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3DOF Flight Trajectory Simulation of a Single Stage Sounding Rocket

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Abstract— In this paper, the 3 degree of freedom (3DOF) simulation of the basic flight parameters of a single stage sounding rocket developed at the Centre for Space Transport and Propulsion is discussed. The purpose of a lauching a sounding rocket rocket at the centre is to place a payload to a particular altitude. In order to determine the accuracy of getting to a targeted altitude using a particular rocket, a model of the system has to be developed. This model reduces the time and cost of making different prototypes of the rocket. The Centre for Space Transport and Propulsion designed a 3DOF model in the Matlab and Simulink environment to simulate the flight trajectory of the rocket TMR-1A. The results obtained via simulation are compared with the real parameters obtained via the onboard data acquisition system, to determine the integrity of the model used for the study. The model showed high degree of accuracy when simulated results are compared with real experimental data. However it was observed that the integrity of the model is affected by the accuracy of the value of drag coefficient, hence future work can be focused on more accurate determination of the drag coefficient.

Index Terms— aerodynamics, data acquisition system, flight trajectory, modeling, simulation, solid propellant, sounding rocket,

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1 Introduction

A sounding rocket sometimes called a research rocket is an instrument-carrying rocket designed to take measurement and perform scientific experiments during its sub-orbital flight. Typical sounding rocket is powered by either a solid-fuel rocket motor, liquid or hybrid rocket engine. Apart from the propulsive system, sounding rocket comprises of a payload (data processing system) either with real-time radio downlink or on-board saving during the flight duration. These Sounding rockets are advantageous for some experimental researches due to their low cost and ability to conduct researches in areas inaccessible to either balloons or satellites. They are also used as test beds for expensive equipments for risky orbital spaceflight missions. The smaller size of a sounding rocket makes it amenable for launching from a temporary site for field studies at remote locations, even in the middle of the ocean, if fired from a ship. Depending on the target altitude, a sounding rocket can be single stage for a low altitude and multiple stages for a higher altitude.

From conceptual phase to launch, the operations required to design, fabricate, test, integrate and launch a sounding rocket to a pre-determined altitude are extremely complex. The objective of any rocket of a particular mass is to place a payload to a pre-determined altitude using a matching mass of propellant. In order to evaluate the accuracy of a designed sounding rocket to reach a particular altitude before fabrication, it is preferable to develop a model using powerful software packages to simulate the flight trajectory parameters.

A sounding rocket belongs to the class of flexible structure that is vulnerable to external disturbances. The sequence of designing the model using software packages reduces the cost of making multiple prototypes and also the time from analysis to complete fabrication. The designed model has its own disadvantages, arising from differences between the model and the actual system. The following parameters are dynamic throughout the atmospheric flight duration of any rocket namely; drag coefficient, air density and acceleration due to gravity and are fully represented in the mathematical model. The only assumption is the neglective atmospheric wind velocity. The budget for a single stage sounding rocket is relatively cheap compare to high-tech rockets. The advent of low cost, MEMS accelerometers and gyroscopes necessitated the use of integrated navigation systems in modern rockets.

This paper therefore presents a model of the flight path trajectory of single stage solid-fuel sounding rocket - TMR-1A designed and constructed at the Centre for Space Transport and Propulsion, at Epe, Lagos, Nigeria.

The following parameters are dynamic throughout the flight duration - drag coefficient, air density and acceleration due to gravity. The effect of the atmospheric wind velocity is also neglected in the model.

2 STRUCTURE OF TMR-1A

The overall structure of TMR-1A is depicted in Figure 1. The structure comprises of two aluminium central tubes –body tube 1 and body tube 2 of 200mm internal diameter. Four fins are attached at the aft of body tube 2 to ensure rocket stability and a parabolic nose cone at the fore of body tube 1 to reduce aerodynamic drag during flight. Aluminium is chosen because

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of its low cost, availability and light weight, though better and more expensive composite materials are now used for sounding rockets structures. Design parameters of the rocket are given in Table 1. The total mass of the rocket was found to be 24Kg.

