Date – 23-09-2019

Experiment 6

Study of Root Locus Plot in Matlab

Aim: To study following:

- Stabilty analysis of linear process by gain variation method and by root locus method of the given TF.

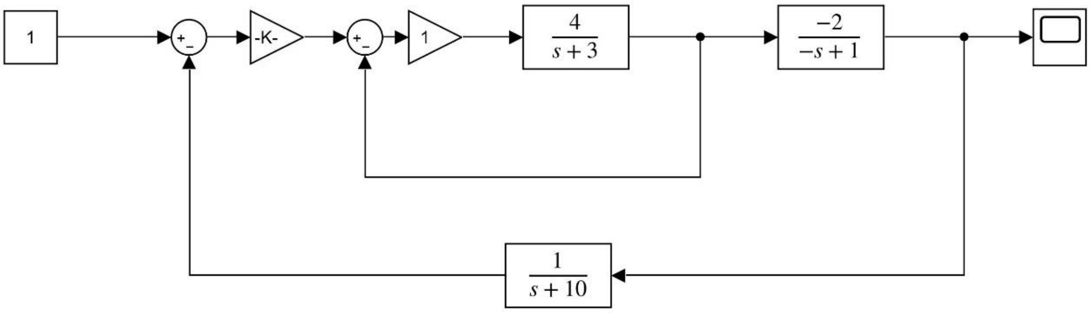

Fig 1: Given Transfer Function

- Stability analysis of linear process by gain variation on kit.

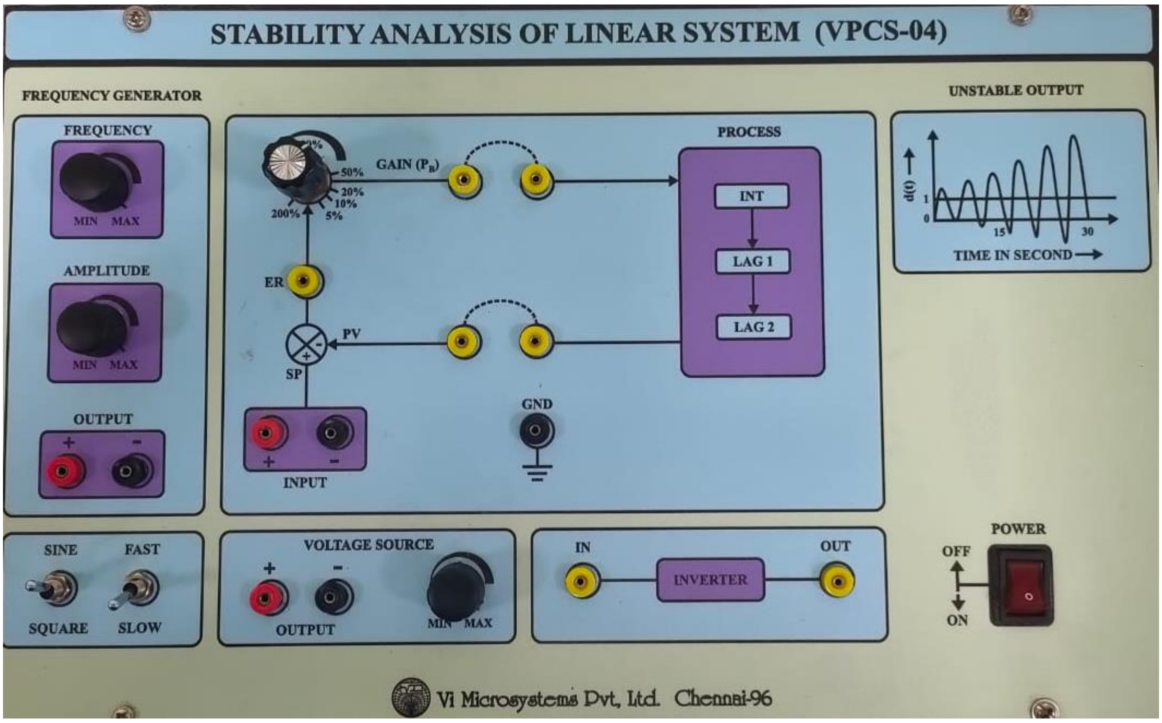

Fig 2: Controller box

Software: MATLAB 2018a

Apparatus:

- Stability analysis kit(VPCS-04)
- Digital Storage Oscilloscope
- Connecting probes

Theory:

Root Locus: The root locus of a feedback system is the graphical representation in the complex s-plane of the possible locations of its closed-loop poles for varying values of a certain system parameter. The root locus is a graphical representation in s-domain and it is symmetrical about the real axis. Because the open loop poles and zeros exist in the s-domain having the values either as real or as complex conjugate pairs. On a root-locus graph, all the poles move towards a zero. Only one pole may move towards one zero, and this means that there must be the same number of poles as zeros.

Let the Transfer Function of a system be G(s) with unity feedback

Characteristic equation is $1 + P(s) = 1 + G(s) \times H(s) = 0$
Pole Zero form: The characteristic equation in pole zero form can be written as

$$1 + G(s) \times H(s) = 1 + \frac{k * \prod_{i=1}^{m}(s+z_i)}{\prod_{j=1}^{n}(s+p_j)} = \mathbf{0} \text{ (m < n)}$$

Angle Criterion is given by

$$\sum_{i=1}^{m} \angle(\mathbf{s}+\mathbf{z_i}) - \sum_{\mathbf{j}=\mathbf{1}}^{\mathbf{n}} \angle(\mathbf{s}+\mathbf{p_j}) = \pm(\mathbf{2q}+\mathbf{1})\boldsymbol{\pi}$$

Where q is an integer.

Magnitude Criterion is given by

$$\left|\frac{k \times \prod_{i=1}^{m}(s+z_i)}{\prod_{j=1}^{n}(s+p_j)}\right| = 1$$

Points satisfying angle criterion forms the root locus. The value of gain corresponding to root can be determined by the magnitude criterion.
Rules:

- The root locus is symmetrical about the real axis(σ-axis).
- When k increases from zero to infinity, each branch of root locus originates from open-loop pole with k = 0 and terminates either on an open-loop zero or on infinity with k = ∞. The number of branches terminating on infinity equals the number of open-loop poles minus zeros(n-m).
- A point on the real axis lies on the locus if the number of open-loop poles plus zeros on the real axis to the right of this point is odd.
- Angles of (n-m) asymptotes are given by

$$\phi_A = \frac{(2q+1)\pi}{n-m}; \text{ q = 0,1,..,(n-m-1)}$$

- The asymptotes cross the real axis at a point called centroid. Its abscissa is given by

$$\sigma = \frac{\Sigma real\ parts\ of\ poles - \Sigma real\ parts\ of\ zeros}{no.of\ poles(n) - no.of\ zeros(m)}$$

- The breakaway points(points at which multiple roots of the characteristic equation occurs) of the root locus are the solutions of $dk/ds = 0$. The angle criterion of the breakaway point must be satisfied. The root locus branches must approach or leave the breakaway point on the real axis at an angle of $\pm\pi/r$where r is the number of branches approaching or leaving the point.
- The angle of departure from an open-loop pole is given by

$$\phi_p = \pm\pi(2+1) + \phi; \text{ q = 0,1,2,..}$$

Where φ is the net angle contribution, at this pole, of all other open-loop poles and zeros.

- The **angle of arrival** at an open-loop zero is given by

$$\phi_p = \pm\pi(2+1) - \phi; \text{ q} = 0,1,2,..$$

Where φ is the net angle contribution, at this zero, of all other open-loop poles and zeros.

$$\phi = \Sigma\theta_p - \Sigma\theta_z$$

- The intersection of root locus branches with the imaginary axis can be determined using the Routh criterion.

rlocus:

Syntax: `rlocus(sys)`

Description: rlocus computes the root locus of a SISO open-loop model. The root locus gives the closed-loop pole trajectories as a function of the feedback gain *k* (assuming negative feedback). Root loci are used to study the effects of varying feedback gains on closed-loop pole locations. In turn, these locations provide indirect information on time and frequency responses.

