Date – 18-09-2019

Assignment 1

Aim: To find the stability of a closed loop system using Routh array for a general n^{th} order system and test on system having characteristic equation:

- $s^3 + 12s^2 + 47s + 52$
- $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 6$
- $s^5 + 6s^4 + 11s^3 + 6s + 100$

Software: Matlab 2018a

Theory: In control system theory, the **Routh–Hurwitz stability criterion** is a mathematical test that is a necessary and sufficient condition for the stability of a linear time invariant (LTI) control system.

A tabular method can be used to determine the stability when the roots of a higher order characteristic polynomial are difficult to obtain. For an n^{th}-degree polynomial

$$G(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$$

The table has n+1 rows, n/2 columns and the following structure:

a_n	a_{n-2}	a_{n-4}	$\cdots$
a_{n-1}	a_{n-3}	a_{n-5}	$\cdots$
b_n	b_{n-1}	b_{n-2}	$\cdots$
c_n	c_{n-1}	c_{n-2}	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$

Where the b_i and c_i are as follow:

$$b_i = \frac{a_{n-1} * a_{n-2*i} - a_n * a_{n-(2*i+1)}}{a_{n-1}}$$

$$c_i = \frac{b_n * a_{n-(2i+1)} - a_{n-1} * b_{i-1}}{b_n}$$

for stability, all the elements in the first column of the Routh array must be positive. So the conditions that must be satisfied for stability of the given system as follows

$$a_n > 0,\ a_{n-1} > 0, b_n > 0, c_n > 0, \ldots$$

Sometimes the presence of poles on the imaginary axis creates a situation of marginal stability. In that case the coefficients of the "Routh array" in a whole row become zero and thus further solution of the polynomial for finding changes in sign is not possible. Then another approach comes into play. The row of polynomial which is just above the row containing the zeroes is called the "auxiliary polynomial".

In such a case the auxiliary polynomial is A(s) which is again equal to zero. The next step is to differentiate the above equation which yields the following polynomial B(s). The coefficients of the row containing zero now become the coefficient of equation B(s). The process of Routh array is proceeded using these values which yield two points on the imaginary axis. These two points on the imaginary axis are the prime cause of marginal stability.

Matlab Code:

```
clear all;
close all;
clc
coeff = input('input vector of your system coefficients: ');
ceoffLength = length(coeff);
rTableColumn = round(ceoffLength/2);
rTable = zeros(ceoffLength,rTableColumn);
rTable(1,:) = coeff(1,1:2:ceoffLength);
if (rem(ceoffLength,2) ~= 0)
    rTable(2,1:rTableColumn - 1) = coeff(1,2:2:ceoffLength);
else
    rTable(2,:) = coeff(1,2:2:ceoffLength);
end
%% Calculate Routh Array table's rows
epss = 0.01;
for i = 3:ceoffLength
       if rTable(i-1,:) == 0
        order = (ceoffLength - i);
        cnt1 = 0;
        cnt2 = 1;
        for j = 1:rTableColumn - 1
            rTable(i-1,j) = (order - cnt1) * rTable(i-2,cnt2);
            cnt2 = cnt2 + 1;
            cnt1 = cnt1 + 2;
        end
    end

    for j = 1:rTableColumn - 1
        firstElemUpperRow = rTable(i-1,1);
        rTable(i,j) = ((rTable(i-1,1) * rTable(i-2,j+1)) - ....
            (rTable(i-2,1) * rTable(i-1,j+1))) / firstElemUpperRow;
    end
    if rTable(i,1) == 0
        rTable(i,1) = epss;
    end
end
%%  Compute number of right hand side poles(unstable poles)
%   Initialize unstable poles with zero
unstablePoles = 0;
%   Check change in signs
for i = 1:ceoffLength - 1
    if sign(rTable(i,1)) * sign(rTable(i+1,1)) == -1
        unstablePoles = unstablePoles + 1;
    end
end
fprintf('\n Routh Array Table:\n')
rTable
if unstablePoles == 0
    fprintf('it is a stable system! \n')
```

```
else
    fprintf('it is an unstable system!\n')
end
fprintf('\n Number of right hand side poles =%2.0f\n',unstablePoles)
```

Results:

```
(a)input vector of your system coefficients: [1 12 47 52]
     Routh Array Table:
     rTable =
         1.0000   47.0000
        12.0000   52.0000
        42.6667         0
        52.0000         0
     it is a stable system!
     Number of right hand side poles = 0

(b) input vector of your system coefficients: [1 2 8 12 20 16 6]
     Routh Array Table:
     rTable = 1.0e+03 *

         0.0010    0.0080    0.0200    0.0060
         0.0020    0.0120    0.0160         0
         0.0020    0.0120    0.0060         0
         0.0000    0.0100         0         0
        -1.9880    0.0060         0         0
         0.0100         0         0         0
         0.0060         0         0         0
     it is an unstable system!

 Number of right hand side poles = 2

(c) input vector of your system coefficients: [1 6 11 6 100]
     Routh Array Table:
     rTable =
          1     11    100
          6      6      0
         10    100      0
        -54      0      0
        100      0      0
     it is an unstable system!
     Number of right hand side poles = 2
```

Conclusion: Using Routh Array method we can find the stability of a system on the basis of first column obtained from table. Also no of right hand side poles were determined.