Table 1
Rocket Parameters

Parameters	Description	Value(m)
D	Diameter of rocket body tube	0.2
h	Total height of rocket	2.6
Ln	Length of nose cone	0.025
Xr	Distance from fin root leading edge to fin tip leading edge parallel to body	0.020
Xp	Distance from tip of nose to front of transition	0.035
Lt	Length of transition	0.010
dr	Diameter at rear of transition	0.0202
df	Diameter at front transition	0.025
Cr	Fin root chord	0.040
Ct	Fin tip chord	0.020
S	Fin semi-span	0.035
Rb	Radius of body rear end	0.010
L_f	Length of the fin mid-chord line	0.04
X_b	Distance from nose tip to fin root chord leading edge	2.34

3 PROPULSIVE SYSTEM OF TMR-1A

The propulsive system of TMR-1A is made of a solid rocket motor comprising a nozzle, combustion chamber and a bulkhead. The combustion chamber is a mild steel pipe of length 485mm, 127mm internal diameter and 3mm thick. It is threaded at both ends for the attachment of both the bulkhead and the nozzle. The bulkhead and the nozzle are fabricated from solid mild steel material; these parts are designed following [4]. The diagram of the assembled solid rocket motor is depicted in Figure 2.

A sugar-based propellant is usually a hybrid composition of one of the common sugars and the oxidizer (potassium nitrate). In this study, the propellant used is a combination of potassium nitrate and sucrose (KNSU) and produced through re-crystallization process. The structural composition comprises of two bates with a hollow core grain geometry. The characterization was done with a Propellant Performance Evaluation Program (ProPep3) as shown in Table 2. The combustion process is governed by the following chemical equation viz;

$$C_{12}H_{22}O_{11} + 8.9KNO_3 + 2.6C \rightarrow 9.65CO_2 + 1.75CO + 4.56H_2O + 5.2H_2 + 4.45N_2 + 3.21K_2CO_3 + 2.48KOH \quad (1)$$