Controller Box: A proportional–integral–derivative controller (PID controller. or three-term controller) is a control loop mechanism employing feedback that is widely used in industrial control systems and a variety of other applications requiring continuously modulated control. A PID controller continuously calculates an *error value* e(t) as the difference between the desired setpoint (SP) and a measured process variable (PV) and applies a correction based on proportional , integral , and derivative terms (denoted *P* , *I* , and *D* respectively)

Matlab Code:

```
clc; clear all; close all;
s = tf('s');
g1 = 0.5;
g2 = 4/(s+3);
g3 = -2/(-s+1);
h = 1/(s+10);
Y = series(feedback(series(g1,g2),1),g3);
rlocus(Y*h);
```

Results:

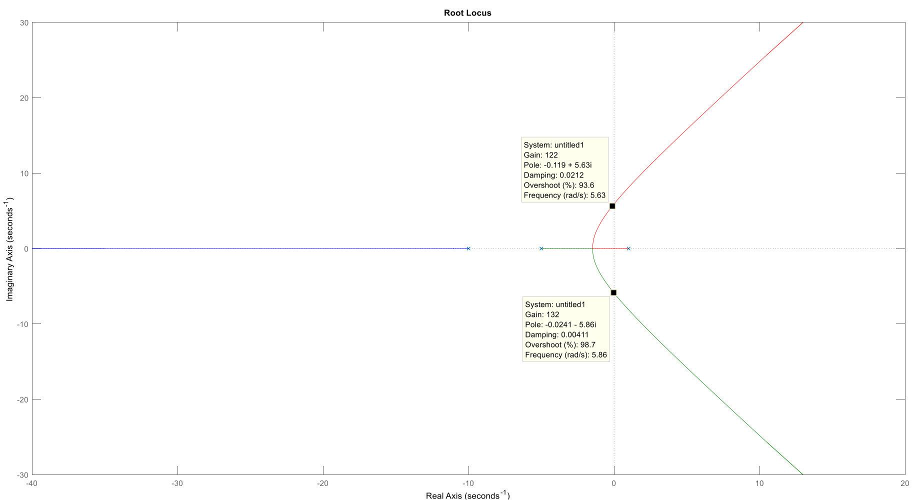

Fig 3: Root Locus

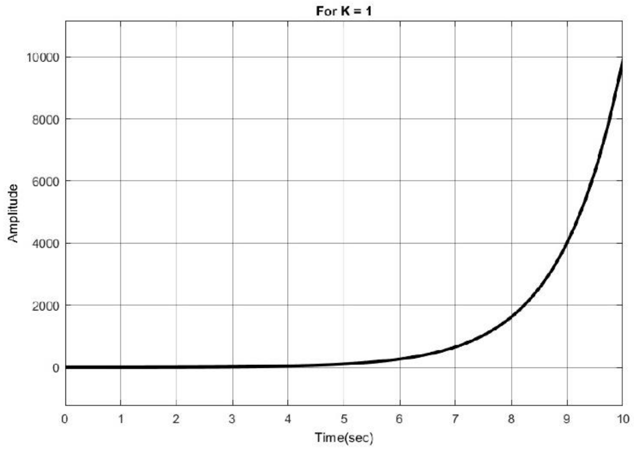

Fig 4: For k not in root locus

