}\ ({r_w}^2/b^2)$$
$$k_2 = \{m\ (b^2/4 + d^2) - \bar{I}\}\ ({r_w}^2/b^2)$$
$$C_R = S^T(M\dot{S} + CS) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},\quad q_R = [\theta_r\ \theta_l]^T.$$

As WMR moves on ground, the gravity vector $g(q)$ and the potential function would certainly be zero, which implies that M_R, C_R satisfies Properties 1 and 3 ([30], [36]). The main implication of system Property 1 is to hold $e_f^T(\dot{M} - 2C)e_f = 0$, and this can be easily verified from M_R and C_R. The WMR dynamics (52) is based on the rolling without slipping assumption, and hence, the term $f(\dot{q}_R)$ is omitted. However, in practical circumstances, a WMR is always subjected to uncertainties, such as friction, slip, skid, and external disturbance. Hence, the system dynamics (53) is modified as

$$M_R \ddot{q}_R + f(\dot{q}_R) + d_s = \tau \tag{54}$$

where $f(\dot{q}_R)$ and d_s are considered to be the unmodeled dynamics and disturbance, respectively. *The ASRC framework does not require any knowledge of M_R, f and d_s.* Since Coriolis component is zero, the ASRC algorithm applied to the WMR is based on the control laws (13) and (45)–(49). It is to be noted that $S(q)$ is only used for coordinate

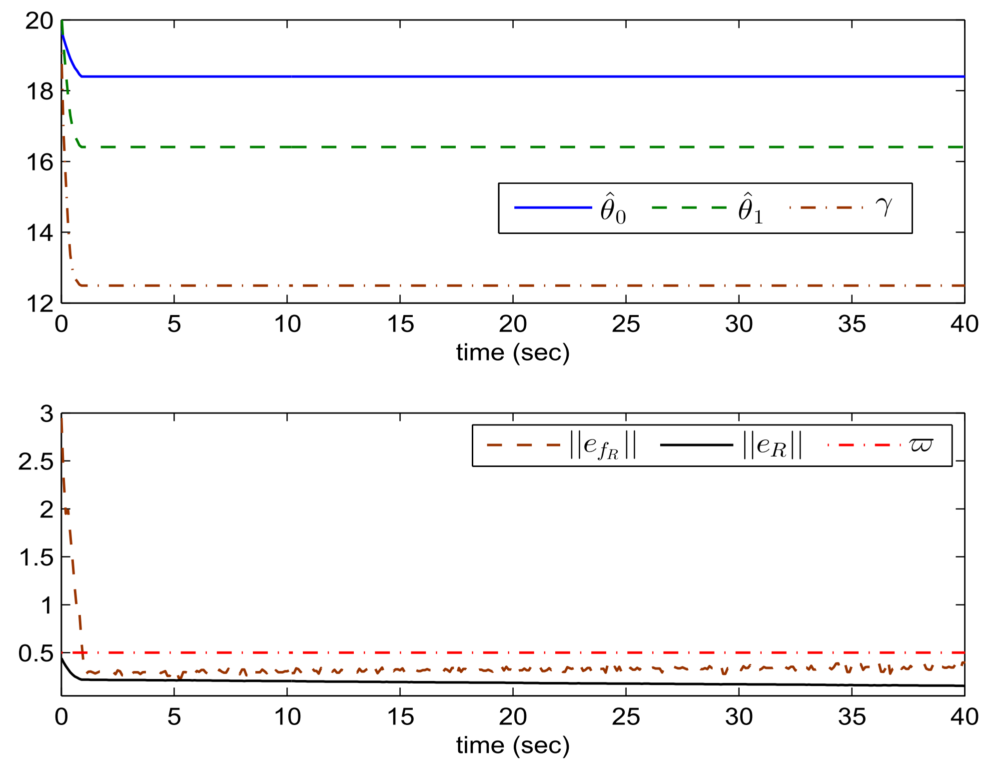

Fig. 5. Switching gain evaluation of ASRC.

transformation and WMR pose (x_c, y_c, φ) representation and not for control law design.

Hence, the objective is to apply ASRC and ASMC to the reduced order WMR system (54) to track a desired $q_R^d(t)$, which in effect track a desired $q^d(t)$ through $q^d(t) = S(q)q_R^d(t)$. To illustrate the fact: one can direct a WMR to move in a specified circular path by designing two suitable different and fixed wheel velocities or in a Lawn–Mower path by applying approximated square-wave velocity profile to the wheels [26].