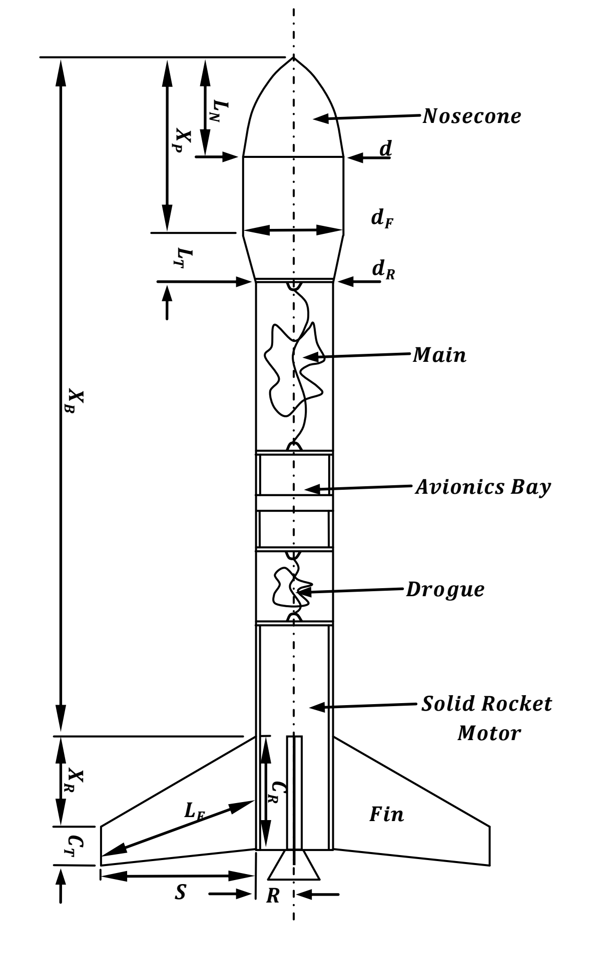

Fig. 1. Structure of TMR-1A

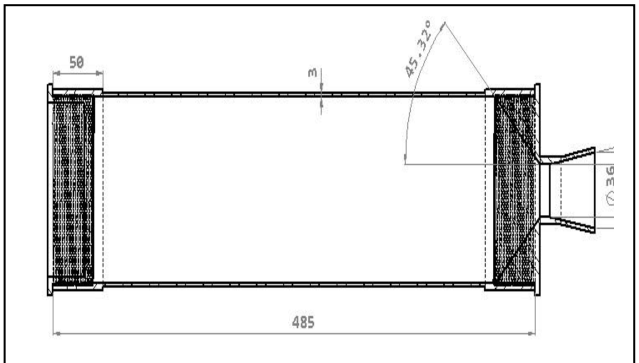

Fig. 2. Section-view of the solid rocket motor

The value of the estimated thrust obtained via ProPep3 need to be verified using static test stand shown in Figure 3 below. The static test stand consists of DI-145 USB data acquisition starter kit and LC 500 load cell from Aerocon Systems. The value of the thrust curve generated is saved in Excel format and the file is used in the Lookup Table used in the modelling the flight trajectory in Matlab/Simulink environment. The thrust profile is shown in Figure 4.

Table 2
Propellant Charaterization

Parameters	Value
Total mass of propellant, Kg	4.3
Propellant density, Kg/m³	1896
Propellant temperature, K	1611
Outer bate diameter, m	0.105
Each bate length, m	0.151
Number of bates	2
Total bates length, m	0.302
Core diameter, mm	38
Chamber pressure, MPa	1.030888
Maximum expected chamber pressure, MPa	1.03
Specific impulse, s	150
Burn time, s	3.7
Mass flow rate, Kg/s	1.07
Estimated thrust, N	1718

Fig. 3. Static Test Stand

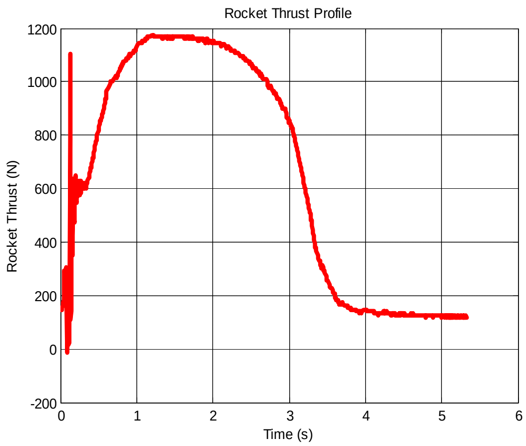

Fig. 4. Rocket Thrust Profile

4 SCIENTIC PAYLOAD AND RECOVERY SYSTEM

The onboard payload comprises of a Jolly Logic Altimeter Two, a cheap lightweight scientific payload popular with amateur rocket flyers. The Altimeter Two is equipped with a 3-axis accelerometer that can measure up to 24Gs of acceleration in each of the body-fixed axes at maximum altitude of 9000m above sea level. In addition, it has the capacity to compute the following flight data: top speed, engine burn time, ejection altitude, peak and average acceleration values and total flight duration. The altimeter is mounted in the interior compactment of the avionics bay and can be orientated in any direction to measure the enumerated flight parameters. Within the avionics bay, three or four tiny holes are usually drilled and spaced effectively in a manner that does not eclipse the openings or the holes preparatory to rocket systems integration before launch. The recovery system consists of two parachutes – the main and drogue parachutes. The drogue parachute is deployed after apogee whilst the main parachute is expected to be deployed at a lower altitude as the rocket descends. The ejection time of the drogue chute is two seconds after apogee, while the main chute is programmed to deploy four seconds after. Before the parachutes are loaded into their different compartments, as depicted in Figure 2, a static ejection test was carried out to verify if the amount of black powder used is enough to eject the chutes. Picture of the static ejection test is depicted in Figure 5 below.

Fig. 4. Static Ejection Test Scene

5 MATHEMATICAL MODELLING OF THE SOUNDING ROCKET

The mathematical model of the sounding rocket includes the complete specification of all the ordinary differential equations (ODEs) and auxiliary equations for the system. The predictive power of a mathematical model depends on its ability to correctly identify the dominant controlling factors and their influences, not upon its completeness or complexity. A model of limited known applicability, is often more useful than a more complete model.