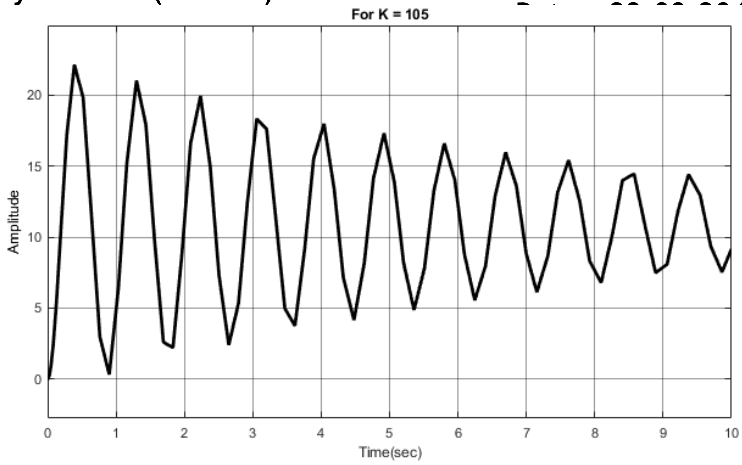

Fig 5: For K in root locus

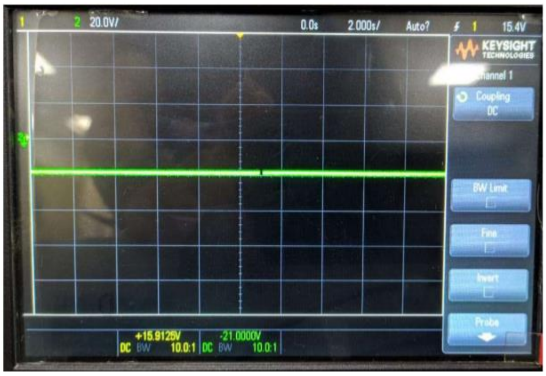

Fig 6: Stable Response

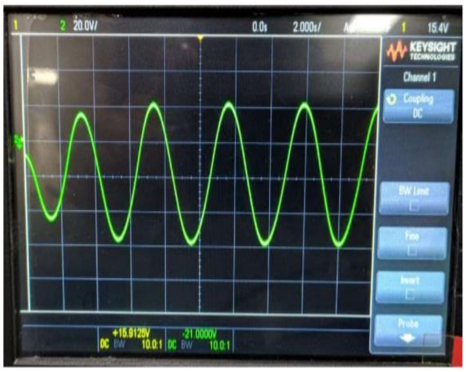

Fig 7: Unstable Stable Response

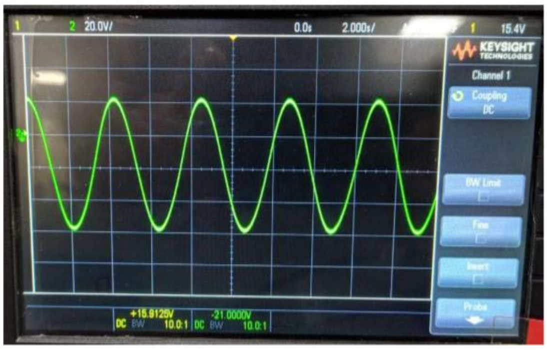

Fig 8 Marginally Stable Response

Conclusion:

For Root Locus

- After locating k on the Root Locus, the system stabilizes, and stable response is obtained
- On a root-locus graph, all the poles move towards a zero. Only one pole may move towards one zero, and this means that there must be the same number of poles as zeros.
- Range of k for stability = ($13 \leq gain \leq 135$)

For Controller Box

- On increasing k, the systems tends to get unstable.
- Response of such a system shows increasing amplitude.

Assignment 6a

Aim: To Study Root Locus of a system with PID controller in MATLAB.