A. *Experimental Scenario*

The WMR is directed to follow a circular path using the following desired trajectories:

$$\theta_r^d = (4t + \pi/10) \text{ rad}, \;\; \theta_l^d = (3t + \pi/10) \text{ rad}.$$

PIONEER 3 uses two quadrature incremental encoders (500 ppr) and always starts from $\theta_r(t_0) = \theta_l(t_0) = 0$ and the initial wheel position error $(\pi/10, \pi/10)$ rad helps to realize the error convergence ability of the controllers. The desired WMR pose $(x_c^d, y_c^d, \varphi^d)$ and actual WMR pose (x_c, y_c, φ) can be determined from $q^d(t) = S(q)q_R^d(t)$ using $(\dot{\theta}_r^d, \dot{\theta}_l^d)$ and $(\dot{\theta}_r, \dot{\theta}_l)$ (obtained from encoder), respectively, with $r_w = 0.097$ m, $b = 0.381$ m, and $d = 0.02$ m (supplied by the manufacturer). The control laws for both ASRC and ASMC are written in VC++ environment. Considering the hardware response time, the sampling interval is selected as 20 ms for all the controllers. Furthermore, to create a dynamic payload variation, a 3.5-kg payload is added (kept for 5 s) and removed (for 5 s) periodically on the robotic platform at different places.

The controller parameters for ASRC are selected as: $G = \Omega = I$, $\varpi = 0.5$, $\hat{\theta}_i(t_0) = \gamma(t_0) = 20$, $\alpha_i = \alpha_3 = 10$ $\forall\, i = 0, 1$, $\beta = 0.1$, $\varsigma = 10$. Furthermore, the controller parameters for ASMC are selected as $s = e_f$, $\bar{K} = 10$, $K(t_0) = 35$, $\epsilon = 0.5$.

B. *Experimental Results and Comparison*

The path tracking performance of ASRC is shown in Fig. 2 while following the desired circular path. The tracking performance comparison of ASRC with ASMC is shown in Fig. 3 in terms of E_p (defined by the Euclidean distance in x_c, y_c error) and $||\tau||$. ASMC framework is built on the assumption that uncertainties are upper bounded by an unknown constant (i.e., $\theta_1^* = \theta_2^* = 0$ for general EL systems and $\theta_1^* = 0$ for WMR as $C_R = 0$). This assumption is restrictive in nature for EL systems, and the switching gain is thus insufficient to provide the necessary robustness. As a matter of fact, ASRC provides better tracking accuracy over ASMC.

To evaluate the benefit of the proposed adaptive–robust law, the evaluation of switching gain for ASMC and ASRC is provided in Figs. 4 and 5, respectively. Fig. 4 reveals that K, the switching gain of ASMC, increases even when $||s||$ approaches toward $||s|| = 0$ during the time $t = 0-1.2$ s. This is due to the fact that K does not decrease unless $||s|| < \epsilon$, and thus invites the *overestimation* problem. On the other hand for ASRC, it can be seen from Fig. 5 that all the gains γ, $\hat{\theta}_0, \hat{\theta}_1$ decrease when $||e_R||$ $(e_R = q_R - q_R^d)$ decreases during $t = 0-1$ s when $||e_{f_R}|| \geq \varpi$ $(||e_{f_R}|| = \dot{e}_R + \Omega e_R)$. So, ASRC overcomes the overestimation problem, which is encountered in ASMC. Furthermore, K decreases monotonically for time durations $t = 1.2 - 38.5$ s, when $||s|| < \epsilon$. This monotonic decrement makes K insufficient to tackle uncertainties at certain time creating *underestimation* problem. As a result,

TABLE II
PERFORMANCE OF ASRC FOR VARIOUS ϖ VALUES

	$\varpi = 0.5$	$\varpi = 0.3$	$\varpi = 0.1$
RMS (root mean squared) E_p (m)	0.053	0.0421	0.0362
RMS $\|\|\tau\|\|$ (Nm)	89.78	75.46	67.13

$||s||$ starts increasing again for $t > 38.5$ s, leading to poor tracking accuracy and K increases again when $||s|| \geq \epsilon$. Gains of ASRC, on the contrary, stay unchanged for $t > 1$ s when the gains are sufficient to keep $||e_{f_R}|| < \varpi$ avoiding any underestimation problem. Evaluation of $\hat{\rho}$ is shown in the supplementary [37]. While reduction of ϵ would cause more overestimation problem for ASMC, Table II shows the improved tracking accuracy of ASRC with reduced ϖ (other control parameters are kept unchanged) while chattering is absent in control input. Controller performances with low initial gain condition are demonstrated in the supplementary [37].

IV. CONCLUSION

A novel ASRC law is proposed for a class of uncertain EL systems where the upper bound of uncertainty possesses an LIP structure. The benefit of the ASRC lies in the fact that it is independent of the nature of the uncertainty and can negotiate uncertainties, which can be LIP or NLIP. ASRC does not presume the overall uncertainty to be bounded by a constant and avoids putting prior restriction on the states. Moreover, the proposed adaptive law alleviates the overestimation–underestimation problem of switching gain. The experimental results validate the efficacy of the proposed control law in comparison with the existing ASMC. The future work would be to extend the ASRC law for systems with unmatched disturbances as well as for underactuated systems.

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