The ordinary differential equations (ODEs) commonly represent conservation equations for mass, momentum and energy while the auxiliary equations are required to complete the ODEs. Examples of auxiliary equations are the flexibility model of a flexible flying aerospace vehicle and turbulence modelling equations in fluid dynamics. In this study, the sounding rocket is considered to be a rigid body while the flexibility model and other auxiliary equations are neglected for simplicity.

The major forces acting on a dynamic sounding rocket are aerodynamic forces, propulsive force (thrust) from the solid rocket motor and gravitational force. It is conventional in aerodynamics, to resolve the sum of the normal and tangential forces that act on the surface due to the fluid motion around a vehicle into three components along axes parallel and perpendicular to the free-stream direction. Basically, in the dynamics of sounding rockets, there are three aerodynamic forces acting in the body-fixed axes due to the effect of lift, drag and side forces. These aerodynamic forces are commonly defined in terms of dimensionless coefficients, the flight dynamic pressure, and a reference area. The aerodynamic coefficients depend on rocket geometries and specified Mach numbers. These coefficients are determined by approximate formulae and are defined more precisely using experimental data and powerful software packages.

Due to the fact that the aerodynamic forces act at the centre of pressure of the system and not at the centre of gravity, moments are produced. There are basically three categories of aerodynamic moments that are produced namely: aerodynamic moments due to the angle of attack α and side slip angle β; aerodynamic damping moments due to roll, pitch and yaw rate; aerodynamic moments due to fin deflection (for rocket using fin deflection mechanism for trajectory control). In a 3-degree-of-freedom motion, only the linear motion along the body-fixed axes is considered. The rotational motions along these axes are neglected. In this study, the sounding rocket is flown in the vertical direction, therefore effect of the lift and side forces are minimal and can be neglected. The aerodynamic moments due to roll, pitch and yaw rate are neglected. The rocket is not equipped with any type of controller, hence the aerodynamic moments due to fin deflection are neglected. Due to the vertical launch, the aerodynamic moments due to the angle of attack and side slip angle are also neglected.

Typical suborbital trajectory of a sounding rocket has a parabolic flight profile that can be divided into three phases: powered phase, coast phase and recovery. The powered phase ends at propellant burnout time; the coast phase starts at propellant burnout time and ends at apogee while the recovery starts from apogee to ground impact.

The first two phases determine the apogee of a given rocket. For the two phases, the ordinary differential equations of motion are given following [7]:

$$a = \frac{(F-D)}{m(t)} - g \quad (2)$$

$$v = \int \frac{da}{dt} \quad (3)$$

$$h = \int \frac{dv}{dt} \quad (4)$$

$$m(t) = m_0(\frac{f_p}{t_b} t) \quad (5)$$

$$k = \frac{\pi}{8} \rho_{air} C_d d^2 \quad (6)$$

$$D = kv|v| \quad (7)$$

where, F is motor thrust, D – drag force, g – acceleration due to gravity, m_0 – initial rocket mass, f_p – propellant mass fraction, t_b – burn time, d – rocket diameter, C_d – drag coefficient, v – velocity of rocket, h – rocket altitude, a – rocket acceleration and t – time.

Following [5], an approximated value of the drag coefficient for parabolic nose cone was calculated using the following formula viz;

$$C_d = \log(1 + 4\gamma^2) / \gamma^2 \quad (8)$$

where, $\gamma = l_n/(0.5d_f)$ is the nose cone aspect ratio.

This value of drag coefficient is computed as 0.31 and valid only for a subsonic flight speed of the rocket.

The behaviour of the drag coefficient in transonic and supersonic zones is quite different. Following [9], the interpolation of the aerodynamic drag coefficient for TMR-1A was obtained for Mach numbers ranging from 0 to 7 (see Figure 6).