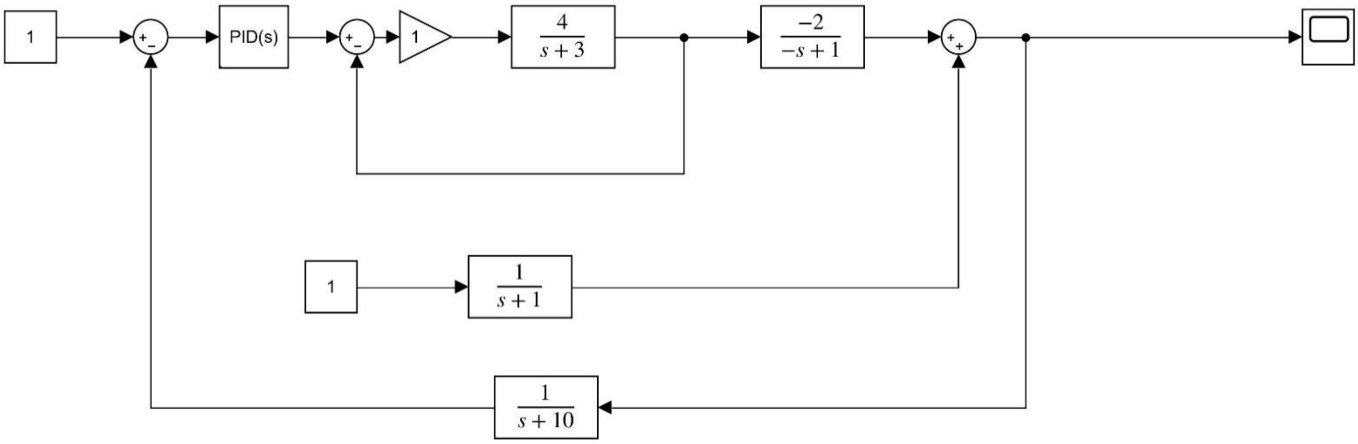

Fig – 1 Given System with Disturbance

Software: Matlab 2018a

Theory: In control system theory, the Routh–Hurwitz stability criterion is a mathematical test that is a necessary and sufficient condition for the stability of a linear time-invariant (LTI) control system. Stability can be characterized by considering a sign of real parts of the roots of the characteristic polynomial of a linear system. If all entries in the routh array are positive, implies a stable system, negative entry implies instability, 0 implies threshold condition.

Matlab Code:

```
close all;
clear; clc;
k = [10, 20, 50, 100];
process = tf(1, [1, 1, 0]);

for i = 1:4
    controller = tf([11, k(i)],1);
    system = feedback(series(controller, process), 1);
    title(sprintf('Without disturbance k = %d', k(i)));
    subplot(4, 3, 1 + 3*(i-1))
    rlocus(system);
    stepinfo(system)

    controller = tf([11, k(i)],1);
    D_system = feedback(process, controller);
    title(sprintf('Only disturbance k = %d', k(i)));
    subplot(4,3, 2 + 3*(i-1))
    rlocus(D_system);
    stepinfo(D_system)

    controller = tf([11, k(i)],1);
    D_system = feedback(process, controller);
    system = feedback(series(controller, process), 1);
    title(sprintf('With disturbance k = %d', k(i)));
    subplot(4,3, 3 + 3*(i-1))
    rlocus(system + D_system);
    stepinfo(system + D_system)
end
```

Results:

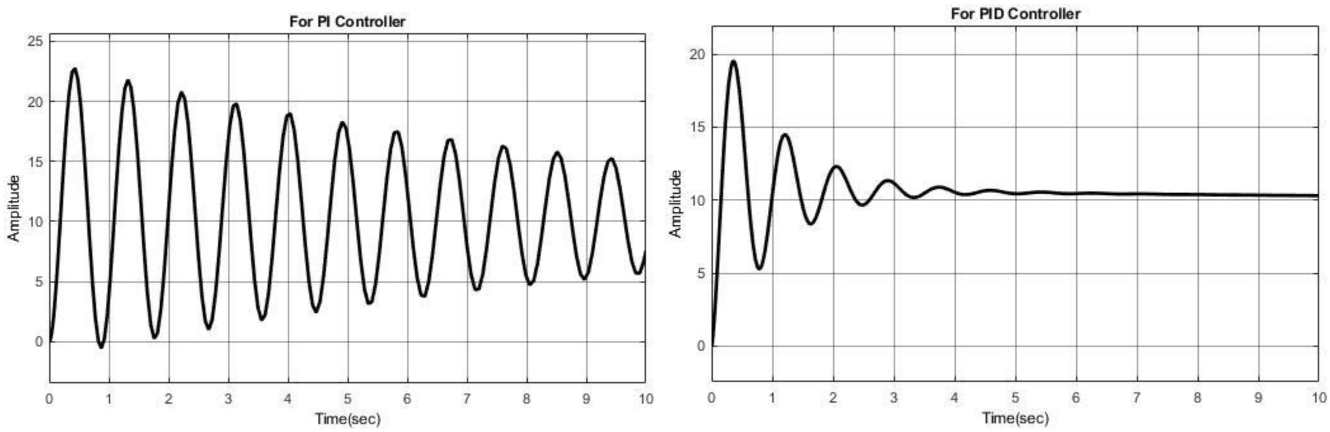

Fig 2 – PI and PID Controller Response on SIMULINK

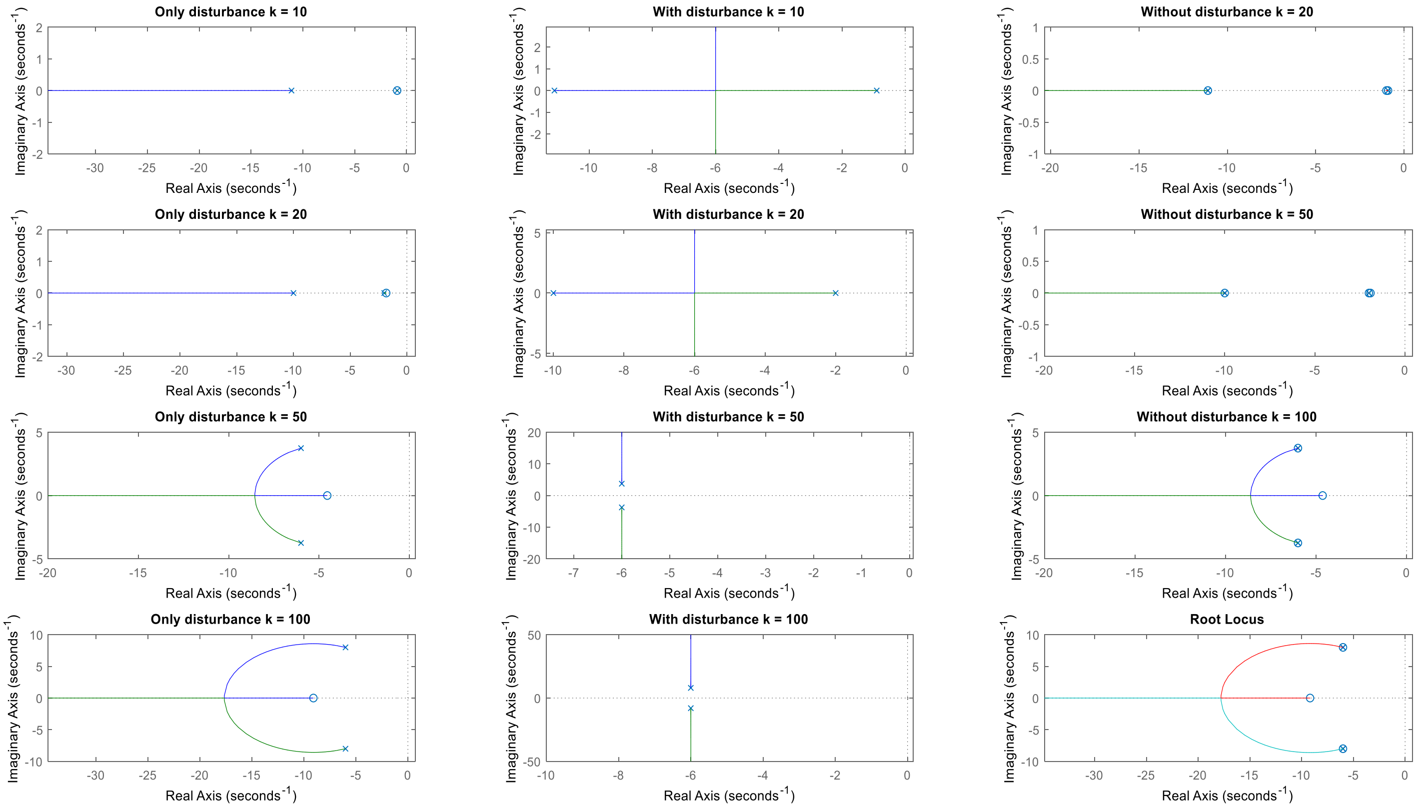

Fig-3: Root Locus of system with and without Disturbance

Conclusion:

- After locating k on the Root Locus, the system stabilizes, and stable response is obtained.
- Step response was studies for different values of disturbances.