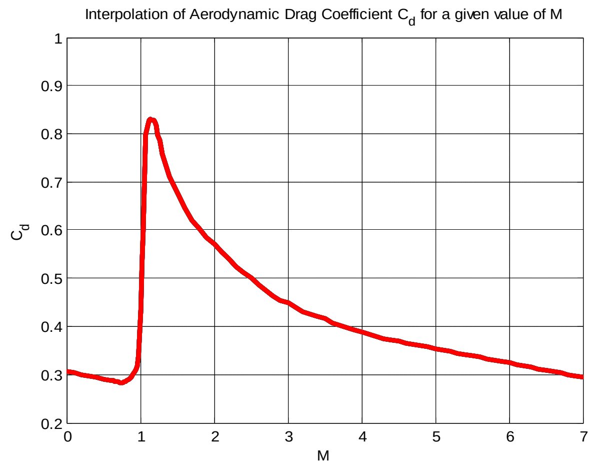

Fig. 6. Aerodynamic drag coefficient versus Mach number.

The ordinary differential equations for both phases are solved numerically in the MatLab/Simulink environment using 4th order Runge-Kutta method. The model designed in the Simulink environment for the flight trajectory simulation is illustrated in Figure 7. In the model, the dynamic parameters are modeled using in-built models in the Simulink Library browser. The value of the drag coefficient used is not the exact value if the rocket is placed inside a wind tunnel for an experimental evaluation. Therefore in order to examine the effect of the values of the drag coefficient on the parameters of the rocket, four additional values of C_d (0.4, 0.6, 0.8, 1) were used for the simulation.

6 Results of Simulation

Based on the simulation, the apogee of the rocket is greatly influenced by the magnitude of the drag coefficient. The simulated results for the computed drag coefficient (0.31) are given in Figure 8, 9 &10. The results of the simulated values for C_d of 0.4, 0.6, 0.8 & 1 are depicted in Figures (11-13). It was observed from the plots that the greater the value of C_d, the lower the apogee, velocity and acceleration of the rocket.

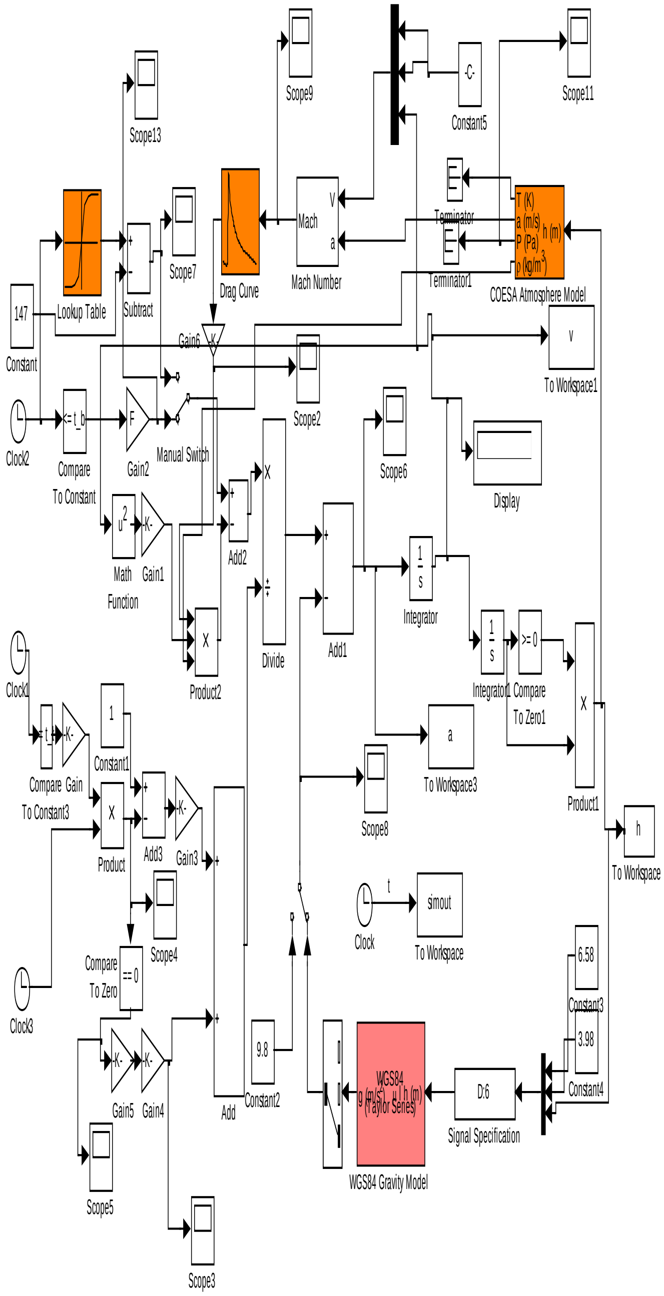

Fig. 7. Simulink Model of Flight Trajectory.

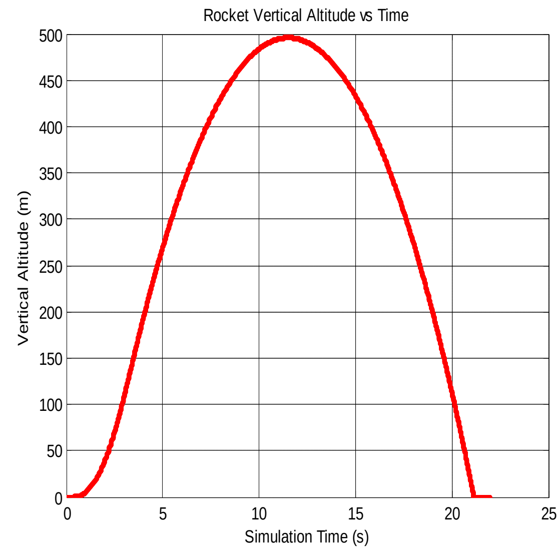

Fig. 8. The graph of Rocket vertical altitude versus time

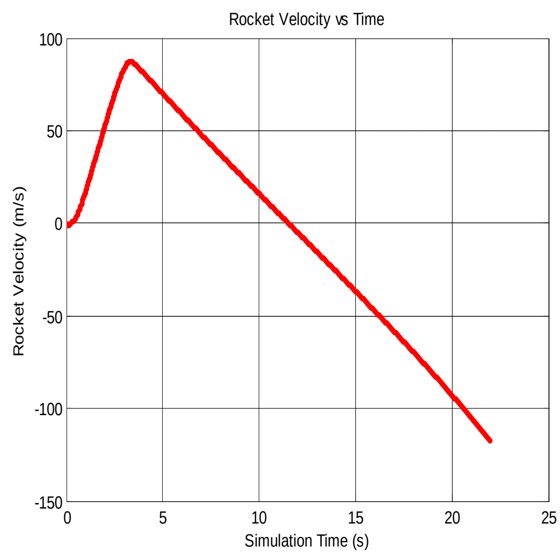

Fig. 9. The graph of rocket velocity versus time

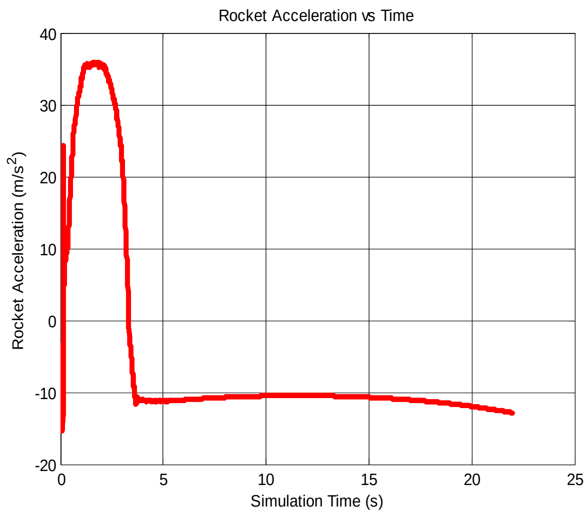

Fig. 10. The graph of rocket acceleration versus time

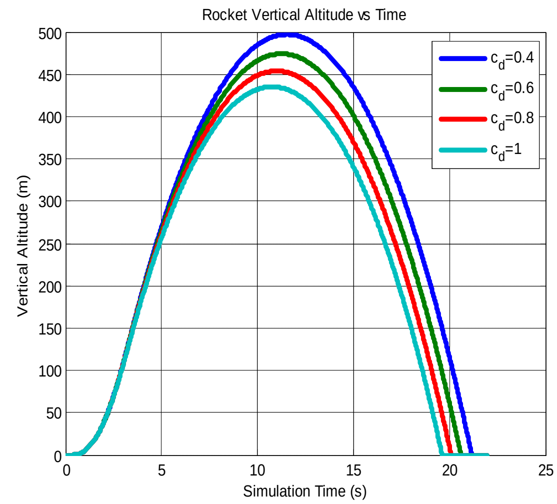

Fig. 11. The effect of C_d on rocket vertical altitude

Table 3
Comparison of Simulated and Measured Parameters

PARAMETERS	SIMULATED	MEASURED	% ERROR
H(MAX) (M)	508.8	480	6
V(MAX) (M/S)	88.1	84	4.9
A(MAX) (M/S^2)	36.15	34	6.3

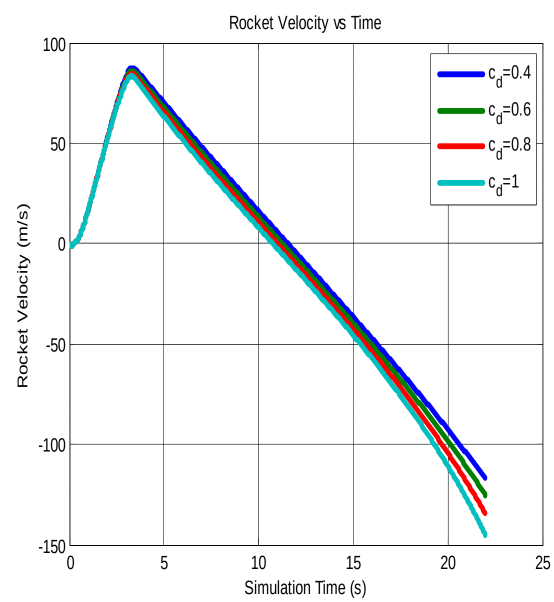

Fig. 12. The effect of C_d on rocket velocity

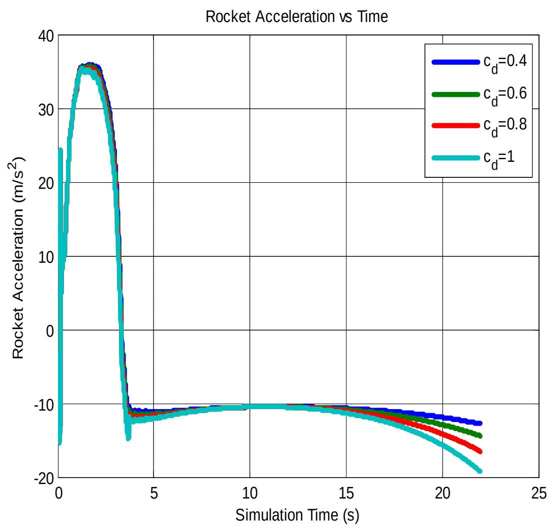

Fig. 13. The effect of C_d on rocket acceleration

Conclusion

The conceptualised mathematical model in this study is assumed to be an approximation of the actual system. The level of accuracy of the model can only be ascertain after the flight parameters of the actual constructed system are compared with the simulated values. The 3-degree-of-freedom simulation of TMR-1A showed a high degree of accuracy (as illustrated in Table 3). The percentage variations of simulated results from the measured values are maginal. The simulation results based on different values of drag coefficient indicated that drag coefficient plays a very vital role in the apogee determination, and hence the accuracy of the mathematical model. It is therefore recommended that future study can be focused to more accurate determination of the value of the drag coefficient for sounding rocket trajectory studies.

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